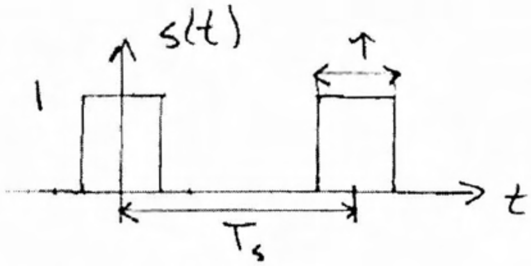
# Chapter 3

**3-1**




$$d = \frac{\tau}{T_s}$$

$$s(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega_s t)$$

$$a_0 = \frac{1}{T_s}\int_0^{T_s} s(t)\,dt = \frac{\tau}{T_s} = d$$

$$a_n = \frac{2}{T_s}\int_{-T_s/2}^{T_s/2} s(t)\cos(n\omega_s t)\,dt$$

$$= \frac{2}{T_s}\int_{-\tau/2}^{\tau/2} \cos\left(\frac{n2\pi}{T_s}t\right)dt$$

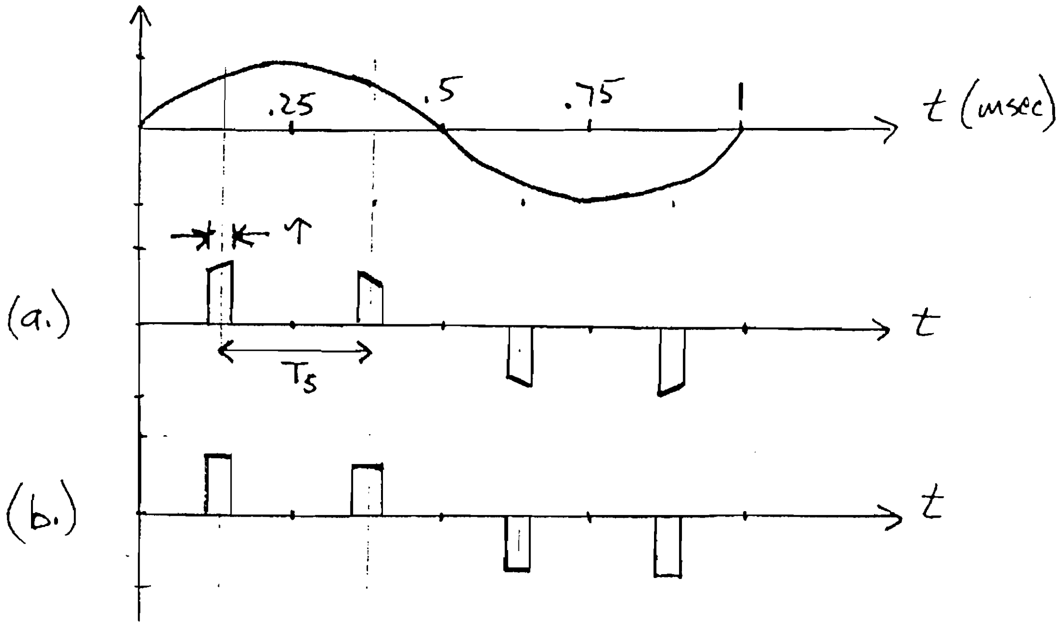
$$= \frac{2}{T_s}\;\frac{\sin\left(\frac{n2\pi}{T_s}t\right)}{n2\pi/T_s}\Bigg|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \qquad \sin(-x) = -\sin(x)$$

$$= \frac{1}{n\pi}\left[\sin(n\pi\tau/T_s) - \sin(-n\pi\tau/T_s)\right]$$

$$= \frac{1}{n\pi}\left[2\sin(n\pi\tau/T_s)\right]$$

$$= \frac{2d\,\sin(n\pi d)}{(n\pi d)}$$

## 3-2




## 3-3

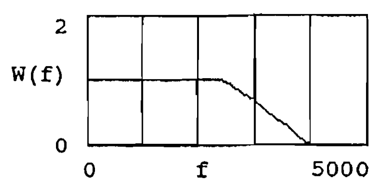
$A := 1 \qquad f1 := 2500 \qquad f2 := 4000$

$f := 0, 200 \,..\, 5000$

$W1(x) := \text{if}(|x| < f1, A, 0)$

$$W2(x) := \left[\frac{-A}{f2 - f1}\right] \cdot (|x| - f2) \cdot (\Phi(|x| - f1) - \Phi(|x| - f2))$$

$W(x) := W1(x) + W2(x)$




$fs := 10000 \qquad \tau := 50\ 10^{-6} \qquad Ts := \frac{1}{fs} \qquad d := \frac{\tau}{Ts}$

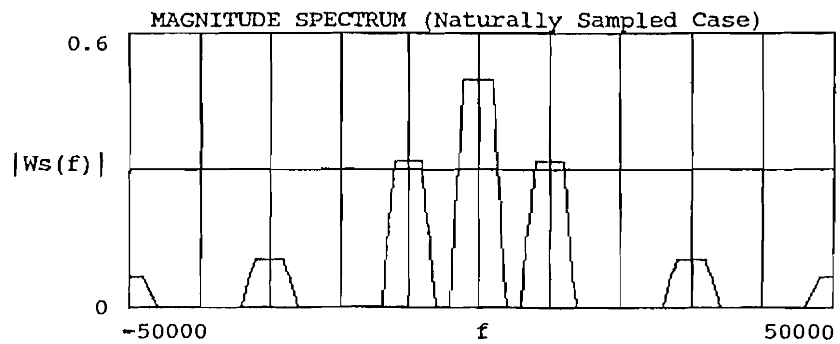
Naturally-sampled PAM

$n := -5, -4 \,..\, 5 \qquad Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$

$f := -50000, -48000 \,..\, 50000$

$$Ws(f) := d \cdot \sum_{n} (Sa(\pi\ n\ d)) \cdot W(f - n \cdot fs)$$

3-3 Cont'd.



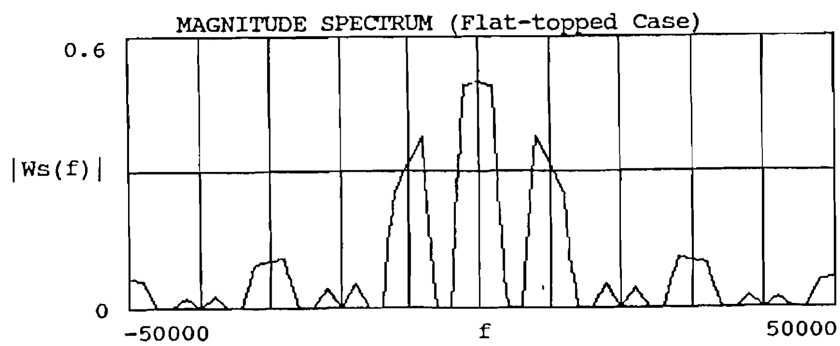

**3-4**

Flat-topped PAM

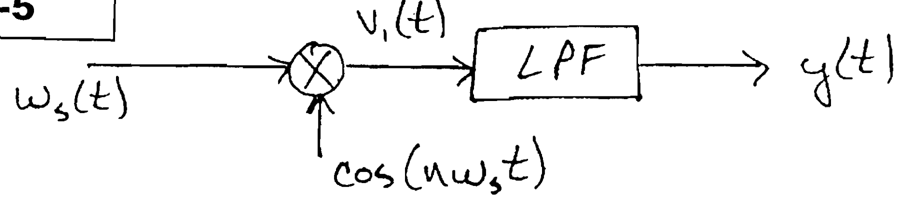
$$H(f) := \tau\ Sa(\pi \cdot \tau\ f)$$

$$Ws(f) := \left[\frac{1}{Ts}\right] \cdot H(f) \cdot \sum_{n} W(f - n\ fs)$$

See next screen for plot




**3-5**



3-5 Cont'd (a.)

$$w_s(t) = w(t)\left[d + 2d\sum_{k=1}^{\infty}\frac{\sin(k\pi d)}{k\pi d}\cos(k\omega_s t)\right]$$

$v_1(t) = w_s(t)\cos(n\omega_s t)$ Deleted by LPF Deleted by LPF

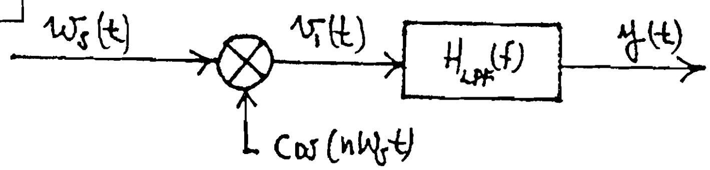
$$= w(t)\left\{d\,\cancel{\cos(n\omega_s t)} + 2d\sum_{\substack{k=1\\k\neq n}}^{\infty}\frac{\sin(k\pi d)}{k\pi d}\cancel{\cos(k\omega_s t)\cos(n\omega_s t)}\right.$$

$$+ 2d\frac{\sin(n\pi d)}{n\pi d}\underbrace{\cos^2(n\omega_s t)}_{\frac{1}{2}+\frac{1}{2}\cancel{\cos(2n\omega_s t)}}$$ Deleted by LPF

$$\Rightarrow y(t) = \frac{d\sin(n\pi d)}{n\pi d}w(t) = C\,w(t)$$

(b) $$C = \frac{d\sin(n\pi d)}{n\pi d}$$

**3-6**




$$v_1(t) = w_s(t)\cos(n\omega_s t) \longleftrightarrow V_1(f) = W_s(f) * \tfrac{1}{2}\left[\delta(f-nf_s)+\delta(f+nf_s)\right]$$

Using (3-10), we get

$$V_1(f) = \frac{1}{T_s}H(f)\sum_{k=-\infty}^{\infty}W(f-kf_s) * \frac{1}{2}\left[\delta(f-nf_s)+\delta(f+nf_s)\right]$$

$$= \frac{1}{T_s}H(f)\sum_{k=-\infty}^{\infty}\left[W(f-nf_s-kf_s) + W(f+nf_s-kf_s)\right]$$

$$= \frac{1}{T_s}H(f)\sum_{k=-\infty}^{\infty}\left[W(f-(n+k)f_s) + W(f-(k-n)f_s)\right]$$

$$\Rightarrow V_1(f) = \frac{2}{T_s}H(f)W(f) + \frac{1}{T_s}H(f)\underbrace{\left[\sum_{\substack{k=-\infty\\k\neq -n}}^{\infty}W(f-(n-k)f_s) + \sum_{\substack{k=-\infty\\k\neq n}}^{\infty}W(f-(k-n)f_s)\right]}_{\text{Does not pass through } H_{LPF}(f)}$$

