CHAPTER 2 SOLUTIONS

Problem 2.1

From Ohm's law, the current I_1 through R_1 is given by

$$I_1 = \frac{V}{R_1} = \frac{6V}{3k\Omega} = \frac{6V}{3000\Omega} = 0.002A = 2mA$$

Notice that 1 V/1 kΩ = 1 mA.
From Ohm's law, the current I_2 through R_2 is given by

$$I_2 = \frac{V}{R_2} = \frac{6V}{6k\Omega} = \frac{6V}{6000\Omega} = 0.001A = 1mA$$

Problem 2.2

From Ohm's law, the current I_1 through R_1 is given by

$$I_1 = \frac{V_1}{R_1} = \frac{2.4V}{800\Omega} = 0.003A = 3mA$$

From Ohm's law, the current I_2 through R_2 is given by

$$I_2 = \frac{V_2}{R_2} = \frac{3.6V}{2k\Omega} = 1.8mA$$

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{3.6V}{3k\Omega} = 1.2mA$$

Problem 2.3

From Ohm's law, the current I_1 through R_1 is given by

$$I_1 = \frac{V_1}{R_1} = \frac{2.4V}{4k\Omega} = 0.6mA = 600\,\mu A$$

From Ohm's law, the current I_2 through R_2 is given by

$$I_2 = \frac{V_1}{R_2} = \frac{2.4V}{6k\Omega} = 0.4mA = 400\,\mu A$$

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_2}{R_2} = \frac{1.2\,V}{1.8\,k\Omega} = \frac{2}{3}\,mA = 0.6667\,mA = 666.5557\,\mu A$$

From Ohm's law, the current I_4 through R_4 is given by

$$I_4 = \frac{V_2}{R_4} = \frac{1.2\,V}{6\,k\Omega} = 0.2\,mA = 200\,\mu A$$

From Ohm's law, the current I_5 through R_5 is given by

$$I_5 = \frac{V_2}{R_5} = \frac{1.2\,V}{9\,k\Omega} = \frac{2}{15}\,mA = 0.1333\,mA = 133.3333\,\mu A$$

Problem 2.4

From Ohm's law, the voltage across R_2 is given by

$V_o = R_2I_2 = 6$ kΩ × 1.2 mA = 6000 × 0.0012 = 7.2 V

Notice that 1 kΩ × 1 mA = 1 V.
From Ohm's law, the current I_1 through R_1 is given by

$$I_1 = \frac{V_1}{R_1} = \frac{2.8\,V}{1.4\,k\Omega} = 2\,mA$$

From Ohm's law, the voltage across R_2 is given by

$V_o = R_2I_2 = 6$ kΩ × 1.2 mA = 6000 × 0.0012 = 7.2 V

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_o}{R_3} = \frac{7.2\,V}{9\,k\Omega} = 0.8\,mA = 800\,\mu A$$

Problem 2.5

From Ohm's law, the voltage across R_4 is given by

$V_o = R_4I_4 = 18$ kΩ × 0.2 mA = 18000 × 0.0002 = 3.6 V

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_o}{R_3} = \frac{3.6V}{6k\Omega} = 0.6\,mA = 600\,\mu A$$

Problem 2.6

From Ohm's law, the voltage across R_4 is given by

$V_o = R_4 I_4 = 8\ k\Omega \times 0.4\ mA = 8000 \times 0.0004 = 3.2\ V$

From Ohm's law, the current I_2 through R_2 is given by

$$I_2 = \frac{V_o}{R_2} = \frac{3.2V}{3k\Omega} = \frac{16}{15}mA = 1.06667\,mA$$

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_o}{R_3} = \frac{3.2V}{6k\Omega} = \frac{16}{30}mA = 0.53333\,mA = 533.3333\,\mu A$$

Problem 2.7

From Ohm's law, the voltage across R_3 is given by

$V_o = R_3 I_3 = 42\ k\Omega \times (1/12)\ mA = 42/12\ V = 3.5\ V$

From Ohm's law, the resistance value R_2 is given by

$$R_2 = \frac{V_o}{I_2} = \frac{3.5V}{\frac{7}{60}mA} = 30\,k\Omega$$

1 V/1 mA = 1 kΩ

Problem 2.8

The power on R_1 is

$P_{R_1} = I^2 R_1 = (2\times10^{-3})^2 \times 2000 = 4\times10^{-6}\times2\times10^{3} = 8\times10^{-3}W = 8\,mW$ (absorbed)

The power on R_2 is

$P_{R_2} = I^2 R_1 = (2\times10^{-3})^2 \times 3000 = 4\times10^{-6}\times3\times10^{3} = 12\times10^{-3}W = 12\,mW$ (absorbed)

The power on V_s is

$$P_{V_s} = -IV_s = -2\times10^{-3}\times10 = -20\times10^{-3}W = -20\,mW \text{ (released)}$$

Total power absorbed = 20 mW = total power released

Problem 2.9

The power on R_1 is

$$P_{R_1} = \frac{V_o^2}{R_1} = \frac{4.8^2}{8000} = 2.88\times10^{-3}W = 2.88\,mW \text{ (absorbed)}$$

The power on R_2 is

$$P_{R_2} = \frac{V_o^2}{R_2} = \frac{4.8^2}{12000} = 1.92\times10^{-3}W = 1.92\,mW \text{ (absorbed)}$$

The power on V_s is

$$P_{I_s} = -I_sV_o = -1\times10^{-3}\times4.8 = -4.8\times10^{-3}W = -4.8\,mW \text{ (released)}$$

Problem 2.10

From Ohm's law, current I_1 is given by

$$I_1 = \frac{20V - 15V}{R_1} = \frac{5V}{0.5\,k\Omega} = 10\,mA$$

From Ohm's law, current I_2 is given by

$$I_2 = \frac{20V - 10V}{R_2} = \frac{10V}{2\,k\Omega} = 5\,mA$$

From Ohm's law, current I_3 is given by

$$I_3 = \frac{10V - 0V}{R_3} = \frac{10V}{1\,k\Omega} = 10\,mA$$

From Ohm's law, current I_4 is given by

$$I_4 = \frac{10V - 15V}{R_4} = \frac{-5V}{1k\Omega} = -5\,mA$$

Problem 2.11

From Ohm's law, current *i* is given by

$$i = \frac{10V - 8V}{R_3} = \frac{2V}{2k\Omega} = 1\,mA$$

From Ohm's law, current I_1 is given by

$$I_1 = \frac{12V - 10V}{R_1} = \frac{2V}{1k\Omega} = 2\,mA$$

From Ohm's law, current I_2 is given by

$$I_2 = \frac{10V - 5V}{R_2} = \frac{5V}{5k\Omega} = 1\,mA$$

From Ohm's law, current I_3 is given by

$$I_3 = \frac{12V - 8V}{R_4} = \frac{4V}{2k\Omega} = 2\,mA$$

From Ohm's law, current I_4 is given by

$$I_4 = \frac{8V - 5V}{R_5} = \frac{3V}{3k\Omega} = 1\,mA$$

From Ohm's law, current I_5 is given by

$$I_5 = \frac{8V}{R_6} = \frac{8V}{4k\Omega} = 2\,mA$$

Problem 2.12

Application of Ohm's law results in

$$I_1 = \frac{34V - 24V}{R_1} = \frac{10V}{2k\Omega} = 5\,mA$$

$$I_2 = \frac{24V - 10V}{R_2} = \frac{14V}{2k\Omega} = 7\,mA$$

$$I_3 = \frac{24V - 28V}{R_3} = \frac{-4V}{2k\Omega} = -2\,mA$$

$$I_4 = \frac{34V - 28V}{R_4} = \frac{6V}{0.6k\Omega} = 10\,mA$$

$$I_5 = \frac{28V - 10V}{R_5} = \frac{18V}{6k\Omega} = 3\,mA$$

$$I_6 = \frac{28V}{R_6} = \frac{28V}{5.6k\Omega} = 5\,mA$$

$$I_7 = \frac{10V}{R_7} = \frac{10V}{1k\Omega} = 10\,mA$$

Problem 2.13

The total voltage from the four voltage sources is

$V = V_{s1} + V_{s2} + V_{s3} + V_{s4} = 9\text{ V} + 2\text{ V} - 3\text{ V} + 2\text{ V} = 10\text{V}$

The total resistance from the five resistors is

$R = R_1 + R_2 + R_3 + R_4 + R_5 = 3\text{ k}\Omega + 5\text{ k}\Omega + 4\text{ k}\Omega + 2\text{ k}\Omega + 4\text{ k}\Omega = 18\text{ k}\Omega$

The current through the mesh is

$$I = \frac{V}{R} = \frac{10V}{18000\Omega} = \frac{5}{9}mA = 0.5556\,mA$$

From Ohm's law, the voltages across the five resistors are given respectively

$V_1 = R_1 I = 3\times5/9\text{ V} = 15/9\text{ V} = 5/3\text{ V} = 1.6667\text{ V}$

$V_2 = R_2 I = 5\times5/9\text{ V} = 25/9\text{ V} = 2.7778\text{ V}$

$V_3 = R_3 I = 4\times5/9\text{ V} = 20/9\text{ V} = 2.2222\text{ V}$

$V_4 = R_4 I = 2\times5/9\text{ V} = 10/9\text{ V} = 1.1111\text{ V}$

$V_5 = R_5I = 4\times5/9$ V = 20/9 V = 2.2222 V

Problem 2.14

Radius is $r = d/2 = 0.2025$ mm $= 0.2025 \times 10^{-3}$ m
$A = \pi r^2 = 1.28825\times10^{-7}$ m^2

(a)

$$R = \frac{\ell}{\sigma A} = \frac{20}{5.69\times10^7\times\pi\times(0.2025\times10^{-3})^2} = 2.7285\,\Omega$$

(b)

$$R = \frac{\ell}{\sigma A} = \frac{200}{5.69\times10^7\times\pi\times(0.2025\times10^{-3})^2} = 27.2846\,\Omega$$

(c)

$$R = \frac{\ell}{\sigma A} = \frac{2000}{5.69\times10^7\times\pi\times(0.2025\times10^{-3})^2} = 272.8461\,\Omega$$

(d)

$$R = \frac{\ell}{\sigma A} = \frac{20000}{5.69\times10^7\times\pi\times(0.2025\times10^{-3})^2} = 2728.4613\,\Omega$$

Problem 2.15

From Ohm's law, the voltage across R_2 is given by

$V_2 = I_2R_2 = 3$ mA × 2 kΩ = 6 V

From Ohm's law, the current through R_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{6V}{3k\Omega} = 2mA$$

According to KCL, current I_1 is the sum of I_2 and I_3. Thus, we have

$I_1 = I_2 + I_3 = 3$ mA + 2 mA = 5 mA

The voltage across R_1 is given by

$V_1 = R_1I_1 = 1$ kΩ × 5 mA = 5 V

Problem 2.16

From Ohm's law, the currents I_2, I_3, and I_4 are given respectively by

$$I_2 = \frac{V_2}{R_2} = \frac{6V}{2k\Omega} = 3mA$$

$$I_3 = \frac{V_2}{R_3} = \frac{6V}{3k\Omega} = 2mA$$

$$I_4 = \frac{V_2}{R_4} = \frac{6V}{6k\Omega} = 1mA$$

From KCL, current I_1 is the sum of I_2, I_3, and I_4. Thus, we have

$I_1 = I_2 + I_3 + I_4 = 3$ mA + 2 mA + 1 mA = 6 mA

The voltage across R_1 is given by

$V_1 = R_1 I_1 = 1$ kΩ × 6 mA = 6 V

Problem 2.17

From Ohm's law, we have

$V_2 = R_4 I_4 = 1$ mA × 6 kΩ = 6 V

From Ohm's law, the current through R_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{6V}{3k\Omega} = 2mA$$

From KCL, I_2 is the sum of I_3 and I_4. Thus,

$I_2 = I_3 + I_4 = 3$ mA

From KCL, I_1 is given by

$I_1 = I_s - I_2 = 2$ mA

From Ohm's law, the voltage across R_1 is

$V_1 = R_1 I_1 = 4.5$ kΩ × 2 mA = 9 V

Problem 2.18

From Ohm's law, we have

$$I_3 = \frac{V_o}{R_3} = \frac{8V}{2k\Omega} = 4mA$$

$$I_4 = \frac{V_o}{R_4} = \frac{8V}{4k\Omega} = 2mA$$

$$I_1 = \frac{V_s - V_o}{R_1} = \frac{12V - 8V}{1k\Omega} = \frac{4V}{1k\Omega} = 4mA$$

$$I_2 = \frac{V_s - V_o}{R_2} = \frac{12V - 8V}{2k\Omega} = \frac{4V}{2k\Omega} = 2mA$$

