Chapter 3
Physical Capital

***Note*:** Special icons in the margin identify problems requiring a computer or calculator and those requiring calculus .

■ Solutions to Problems

1. The key characteristics of physical capital are that it is productive, it is produced, its use is limited, it can earn a return, and it wears out.
   a. A delivery truck is physical capital because it is productive, i.e., by allowing a delivery man to drive instead of walk it increases his output (deliveries made); it has been produced itself; it is rival in its use, i.e., only one person can drive it to use it at a time; it can earn a return if rented out; and it suffers wear and tear (depreciation) for the period of its use.
   b. Milk is not physical capital because it is not productive. (To be slightly technical, milk *is* used in making other things—for example, cheese. But in this case milk is a raw material rather than a factor of production. In economic terms, "cheese production" is the value added resulting from using capital and labor to turn milk into cheese.)
   c. Farmland is not physical capital because even though it allows a worker to produce more output, it has not been produced itself.
   d. The Pythagorean Theorem is not physical capital because it is non-rival in its use. That is, an unlimited amount of people can use it at the same time.

2. To find the steady-state value of the country, we refer to Equation (3.3) on page 63.

$$y_{ss} = A^{\frac{1}{1-\alpha}}\left(\frac{\gamma}{\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Plugging in values: $A = 1$, $\alpha = 0.5$, $\gamma = 0.5$, and $\delta = 0.05$, we get:

$$y_{ss} = 1^{\frac{1}{1-0.5}}\left(\frac{0.5}{0.05}\right)^{\frac{0.5}{1-0.5}}.$$

Simplifying the above equation, we get $y_{ss} = 10$.

To find the current output per worker, we substitute in $k = 400$ into the production function to get:

$$y = k^{\frac{1}{2}} = 400^{\frac{1}{2}} = 20.$$

That is, the current output is 20 whereas the steady-state output level is 10. Therefore, we conclude that $y > y_{ss}$ so the country is above its steady-state level of output per worker.

3. Answers will vary.
4. This problem is solved on Page 64. If α =1/3, the ratio of the steady-state output of Country 1 and Country 2 is 1:2; if α =2/3, 1:16.

5. Since we know productivity, *A*, and depreciation, δ, are the same, we know that they will cancel out in our steady state ratio analysis. Therefore, with α =1/3, our equation of interest boils down to

$$\frac{y_{1,ss}}{y_{2,ss}} = \left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{2}},$$

for all three pairs of countries.

a. Using a subscript *T* for Thailand and a subscript *B* for Bolivia, we rewrite the previous equation for Thailand and Bolivia as

$$\frac{y_{T,ss}}{y_{B,ss}} = \left(\frac{\gamma_T}{\gamma_B}\right)^{\frac{1}{2}}.$$

Substituting in $\gamma_T = 0.352$ and $\gamma_B = 0.126,$ we get the steady state ratio to be:

$$\frac{y_{T,ss}}{y_{B,ss}} = \left(\frac{0.352}{0.126}\right)^{\frac{1}{2}} \approx 1.67.$$

The actual ratio is,

$$\frac{y_T}{y_B} = \left(\frac{\$13,279}{\$8,202}\right) \approx 1.62.$$

Therefore, the Solow Model does a good job in predicting relative income for Thailand and Bolivia.

b. Using a subscript *N* for Nigeria and a subscript *T* for Turkey, we rewrite the previous equation, with $\gamma_N = 0.064$ and $\gamma_T = 0.163$ to get,

$$\frac{y_{N,ss}}{y_{T,ss}} = \left(\frac{\gamma_N}{\gamma_T}\right)^{\frac{1}{2}} = \left(\frac{0.064}{0.163}\right)^{\frac{1}{2}} \approx 0.63.$$

The actual ratio is,

$$\frac{y_N}{y_T} = \left(\frac{\$6,064}{\$29,699}\right) \approx 0.20.$$

Therefore, the Solow Model does a poor job in predicting relative income for Nigeria and Turkey.

c. Using a subscript *J* for Japan and a subscript *N* for New Zealand, we rewrite the previous equation, with $\gamma_J = 0.299$ and $\gamma_N = 0.186$ to get,

$$\frac{y_{J,ss}}{y_{N,ss}} = \left(\frac{\gamma_J}{\gamma_N}\right)^{\frac{1}{2}} = \left(\frac{0.299}{0.186}\right)^{\frac{1}{2}} \approx 1.27.$$

The actual ratio is,

$$\frac{y_J}{y_N} = \left(\frac{\$57,929}{\$49,837}\right) \approx 1.16.$$

Therefore, the Solow Model does a good job in predicting relative income for Japan and New Zealand.

6. If output per worker is rising in Country X and output per worker is falling in Country Y, we can be assured that both countries are not in their respective steady states. Instead, they are converging to their respective steady states. In addition, for Country X and Country Y, we are given information that depreciation, productivity, and output per worker are identical. By the process of elimination, the only difference between the countries can and must be the level of capital stocks. Capital stock levels follow the process:

$$\Delta k_i = \gamma_i f(k_i) - \delta_i k_i.$$

As such, we can conclude that differences in investment rates are responsible for the divergence in output per worker. Specifically, a rise in output per worker for Country X and a fall in output per worker for Country Y imply that the ratio of steady-state output per worker is,

$$\frac{y_{X,ss}}{y_{Y,ss}} > 1; \quad \text{therefore,} \left(\frac{\gamma_X}{\gamma_Y}\right)^{\frac{\alpha}{1-\alpha}} > 1; \quad \text{implying, } \gamma_X > \gamma_Y.$$

Consequently, we can determine that Country X has a higher investment rate than that of Country Y, and these differences account for the falling and rising levels of output per worker.