Use (3-11)

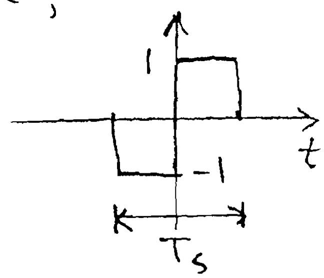
$$\Rightarrow Y(f) = \frac{2}{T}H(f)W(f)H_{LPF}(f) = 2d\frac{\sin(\pi\tau f)}{\pi\tau f}W(f)H_{LPF}(f) = C\,W(f)$$ Set

$$\Rightarrow H_{LPF}(f) = \begin{cases}\dfrac{C}{2d}\dfrac{\pi\tau f}{\sin\pi\tau f}, & |f| < f_{co}\\ 0, & f \text{ elsewhere}\end{cases}$$ where $B < f_{co} < f_s - B$

**3-7** Using (3-10)

$$W_s(f) = \frac{H(f)}{T_s} \sum_{k=-\infty}^{\infty} W(f - kf_s)$$

where $H(f)$ is the spectrum of the Manchester encoded pulse, $h(t)$.




Thus

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$= \int_{-T_s/2}^{0} (-1) e^{-j\omega t} dt + \int_{0}^{T_s/2} (1) e^{-j\omega t} dt$$

$$= \frac{j}{\omega}\left[ -e^{-j\omega t}\Big|_{-T_s/2}^{0} + e^{-j\omega t}\Big|_{0}^{T_s/2} \right]$$

$$= \frac{-j}{\omega}\left[ 2 - 2\left( \cancelto{\cos\frac{\omega T_s}{2}}{\frac{e^{j\omega T_s/2} + e^{-j\omega T_s/2}}{2}} \right) \right]$$

$$H(f) = -j T_s \frac{\left(1 - \cos \omega T_s/2\right)}{\omega T_s/2}$$

3-7 Cont'd.

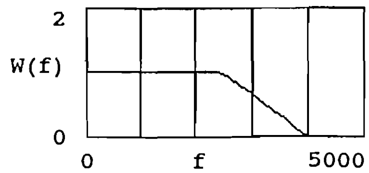
$A := 1 \qquad f1 := 2500 \qquad f2 := 4000$

$f := 0, 200 \,..\, 5000$

$W1(x) := \text{if}(|x| < f1, A, 0)$

$$W2(x) := \left[\frac{-A}{f2 - f1}\right] (|x| - f2)\,(\Phi(|x| - f1) - \Phi(|x| - f2))$$

$W(x) := W1(x) + W2(x)$




$fs := 10000$

$$Ts := \frac{1}{fs}$$

$j := \sqrt{-1}$

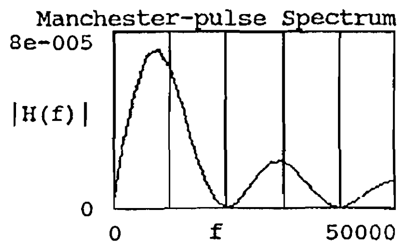
$n := -5, -4 \,..\, 5$

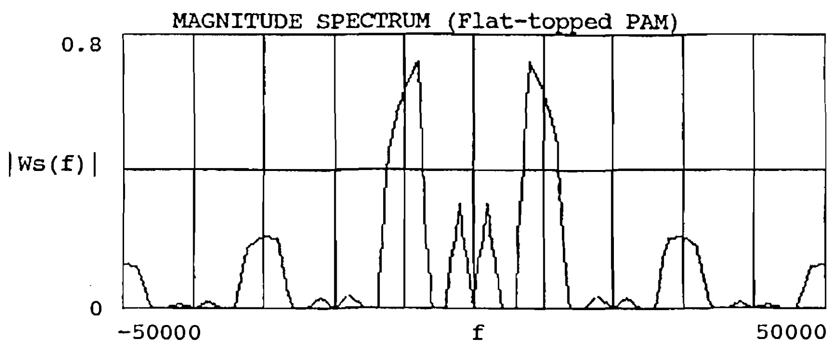
$$Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

$f := -50000, -48000 \,..\, 50000$

$$H(f) := j \cdot Ts \cdot Sa\left[\pi \cdot f \frac{Ts}{2}\right] \cdot \sin\left[2\,\pi \cdot f \cdot \frac{Ts}{4}\right]$$

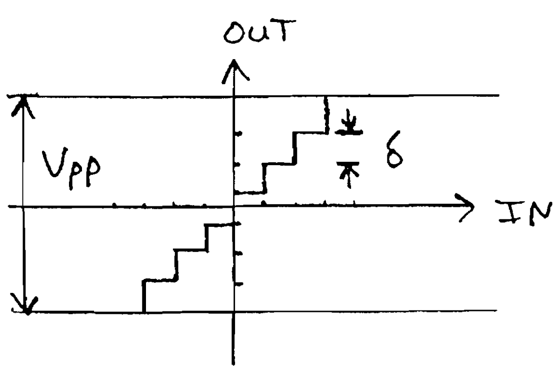
$$Ws(f) := \left[\frac{1}{Ts}\right] \cdot H(f) \cdot \sum_{n} W(f - n\,fs)$$




**3-8** Binary PCM

$M = 2^n$ levels



3-8. Cont'd for $|n_q| \le \frac{P}{100} V_{pp}$

we need

$$\text{step size} = \delta = \frac{V_{pp}}{M} \le \frac{2P}{100} V_{pp}$$

$$\frac{1}{M} \le \frac{P}{50}$$

$$2^n = M \ge \frac{50}{P}$$

$$n \ge \log_2\left(\frac{50}{P}\right)$$

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)} = \log_a(b)\log_b(x)$$

$$\ge \log_2(10)\log_{10}\left(\frac{50}{P}\right)$$

**3-9** (a.) $f_s = 2B = 2(100) = \underline{\underline{200 \text{ samples/sec}}}$

(b.) Using the results given in prob. 3-8.

$$n \ge 3.32\log_{10}\left(\frac{50}{P}\right) = 3.32\log_{10}\left(\frac{50}{0.1}\right)$$

$$= 8.96$$

$$\underline{\underline{n = 9 \text{ bits/word}}}$$

(c.) $R = \left(\frac{n \text{ bits}}{\text{word}}\right)\left(\frac{f_s \text{ words}}{\text{sec}}\right) = 200(9) = \underline{\underline{1.8 \frac{\text{k bits}}{\text{sec}}}}$

(d.) For binary PCM $D = R$

eq. (3-74) $D = \frac{2B}{1+r}$, for $B_{min}$, $r = 0$

$$\Rightarrow B = \frac{D}{2} = \underline{\underline{900 \text{ Hz}}}$$

**3-10**

$$\left(\frac{S}{N}\right) = M^2 = 10^{\frac{30}{10}} = 1000 \Rightarrow M = \sqrt{1000} = 31.6$$

(30 dB)

$\Rightarrow$ Use $M = 32 = 2^5$ $\Rightarrow$ Need $\underline{\underline{5 \text{ bits/sample} = n}}$

$$R = f_s n = \left(8 \frac{\text{k samples}}{\text{sec}}\right)\left(\frac{5 \text{ bits}}{\text{sample}}\right) = 40 \frac{\text{k bits}}{\text{sec}}$$

$$\Rightarrow R = \left(\frac{40 \text{ k bits}}{\text{sec}}\right)\left(\frac{\text{byte}}{8 \text{ bits}}\right) = 5 \frac{\text{k bytes}}{\text{sec}}$$

$$T = \frac{700 \text{ M bytes}}{5 \frac{\text{k bytes}}{\text{sec}}} = \frac{700 \times 10^6}{5 \times 10^3} \text{sec} = 140 \times 10^3 \text{ sec}$$

$$\Rightarrow \underline{\underline{T = 140 \times 10^3 \text{ sec}}} = \frac{140 \times 10^3 \text{ sec}}{60 \text{ sec/min}} = \underline{\underline{2{,}333 \text{ min} = 38 \text{ hrs}, 53 \text{ min}}}$$

**3-11**

Using (3-18),

(a) $\left(\frac{S}{N}\right)_{dB\ peak} = 6.02n + 4.77 = 55$ (set) $\Rightarrow n = 8.34 \Rightarrow$ Use $\underline{\underline{n = 9 \text{ bits}}}$ Word length

$M = 2^n = 2^9 = \underline{\underline{512 \text{ quantizing steps}}}$

(b) $f_s = 2 f_{analog} = 2(4.2 \text{ MHz}) = 8.4 \text{ Msamples/sec}$

$$R = f_s n = \left(8.4 \frac{\text{Msamples}}{\text{sec}}\right)\left(\frac{9 \text{ bits}}{\text{sample}}\right) = \underline{\underline{75.6 \frac{\text{M bits}}{\text{sec}}}}$$

For rectangular pulse shape,

$\underline{\underline{B_{null} = R = 75.6 \text{ MHz}}}$

**3-12**

(a.) $f_s \geq 2 B_{analog} = 2(20 \text{ kHz}) = 40 \frac{\text{k samples}}{\text{sec}}$

For 8X oversampling of the <u>recovered</u> PCM signal
(used to increase $f_s$ 8X and simplify LPF requirements)

$\Rightarrow f_{8x} = 8 f_s = 320$ k samples/sec

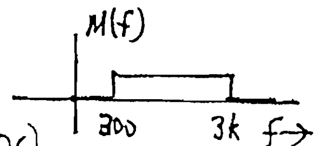
$$B_{null} = R = n f_{8x} = \left(16 \frac{\text{bits}}{\text{sample}}\right)\left(320 \frac{\text{k samples}}{\text{sec}}\right) = \underline{\underline{5.12 \text{ MHz}}}$$

(b) Using (3-18)