As a check, $I_1 + I_2 = I_3 + I_4 = 6$ mA

Problem 2.19

From Ohm's law, we have

$$I_3 = \frac{V_4}{R_4} = \frac{5V}{2.5k\Omega} = 2mA$$

$V_3 = R_3 I_3 = 2$ kΩ × 2 mA = 4 V

$V_2 = V_3 + V_4 = 4$ V + 5 V = 9 V

$$I_2 = \frac{V_2}{R_2} = 3\frac{9V}{4k\Omega} = 3mA$$

From KCL, we have

$I_1 = I_2 + I_3 = 5$ mA

From Ohm's law, we get

$V_1 = R_1 I_1 = 1$ kΩ × 5 mA = 5 V

Problem 2.20

Application of KCL at node *a* yields

$I_s = I_1 + I_2 + I_3$

Solving for I_2, we obtain

$I_2 = I_s - I_1 - I_3 = 10\text{ mA} - 5\text{ mA} - 2\text{ mA} = 3\text{ mA}$

Application of KCL at node *b* yields

$I_1 + I_2 = I_4 + I_5$

Solving for I_5, we obtain

$I_5 = I_1 + I_2 - I_4 = 5\text{ mA} + 3\text{ mA} - 2\text{ mA} = 6\text{ mA}$

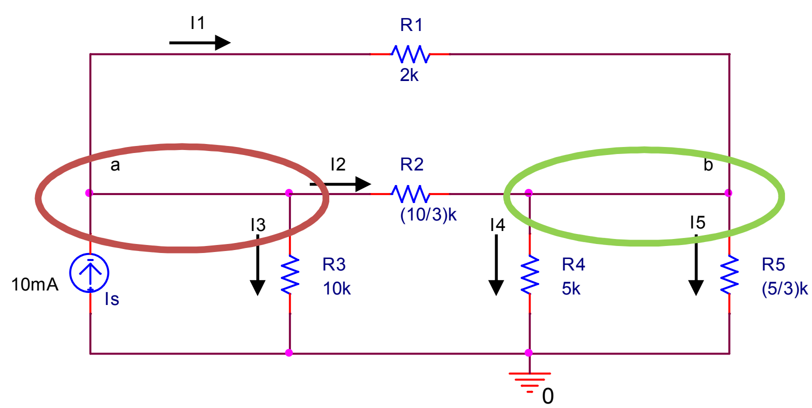


Figure S2.20

Problem 2.21

Application of KCL at node *b* yields

$I_s = I_2 + I_3$

Solving for I_2, we obtain

$I_2 = I_s - I_3 = 15\text{ mA} - 10\text{ mA} = 5\text{ mA}$

Application of KCL at node *a* yields

$I_4 = I_2 - I_1 = 5\text{ mA} - 2\text{ mA} = 3\text{ mA}$

Application of KCL at node *c* yields

$I_5 = I_1 + I_3 = 2\text{ mA} + 10\text{ mA} = 12\text{ mA}$

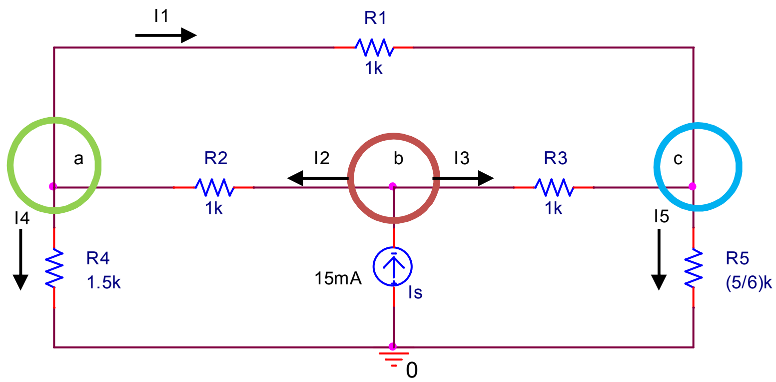


Figure S2.21

Problem 2.22

Application of KCL at node *b* yields

$I_1 = I_s - I_4 = 20 \text{ mA} - 10 \text{ mA} = 10 \text{ mA}$

Application of KCL at node *a* yields

$I_2 = I_1 - I_3 = 10 \text{ mA} - 5 \text{ mA} = 5 \text{ mA}$

Application of KCL at node *c* yields

$I_6 = I_3 + I_4 - I_5 = 5 \text{ mA} + 10 \text{ mA} - 5 \text{ mA} = 10 \text{ mA}$

Application of KCL at node *d* yields

$I_7 = I_2 + I_5 = 5 \text{ mA} + 5 \text{ mA} = 10 \text{ mA}$

Problem 2.23

Application of KCL at node *d* yields

$I_2 = 13 - 10 = 3 \text{ A}$

Application of KCL at node *a* yields

$I_1 = I_2 - 2 = 3 - 2 = 1 \text{ A}$

Application of KCL at node *b* yields

$I_3 = - I_1 - 5 = - 1 - 5 = - 6 \text{ A}$

Application of KCL at node *c* yields

$I_5 = - 2 - 10 = - 12$ A

Application of KCL at node *e* yields

$I_4 = - I_3 - 13 = - (-6) - 13 = - 7$ A

Problem 2.24

Summing the voltage drops around mesh 1 in the circuit shown in Figure S2.24 in the clockwise direction, we obtain

$-V_1 + V_{R1} + V_{R3} = 0$

Since $V_1 = 30V$ and $V_{R1} = 10V$, this equation becomes

$-30 + 10 + V_{R3} = 0$

Thus,

$V_{R3} = 30-10 = 20V$.

Summing the voltage drops around mesh 2 in the circuit shown in Figure S2.11 in the clockwise direction, we obtain

$-V_{R3} + V_{R2} + V_{R4} = 0$

Since $V_{R3} = 20V$ and $V_{R4} = 15V$, this equation becomes

$-20 + V_{R2} + 15 = 0$

Thus,

$V_{R2} = 20-15 = 5V$.

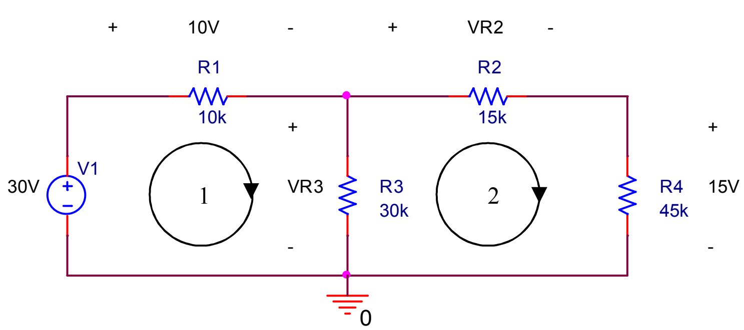


Figure S2.24

Problem 2.25

Consider the loop consisting of V_1, R_1 and R_5, shown in the circuit shown in Figure S2.25. Summing the voltage drops around this loop in the clockwise direction, we obtain

$-V_1 + V_{R1} + V_{R5} = 0$

Since $V_1 = 20V$ and $V_{R1} = 10V$, this equation becomes

$-20 + 10 + V_{R5} = 0$

Thus,

$V_{R5} = 20 - 10 = 10V$.

In the mesh consisting of R_4, R_3 and R_5, shown in the circuit shown in Figure S2.25, summing the voltage drops around this mesh in the clockwise direction, we obtain

$-V_{R4} + V_{R3} + V_{R5} = 0$

Since $V_{R3} = 5V$ and $V_{R5} = 10V$, this equation becomes

$- V_{R4} + 5 + 10 = 0$

Thus,

$V_{R4} = 5+10 = 15V$.

In the mesh consisting of V_1, R_2 and R_4, shown in the circuit shown in Figure S2.25, summing the voltage drops around this mesh in the clockwise direction, we obtain

$-V_1 + V_{R2} + V_{R4} = 0$

Since $V_1 = 20V$ and $V_{R4} = 15V$, this equation becomes

$- 20 + V_{R2} + 15 = 0$

Thus,

$V_{R2} = 20 - 15 = 5V$.

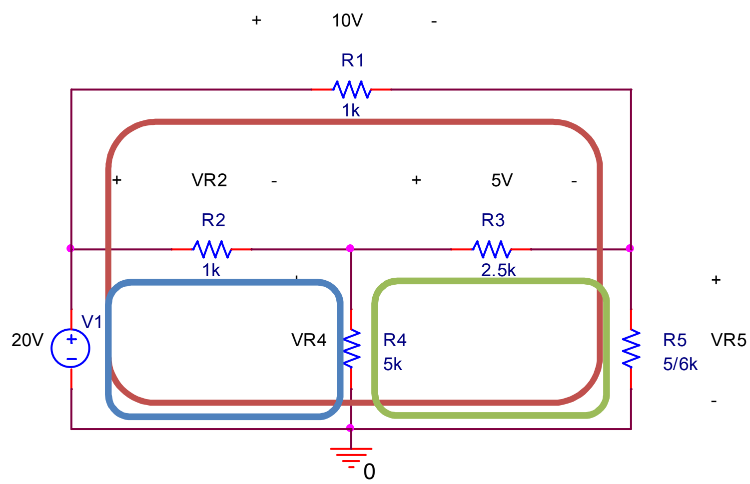


Figure S2.25

Problem 2.26

In the mesh consisting of R_1, R_3 and R_4, upper left in the circuit shown in Figure S2.26, summing the voltage drops around this mesh in the clockwise direction, we obtain

$V_{R1} + V_{R3} - V_{R4} = 0$

Since $V_{R1} = 5V$ and $V_{R3} = 5V$, this equation becomes

$5 + 5 - V_{R4} = 0$

Thus,

$V_{R4} = 5 + 5 = 10V.$

In the mesh consisting of V_1, R_4 and R_6, lower left in the circuit shown in Figure S2.26, summing the voltage drops around this mesh in the clockwise direction, we obtain

$-V_1 + V_{R4} + V_{R6} = 0$

Since $V_1 = 20V$ and $V_{R4} = 10V$, this equation becomes

$- 20 + 10 + V_{R6} = 0$

Thus,

$V_{R6} = 20-10 = 10V.$

In the mesh consisting of R_3, R_2 and R_5, upper right in the circuit shown in Figure S2.26, summing the voltage drops around this mesh in the clockwise direction, we obtain

$-V_{R3} + V_{R2} - V_{R5} = 0$

Since $V_{R3} = 5V$ and $V_{R5} = 5V$, this equation becomes

$-5 + V_{R2} - 5 = 0$

Thus,

$V_{R2} = 5 + 5 = 10V$.

In the mesh consisting of R_6, R_5 and R_7, lower right in the circuit shown in Figure S2.26, summing the voltage drops around this mesh in the clockwise direction, we obtain

$-V_{R6} + V_{R5} + V_{R7} = 0$

Since $V_{R6} = 10V$ and $V_{R5} = 5V$, this equation becomes

$-10 + 5 + V_{R7} = 0$

Thus,

$V_{R7} = 10 - 5 = 5V$.

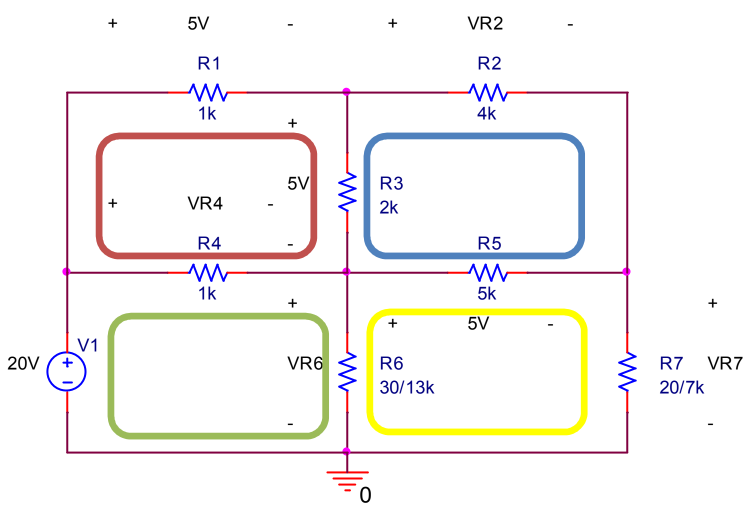


Figure S2.26

Problem 2.27

From Ohm's law, the current I_5 is given by

$$I_5 = \frac{V_5}{R_5} = \frac{6V}{1k\Omega} = 6\,mA$$

From Ohm's law, the current I_1 is given by

$$I_1 = \frac{V_s - V_5}{R_1} = \frac{16V - 6V}{5k\Omega} = \frac{10V}{5k\Omega} = 2mA$$

From KCL, we have

$I_3 = I_5 - I_1 = 6$ mA - 2 mA = 4 mA

The voltage across R_3 is

$V_3 = R_3 I_3 = 1$ kΩ × 4 mA = 4 V

From KVL, the voltage across R_4 is given by

$V_4 = V_3 + V_5 = 4$ V + 6 V = 10 V

The current through R_4 is given by

$$I_4 = \frac{V_4}{R_4} = \frac{10V}{5k\Omega} = 2mA$$

From KCL, current I_2 is given by

$I_2 = I_3 + I_4 = 4$ mA + 2 mA = 6 mA

Problem 2.28

The voltage across R_3 is given by

$V_2 = R_3 I_3 = 4$ kΩ × 2 mA = 8 V

From Ohm's law, current I_4 is given by

$$I_4 = \frac{V_2}{R_4} = \frac{8V}{2k\Omega} = 4mA$$

From KCL, the current through R_2 is given by

$I_2 = I_3 + I_4 = 2$ mA + 4 mA = 6 mA

From KCL, the current through R_1 is given by

$I_1 = I_s - I_2 = 8$ mA - 6 mA = 2 mA

The voltage across R_1 is given by

$V_1 = R_1I_1 = 7\ k\Omega \times 2\ mA = 14\ V$

Problem 2.29

The voltage across R_1 is given by

$V_1 = R_1I_1 = 5\ k\Omega \times 1\ mA = 5\ V$

From KCL, the current through R_2 is given by

$I_2 = I_s - I_1 = 5\ mA - 1\ mA = 4\ mA$

From KVL, V_2 is given by

$V_2 = V_1 - R_2I_2 = 5\ V - 0.5\ k\Omega \times 4\ mA = 5\ V - 2\ V = 3\ V$

From Ohm's law, current I_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{3V}{1k\Omega} = 3\,mA$$