7. a. First we find the steady-state level of capital per worker. Using the values for investment, γ = 0.25, depreciation, $\delta = 0.05$, productivity, $A = 1$, and $\alpha = 0.5$, we get,

$$k_{ss} = \left(\frac{A\gamma}{\delta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{(1)(0.25)}{0.05}\right)^{\frac{1}{1-0.5}} = 5^2 = 25.$$

That is, the steady-state level of capital per worker is 25. Plugging in k_{ss} into the production function we get the steady-state level of output per worker to be:

$$y_{ss} = k_{ss}^{\frac{1}{2}} = (25)^{\frac{1}{2}} = 5.$$

That is, the steady-state level of output per worker is 5.

b. For year 2, using 16.2 as the value for capital per worker, calculate output, y, followed by investment γy, depreciation δk, and then change in capital stock. Add the value for change in capital stock to 16.2, the value for capital per worker in year 2, to get capital per worker for year 3. Use year 3 capital to obtain all the values for year 3 and continue up to year 8. The filled in table is below.

Year	**Capital**	**Output**	**Investment**	**Depreciation**	**Change in Capital Stock**
1	16.00	4.00	1.00	0.08	0.20
2	16.20	4.02	1.01	0.81	0.20
3	16.40	4.05	1.01	0.82	0.19
4	16.59	4.07	1.02	0.83	0.19
5	16.78	4.10	1.02	0.84	0.19
6	16.96	4.12	1.03	0.85	0.18
7	17.14	4.14	1.04	0.86	0.18
8	17.32	4.16	1.04	0.87	0.17

c. The growth rate of output between years 1 and 2 is given by:

$$g = \left(\frac{y_2}{y_1}\right) - 1 = \left(\frac{4.02}{4}\right) - 1 = 0.005.$$

That is, output per worker grew at a rate of 0.5 percent between years 1 and 2. (Using exact values, the growth rate is approximately 0.62 percent for years 1 and 2.)

d. The growth rate of output between years 7 and 8 is given by:

$$g = \left(\frac{y_8}{y_7}\right) - 1 = \left(\frac{4.16}{4.14}\right) - 1 = 0.0048.$$

That is, output per worker grew at a rate of 0.48 percent between years 7 and 8. (Using exact values, the growth rate is approximately 0.52 percent for years 7 and 8.)

e. The speed of growth has changed from 0.50 percent to 0.48 percent implying that growth has slowed down at a rate of 4 percent. Thus, as a country reaches their steady-state value, the rate of growth slows.

8. First, in a steady-state level that maximizes consumption per worker, the change in capital stock will be zero. That is,

$$\Delta k = 0 = \gamma f(k) - \delta k.$$

Rearranging the previous equation, we know that investment must equal depreciation.

$$\gamma f(k) = \delta k.$$

Second, given that any output not saved is consumed, we can write an equation for consumption as,

$$C = y - \gamma f(k) = A(k)^{\alpha} - \delta k.$$

In the last part of the previous equation, we replace savings with depreciation and write output in functional form. In this form, we are able to take the derivative to find the necessary condition that will guarantee consumption maximization. Taking the derivative with respect to *k* and rearranging,

$$\frac{d}{dk}C = \frac{d}{dk}A(k)^{\alpha} - \delta k = \alpha A(k)^{\alpha-1} - k.$$

$$aA(k)^{\alpha-1} = \delta.$$

That is, the marginal product of capital must equal the rate of depreciation. Combining the consumption maximization condition $(aA(k)^{\alpha-1} = \delta.)$ with the steady-state condition $(\gamma f(k) = \delta k.)$, we get:

$$\text{saving} = \gamma y = \gamma f(k) = \delta k = \alpha A(k)^{\alpha-1} \times k = \alpha A(k)^{\alpha} = \alpha f(k) = \alpha y.$$

Therefore, it is easy to see the γ must equal α by the above string of equalities. In any steady-state level of consumption per worker, the investment/saving rate must equal the value α.

9. To find the steady-state value of the country, we refer to Equation (3.3) on page 63.

$$y_{ss} = A^{\frac{1}{1-\alpha}} \left(\frac{\gamma}{\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Plugging in values: $A = 1$, $\alpha = 0.5$, $\gamma = 0.2$ and $\delta = 0.02$, we get:

$$y_{ss} = 1^{\frac{1}{1-0.5}} \left(\frac{0.2}{0.02} \right)^{\frac{0.5}{1-0.5}}.$$

Simplifying the above equation, we get $y_{ss,1} = 10$.

Plugging in values: $A = 1$, $\alpha = 0.5$, $\gamma = 0.4$ and $\delta = 0.02$, we get:

$$y_{ss} = 1^{\frac{1}{1-0.5}} \left(\frac{0.4}{0.02} \right)^{\frac{0.5}{1-0.5}}.$$

Simplifying the above equation, we get $y_{ss,2} = 20$.

We can verify that each steady-state income level corresponds to the appropriate investment rate. From the figure, we can see that the upper steady state is stable, while the lower steady state is stable from one side but unstable from the other. That is, if initial income is less than 10 (capital stock less than 100), then the economy will go to the steady state with y=10, k=100. If the economy starts off k>100, it will go to the steady state with y=20, k=400 (note, figure is not drawn to scale).