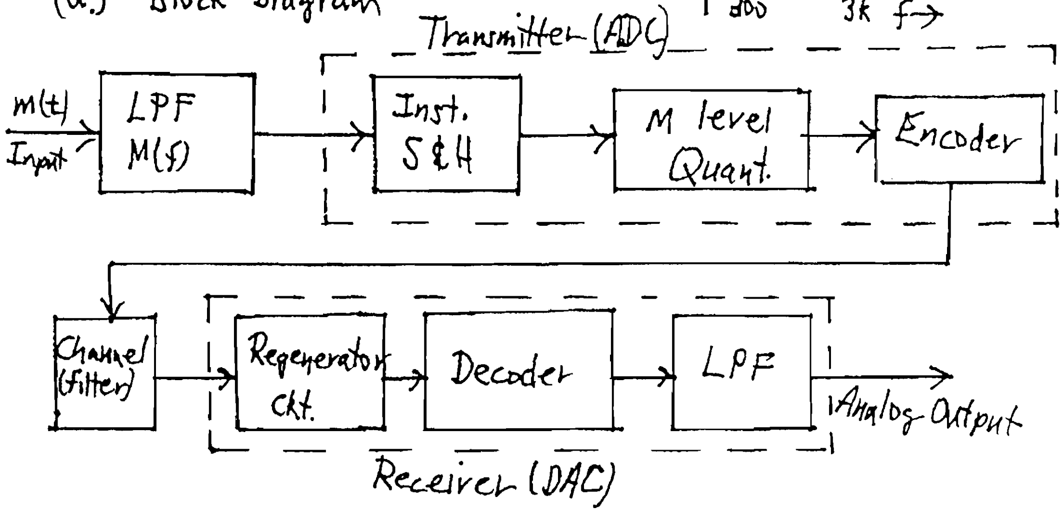
$$\left(\frac{S}{N}\right)_{peak} = 6.02n + 4.77 \text{dB} = 6.02(15) + 4.77 = \underline{\underline{94.77 \text{dB}}}$$

**3-13** $(44.1\text{ k samples/sec})(16\text{ bits/sample}) = 705.6\text{ kb/s}$ for MONO
$\Rightarrow \underline{\underline{1{,}411\text{ kb/s}}}$ for Stereo

**3-14** PCM system




(a.) Block Diagram




(b.) Assume $P_e = 0 \Rightarrow$ Use (3-18).

$$\left(\frac{S}{N}\right)_{dB\ peak} = 6.02n + 4.77 = 30\text{dB} \Rightarrow n = 4.19$$

$\Rightarrow$ Use $n = 5 + \text{parity} = \underline{\underline{6}}$ (If no parity, $n = 5$ bits)

$$\Rightarrow B_{null} = R = nf_s = (6)\left(7k\frac{\text{sample}}{\text{sec}}\right) = \underline{\underline{42\text{ kHz}}}\ \text{(with parity)}$$

$$B_{null} = (5)(7) = \underline{\underline{35\text{ kHz}}}\ \text{(No parity)}$$

(c) If the audio signal is a voice signal it is likely to have a large peak-to-average voltage ratio. Therefore, a non-uniform quantizer with smaller step sizes near the zero level will reduce the quantizing noise. A $\mu = 255$ non-uniform characteristic is typically used in the U.S. See the textbook for further discussion.

**3-15** For the case of (3-17b) with $P_e = 0$,

$\left(\frac{S}{N}\right) = M^2 \Rightarrow$ For no more than 0.1% error $\left(\frac{S}{N}\right) = 0.999M^2$

Using (3-16b) we get

$$0.999\cancel{M^2} = \frac{\cancel{M^2}}{1+4(M^2-1)P_e}$$

$$\Rightarrow P_e = \frac{1.001\times10^{-3}}{4(M^2-1)} = \frac{2.5025\times10^{-4}}{M-1}$$

For: $M=4$, $P_e \le 1.668\times10^{-5}$
$M=8$, $P_e \le 3.972\times10^{-6}$
$M=16$, $P_e \le 9.814\times10^{-9}$

As the number of levels is increased, the approximation becomes more constrained.

**3-16** (a.) $P_e = 10^{-4}$ $\frac{S}{N} \ge 30\,dB$

Eq. (3-16) $\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left[\frac{3m^2}{1+4(m^2-1)P_e}\right]$

$m = 2^n$ levels

for $n=4$: $\left(\frac{S}{N}\right)_{OUT} = 28.4\,dB$

for $\underline{\underline{n=5}}$: $\left(\frac{S}{N}\right)_{OUT} = 33.4\,dB$

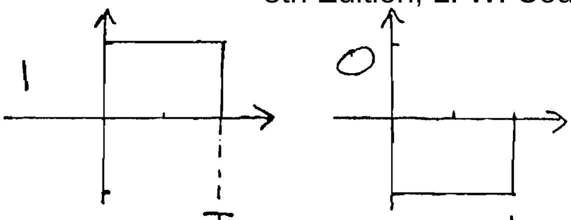
$\longrightarrow m = 2^5 = \underline{\underline{32\text{ levels}}}$

(b.) $f_s = 2(2.7\,KHz) = 5.4\,k\,\frac{\text{samples}}{\text{sec}}$

The first zero-crossing of the $\frac{\sin x}{x}$ type spectrum is:

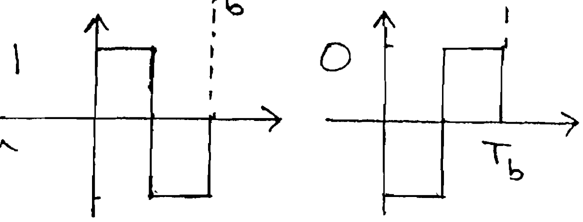
$$B = \frac{n}{T_s} = nf_s = 5(5.4k) = \underline{\underline{27\,KHz = B}}$$

**3-17** polar –

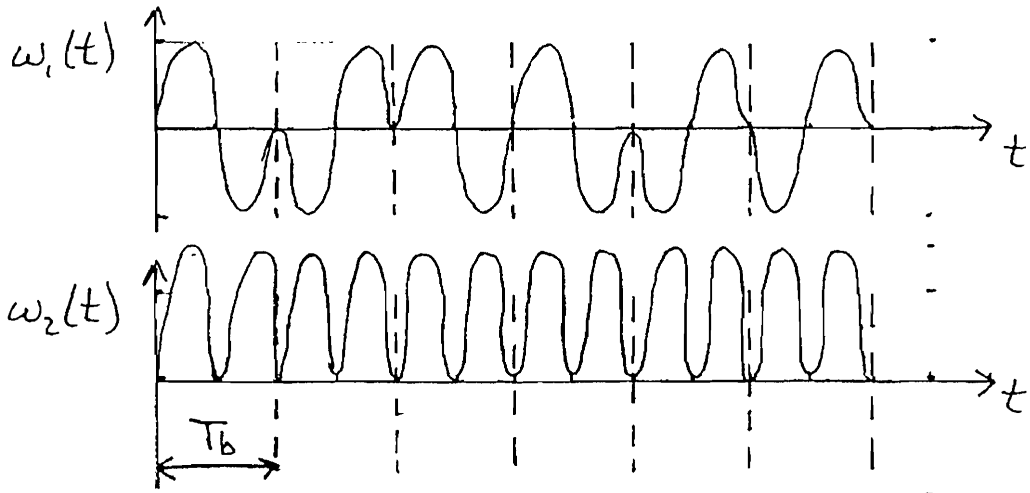


See Fig. 3-20 for waveforms

Manchester –



Waveforms shown below:



fund. freq. of $w_2(t) = \frac{2}{T_b} \neq f_0$

$\therefore$ Synch. shown in Fig. 3-17 will not work for Manchester format unless the N.B.F. $(H(f))$ is tuned to $2f_0$.

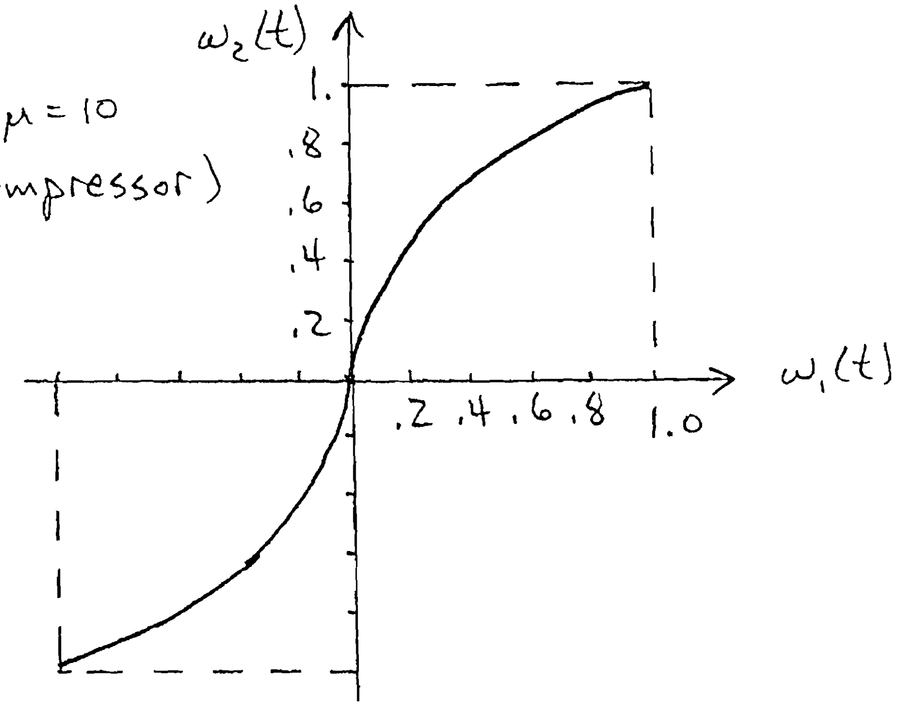
If this change is made:
$w_4(t)$ will consist of pulses $\frac{T_b}{2}$ sec. apart, and must be passed through a frequency divider ckt. to produce pulses at the proper spacing – $T_b$.