From Ohm's law, current I_4 is given by

$$I_4 = \frac{V_2}{R_4} = \frac{3V}{3k\Omega} = 1\,mA$$

Problem 2.30

Application of KVL around the outer loop yields

$- 2 - V_1 - 3 = 0$

Solving for V_1, we obtain

$V_1 = -5\ V$

Application of KVL around the top mesh yields

$- V_1 - 4 + V_2 = 0$

Solving for V_2, we obtain

$V_2 = V_1 + 4 = -1\ V$

Application of KVL around the center left mesh yields

$- V_2 + 5 - V_3 = 0$

Solving for V_3, we obtain

$V_3 = - V_2 + 5 = 6$ V

Application of KVL around the center right mesh yields

$- 5 + 4 + V_4 = 0$

Solving for V_4, we obtain

$V_4 = 5 - 4 = 1$ V

Application of KVL around the bottom left mesh yields

$- 2 + V_3 - V_5 = 0$

Solving for V_5, we obtain

$V_5 = - 2 + 6 = 4$ V

Problem 2.31

Application of KVL around the outer loop yields

$- 3 - V_1 = 0$

Solving for V_1, we obtain

$V_1 = - 3$ V

Application of KVL around the lower left mesh yields

$- 3 + V_2 - 1 = 0$

Solving for V_2, we obtain

$V_2 = 3 + 1 = 4$ V

Application of KVL around the lower right mesh yields

$1 - V_5 = 0$

Solving for V_5, we obtain

$V_5 = 1$ V

Application of KCL at node *a* yields

$I_1 = 2 + 2 = 4A$

Application of KCL at node *b* yields

$I_4 = 2 + 3 = 5A$

Problem 2.32

Resistor R_1 is in series to the parallel combination of R_2 and R_3. Thus, the equivalent resistance R_{eq} is given by

$$R_{eq} = R_1 + \left(R_2 \parallel R_3\right) = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 2000 + \frac{4000 \times 12000}{4000 + 12000}$$
$$= 2000 + \frac{48,000,000}{16,000} = 2000 + 3000 = 5000\Omega = 5k\Omega$$

Instead of ohms (Ω), we can use kilo ohms (kΩ) to simplify the algebra:

$$R_{eq} = R_1 + \left(R_2 \parallel R_3\right) = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 2k + \frac{4k \times 12k}{4k + 12k} = 2k + \frac{48k^2}{16k} = 2k + 3k = 5k\Omega$$

If all the resistance values are in kΩ, k can be removed during calculations, and represent the answer in kΩ as shown below.

$$R_{eq} = R_1 + \left(R_2 \parallel R_3\right) = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 2 + \frac{4 \times 12}{4 + 12} = 2 + \frac{48}{16} = 2 + 3 = 5k\Omega$$

Problem 2.33

Resistors R_1 and R_2 are in parallel, and resistors R_3 and R_4 are in parallel. The equivalent resistance is the sum of $R_1 \parallel R_2$ and $R_3 \parallel R_4$.

$$R_{eq} = \left(R_1 \parallel R_2\right) + \left(R_3 \parallel R_4\right) = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = \frac{10 \times 40}{10 + 40} + \frac{8 \times 56}{8 + 56} = \frac{400}{50} + \frac{448}{64} = 8 + 7 = 15\,k\Omega$$

Problem 2.34

The equivalent resistance is the sum of R_1 and the parallel combination of R_2, R_3, and R_4.

$$R_{eq} = R_1 + \left(R_2 \parallel R_3 \parallel R_4\right) = R_1 + \frac{1}{\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = 5 + \frac{1}{\frac{1}{30}+\frac{1}{60}+\frac{1}{5}} = 5 + \frac{1}{\frac{2}{60}+\frac{1}{60}+\frac{12}{60}}$$

$$= 5 + \frac{60}{15} = 5 + 4 = 9\,k\Omega$$

Problem 2.35

The equivalent resistance of the parallel combination of R_4 and a short circuit (0Ω) is given by

$$R_4 \parallel 0 = \frac{20\times 0}{20+0} = \frac{0}{20} = 0\,\Omega$$

The equivalent resistance is the sum of R_1 and the parallel combination of R_2 and R_3.

$$R_{eq} = R_1 + \left(R_2 \parallel R_3\right) = R_1 + \frac{R_2R_3}{R_2+R_3} = 12 + \frac{99\times 22}{99+22} = 12 + \frac{2178}{121} = 12 + 18 = 30\,k\Omega$$

Problem 2.36

The equivalent resistance R_a of the series connection of three resistors R_4, R_5, and R_6 is

$R_a = R_4 + R_5 + R_6 = 25 + 20 + 33 = 78\ k\Omega$

The equivalent resistance R_b of the parallel connection of R_3 and R_a is

$$R_b = R_3 \parallel R_a = \frac{R_3R_a}{R_3+R_a} = \frac{39\times 78}{39+78} = \frac{3042}{117} = 26\,k\Omega$$

The equivalent resistance R_{eq} of the circuit shown in Figure P2.5 is the sum of R_1, R_b, and R_2:

$R_{eq} = R_1 + R_b + R_2 = 10 + 26 + 14 = 50\ k\Omega$

Problem 2.37

The resistors R_1 and R_2 are connected in parallel. Let R_a be $R_1 \parallel R_2$. Then, we have

$$R_a = R_1 \parallel R_2 = \frac{R_1R_2}{R_1+R_2} = \frac{50\times 75}{50+75} = \frac{50\times 75}{125} = \frac{50\times 3}{5} = 10\times 3 = 30\,k\Omega$$

The resistors R_3 and R_4 are connected in parallel. Let R_b be $R_3 \parallel R_4$. Then, we have

$$R_b = R_3 \parallel R_4 = \frac{R_3 R_4}{R_3 + R_4} = \frac{55\times 66}{55+66} = \frac{5\times 66}{5+6} = \frac{5\times 66}{11} = 5\times 6 = 30\,k\Omega$$

The equivalent resistance R_{eq} of the circuit shown in Figure P2.6 is given by the sum of R_a and R_b:

$R_{eq} = R_a + R_b = 30\ \text{k}\Omega + 30\ \text{k}\Omega = 60\ \text{k}\Omega$

MATLAB

```
clear all;
R1=50000;R2=75000;R3=55000;R4=66000;
Req=P([R1,R2])+P([R3,R4])

Answer:
Req =
       60000
```

Problem 2.38

The equivalent resistance R_{eq} can be found by combining resistances from the right side of the circuit and moving toward the left. Since R_7, R_8, and R_9 are connected in series, we have

$R_a = R_7 + R_8 + R_9 = 15 + 19 + 20 = 54\ \text{k}\Omega$

Let R_b be the equivalent resistance of the parallel connection of R_6 and R_a. Then we have

$$R_b = R_6 \parallel R_a = \frac{R_6 \times R_a}{R_6 + R_a} = \frac{27\times 54}{27+54} = \frac{1\times 54}{1+2} = \frac{54}{3} = 18\,k\Omega$$

Let R_c be the sum of R_4, R_b, and R_5. Then, we have

$R_c = R_4 + R_b + R_5 = 6 + 18 + 4 = 28\ \text{k}\Omega$.

Let R_d be the equivalent resistance of the parallel connection of R_3 and R_c. Then, we have

$$R_d = R_3 \parallel R_c = \frac{R_3 \times R_c}{R_3 + R_c} = \frac{21\times 28}{21+28} = \frac{3\times 28}{3+4} = \frac{3\times 28}{7} = 12\,k\Omega$$

The equivalent resistance R_{eq} is the sum of R_1, R_d, and R_2. Thus, we have

$R_{eq} = R_1 + R_d + R_2 = 3 + 12 + 5 = 20\ \text{k}\Omega$

MATLAB

```
clear all;
R1=3000;R2=5000;R3=21000;R4=6000;R5=4000;R6=27000;R7=15000;R8=19000;R9=20000;
Req=R1+R2+P([R3,R4+R5+P([R6,R7+R8+R9])])

Answer:
Req =
       20000
```

Problem 2.39

Let R_a be the equivalent resistance of the parallel connection of R_5 and R_6. Then, we have

$$R_a = R_5 \| R_6 = \frac{R_5 \times R_6}{R_5 + R_6} = \frac{20 \times 20}{20 + 20} = \frac{1 \times 20}{1+1} = \frac{20}{2} = 10\,k\Omega$$

Let R_b be the equivalent resistance of the series connection of R_4 and R_a. Then, we have

$R_b = R_4 + R_a = 10 + 10 = 20$ kΩ.

Let R_c be the equivalent resistance of the parallel connection of R_3 and R_b. Then, we have

$$R_c = R_3 \| R_b = \frac{R_3 \times R_b}{R_3 + R_b} = \frac{20 \times 20}{20 + 20} = \frac{1 \times 20}{1+1} = \frac{20}{2} = 10\,k\Omega$$

Let R_d be the equivalent resistance of the series connection of R_2 and R_c. Then, we have

$R_d = R_2 + R_c = 10 + 10 = 20$ kΩ.

The equivalent resistance R_{eq} of the circuit shown in Figure P3.8 is the parallel connection of R_1 and R_d. Thus, we get

$$R_{eq} = R_1 \| R_d = \frac{R_1 \times R_d}{R_1 + R_d} = \frac{20 \times 20}{20 + 20} = \frac{1 \times 20}{1+1} = \frac{20}{2} = 10\,k\Omega$$

MATLAB

```
clear all;
R1=20000;R2=10000;R3=20000;R4=10000;R5=20000;R6=20000;
Req=P([R1,R2+P([R3,R4+P([R5,R6])])])

Answer:
Req =
       10000
```

Problem 2.40

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{1}{\frac{1}{2000} + \frac{1}{5000} + \frac{1}{4000} + \frac{1}{3000}} = \frac{60000}{30 + 12 + 15 + 20}$$

$$= \frac{60000}{77} = 779.2208\Omega$$

```
>> R1=2000;R2=5000;R3=4000;R4=3000;
>> Req=P([R1,R2,R3,R4])
Req =
     7.792207792207792e+02
```

Problem 2.41

Let $R_9 = R_2 || R_3 || R_4$, $R_{10} = R_6 || R_7 || R_8$, and $R_{11} = R_9 + R_5 + R_{10}$. Then, $R_{eq} = R_1 || R_{11}$.

$$R_9 = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{1}{\frac{1}{1000} + \frac{1}{2700} + \frac{1}{2000}} = 534.6535\,\Omega$$

$$R_{10} = \frac{1}{\frac{1}{R_6} + \frac{1}{R_7} + \frac{1}{R_8}} = \frac{1}{\frac{1}{2000} + \frac{1}{1500} + \frac{1}{6000}} = 750\,\Omega$$

$R_{11} = R_9 + R_5 + R_{10} = 3.7837\ \text{k}\Omega$

$$R_{eq} = \frac{R_1 R_{11}}{R_1 + R_{11}} = 2.877215\,k\Omega$$

```
clear all;
R1=12000;R2=1000;R3=2700;R4=2000;R5=2500;R6=2000;R7=1500;R8=6000;
R9=P([R2,R3,R4])
R10=P([R6,R7,R8])
R11=R9+R5+R10
Req=P([R1,R11])

Answer:
Req =
     2.877214991375255e+03
```

Problem 2.42

Let $R_6 = R_1 || R_2$, $R_7 = R_3 || R_4$. Then we have

$$R_6 = \frac{R_1 R_2}{R_1 + R_2} = 571.4286\ \Omega$$

$$R_7 = \frac{R_3 R_4}{R_3 + R_4} = 1.66667 \text{ k}\Omega$$

$R_{eq} = R_6 + R_7 + R_5 = 2.7381 \text{ k}\Omega$

```
clear all;
R1=600;R2=12000;R3=2000;R4=10000;R5=500;
R6=P([R1,R2])
R7=P([R3,R4])
Req=R6+R7+R5

Answer:
Req =
     2.738095238095238e+03
```

Problem 2.43

Let $R_9 = R_3||R_4$, $R_{10} = R_5||R_6$, $R_{11} = R_7||R_8$, $R_{12} = R_2 + R_9$, $R_{13} = R_{10} + R_{11}$. Then, $R_{eq} = R_1 + (R_{12}||R_{13})$.

$$R_9 = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{60k \times 20k}{60k + 20k} = \frac{1200k}{80} = 15k\Omega$$

$$R_{10} = \frac{R_5 \times R_6}{R_5 + R_6} = \frac{10k \times 15k}{10k + 15k} = \frac{150k}{25} = 6k\Omega$$

$$R_{11} = \frac{R_7 \times R_8}{R_7 + R_8} = \frac{20k \times 30k}{20k + 30k} = \frac{600k}{50} = 12k\Omega$$

$R_{12} = R_2 + R_9 = 3\text{k}\Omega + 15\text{k}\Omega = 18\text{k}\Omega$

$R_{13} = R_{10} + R_{11} = 6\text{k}\Omega + 12\text{k}\Omega = 18\text{k}\Omega$

$R_{eq} = R_1 + (R_{12}||R_{13}) = 6\text{k}\Omega + (18\text{k}\Omega||18\text{k}\Omega) = 6\text{k}\Omega + 9\text{k}\Omega = 15\text{k}\Omega$

```
clear all;
R1=6000;R2=3000;R3=60000;R4=20000;R5=10000;R6=15000;R7=20000;R8=30000;
R9=P([R3,R4])
R10=P([R5,R6])
R11=P([R7,R8])
R12=R2+R9
R13=R10+R11
Req=R1+P([R12,R13])

Req =
       15000
```

Problem 2.44

Let $R_6 = R_4||R_5$, $R_7 = R_3 + R_6$, $R_8 = R_2||R_7$. Then, $R_{eq} = R_1 + R_8$.

$$R_6 = \frac{R_4 \times R_5}{R_4 + R_5} = \frac{2k \times 3k}{2k + 3k} = \frac{6k}{5} = 1.2k\Omega$$

$R_7 = R_3 + R_6 = 1.8k\Omega + 1.2k\Omega = 3k\Omega$

$$R_8 = \frac{R_2 \times R_7}{R_2 + R_7} = \frac{7k \times 3k}{7k + 3k} = \frac{21k}{10} = 2.1k\Omega$$

$R_{eq} = R_1 + R_8 = 0.9k\Omega + 2.1k\Omega = 3k\Omega$

```
clear all;
R1=900;R2=7000;R3=1800;R4=2000;R5=3000;
Req=R1+P([R2,R3+P([R4,R5])])

Answer:
Req =
        3000
```

Problem 2.45

Let $R_8 = R_6||R_7$, $R_9 = R_4 + R_5 + R_8$. Then, $R_{eq} = R_1||R_2||R_3||R_9$.

$$R_8 = \frac{R_6 \times R_7}{R_6 + R_7} = \frac{20k \times 80k}{20k + 80k} = \frac{1600k}{100} = 16k\Omega$$

$R_9 = R_4 + R_5 + R_8 = 10k\Omega + 4k\Omega + 16k\Omega = 30k\Omega$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_9}} = \frac{1}{\frac{1}{4000} + \frac{1}{10000} + \frac{1}{30000} + \frac{1}{30000}} = 2.4k\Omega$$

```
clear all;
R1=4000;R2=10000;R3=30000;R4=10000;R5=4000;R6=20000;R7=80000;
Req=P([R1,R2,R3,R4+R5+P([R6,R7])])

Req =
        2400
```

Problem 2.46

Let $R_8 = R_3||R_4$, $R_9 = R_6||R_7$, $R_{10} = R_8 + R_5 + R_9$. Then, $R_{eq} = R_1||R_2||R_{10}$.

$$R_8 = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{10k \times 10k}{10k + 10k} = \frac{100k}{20} = 5k\Omega$$

$$R_9 = \frac{R_6 \times R_7}{R_6 + R_7} = \frac{10k \times 15k}{10k + 15k} = \frac{150k}{25} = 6k\Omega$$

$R_{10} = R_8 + R_5 + R_9 = 5k\Omega + 4k\Omega + 6k\Omega = 15k\Omega$

$$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_{10}}} = \frac{1}{\frac{1}{3000}+\frac{1}{10000}+\frac{1}{15000}} = \frac{30000}{\frac{30000}{3000}+\frac{30000}{10000}+\frac{30000}{15000}} = \frac{30000}{15} = 2k\Omega$$

```
clear all;
R1=3000;R2=10000;R3=10000;R4=10000;R5=4000;R6=10000;R7=15000;
Req=P([R1,R2,P([R3,R4])+R5+P([R6,R7])])

Req =
        2000
```

Problem 2.47

The voltage from the voltage source is divided into V_1 and V_2 in proportion to the resistance values. Thus, we have

$$V_1 = \frac{R_1}{R_1+R_2}V_s = \frac{2.5}{2.5+7.5}20\ V = \frac{1}{4}20\ V = 5\ V$$

$$V_2 = \frac{R_2}{R_1+R_2}V_s = \frac{7.5}{2.5+7.5}20\ V = \frac{3}{4}20\ V = 15\ V$$

Notice that V_2 can also be obtained from $V_2 = V_S - V_1 = 20 - 5 = 15$ V.

Problem 2.48

The equivalent resistance of the parallel connection of R_2 and R_3 is given by

$$R_4 = R_2 \parallel R_3 = \frac{R_2R_3}{R_2+R_3} = \frac{38k\times 57k}{38k+57k} = \frac{2166}{95}k = 22.8\ k\Omega$$

The voltage V_1 across R_1 is given by

$$V_1 = \frac{R_1}{R_1+R_4}V_s = \frac{27.2}{27.2+22.8}25\ V = \frac{27.2}{50}25\ V = \frac{27.2}{2}\ V = 13.6\ V$$

The voltage V_2 across R_2 and R_3 is given by

$$V_2 = \frac{R_4}{R_1+R_4}V_s = \frac{22.8}{27.2+22.8}25\ V = \frac{22.8}{50}25\ V = \frac{22.8}{2}\ V = 11.4\ V$$

Notice that V_2 can also be obtained from $V_2 = V_S - V_1 = 25 - 13.6 = 11.4$ V.

Problem 2.49

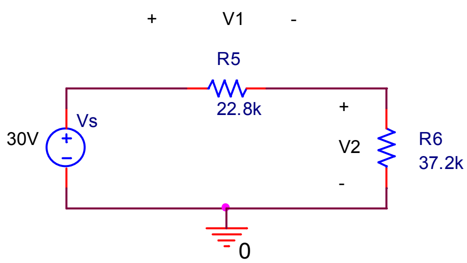
Let R_5 be the equivalent resistance of the parallel connection of R_1 and R_2. Then, we have

$$R_5 = \frac{R_1R_2}{R_1+R_2} = \frac{30k \times 95k}{30k+95k} = \frac{2850}{125}k = 22.8\ k\Omega$$

Let R_6 be the equivalent resistance of the parallel connection of R_3 and R_4. Then, we have

$$R_6 = \frac{R_3R_4}{R_3+R_4} = \frac{62k \times 93k}{62k+93k} = \frac{5766}{155}k = 37.2\ k\Omega$$

The circuit reduces to



The voltage V_1 across R_5 is given by

$$V_1 = \frac{R_5}{R_5+R_6}V_S = \frac{22.8}{22.8+37.2}\times 30\ V = \frac{22.8}{60}\times 30\ V = \frac{22.8}{2}\ V = 11.4\ V$$

The voltage V_2 across R_6 is given by

$$V_2 = \frac{R_6}{R_5+R_6}V_s = \frac{37.2}{22.8+37.2}\times 30\ V = \frac{37.2}{60}\times 30\ V = \frac{37.2}{2}\ V = 18.6\ V$$

Notice that V_2 can also be obtained from $V_2 = V_S - V_1 = 30 - 11.4 = 18.6$ V.

Problem 2.50

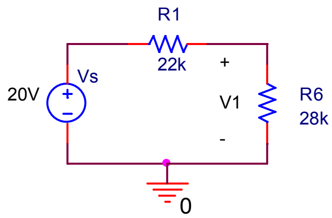
Let R_5 be the combined resistance of the series connection of R_3 and R_4. Then, we have

$R_5 = R_3 + R_4 = 24\ k\Omega + 60\ k\Omega = 84\ k\Omega$.

Let R_6 be the equivalent resistance of the parallel connection of R_2 and R_5. Then, R_6 is given by

$$R_6 = R_2 \,||\, R_5 = \frac{R_2R_5}{R_2+R_5} = \frac{42k \times 84k}{42k+84k} = \frac{3528}{126}k = 28\ k\Omega$$

The circuit reduces to



The voltage V_1 across R_6 is given by

$$V_1 = \frac{R_6}{R_1 + R_6} V_s = \frac{28}{22+28} \times 20\ V = \frac{28}{50} \times 20\ V = \frac{56}{5}\ V = 11.2\ V$$

The voltage V_1 is split between R_3 and R_4 in proportion to the resistance values. Applying the voltage divider rule, we have

$$V_2 = \frac{R_4}{R_3 + R_4} V_1 = \frac{60}{24+60} \times 11.2\ V = \frac{60}{84} \times 11.2\ V = 8\ V$$

Problem 2.51

Let R_6 be the equivalent resistance of the parallel connection of R_4 and R_5. Then, we have

$$R_6 = \frac{R_4 R_5}{R_4 + R_5} = \frac{22k \times 99k}{22k + 99k} = \frac{2178}{121} k = 18\ k\Omega$$

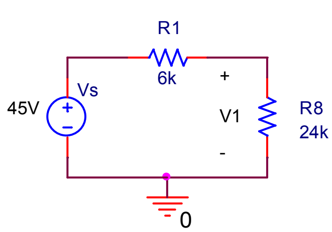
Let R_7 be the equivalent resistance of the series connection of R_3 and R_6. Then, we have

$R_7 = R_3 + R_6 = 70$ kΩ + 18 kΩ = 88 kΩ.

Let R_8 be the equivalent resistance of the parallel connection of R_2 and R_7. Then, we have

$$R_8 = \frac{R_2 R_7}{R_2 + R_7} = \frac{33k \times 88k}{33k + 88k} = \frac{2904}{121} k = 24\ k\Omega$$

The circuit reduces to



The voltage V_1 across R_8 is given by

$$V_1 = \frac{R_8}{R_1 + R_8} V_s = \frac{24}{6+24} \times 45\ V = \frac{24}{30} \times 45\ V = \frac{72}{2}\ V = 36\ V$$

The voltage across R_1 is given by

$$V_{R1} = \frac{R_1}{R_1 + R_8} V_s = \frac{6}{6+24} \times 45\ V = \frac{6}{30} \times 45\ V = \frac{18}{2}\ V = 9\ V$$

The voltage V_1 is split between R_3 and R_6 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_2 = \frac{R_6}{R_3 + R_6} V_1 = \frac{18}{70+18} \times 36\ V = \frac{18}{88} \times 36\ V = \frac{81}{11}\ V = 7.3636\ V$$

The voltage across R_3 is given by

$$V_{R3} = \frac{R_3}{R_3 + R_6} V_1 = \frac{70}{70+18} \times 36\ V = \frac{70}{88} \times 36\ V = \frac{315}{11}\ V = 28.6364\ V$$

Problem 2.52

Let R_8 be the equivalent resistance of the parallel connection of R_6 and R_7. Then, we have

$$R_8 = \frac{R_6 R_7}{R_6 + R_7} = \frac{6 \times 12}{6+12} k = \frac{72}{18} k = 4\ k\Omega$$

Let R_9 be the equivalent resistance of the series connection of R_5 and R_8. Then, we have

$R_9 = R_5 + R_8 = 5\ k\Omega + 4\ k\Omega = 9\ k\Omega$.

Let R_{10} be the equivalent resistance of the parallel connection of R_4 and R_9. Then, we have

$$R_{10} = \frac{R_4 R_9}{R_4 + R_9} = \frac{18 \times 9}{18+9} k = \frac{162}{27} k = 6\ k\Omega$$

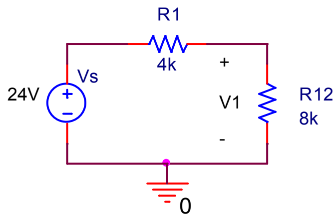
Let R_{11} be the equivalent resistance of the series connection of R_3 and R_{10}. Then, we have

$R_{11} = R_3 + R_{10} = 4\ k\Omega + 6\ k\Omega = 10\ k\Omega$.

Let R_{12} be the equivalent resistance of the parallel connection of R_2 and R_{11}. Then, we have

$$R_{12} = \frac{R_2 R_{11}}{R_2 + R_{11}} = \frac{40 \times 10}{40+10} k = \frac{400}{50} k = 8\ k\Omega$$

The circuit reduces to



The voltage V_1 across R_{12} is given by

$$V_1 = \frac{R_{12}}{R_1 + R_{12}} V_s = \frac{8}{4+8} \times 24\ V = \frac{8}{12} \times 24\ V = 16\ V$$

The voltage across R_1 is given by

$$V_{R1} = \frac{R_1}{R_1 + R_{12}} V_s = \frac{4}{4+8} \times 24\ V = \frac{4}{12} \times 24\ V = 8\ V$$

The voltage V_1 is split between R_3 and R_{10} in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_2 = \frac{R_{10}}{R_3 + R_{10}} V_1 = \frac{6}{4+6} \times 16\ V = \frac{6}{10} \times 16\ V = \frac{48}{5}\ V = 9.6\ V$$

The voltage across R_3 is given by

$$V_{R3} = \frac{R_3}{R_3 + R_{10}} V_1 = \frac{4}{4+6} \times 16\ V = \frac{4}{10} \times 16\ V = \frac{32}{5}\ V = 6.4\ V$$

The voltage V_2 is split between R_5 and R_8 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_3 = \frac{R_8}{R_5 + R_8} V_2 = \frac{4}{5+4} \times 9.6\ V = \frac{4}{9} \times 9.6\ V = \frac{12.8}{3}\ V = 4.2667\ V$$

The voltage across R_5 is given by

$$V_{R5} = \frac{R_5}{R_5 + R_8} V_2 = \frac{5}{5+4} \times 9.6\ V = \frac{5}{9} \times 9.6\ V = \frac{16}{3}\ V = 5.3333\ V$$

Problem 2.53

Let R_7 be the equivalent resistance of the parallel connection of R_4, R_5 and R_6. Then, we have

$$R_7 = \frac{1}{\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}} = \frac{1}{\frac{1}{30} + \frac{1}{36} + \frac{1}{45}} k = 12\ k\Omega$$

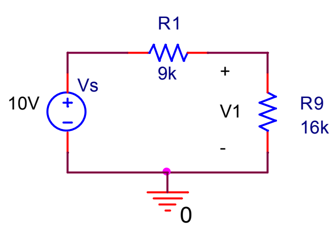
Let R_8 be the equivalent resistance of the series connection of R_3 and R_7. Then, we have

$R_8 = R_3 + R_7 = 8\ k\Omega + 12\ k\Omega = 20\ k\Omega$.

Let R_9 be the equivalent resistance of the parallel connection of R_2 and R_8. Then, we have

$$R_9 = \frac{R_2 R_8}{R_2 + R_8} = \frac{80 \times 20}{80 + 20} k = \frac{1600}{100} k = 16\ k\Omega$$

The circuit reduces to



The voltage V_1 across R_9 is given by

$$V_1 = \frac{R_9}{R_1 + R_9} V_s = \frac{16}{9+16} \times 10\ V = \frac{16}{25} \times 10\ V = \frac{32}{5}\ V = 6.4\ V$$

The voltage across R_1 is given by

$$V_{R1} = \frac{R_1}{R_1 + R_9} V_s = \frac{9}{9+16} \times 10\ V = \frac{9}{25} \times 10\ V = \frac{18}{5}\ V = 3.6\ V$$

The voltage V_1 is split between R_3 and R_7 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_2 = \frac{R_7}{R_3 + R_7} V_1 = \frac{12}{8+12} \times 6.4\ V = \frac{12}{20} \times 6.4\ V = \frac{96}{25}\ V = 3.84\ V$$

The voltage across R_3 is given by

$$V_{R3} = \frac{R_3}{R_3 + R_7} V_1 = \frac{8}{8+12} \times 6.4\ V = \frac{8}{20} \times 6.4\ V = \frac{64}{25}\ V = 2.56\ V$$

Problem 2.54

Let R_9 be the equivalent resistance of the parallel connection of R_6, R_7 and R_8. Then, we have

$$R_9 = \frac{1}{\frac{1}{\mathrm{R}_6} + \frac{1}{\mathrm{R}_7} + \frac{1}{\mathrm{R}_8}} = \frac{1}{\frac{1}{18} + \frac{1}{27} + \frac{1}{54}} k = 9\ k\Omega$$

Let R_{10} be the equivalent resistance of the series connection of R_5 and R_9. Then, we have

$R_{10} = R_5 + R_9 = 6\ k\Omega + 9\ k\Omega = 15\ k\Omega$.