**3-18**

(a.) $$|w_2(t)| = \frac{\ln\left(1+\mu|w_1(t)|\right)}{\ln(1+\mu)}$$

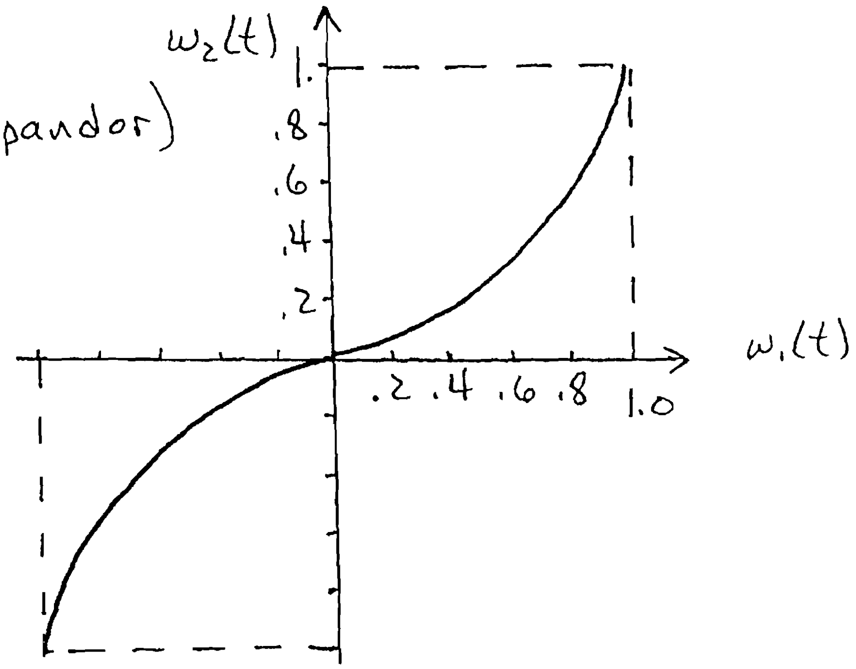
let $w_1(t) = \frac{v_{in}(t)}{5}$ ; $w_2(t) = \frac{v_{OUT}(t)}{5}$

$\mu = 10$
(compressor)



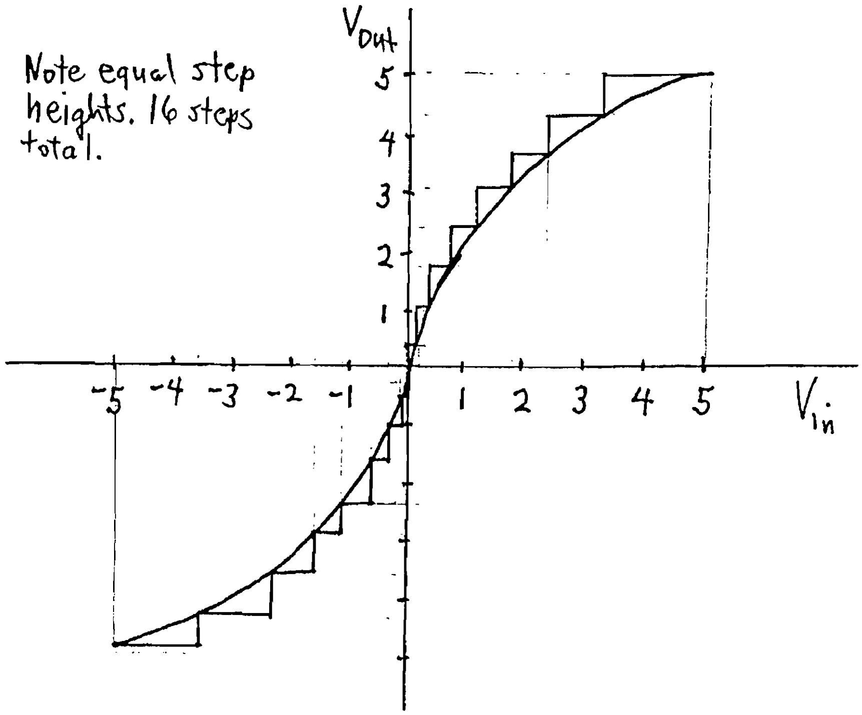

(b.)

(expandor)



3-18. Continued

(c.) 16-level non-uniform Quantization




## 3-19

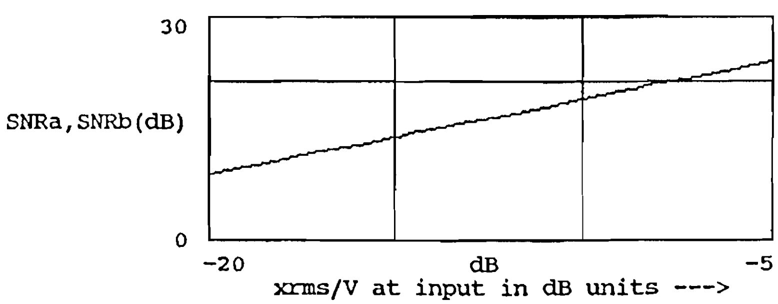
dB := -20,-19 ..-5 n := 4

(a) Using (3-25) and (3-26b), $\mu := 10$

$$SNRa := 6.02\cdot n + 4.77 - 20\cdot \log(\ln(1 + \mu))$$

(b) Using (3-25) and (3-26a),

$$SNRb(dB) := 6.02\cdot n + 4.77 + dB$$

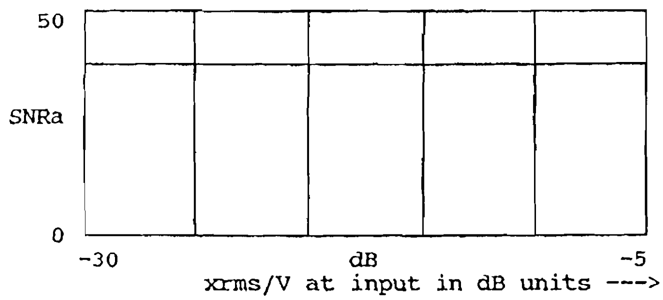


**3-20**

dB := -30,-29 ..-5    M := 256    $n := \dfrac{\log(M)}{\log(2)}$

$\mu$ := 255    n = 8

SNRa := 6.02 n + 4.77 - 20 log(ln(1 + $\mu$))




**3-21**

(a) $L = 2^{\ell} = 16 \quad \Rightarrow \quad \ell = 4$ bits/level

(b) $D = \dfrac{N}{T} = \dfrac{1\ \text{symbol}}{0.8 \times 10^{-3}\ \text{sec}} = 1{,}250$ baud

(c) $R = \ell D = 4(1{,}250) = 5$ kbits/sec

**3-22**

$R = \ell D = (3\ \text{bits/symbol})\left(10.76\ \dfrac{\text{Msymbols}}{\text{sec}}\right) = 32.28$ Mbits/sec

$8 = 2^3 \Rightarrow \ell = 3$

**3-23**

(a) $D = \frac{R}{\ell} = \frac{1{,}500k}{4} = 375 \text{ k baud}$

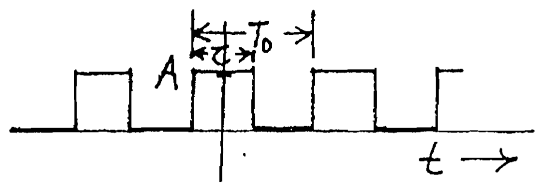
$$B \geq \frac{1}{2}\frac{N}{T} = \frac{1}{2}D = \frac{1}{2}(375k) = \underline{\underline{187.5 \text{ kHz}}} \text{ minimum BW}$$

(b)

$$D = \frac{1{,}500}{8} = 187.5 \text{ k baud}$$

$$B \geq \frac{1}{2}D = \frac{1}{2}(187k) = \underline{\underline{93.75 \text{ kHz}}} \text{ minimum BW}$$

**3-24** For alternating data the waveform is periodic where $T_0 = 2T_b$.




From (2-109) the spectrum is

$$W(f) = \sum_{-\infty}^{\infty} c_n \, \delta(f - nf_0)$$

Where

$$c_n = \frac{1}{T_0}\int_a^{a+T_0} w(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T_0}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{A}{T_0}\left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{2A}{T_0}\left(\frac{e^{-jn\omega_0\tau/2} + e^{+jn\omega_0\tau/2}}{-2jn\omega_0}\right)$$

$\sin(n\omega_0\tau/2)$

$$\Rightarrow c_n = \frac{\cancel{2}A\tau}{T_0\cancel{2}}\,\frac{\sin(\omega_0 n\tau/2)}{\frac{n\omega_0\tau}{2}} = \frac{A}{2}\left(\frac{\tau}{T_b}\right)\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T_b}$$

$$\Rightarrow W(f) = \sum_{n=-\infty}^{\infty}\frac{A}{2}\left(\frac{\tau}{T_b}\right)\left(\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}\right)\delta\left(f - \frac{n}{2}R\right) \quad \text{(A)}$$

where $R = \frac{1}{T_b}$ = bit rate

3-24. Cont'd

Using (A) for NRZ signaling with $\tau = T_b$

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$ Unipolar NRZ (alternating data)

If the data are a sequence of four "1"'s followed by four "0"'s, the waveform would have the same shape except $T_0$ would be 4 times as large. i.e. $T_0 = 8T_b$.