Let R_{11} be the equivalent resistance of the parallel connection of R_4 and R_{10}. Then, we have

$$R_{11} = \frac{R_4 R_{10}}{R_4 + R_{10}} = \frac{30 \times 15}{30 + 15} k = \frac{450}{45} k = 10\ k\Omega$$

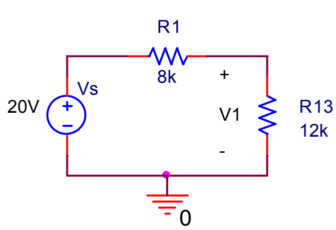
Let R_{12} be the equivalent resistance of the series connection of R_3 and R_{11}. Then, we have

$R_{12} = R_3 + R_{11} = 10\ k\Omega + 10\ k\Omega = 20\ k\Omega$.

Let R_{13} be the equivalent resistance of the parallel connection of R_2 and R_{12}. Then, we have

$$R_{13} = \frac{R_2 R_{12}}{R_2 + R_{12}} = \frac{30 \times 20}{30 + 20} k = \frac{600}{50} k = 12\ k\Omega$$

The circuit reduces to



The voltage V_1 across R_{13} is given by

$$V_1 = \frac{R_{13}}{R_1 + R_{13}} V_s = \frac{12}{8+12} \times 20\ V = \frac{12}{20} \times 20\ V = 12\ V$$

The voltage across R_1 is given by

$$V_{R1} = \frac{R_1}{R_1 + R_{13}} V_s = \frac{8}{8+12} \times 20\ V = \frac{8}{20} \times 20\ V = 8\ V$$

The voltage V_1 is split between R_3 and R_{11} in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_2 = \frac{R_{11}}{R_3 + R_{11}} V_1 = \frac{10}{10+10} \times 12\ V = \frac{10}{20} \times 12\ V = 6\ V$$

The voltage across R_3 is given by

$$V_{R3} = \frac{R_3}{R_3 + R_{11}} V_1 = \frac{10}{10+10} \times 12\ V = \frac{10}{20} \times 12\ V = 6\ V$$

The voltage V_2 is split between R_5 and R_9 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_3 = \frac{R_9}{R_5 + R_9} V_2 = \frac{9}{6+9} \times 6\ V = \frac{9}{15} \times 6\ V = \frac{18}{5}\ V = 3.6\ V$$

The voltage across R_5 is given by

$$V_{R5} = \frac{R_5}{R_5 + R_9} V_2 = \frac{6}{6+9} \times 6\ V = \frac{6}{15} \times 6\ V = \frac{12}{5}\ V = 2.4\ V$$

Problem 2.55

Let R_7 be the equivalent resistance of the parallel connection of R_4, R_5 and R_6. Then, we have

$$R_7 = \frac{1}{\frac{1}{\mathrm{R}_4} + \frac{1}{\mathrm{R}_5} + \frac{1}{\mathrm{R}_6}} = \frac{1}{\frac{1}{30} + \frac{1}{60} + \frac{1}{80}} k = 16\ k\Omega$$

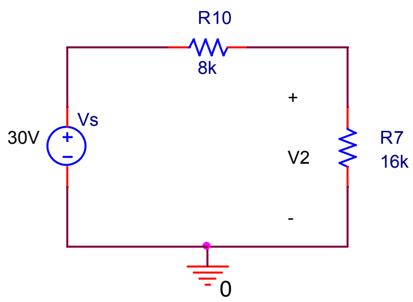
Let R_8 be the equivalent resistance of the series connection of R_1 and R_2. Then, we have

$R_9 = R_1 + R_2 = 10\ k\Omega + 30\ k\Omega = 40\ k\Omega$.

Let R_{10} be the equivalent resistance of the parallel connection of R_3 and R_9. Then, we have

$$R_{10} = \frac{R_3 R_9}{R_3 + R_9} = \frac{10 \times 40}{10+40} k = \frac{400}{50} k = 8\ k\Omega$$

R_{10} is in series with R_7. The circuit reduces to



The voltage V_2 across R_7 is given by

$$V_2 = \frac{R_7}{R_{10} + R_7} V_S = \frac{16}{8+16} \times 30\ V = \frac{16}{24} \times 30\ V = 20\ V$$

The voltage across R_{10} is given by

$$V_{R10} = \frac{R_{10}}{R_{10} + R_7} V_S = \frac{8}{8+16} \times 30\ V = \frac{8}{24} \times 30\ V = 10\ V$$

The voltage V_{R10} is split between R_1 and R_2 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_1 = V_2 + \frac{R_2}{R_1 + R_2} V_{R10} = 20 + \frac{30}{10+30} \times 10\ V = 20 + \frac{30}{40} \times 10\ V = 27.5\ V$$

Problem 2.56

Let R_7 be the equivalent resistance of the parallel connection of $R_2 + R_4$ and $R_3 + R_5$. Then we have

$$R_7 = (R_2 + R_4) \,||\, (R_3 + R_5) = 5k \,||\, 5k = \frac{5k \times 5k}{5k + 5k} = \frac{25k^2}{10k} = 2.5\,k\Omega$$

The voltage Vs is divided across R_1, R_7, and R_6 in proportion to the resistance values. The voltage across R_7 is given by

$$V_{R7} = \frac{R_7}{R_1 + R_7 + R_6} V_S = \frac{2.5}{1+2.5+1.5} \times 10V = \frac{2.5}{5} \times 10V = 5V$$

The voltage across R_1 is given by

$$V_{R1} = \frac{R_1}{R_1 + R_7 + R_6} V_S = \frac{1}{1+2.5+1.5} \times 10V = \frac{1}{5} \times 10V = 2V$$

The voltage across R_6 is given by

$$V_{R6} = \frac{R_6}{R_1 + R_7 + R_6} V_S = \frac{1.5}{1 + 2.5 + 1.5} \times 10V = \frac{1.5}{5} \times 10V = 3V$$

The voltage V_{R7} is divided across R_2 and R_4 in proportion to the resistance values. Thus, we have

$$V_{R2} = \frac{R_2}{R_2 + R_4} V_{R7} = \frac{1}{1+4} \times 5V = \frac{1}{5} \times 5V = 1V$$

$$V_{R4} = \frac{R_4}{R_2 + R_4} V_{R7} = \frac{4}{1+4} \times 5V = \frac{4}{5} \times 5V = 4V$$

The voltage V_{R7} is divided across R_3 and R_5 in proportion to the resistance values. Thus, we have

$$V_{R3} = \frac{R_3}{R_3 + R_5} V_{R7} = \frac{3}{3+2} \times 5V = \frac{3}{5} \times 5V = 3V$$

$$V_{R5} = \frac{R_5}{R_3 + R_5} V_{R7} = \frac{2}{3+2} \times 5V = \frac{2}{5} \times 5V = 2V$$

The voltage at node *a*, V_a, is the sum of V_{R4} and V_{R6}. Thus, we have

$V_a = V_{R4} + V_{R6} = 4V + 3V = 7V$

The voltage at node *b*, V_b, is the sum of V_{R5} and V_{R6}. Thus, we have

$V_b = V_{R5} + V_{R6} = 2V + 3V = 5V$

The voltage V_{ab} is the difference of V_a and V_b, that is,

$V_{ab} = V_a – V_b = 7V – 5V = 2V.$

Problem 2.57

Let $R_7 = R_2||R_3$ and $R_8 = R_5||R_6$. Then, we have

$$R_7 = R_2 \parallel R_3 = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{5k\Omega \times 5k\Omega}{5k\Omega + 5k\Omega} = \frac{25}{10} k\Omega = 2.5k\Omega$$

$$R_8 = R_5 \parallel R_6 = \frac{R_5 \times R_6}{R_5 + R_6} = \frac{2k\Omega \times 8k\Omega}{2k\Omega + 8k\Omega} = \frac{16}{10} k\Omega = 1.6k\Omega$$

The equivalent resistance seen from the voltage source is

$R_{eq} = R_1 + R_7 + R_4 + R_8 = 0.5\ k\Omega + 2.5\ k\Omega + 0.4\ k\Omega + 1.6\ k\Omega = 5\ k\Omega$

From Ohm's law, the current I_1 is given by

$$I_1 = \frac{V_S}{R_{eq}} = \frac{10V}{5k\Omega} = 2mA$$

The voltage drop across R_1 is $I_1R_1 = 2mA \times 0.5k\Omega = 1V$. The voltage V_1 is given by

$V_1 = V_S - I_1R_1 = 10\ V - 1V = 9V$.

Since $R_2 = R_3$, $I_2 = I_3 = I_1/2 = 1mA$. The voltage drop across R_7 is $I_1 \times R_7 = 2mA \times 2.5k\Omega = 5V$. We can get the same voltage drop from $I_2R_2 = I_3R_3 = 5V$. The voltage V_2 is given by

$V_2 = V_1 - 5V = 9V - 5V = 4V$.

The voltage drop across R_4 is $I_1 \times R_4 = 2mA \times 0.4k\Omega = 0.8V$. The voltage V_3 is given by

$V_3 = V_2 - 0.8V = 4V - 0.8V = 3.2V$.

The current through R_5 is given by

$$I_4 = \frac{V_3}{R_5} = \frac{3.2V}{2k\Omega} = 1.6mA$$

The current through R_6 is given by

$$I_5 = \frac{V_3}{R_6} = \frac{3.2V}{8k\Omega} = 0.4mA$$

Problem 2.58

From the current divider rule, the current I_{R1} is given by

$$I_{R1} = \frac{R_2}{R_1 + R_2} I_S = \frac{3}{2+3} \times 10\,mA = 6\,mA$$

Similarly, the current I_{R2} is given by

$$I_{R2} = \frac{R_1}{R_1 + R_2} I_S = \frac{2}{2+3} \times 10\,mA = 4\,mA$$

Problem 2.59

From the current divider rule, the current I_{R1} is given by

$$I_{R1}=\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}I_S=\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}\times 26\,mA=\frac{\frac{1}{2}}{\frac{6}{12}+\frac{4}{12}+\frac{3}{12}}\times 26\,mA=12\,mA$$

Similarly, the currents I_{R2} and I_{R3} are given respectively by

$$I_{R2}=\frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}I_S=\frac{\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}\times 26\,mA=\frac{\frac{1}{3}}{\frac{6}{12}+\frac{4}{12}+\frac{3}{12}}\times 26\,mA=8\,mA$$

$$I_{R3}=\frac{\frac{1}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}I_S=\frac{\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}\times 26\,mA=\frac{\frac{1}{4}}{\frac{6}{12}+\frac{4}{12}+\frac{3}{12}}\times 26\,mA=6\,mA$$

Problem 2.60

Let R_6 be the equivalent resistance of the parallel connection of R_2 and R_3. Then, R_6 is given by

$$R_6=\frac{R_2R_3}{R_2+R_3}=\frac{30k\times 60k}{30k+60k}=\frac{1800}{90}k=20\,k\Omega$$

Let R_7 be the equivalent resistance of the parallel connection of R_4 and R_5. Then, R_7 is given by

$$R_7=\frac{R_4R_5}{R_4+R_5}=\frac{90k\times 180k}{90k+180k}=\frac{180}{3}k=60\,k\Omega$$

Let R_8 be the equivalent resistance of the series connection of R_6 and R_7. Then, R_8 is given by

$R_8 = R_6 + R_7 = 80$ kΩ

The current from the current source I_S is split into I_{R1} and I_{R8} according to the current divider rule. Thus, we have

$$I_{R1}=\frac{R_8}{R_1+R_8}I_S=\frac{80}{20+80}\times 48\,mA=38.4\,mA$$

$$I_{R8} = \frac{R_1}{R_1 + R_8} I_S = \frac{20}{20+80} \times 48\,mA = 9.6\,mA$$

The current I_{R8} is split into I_{R2} and I_{R3} according to the current divider rule. Thus, we have

$$I_{R2} = \frac{R_3}{R_2 + R_3} I_{R8} = \frac{60}{30+60} \times 9.6\,mA = 6.4\,mA$$

$$I_{R3} = \frac{R_2}{R_2 + R_3} I_{R8} = \frac{30}{30+60} \times 9.6\,mA = 3.2\,mA$$

The current I_{R8} is split into I_{R4} and I_{R5} according to the current divider rule. Thus, we have

$$I_{R4} = \frac{R_5}{R_4 + R_5} I_{R8} = \frac{180}{90+180} \times 9.6\,mA = 6.4\,mA$$

$$I_{R5} = \frac{R_4}{R_4 + R_5} I_{R8} = \frac{90}{90+180} \times 9.6\,mA = 3.2\,mA$$

Problem 2.61

$$R_3 \,||\, R_4 = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{4k\Omega \times 6k\Omega}{4k\Omega + 6k\Omega} = \frac{24}{10} k\Omega = 2.4k\Omega$$

$R_5 = R_2 + (R_3||R_4) = 0.6\ k\Omega + 2.4\ k\Omega = 3k\Omega$

The current from the current source, $I_s = 2$ mA, is split between I_1 and I_2 based on the current divider rule.

$$I_1 = I_S \times \frac{R_5}{R_1 + R_5} = 2mA \times \frac{3k\Omega}{7k\Omega + 3k\Omega} = 0.6mA$$

$$I_2 = I_S \times \frac{R_1}{R_1 + R_5} = 2mA \times \frac{7k\Omega}{7k\Omega + 3k\Omega} = 1.4mA$$

The currents I_3 and I_4 are found by applying the current divider rule on R_3 and R_4.

$$I_3 = I_2 \times \frac{R_4}{R_3 + R_4} = 1.4mA \times \frac{6k\Omega}{4k\Omega + 6k\Omega} = 0.84mA$$

$$I_4 = I_2 \times \frac{R_3}{R_3 + R_4} = 1.4mA \times \frac{4k\Omega}{4k\Omega + 6k\Omega} = 0.56mA$$

The voltages V_1 and V_2 are found by applying Ohm's law.