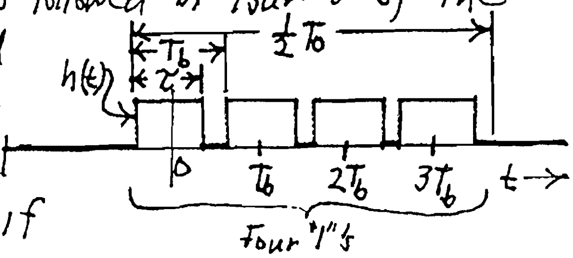
$$\Rightarrow |W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{8}R\right)$$ Unipolar NRZ 4 "1"'s plus 4 "0"'s

**3-25**

Using (A) for RZ signaling with $\tau = \frac{3}{4}T_b$,

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{3}{8}A \left| \frac{\sin\left(\frac{3}{8}n\pi\right)}{\left(\frac{3}{8}n\pi\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$ Unipolar RZ (alternating data)

For RZ with four "1"'s followed by four "0"'s, the periodic waveform would appear as shown where $T_0 = 8T_b$. The mathematical calculations are simplified if (2-112) is used




$$c_n = f_0 H(nf_0)$$

where $h(t)$ is the basic waveform that is repeated to create the periodic waveform (as shown in the figure). $h(t)$ consists of the superposition of four rectangular pulses. Using the time delay theorem of Table 2-1

3-25 Cont'd.

and the rectangular pulse spectrum of Table 2-2

$$H(f) = A\tau \frac{\sin(\pi f \tau)}{\pi f \tau}\left[1 + e^{-j\omega T_b} + e^{-j\omega 2T_b} + e^{-j\omega 3T_b}\right]$$

or

$$c_n = \frac{A\tau}{8T_b}\,\frac{\sin\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

$$\boxed{f = nf_0 = \frac{n}{T_0} = \frac{n}{8T_b}}$$

For RZ with $\tau = \frac{3}{4}T_b$, this becomes

$$c_n = \frac{3}{32}A\left(\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right)\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

Thus, the spectrum for Unipolar RZ with four alternate "1" and "0"'s is

$$|W(f)| = \sum_{n=-\infty}^{\infty}\frac{3}{32}A\left|\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right|\left|1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right|\delta\left(f - \frac{n}{8}R\right)$$

**3-26** In general the PSD is given by (3-36). Now follow the text discussion that leads up to (3-37a). It is seen that for unipolar signaling $R(0) = \frac{A^2}{2}$ and $R(k) = \frac{A^2}{4}$, for $k \neq 0$. Thus,

$$\mathcal{P}(f)_{\text{unipolar}} = \frac{|F(f)|^2}{T_b}\left[\frac{A^2}{2} + 2\frac{A^2}{4}\sum_{k=1}^{\infty}\cos(2\pi k f T_b)\right]$$

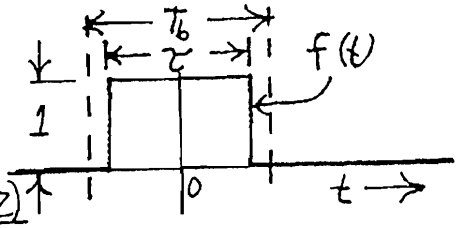
$$= \frac{A^2}{4T_b}|F(f)|^2\left[2 + 2\sum_{1}^{\infty}\cos(2\pi k f T_b)\right]$$

$$= \frac{A^2}{4T_b}|F(f)|^2\left[1 + \sum_{k=-\infty}^{\infty}e^{j k\omega T_b}\right]$$

3-26. Cont'd.

$$\Rightarrow \mathcal{P}_{unipolar}(f) = \frac{A}{4T_b}|F(f)|^2\left[1 + \frac{1}{T_b}\sum_{-\infty}^{\infty}\delta\left(f-\frac{n}{T_b}\right)\right]$$

Now evaluate $F(f)$ for the unipolar pulse shape shown.




$$F(f) = \mathcal{F}[f(t)] = \frac{\tau \sin(\pi f \tau)}{(\pi f \tau)}$$

$$\Rightarrow \mathcal{P}_{unipolar}(f) = \frac{A^2}{4}\left(\frac{\tau^2}{T_b}\right)\left[\frac{\sin(\pi f\tau)}{(\pi f \tau)}\right]^2\left[1 + R\sum_{n=-\infty}^{\infty}\delta(f-nR)\right] \quad (A)$$

where the bit rate is $R = 1/T_b$ and $\tau$ is the pulse width.

(a) For Unipolar NRZ signaling let $\tau = T_b$, (A) becomes

$$\Rightarrow \mathcal{P}_{unipolar\,NRZ}(f) = \frac{A^2}{4}T_b\left(\frac{\sin(\pi f T_b)}{(\pi f T_b)}\right)^2 + \frac{A^2}{4}\delta(f) \quad \text{Unipolar NRZ}$$

because $\frac{\sin(\pi f T_b)}{(\pi f T_b)} = 0$ at $f = nR = \frac{n}{T_b}$ except for $n = 0$.

Note that this result is the same as (3-39b)

(b) For the RZ case of $\tau = \frac{3}{4}T_b$, (A) becomes

$$\mathcal{P}_{unipolar\,RZ}(f) = \frac{9A^2}{64}T_b\left[\frac{\sin(\frac{3}{4}\pi f T_b)}{(\frac{3}{4}\pi f T_b)}\right]^2\left[1 + R\sum_{n=-\infty}^{\infty}\delta(f-nR)\right] \quad \text{Unipolar RZ}$$

where $R = 1/T_b$.

The magnitude spectra for the periodic case, Prob. 3-24, consists only of line spectra. On the other

3-26. Cont'd.

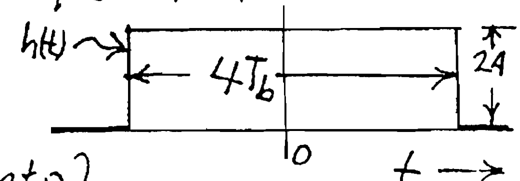
hand, the random data case gives a continuous spectral component in the PSD as well as some disirete lines. The null bandwidth is the same for both cases. That is, for Unipolar NRZ the null bandwidth is $B_{null} = \frac{1}{T_b} = R$ and the spectral efficiency is $\eta = 1$ (bit/sec)/Hz. For Unipolar RZ with $\tau = \frac{3}{4}T_b$, $B_{null} = \frac{4}{3}\frac{1}{T_b} = \frac{4}{3}R$ and $\eta = \frac{3}{4}$ (bits/sec)/Hz.

**3-27**

The solution is identical to that of Prob. 3-22(a) where the DC level is set to zero and the peak-to-peak level is now 2A instead of A. Using the solution to Prob 3-24, we get

$$\underline{\underline{|W(f)| = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} A \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{2}R\right)}}$$

For the case of a sequence consisting of four "1"'s followed by two "0"'s, the calculation is simplified if the technique shown in the solution to prob. 3-25 is used. Using (2-112)




$$c_n = \begin{cases} f_0 H(nf_0) & ; n \neq 0 \\ c_0 & ; n = 0 \end{cases}$$

where $f_0 = \frac{1}{T_0} = \frac{1}{6T_b}$ and $H(f) = \mathcal{F}[h(t)]$ for the $h(t)$ shown.

3-27 Cont'd

The DC level is

$$c_0 = \langle w(t) \rangle = \frac{1}{T_0}\int_{-2T_b}^{4T_b} w(t)\,dt = \frac{1}{6T_b}\left[4AT_b - 2AT_b\right]$$

$$\therefore c_0 = \frac{A}{3}$$

Using the rectangular pulse spectrum shown in Table 2-2,

$$H(f) = 2A\left[4T_b\,\frac{\sin(\pi f 4T_b)}{(\pi f 4T_b)}\right] = 8AT_b\,\frac{\sin(4\pi f T_b)}{(4\pi f T_b)}$$

Thus,

$$c_n = \begin{cases} \dfrac{4A}{3}\,\dfrac{\sin(4\pi f T_b)}{4\pi f T_b}, & n \neq 0 \\[2mm] \dfrac{A}{3}, & n = 0 \end{cases}$$

Using this in (2-109) we get (for polar signaling with a data pattern of four "1"'s followed by two "0"'s )

$$|W(f)| = \frac{A}{3}\delta(f) + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{4}{3}A\left|\frac{\sin(4\pi f T_b)}{(4\pi f T_b)}\right|\delta\!\left(f - \tfrac{1}{6}nR\right)$$

**3-28**

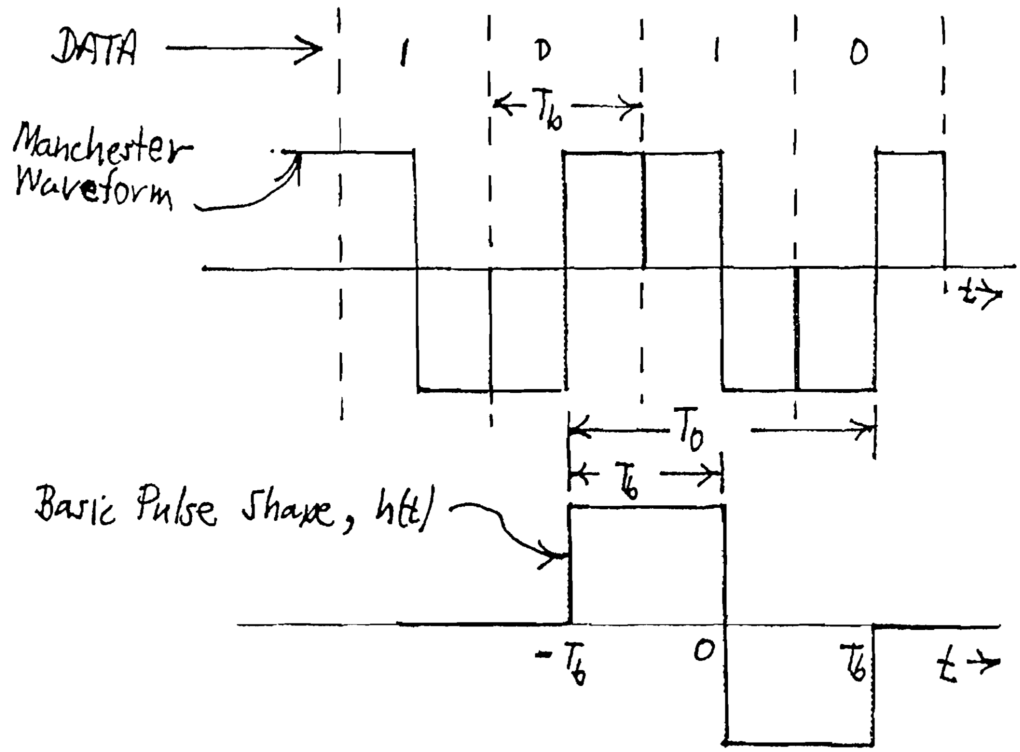
For Manchester signaling, use the pulse shape that is shown on the following page.

$$c_n = f_0 H(nf_0) \quad \text{and} \quad f_0 = \frac{1}{T_0} = \frac{1}{2T_b}$$