$V_1 = I_1 \times R_1 = 0.6\text{mA} \times 7\text{k}\Omega = 4.2\text{V}$

$V_2 = I_3 \times R_3 = 0.84\text{mA} \times 4\text{k}\Omega = 3.36\text{V}$

Problem 2.62

Let R_a be the equivalent resistance of the series connection of R_2 and R_3. Then, we have

$R_a = R_2 + R_3 = 2\text{ k}\Omega + 5\text{ k}\Omega = 7\text{ k}\Omega$

Application of current divider rule yields

$$I_1 = I_s \times \frac{R_a}{R_1 + R_a} = 20\,mA \times \frac{7}{3+7} = 14\,mA$$

$$I_2 = I_s \times \frac{R_1}{R_1 + R_a} = 20\,mA \times \frac{3}{3+7} = 6\,mA$$

Problem 2.63

Let R_a be the equivalent resistance of the parallel connection of R_2 and R_3. Then, we have

$$R_a = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{20 \times 20}{20 + 20}k = 10\,k\Omega$$

Application of voltage divider rule yields

$$V_1 = V_s \times \frac{R_a}{R_1 + R_a} = 50\,V \times \frac{10}{15+10} = 20\,V$$

Application of Ohm's law yields

$$I_2 = \frac{V_1}{R_2} = \frac{20\,V}{20\,k\Omega} = 1\,mA$$

$$I_3 = \frac{V_1}{R_3} = \frac{20\,V}{20\,k\Omega} = 1\,mA$$

From KCL, we have

$I_1 = I_2 + I_3 = 1$ mA + 1 mA = 2 mA

Problem 2.64

Let R_8 be the equivalent resistance of the parallel connection of R_6 and R_7. Then, R_8 is given by

$$R_8 = \frac{R_6 R_7}{R_6 + R_7} = \frac{9k \times 18k}{9k + 18k} = \frac{18}{3}k = 6\,k\Omega$$

Let R_9 be the equivalent resistance of the series connection of R_5 and R_8. Then, R_9 is given by

$R_9 = R_5 + R_8 = 10$ kΩ

Let R_{10} be the equivalent resistance of the parallel connection of R_3, R_4 and R_9. Then, R_{10} is given by

$$R_{10} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_9}} = \frac{1}{\frac{1}{20k} + \frac{1}{20k} + \frac{1}{10k}} = \frac{20k}{4} = 5\,k\Omega$$

Let R_{11} be the equivalent resistance of the series connection of R_2 and R_{10}. Then, R_{11} is given by

$R_{11} = R_2 + R_{10} = 10$ kΩ

The current from the current source I_S is split into I_{R1} and I_{R11} according to the current divider rule. Thus, we have

$$I_{R1} = \frac{R_{11}}{R_1 + R_{11}} I_S = \frac{10}{15 + 10} \times 50\,mA = 20\,mA$$

$$I_{R11} = \frac{R_1}{R_1 + R_{11}} I_S = \frac{15}{15 + 10} \times 50\,mA = 30\,mA$$

Notice that $I_{R2} = I_{R11} = 30$ mA.

The current I_{R11} is split into I_{R3}, I_{R4} and I_{R9} according to the current divider rule. Thus, we have

$$I_{R3} = \frac{\frac{1}{R_3}}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_9}} I_{R11} = \frac{\frac{1}{20}}{\frac{1}{20} + \frac{1}{20} + \frac{1}{10}} \times 30\,mA = \frac{1}{4} \times 30\,mA = 7.5\,mA$$

$$I_{R4} = \frac{\frac{1}{R_4}}{\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_9}} I_{R11} = \frac{\frac{1}{20}}{\frac{1}{20}+\frac{1}{20}+\frac{1}{10}} \times 30\,mA = \frac{1}{4}\times 30\,mA = 7.5\,mA$$

$$I_{R9} = \frac{\frac{1}{R_9}}{\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_9}} I_{R11} = \frac{\frac{1}{10k}}{\frac{1}{20k}+\frac{1}{20k}+\frac{1}{10k}} \times 30\,mA = \frac{2}{4}\times 30\,mA = 15\,mA$$

Notice that $I_{R5} = I_{R9} = 15$ mA.

The current I_{R9} is split into I_{R6} and I_{R7} according to the current divider rule. Thus, we have

$$I_{R6} = \frac{R_7}{R_6+R_7} I_{R9} = \frac{18k}{9k+18k}\times 15\,mA = \frac{2}{3}\times 15\,mA = 10\,mA$$

$$I_{R7} = \frac{R_6}{R_6+R_7} I_{R9} = \frac{9k}{9k+18k}\times 15\,mA = \frac{1}{3}\times 15\,mA = 5\,mA$$

Problem 2.65

Let R_a be the equivalent resistance of the parallel connection of R_1 and R_2. Then, we have

$$R_a = \frac{R_1\times R_2}{R_1+R_2} = \frac{90\times 180}{90+180} = \frac{1\times 180}{1+2} = 60\,\Omega$$

Let R_b be the equivalent resistance of the parallel connection of R_4 and R_5. Then, we have

$$R_b = \frac{R_4\times R_5}{R_4+R_5} = \frac{100\times 150}{100+150} = \frac{2\times 150}{2+3} = 60\,\Omega$$

Let R_c be the equivalent resistance of the series connection of R_a and R_b. Then, we have

$R_c = R_a + R_b = 60\ \Omega + 60\ \Omega = 120\ \Omega$

Application of current divider rule yields

$$I_3 = I_s \times \frac{R_c}{R_3+R_c} = 9.6\,mA\times\frac{120}{360+120} = 2.4\,mA$$

From Ohm's law, the voltage across R_3 is given by

$V_1 = R_3I_3 = 360\ \Omega \times 0.0024\ \text{A} = 0.864\ \text{V}$

Application of voltage divider rule yields

$$V_2 = V_1 \times \frac{R_b}{R_a + R_b} = 0.864V \times \frac{60}{60+60} = 0.432V$$

Application of Ohm's law yields

$$I_1 = \frac{V_1 - V_2}{R_1} = \frac{0.864 - 0.432}{90} = \frac{0.432}{90} = 4.8\,mA$$

$$I_2 = \frac{V_1 - V_2}{R_2} = \frac{0.864 - 0.432}{180} = \frac{0.432}{180} = 2.4\,mA$$

$$I_4 = \frac{V_2}{R_4} = \frac{0.432}{100} = 4.32\,mA$$

$$I_5 = \frac{V_2}{R_5} = \frac{0.432}{150} = 2.88\,mA$$

MATLAB

```
clear all;format long;
R1=90;R2=180;R3=360;R4=100;R5=150;
Is=9.6e-3;
Ra=P([R1,R2])
Rb=P([R4,R5])
Rc=Ra+Rb
I3=Is*Rc/(R3+Rc)
V1=R3*I3
V2=V1*Rb/(Ra+Rb)
I1=(V1-V2)/R1
I2=(V1-V2)/R2
I4=V2/R4
I5=V2/R5

Answers:
Ra =
    60
Rb =
    60
Rc =
   120
I3 =
   0.002400000000000
V1 =
   0.864000000000000
V2 =
   0.432000000000000
I1 =
   0.004800000000000
I2 =
   0.002400000000000
I4 =
```

```
   0.004320000000000
I5 =
   0.002880000000000
```

Problem 2.66

Let R_a be the equivalent resistance of the series connection of R_5 and R_6. Then, we have

$R_a = R_5 + R_6 = 10\ \Omega + 5\ \Omega = 15\ \Omega$

Let R_b be the equivalent resistance of the parallel connection of R_4 and R_a. Then, we have

$$R_b = \frac{R_4 \times R_a}{R_4 + R_a} = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6\,\Omega$$

Let R_c be the equivalent resistance of the series connection of R_3 and R_b. Then, we have

$R_c = R_3 + R_b = 10\ \Omega + 6\ \Omega = 16\ \Omega$

Let R_d be the equivalent resistance of the parallel connection of R_2 and R_c. Then, we have

$$R_d = \frac{R_2 \times R_b}{R_2 + R_b} = \frac{20 \times 16}{20 + 16} = \frac{320}{36} = \frac{80}{9} = 8.8889\,\Omega$$

Let R_e be the equivalent resistance of the series connection of R_1 and R_d. Then, we have

$R_e = R_1 + R_d = 4\ \Omega + 8.8889\ \Omega = 12.8889\ \Omega$

Application of Ohm's law yields

$$I_1 = \frac{V_s}{R_e} = \frac{100}{12.8889} = 7.9786\,A$$

$V_1 = R_1 I_1 = 4 \times 7.9786 = 31.0345\ V$

From KVL, we have

$V_2 = V_s - V_1 = 100 - 31.0345 = 68.9655\ V$

Application of Ohm's law yields

$$I_2 = \frac{V_2}{R_2} = \frac{68.9655\,V}{20\,\Omega} = 3.4483\,A$$

From KCL, we have

$I_3 = I_1 - I_2 = 7.9786 - 3.4483 = 4.3103$ A

From Ohm's law, we have

$V_3 = R_3 I_3 = 10 \times 4.3103 = 43.1034$ V

From KVL, we have

$V_4 = V_2 - V_3 = 68.9655 - 43.1034 = 25.8621$ V

Application of Ohm's law yields

$$I_4 = \frac{V_4}{R_4} = \frac{25.8621V}{10\Omega} = 2.5862A$$

$$I_5 = \frac{V_4}{R_a} = \frac{25.8621V}{15\Omega} = 1.7241A$$

$V_5 = R_5 I_5 = 10 \times 1.7241 = 17.2414$ V

$V_6 = R_6 I_5 = 5 \times 1.7241 = 8.6207$ V

MATLAB

```
clear all;format long;
R1=4;R2=20;R3=10;R4=10;R5=10;R6=5;
Vs=100;
Ra=R5+R6
Rb=P([R4,Ra])
Rc=R3+Rb
Rd=P([R2,Rc])
Re=R1+Rd
I1=Vs/Re
V1=R1*I1
V2=Vs-V1
I2=V2/R2
I3=I1-I2
V3=R3*I3
V4=V2-V3
I4=V4/R4
I5=V4/Ra
V5=R5*I5
V6=R6*I5
SV=-Vs+V1+V3+V5+V6
SI=-I1+I2+I4+I5

Answers:
Ra =
    15
Rb =
   5.999999999999999
Rc =
    16
Rd =
   8.888888888888889
Re =
  12.888888888888889
```

```
I1 =
   7.758620689655173
V1 =
  31.034482758620690
V2 =
  68.965517241379303
I2 =
   3.448275862068965
I3 =
   4.310344827586207
V3 =
  43.103448275862071
V4 =
  25.862068965517231
I4 =
   2.586206896551723
I5 =
   1.724137931034482
V5 =
  17.241379310344819
V6 =
   8.620689655172409
SV =
    -3.552713678800501e-15
SI =
    -1.998401444325282e-15
```

Problem 2.67

Let R_a be the equivalent resistance of the parallel connection of $R_6 = 4\ \Omega$ and $R_7 + R_8 + R_9 = 12\ \Omega$. Then, we have

$$R_a = \frac{4 \times 12}{4 + 12} = \frac{48}{16} = 3\,\Omega$$

Let R_b be the equivalent resistance of the parallel connection of $R_2 = 4\ \Omega$ and $R_3 + R_4 + R_5 = 12\ \Omega$. Then, we have

$$R_b = \frac{4 \times 12}{4 + 12} = \frac{48}{16} = 3\,\Omega$$

Let R_c be the equivalent resistance of the series connection of R_1, R_a, and R_b. Then, we have

$R_c = R_1 + R_a + R_b = 4\ \Omega + 3\ \Omega + 3\ \Omega = 10\ \Omega$

The current through R_1 is

$$I_1 = \frac{V_2}{R_c} = \frac{40\,V}{10\,\Omega} = 4\,A$$

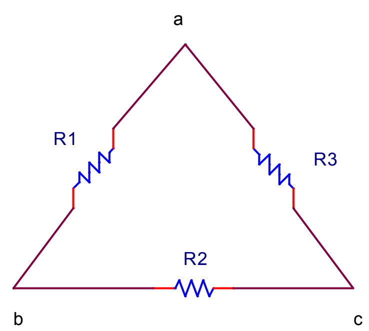
Application of current divider rule yields

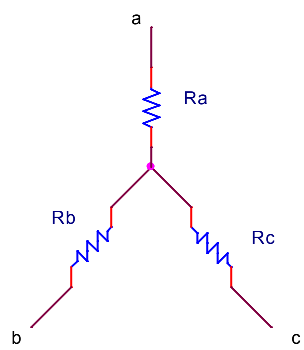
$$I = 4\,A \times \frac{4}{4 + 12} = \frac{16}{16}\,A = 1\,A$$

Problem 2.68

Resistors R_1, R_2, and R_3 are connected in delta. These three resistors can be transformed to wye configuration with resistors R_a, R_b, and R_c using

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}, \; R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}, \; R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$





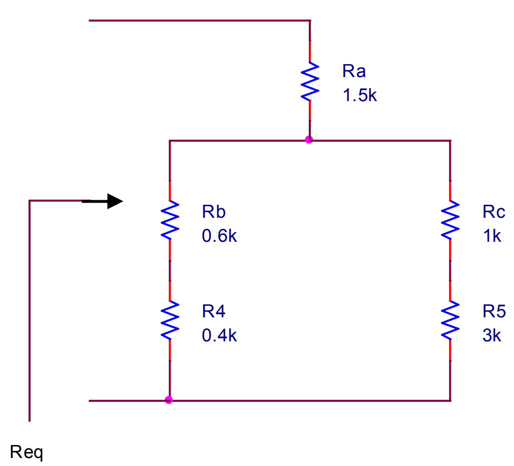
Substituting the values, we obtain

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{3 \times 5}{3+2+5} = \frac{15}{10} = 1.5\,k\Omega$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{3 \times 2}{3+2+5} = \frac{6}{10} = 0.6\,k\Omega$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{2 \times 5}{3+2+5} = \frac{10}{10} = 1\,k\Omega$$

The circuit shown in Figure P2.68 can be redrawn as that shown below.



The sum of R_b and R_4 is 1 kΩ, and the sum of R_c and R_5 is 4 kΩ. These two are connected in parallel. Thus, we have

$$(R_b + R_4) \,||\, (R_c + R_5) = 1 \,||\, 4 = \frac{1 \times 4}{1+4} = \frac{4}{5} = 0.8 k\Omega$$

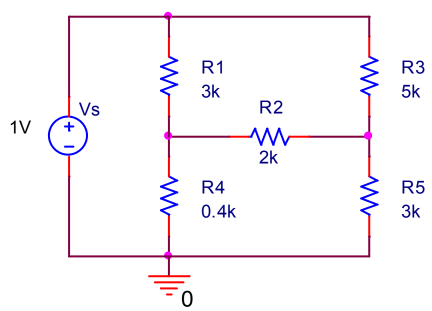
The equivalent resistance R_{eq} is the sum of R_a and $(R_b + R_4) \,||\, (R_c + R_5)$:

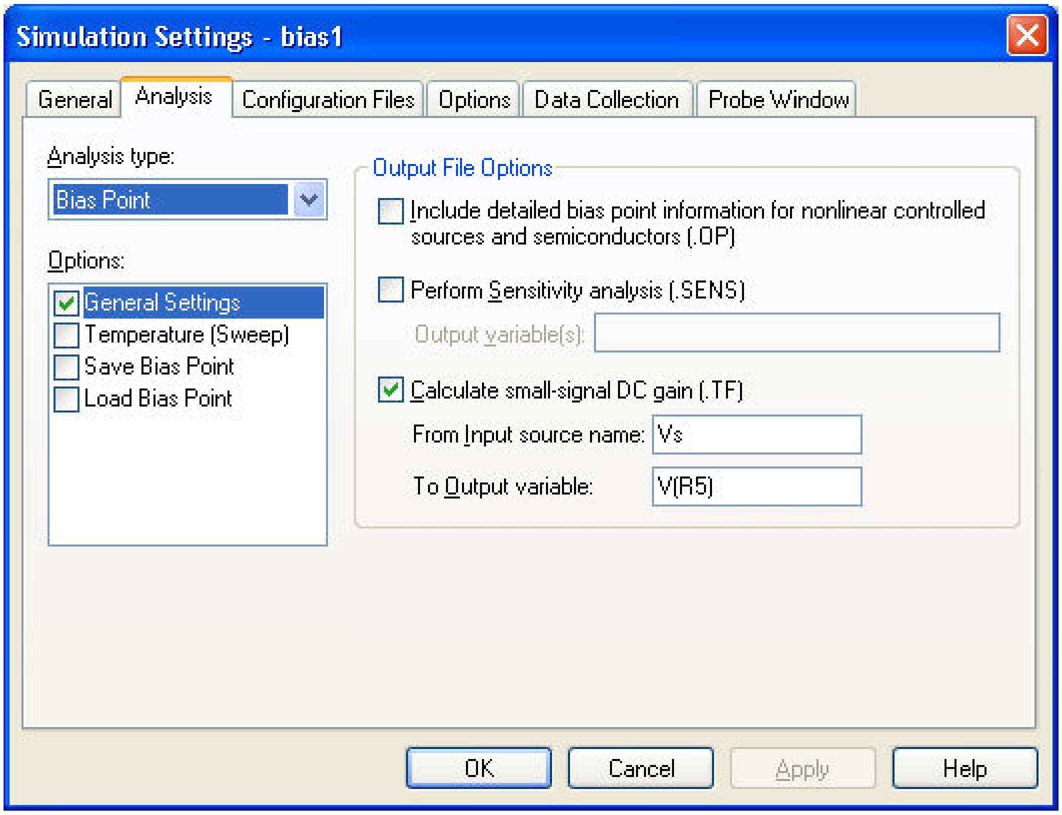
$R_{eq} = R_a + 0.8 = 1.5 + 0.8 = 2.3$ kΩ.

MATLAB

```
>> [Ra,Rb,Rc]=D2Y([3000,2000,5000])
Ra =
        1500
Rb =
   600
Rc =
        1000
>> Req=Ra+P([Rb+400,Rc+3000])
Req =
        2300
```

PSpice

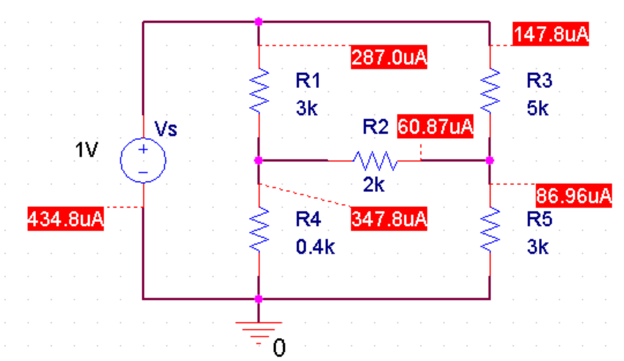




Click on View Simulation Output File. Part of the output file reads

```
****     SMALL-SIGNAL CHARACTERISTICS

      V(R_R5)/V_Vs =  2.609E-01

      INPUT RESISTANCE AT V_Vs =  2.300E+03

      OUTPUT RESISTANCE AT V(R_R5) =  1.043E+03
```

The input resistance is 2.3 kΩ. Alternatively, just run the bias point analysis (uncheck .TF) and display currents.



The current through the voltage source is 434.8μA. The input resistance is given by the ratio of the test voltage 1V to the current. Thus, we have

$$R_{eq} = \frac{1V}{434.8 \times 10^{-6}} = 2.2999\,k\Omega$$

Problem 2.69

The wye-connected resistors R_a, R_b, and R_c can be transformed to delta connected resistors R_1, R_2, and R_3.

$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_c} = \frac{14.4 \times 21.6 + 21.6 \times 12.96 + 14.4 \times 12.96}{12.96} = 60\,k\Omega$$

$$R_2 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_a} = \frac{14.4 \times 21.6 + 21.6 \times 12.96 + 14.4 \times 12.96}{14.4} = 54\,k\Omega$$

$$R_3 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b} = \frac{14.4 \times 21.6 + 21.6 \times 12.96 + 14.4 \times 12.96}{21.6} = 36\,k\Omega$$

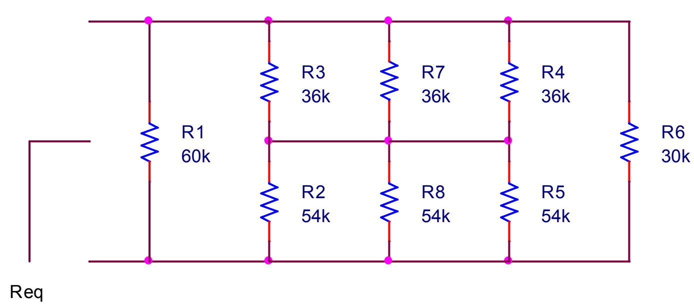
Similarly, the wye-connected resistors R_d, R_e, and R_f can be transformed to delta connected resistors R_4, R_5, and R_6.

$$R_4 = \frac{R_dR_e + R_eR_f + R_dR_f}{R_f} = \frac{9 \times 16.2 + 16.2 \times 13.5 + 9 \times 13.5}{13.5} = 36\,k\Omega$$

$$R_5 = \frac{R_dR_e + R_eR_f + R_dR_f}{R_d} = \frac{9 \times 16.2 + 16.2 \times 13.5 + 9 \times 13.5}{9} = 54\,k\Omega$$

$$R_6 = \frac{R_dR_e + R_eR_f + R_dR_f}{R_e} = \frac{9 \times 16.2 + 16.2 \times 13.5 + 9 \times 13.5}{16.2} = 30\,k\Omega$$

After two wye-delta transformations, the circuit shown in Figure P2.69 is transformed to the circuit shown below.



The equivalent resistance of the parallel connection of R_3, R_7, and R_4 is given by

$$R_g = \frac{1}{\frac{1}{R_3}+\frac{1}{R_7}+\frac{1}{R_7}} = \frac{1}{\frac{1}{36}+\frac{1}{36}+\frac{1}{36}} = \frac{1}{\frac{3}{36}} = \frac{36}{3} = 12\,k\Omega$$

The equivalent resistance of the parallel connection of R_2, R_8, and R_5 is given by

$$R_h = \frac{1}{\frac{1}{R_2}+\frac{1}{R_8}+\frac{1}{R_5}} = \frac{1}{\frac{1}{54}+\frac{1}{54}+\frac{1}{54}} = \frac{1}{\frac{3}{54}} = \frac{54}{3} = 18\,k\Omega$$

Resistors R_g and R_h are connected in series. The equivalent resistance of R_g and R_h is given by

$R_i = R_g + R_h = 12 + 18 = 30$ kΩ.

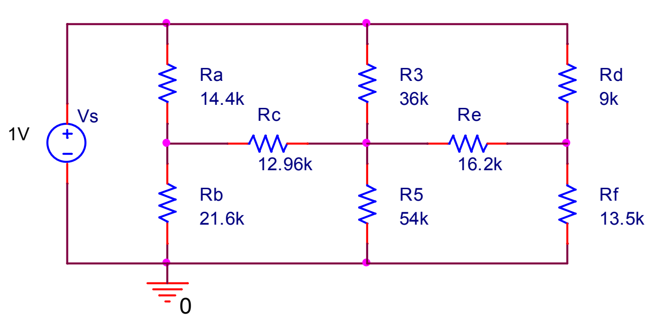
The equivalent resistance R_{eq} of the circuit shown in Figure P2.10 is given by the parallel connection of R_1, R_i, and R_6, that is,

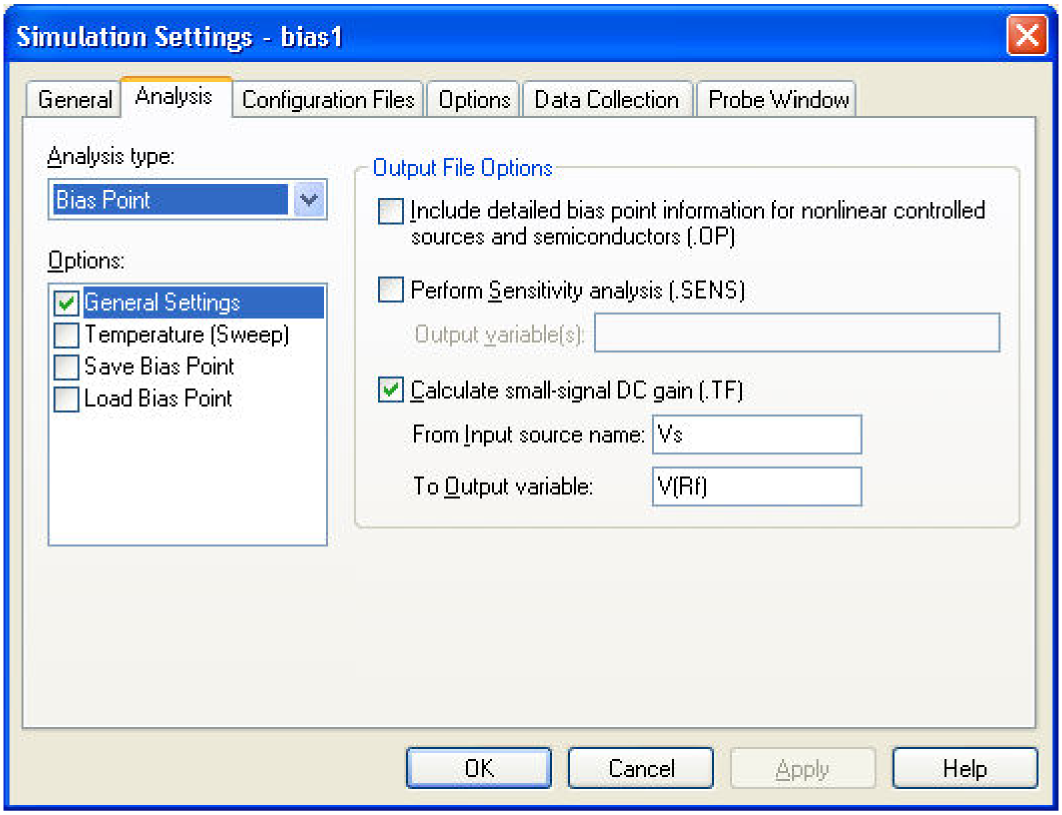
$$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_i}+\frac{1}{R_6}} = \frac{1}{\frac{1}{60}+\frac{1}{30}+\frac{1}{30}} = \frac{1}{\frac{5}{60}} = \frac{60}{5} = 12\,k\Omega$$

MATLAB

```
clear all;
Ra=14400;Rb=21600;Rc=12960;Rd=9000;Re=16200;Rf=13500;R7=36000;R8=54000;
[R1,R2,R3]=Y2D([Ra,Rb,Rc])
[R4,R5,R6]=Y2D([Rd,Re,Rf])
Req=P([R1,R6,P([R3,R7,R4])+P([R2,R8,R5])])

Answer:
Req =
   1.2000e+04
```

PSpice



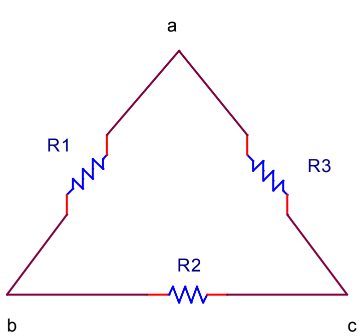


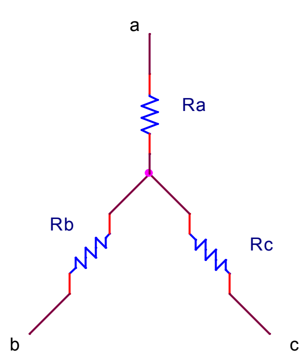
View Simulation Output File.

```
****     SMALL-SIGNAL CHARACTERISTICS

      V(R_Rf)/V_Vs =   6.000E-01

      INPUT RESISTANCE AT V_Vs =   1.200E+04

      OUTPUT RESISTANCE AT V(R_Rf)  =   4.500E+03
```

The input resistance is $R_{eq} = 12\ k\Omega$.