Using Tables 2-1 and 2-2, we get

$$H(f) = \mathcal{F}[h(t)] = A\left[\frac{T_b \sin(\pi f T_b)}{(\pi f T_b)}\right]e^{j\frac{\omega T_b}{2}} - A\left[T_b\,\frac{\sin(\pi f T_b)}{\pi f T_b}\right]e^{-j\frac{\omega T_b}{2}}$$

3-28 Cont'd.




$$\Rightarrow H(f) = j2AT_b\left(\frac{\sin(\pi f T_b)}{(\pi f T_b)}\right)\left(\frac{e^{j\omega T_b/2} - e^{-j\omega T_b/2}}{j2}\right)$$

$$= j2AT_b\left(\frac{\sin(\pi f T_b)}{(\pi f T_b)}\right)\sin(\pi f T_b)$$

Using $f_0 = \frac{1}{T_0} = \frac{1}{2T_b}$, this reduces to

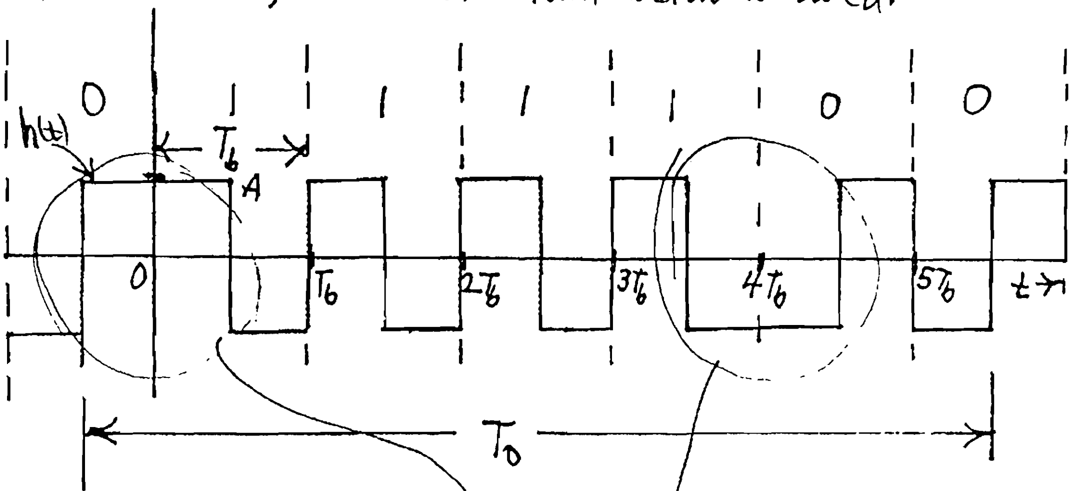
$$c_n = jA\left(\frac{\sin(\frac{\pi}{2}n)}{(\frac{\pi}{2}n)}\right)\sin\left(\frac{\pi}{2}n\right)$$

Using (2-109), we get

$$|W(f)| = \sum_{-\infty}^{\infty}\frac{A}{2}\left|\frac{\sin(\frac{\pi}{2}n)}{(\frac{\pi}{2}n)}\right|\left|\sin(\tfrac{\pi}{2}n)\right|\delta\left(f - \tfrac{n}{2}R\right)$$

3-28 Cont'd

If the test pattern consists of four "1"'s followed by two "0"'s, the $h(t)$ shown below is used.




$$\Rightarrow H(f) = AT_b\left(\frac{\sin(\pi f T_b)}{\pi f T_b}\right)\left[1 - e^{-j\omega 4T_b}\right]$$

$$+\frac{AT_b}{2}\left(\frac{\sin\left(\pi f \frac{T_b}{2}\right)}{\left(\pi f \frac{T_b}{2}\right)}\right)\Big[-e^{-j\omega\frac{3}{4}T_b} + e^{-j\omega\frac{5}{4}T_b} - e^{-j\omega\frac{7}{4}T_b}$$

$$+ e^{-j\omega\frac{9}{4}T_b} - e^{-j\omega\frac{11}{4}T_b} + e^{-j\omega\frac{13}{4}T_b}$$

$$+ e^{-j\omega\frac{19}{4}T_b} - e^{-j\omega\frac{21}{4}T_b}\Big]$$

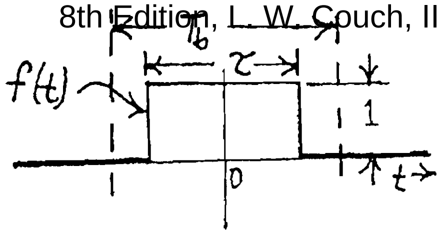
Using (2-109), the magnitude spectrum for this Manchester data waveform is

$$\underline{\underline{|W(f)| = \sum_{-\infty}^{\infty} f_0\,|H(nf_0)|\;\delta(f - nf_0)}}$$

<u>where $H(f)$ is given above</u>

and $f_0 = \frac{1}{T_0} = \frac{1}{6T_b} = \frac{1}{6}R$

**3-29** (a) Substituting (3-40) into (3-36a) the PSD for Polar RZ signaling is




$$\mathcal{P}(f) = \frac{A^2}{T_b} |F(f)|^2$$

where the pulse shape, f(t), is shown in the figure. Thus,

$$F(f) = \mathcal{F}[f(t)] = \tau \frac{\sin(\pi f \tau)}{\pi f \tau}$$

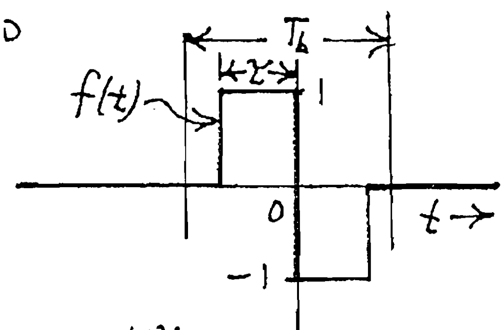
and $$\mathcal{P}(f) = \frac{A^2 \tau^2}{T_b} \left[\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right]^2$$

For the case of $\tau = \frac{1}{2} T_b$, this becomes

$$\underline{\underline{\mathcal{P}(f) = \frac{A^2 T_b}{4} \left[\frac{\sin(\frac{\pi}{2} f T_b)}{(\frac{\pi}{2} f T_b)}\right]^2}}$$

The first-null bandwidth is $B_{null} = \frac{2}{T_b} = 2R$ and the bandwidth efficiency is $\eta = \frac{1}{2}$ (bit/sec)/Hz.

(b) Equation (3-36) can also be used to evaluate the PSD for RZ Manchester signaling where the pulse shape is shown in the figure.




$$F(f) = \tau \left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right) \left[ e^{j\omega \frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}} \right]$$

3-29 (b.) Cont'd

$$\Rightarrow \quad F(f) = j2\ \tau \left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right) \sin\left(\frac{\omega \tau}{2}\right)$$

Using (3-40) and (3-36), the PSD for Manchester signaling is

$$\mathcal{P}(f) = \frac{4A^2\tau^2}{T_b}\left[\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right]^2 \left[\sin(\pi f \tau)\right]^2$$

If $\tau = \frac{1}{4}T_b$, this becomes

$$\underline{\underline{\mathcal{P}(f) = \frac{1}{4}A^2 T_b \left[\frac{\sin\left(\frac{\pi}{4} f T_b\right)}{\left(\frac{\pi}{4} f T_b\right)}\right]^2 \left[\sin\left(\frac{\pi}{4} f T_b\right)\right]^2}}$$

The first-null bandwidth is $B_{null} = \frac{4}{T_b} = 4R$
and the spectral efficiency is $\eta = \frac{1}{4}$ (bits/sec)/Hz.

**3-30**

From (3-44) for Bipolar signaling

$$R(k) = \begin{cases} A^2/2, & k=0 \\ -A^2/4, & k=1 \\ 0, & k \text{ otherwise} \end{cases}$$

Substituting this into (3-36a) the PSD for Bipolar signaling is

$$\mathcal{P}(f) = \frac{|F(f)|^2}{T_b}\left[\frac{A^2}{2} - \frac{A^2}{2}\cos(2\pi f T_b)\right] = \frac{A^2}{2T_b}|F(f)|^2 \underbrace{\left[1 - \cos(2\pi f T_b)\right]}_{2\sin^2(\pi f T_b)}$$

$$\Rightarrow \mathcal{P}(f) = \frac{A^2}{T_b}|F(f)|^2 \sin^2(\pi f T_b)$$

For Bipolar NRZ (i.e. pulse width is $T_b$), $F(f) = T_b \frac{\sin(\pi f T_b)}{\pi f T_b}$

so that the PSD becomes

$$\underline{\underline{\mathcal{P}(f) = A^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \sin^2(\pi f T_b)}} \qquad \text{Bipolar NRZ}$$

3-30. Cont'd.

For Bipolar RZ (i.e. pulse width is $T_b/2$), $F(f) = \frac{T_b}{2}\frac{\sin(\pi f T_b/2)}{\pi f T_b/2}$

so that the PSD becomes

$$\mathcal{P}(f) = \frac{A^2 T_b}{4}\left[\frac{\sin(\pi f T_b/2)}{\pi f T_b/2}\right]^2 \sin^2(\pi f T_b) \quad \text{Bipolar RZ}$$

The PSD's are plotted using MathCAD.

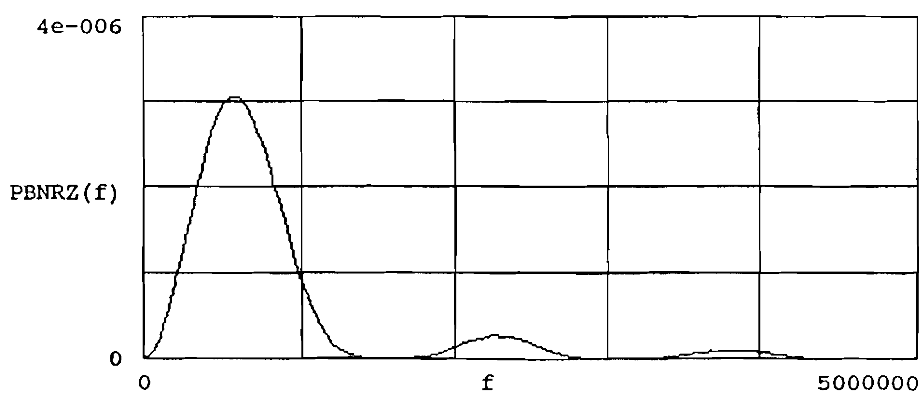
$A := 3$ $\qquad R := 1.544 \cdot 10^6$ $\qquad Tb := \frac{1}{R}$

$Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$

$f := 0, 0.05\ 10^6 .. 5 \cdot 10^6$

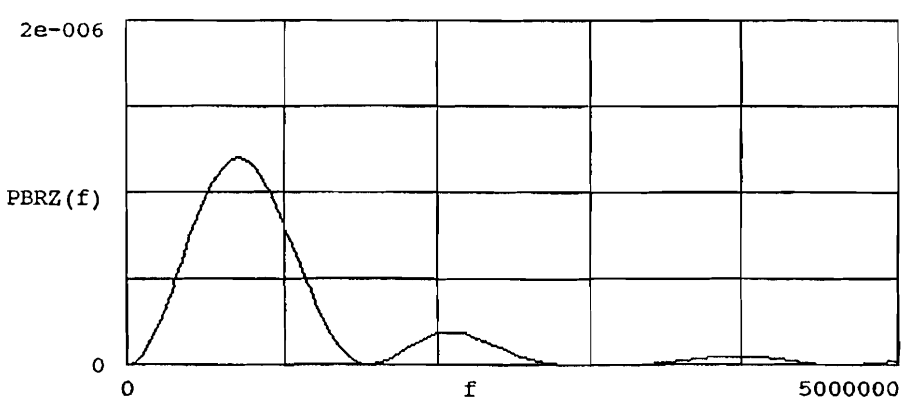
Following the discussion leading up to (3-45), the PSD for Bipolar NRZ is:

$PBNRZ(f) := A^2\ Tb \cdot (Sa(\pi\ f\ Tb))^2 \cdot (\sin(\pi \cdot f \cdot Tb))^2$




Using (3-45) the PSD for Bipolar RZ is:

$PBRZ(f) := A^2 \cdot \left[\frac{Tb}{4}\right] \left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2 (\sin(\pi\ f\ Tb))^2$



**3-31**

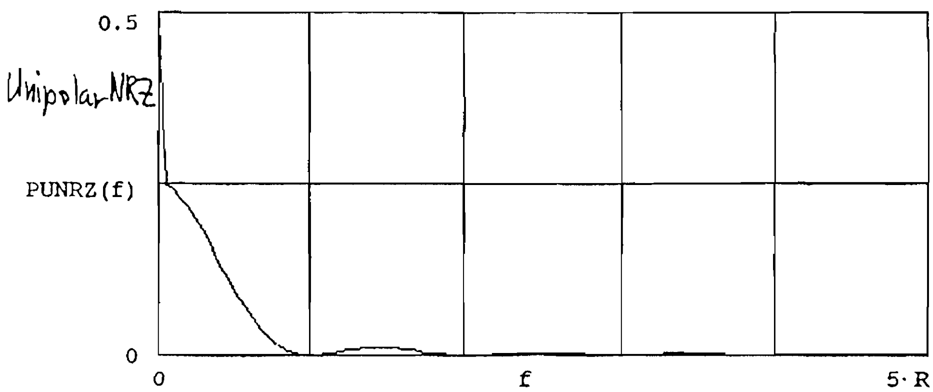
$A := 1 \quad R := 1 \quad f := 0, 0.05 .. 5 \quad Tb := \frac{1}{R}$

$$Sa(x) := if\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

The PSD for Unipolar NRZ is given by (3-39b) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot *infinite* values for the delta functions, so plot the weights of the delta functions instead. Thus (3-39b) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

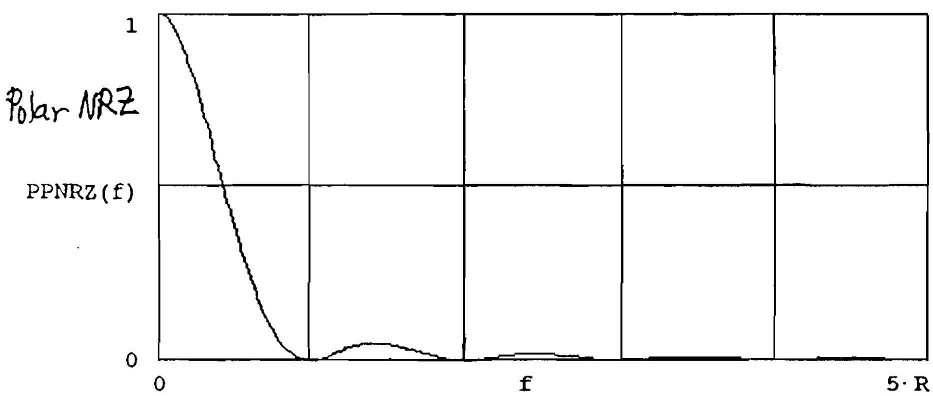
$$PUNRZc(f) := \left[A^2 \cdot \frac{Tb}{4}\right] \cdot (Sa(\pi\ f\ Tb))^2 \qquad PUNRZd(f) := if\left[f \neq 0, 0, \frac{A^2}{4}\right]$$

$$PUNRZ(f) := PUNRZc(f) + PUNRZd(f)$$




Use (3-41) for Polar NRZ spectrum:

$$PPNRZ(f) := \left[A^2\ Tb\right]\ (Sa(\pi\ f \cdot Tb))^2$$




The PSD for Unipolar RZ is given by (3-43) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-43) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

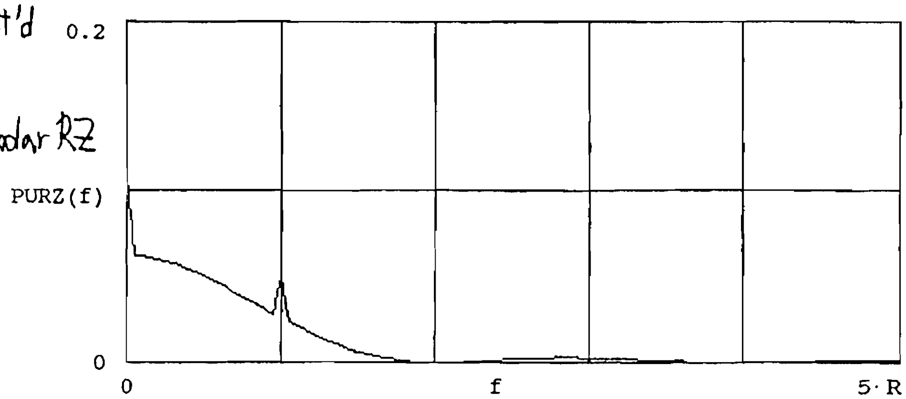
$$PURZc(f) := \left[A^2\ \frac{Tb}{16}\right]\ \left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2$$

$$PURZd(f) := if\left[mod(f,R) \neq 0, 0, \frac{A^2}{16} \cdot \left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2\right]$$

3-31 Cont'd

$$PURZ(f) := PURZc(f) + PURZd(f)$$

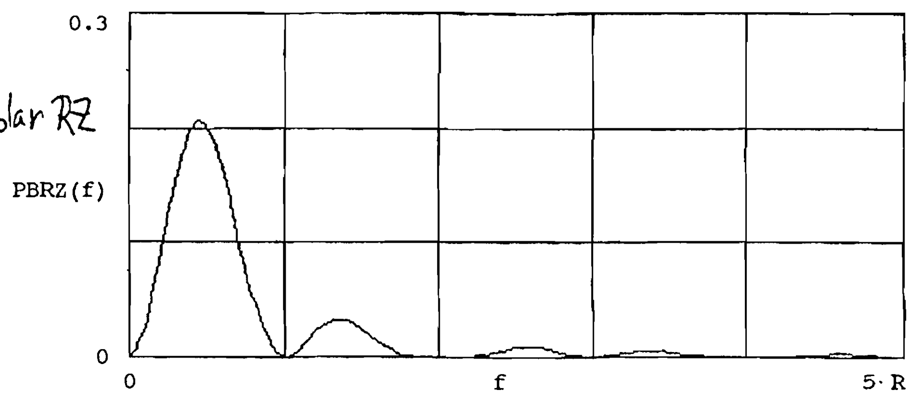
Unipolar RZ




Using (3-45) the PSD for Bipolar RZ is:

$$PBRZ(f) := A^2 \left[\frac{Tb}{4}\right] \cdot \left[Sa\left[\pi\ f \cdot \frac{Tb}{2}\right]\right]^2 \ (\sin(\pi\ f\ Tb))^2$$

Bipolar RZ



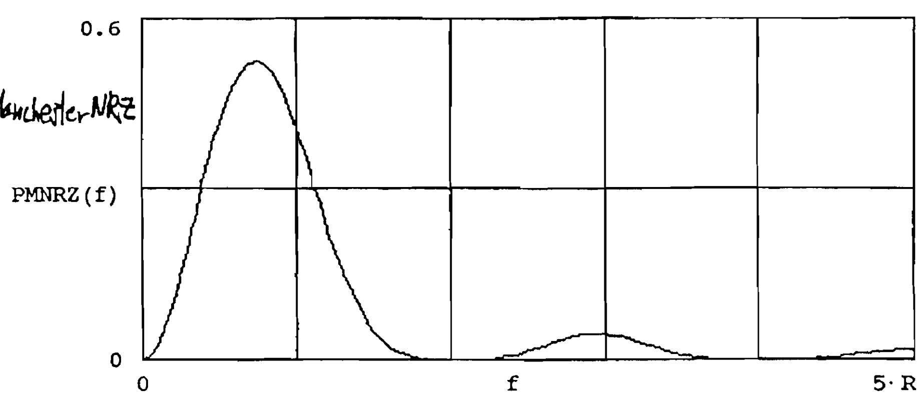

$A := 1 \qquad R := 1 \qquad f := 0, 0.05 \,..5 \qquad Tb := \frac{1}{R}$

$$Sa(x) := if\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

Use (3-46c) for the Manchester NRZ spectrum:

$$PMNRZ(f) := A^2 \cdot Tb \left[Sa\left[\pi \cdot f \cdot \frac{Tb}{2}\right]^2\right] \cdot \left[\sin\left[\pi\ f \cdot \frac{Tb}{2}\right]^2\right]$$

Manchester NRZ



## 3-32

Referring to (3-36), it is seen that this equation is derived in Chapter 6 and that (3-36) is identical to (6-70b). Furthermore, the solution to this problem is given in the text following (6-70b). Equation (6-70e) shows that there will be delta functions in the spectrum at frequencies f=nR if and only if the line code has a non-zero dc value and F(nR) is not zero.