Problem 2.70





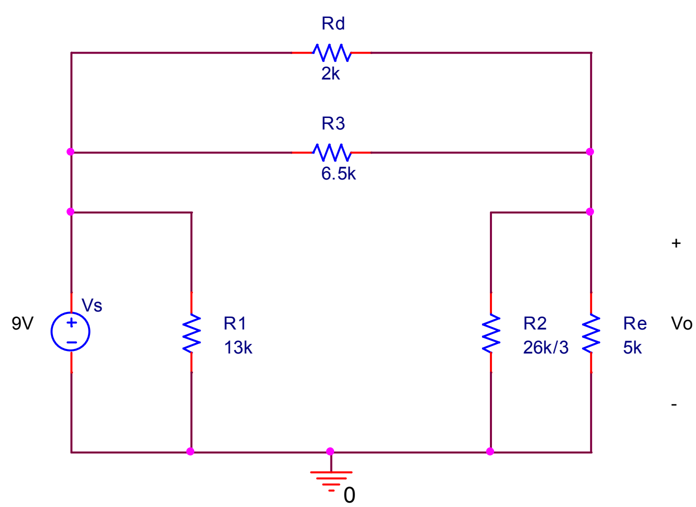
The wye-connected resistors R_a, R_b, and R_c can be transformed to delta connected resistors R_1, R_2, and R_3.

$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_c} = \frac{3\times4+4\times2+3\times2}{2}k = \frac{26}{2}k = 13\,k\Omega$$

$$R_2 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_a} = \frac{3\times4+4\times2+3\times2}{3}k = \frac{26}{3}k = 8.6667\,k\Omega$$

$$R_3 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b} = \frac{3\times4+4\times2+3\times2}{4}k = \frac{26}{4}k = 6.5\,k\Omega$$

The circuit shown in Figure P2.70 can be redrawn as that shown below.



Notice that

$R_3 \,||\, R_d = 1.5294\text{ k}\Omega$, $R_2 \,||\, R_e = 3.1707\text{ k}\Omega$

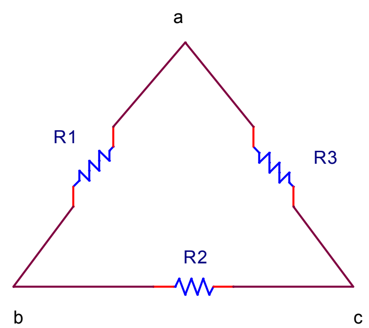
Application of voltage divider rule yields

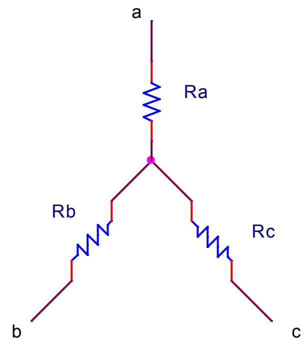
$$V_o = V_s \times \frac{R_2 \,||\, R_e}{R_3 \,||\, R_d + R_2 \,||\, R_e} = 9V \times \frac{3.1707}{1.5294 + 3.1707} = 6.0714V$$

Problem 2.71

Resistors R_1, R_2, and R_3 are connected in delta. These three resistors can be transformed to wye configuration with resistors R_a, R_b, and R_c using

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}, \; R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}, \; R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$





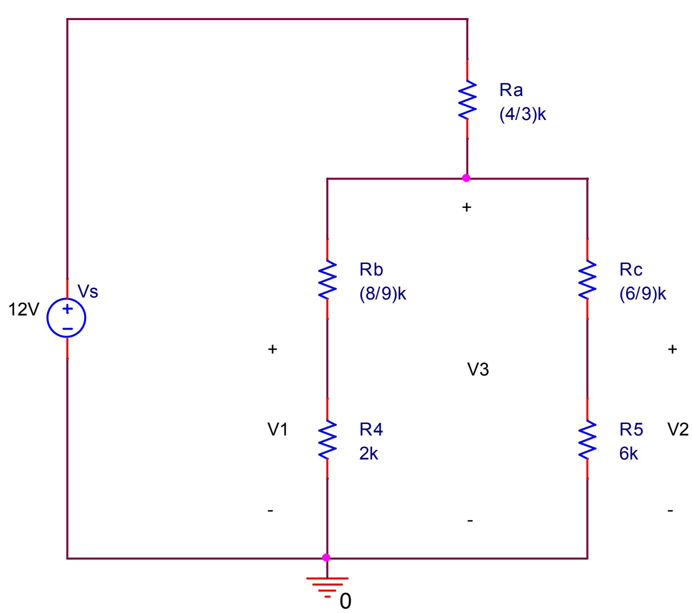
Substituting the values, we obtain

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{4 \times 3}{4+2+3} = \frac{12}{9} = \frac{4}{3} k\Omega = 1.3333 \, k\Omega$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{4 \times 2}{4+2+3} = \frac{8}{9} k\Omega = 0.8889 \, k\Omega$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{2 \times 3}{4+2+3} = \frac{6}{9} k\Omega = 0.6667 \, k\Omega$$

The circuit shown in Figure P2.70 can be redrawn as that shown below.



Let

$R_{10} = R_b + R_4 = 2.8889\ k\Omega$

$R_{11} = R_c + R_5 = 6.6667\ k\Omega$

$$R_{12} = R_{10} \parallel R_{11} = \frac{R_{10} \times R_{11}}{R_{10} + R_{11}} = 2.0115\, k\Omega$$

V_3 = voltage across R_{10} and R_{11}.

Application of voltage divider rule yields

$$V_3 = V_s \times \frac{R_{12}}{R_a + R_{12}} = 9\,V \times \frac{2.0115}{1.3333 + 2.0115} = 7.2222\,V$$

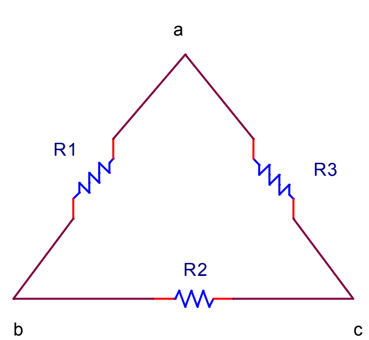
$$V_1 = V_3 \times \frac{R_4}{R_{10}} = 7.2222\,V \times \frac{2}{2.8889} = 5\,V$$

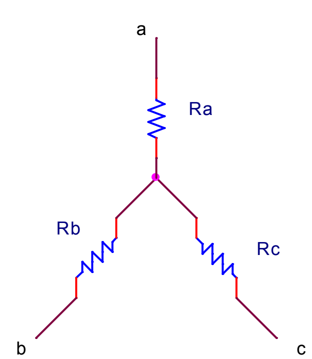
$$V_2 = V_3 \times \frac{R_5}{R_{11}} = 7.2222\,V \times \frac{6}{6.6667} = 6.5\,V$$

Problem 2.72

Resistors R_1, R_2, and R_3 are connected in delta. These three resistors can be transformed to wye configuration with resistors R_a, R_b, and R_c using

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3},\ R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3},\ R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$





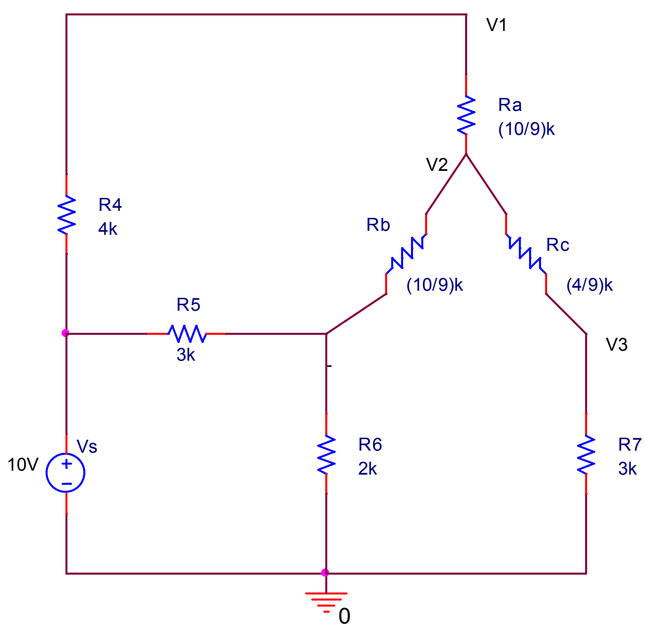
Substituting the values, we obtain

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{5 \times 2}{5+2+2} = \frac{10}{9} k\Omega = 1.1111 k\Omega$$

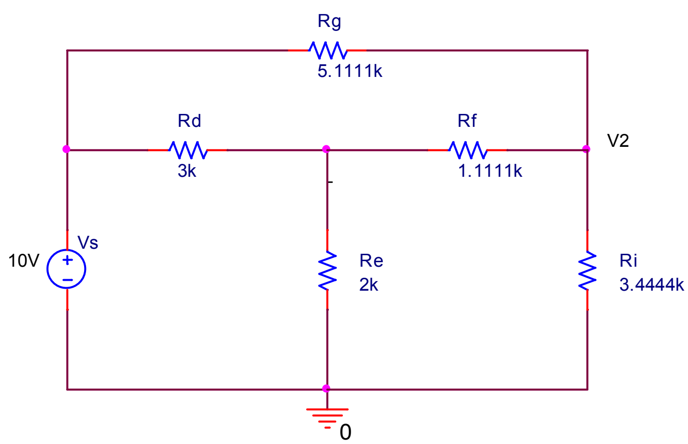
$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{5 \times 2}{5+2+2} = \frac{10}{9} k\Omega = 1.1111 k\Omega$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{2 \times 2}{5+2+2} = \frac{4}{9} k\Omega = 0.4444 k\Omega$$

The circuit shown in Figure P2.70 can be redrawn as that shown below.



This circuit can be redrawn as



Notice that

$R_g = R_4 + R_a = 5.1111\ k\Omega$

$R_d = R_5 = 3\ k\Omega$

$R_e = R_6 = 2\ k\Omega$

$R_f = R_b = 1.1111\ k\Omega$

$R_i = R_7 + R_c = 3.4444\ k\Omega$

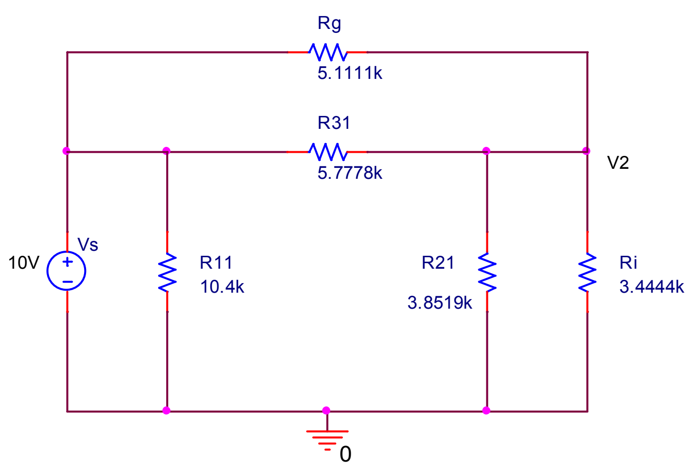
Converting the wye configuration R_d, R_e, R_f to delta configuration, we obtain

$$R_{11} = \frac{R_d R_e + R_e R_f + R_d R_f}{R_f} = \frac{3\times 2 + 2\times 1.1111 + 3\times 1.1111}{1.1111} k = 10.4 k\Omega$$

$$R_{21} = \frac{R_d R_e + R_e R_f + R_d R_f}{R_d} = \frac{3\times 2 + 2\times 1.1111 + 3\times 1.1111}{3} k = 3.8519 k\Omega$$

$$R_{31} = \frac{R_d R_e + R_e R_f + R_d R_f}{R_e} = \frac{3\times 2 + 2\times 1.1111 + 3\times 1.1111}{2} k = 5.7778 k\Omega$$

The circuit with R_{11}, R_{21}, and R_{31} is shown below.



Let $R_{51} = R_g \parallel R_{31}$ and $R_{52} = R_i \parallel R_{21}$. Then, we have

$$R_{51} = \frac{R_g \times R_{31}}{R_g + R_{31}} = \frac{5.1111\times 5.7778}{5.1111 + 5.7778} k = 2.712 k\Omega$$

$$R_{52} = \frac{R_i \times R_{21}}{R_i + R_{21}} = \frac{3.4444 \times 3.8519}{3.4444 + 3.8519} k = 1.8184\,k\Omega$$

Application of voltage divider rule yields

$$V_2 = V_s \times \frac{R_{52}}{R_{51} + R_{52}} = 10\,V \times \frac{1.8184}{2.712 + 1.8184} = 4.0137\,V$$

Application of voltage divider rule yields

$$V_1 = V_2 + \left(V_s - V_2\right) \times \frac{R_a}{R_4 + R_a} = 4.0137\,V + 5.9863\,V \times \frac{1.1111}{5.1111} = 5.3151\,V$$