If the line code is designed to have a delta function at f=R or some harmonic of R, then a phase-locked loop can be used to extract the bit sync signal as discussed in Section 3-5 (Bit Synchronizers).

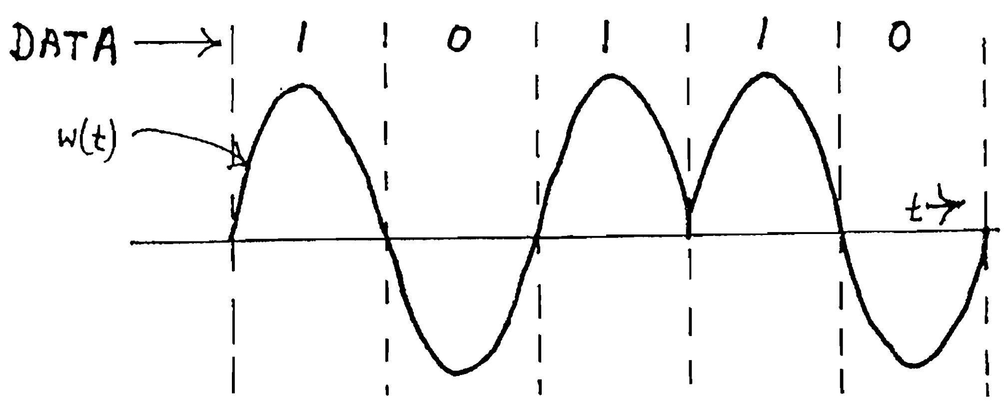
## 3-33

(a.) $$w(t) = A \sum_{n=-\infty}^{\infty} a_n f(t - nT_b)$$

where $a_n = \pm 1$ random data

$$\Rightarrow w(t) = A \sum_{-\infty}^{\infty} a_n \begin{cases} \cos\left(\frac{\pi}{T_b}(t - nT_b)\right), & |t - nT_b| < \frac{T_b}{2} \\ 0, & t \text{ elsewhere} \end{cases}$$

This is plotted in the figure below for a random data pattern.



3-33 Cont'd

(b) Using (3-40) in (3-36a) the PSD is given by

$$\mathcal{P}(f) = \frac{A^2}{T_b} |F(f)|^2 \quad \text{where } F(f) = \mathcal{F}[f(t)]. \quad (1)$$

Now, evaluate $F(f)$:

$$F(f) = \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \cos\left(\frac{\pi t}{T_b}\right) e^{-j2\pi f t}\, dt$$

$$= \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \cos\left(\frac{\pi t}{T_b}\right)\cos(2\pi f t)\,dt - j\underbrace{\int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \overbrace{\underbrace{\cos\left(\frac{\pi t}{T_b}\right)}_{\text{even}}\underbrace{\sin(2\pi f t)}_{\text{odd}}}^{\text{odd}}\,dt}_{0}$$

$$= \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \frac{1}{2}\cos\left[2\pi\left(\frac{1}{2T_b}+f\right)t\right]dt + \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \frac{1}{2}\cos\left[2\pi\left(\frac{1}{2T_b}-f\right)t\right]dt$$

$$= \left[\frac{\sin\left[2\pi\left(\frac{1}{2T_b}+f\right)t\right]}{2\pi\left(\frac{1}{2T_b}+f\right)}\right]\Bigg|_0^{\frac{T_b}{2}} + \left[\frac{\sin\left[2\pi\left(\frac{1}{2T_b}-f\right)t\right]}{2\pi\left(\frac{1}{2T_b}-f\right)}\right]\Bigg|_0^{\frac{T_b}{2}}$$

$$\boxed{\text{Integrals are even} \Rightarrow \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \sim dt = 2\int_0^{\frac{T_b}{2}} \sim dt}$$

$$\Rightarrow F(f) = \left[\frac{\sin\left[2\pi\left(\frac{1}{2T_b}+f\right)\frac{T_b}{2}\right]}{2\pi\left(\frac{1}{2T_b}+f\right)} + \frac{\sin\left[2\pi\left(\frac{1}{2T_b}-f\right)\frac{T_b}{2}\right]}{2\pi\left(\frac{1}{2T_b}-f\right)}\right]$$

$$\sin\left[2\pi\left(\tfrac{1}{2T_b}+f\right)\tfrac{T_b}{2}\right] \rightarrow \cos(\pi f T_b), \qquad \sin\left[2\pi\left(\tfrac{1}{2T_b}-f\right)\tfrac{T_b}{2}\right] \rightarrow \cos(\pi f T_b)$$

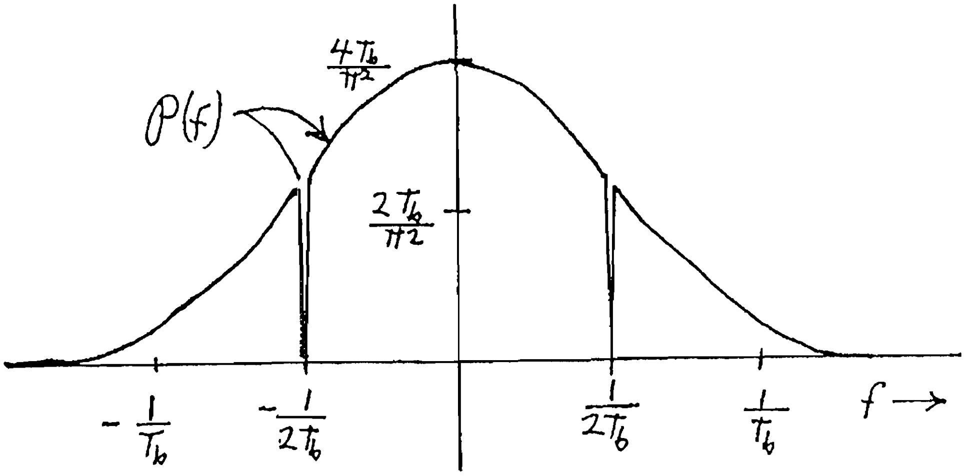
$$= \frac{\left(\frac{\pi}{T_b} - 2\pi f\right)\cos(\pi f T_b) + \left(\frac{\pi}{T_b} + 2\pi f\right)\cos(\pi f T_b)}{(2\pi)^2\left(\frac{1}{2T_b}+f\right)\left(\frac{1}{2T_b}-f\right)}$$

$$\Rightarrow F(f) = \frac{2T_b}{\pi}\,\frac{\cos(\pi f T_b)}{1-(2T_b f)^2}$$

Substituting this into (1), the PSD is

$$\underline{\underline{\mathcal{P}(f) = \frac{4T_b A^2}{\pi^2}\left(\frac{\cos(\pi f T_b)}{1-(2T_b f)^2}\right)^2}}$$

3-33.(b) Cont'd. This result is plotted below.




(c) The first-null bandwidth is $B_{null} = \frac{1}{2T_b} = \frac{1}{2}R$ and the spectral efficiency is $\eta = 2$ (bits/sec)/Hz.

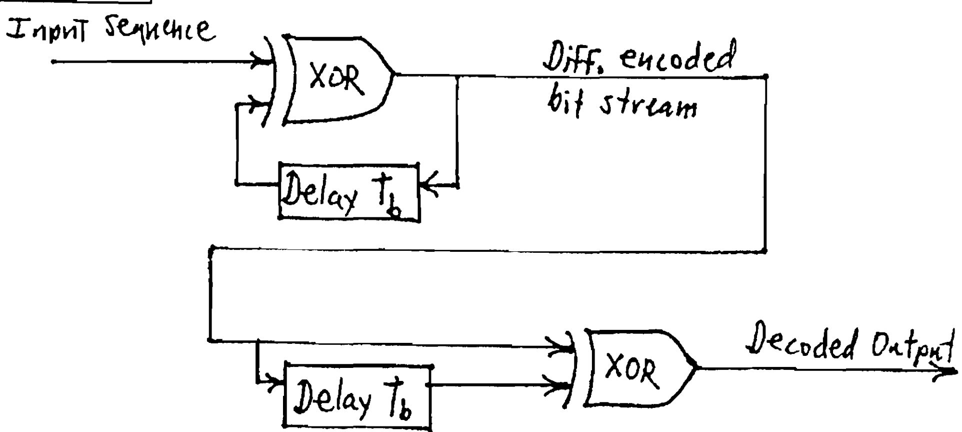
**3-34**

$e_k = d_k \oplus e_{k-1}$

Reference digit

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Input Sequence | | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| Encoded Sequence (1) | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| Encoded Sequence (2) | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |

**3-35**




Reference digit

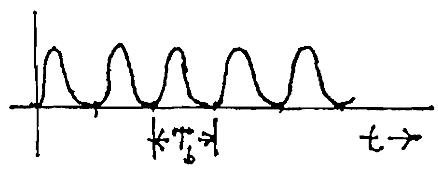
Input sequence 0 0 1 1 1 1 0 1 0 0 0 1

Encoded sequence 1 1 1 0 1 0 1 1 0 0 0 0 1

Decoded sequence 0 0 1 1 1 1 0 1 0 0 0 1

## 3-36

A possible regenerative repeater for a Polar RZ line code would be almost identical to the Unipolar NRZ repeater shown in Fig. 3-19 except that a monostable output circuit would be required to produce the permitted binary levels of $+V_0$ and $-V_0$ instead of $+V_0$ and 0. A bit synchronizer similar to the square-law synchronizer shown in Fig. 3-20 could be used. In this case the output of the square-law device would be of the Unipolar RZ type and this waveform would be exactly periodic (except for noise) as shown. consequently, we would expect the PSD of this waveform to contain only delta functions. To show that this is the case, use (3-36) where the $a_n$ can take on only one value, call it $+A$, with the probability of 1. Thus,




$$\mathcal{P}(f) = \frac{A^2|F(f)|^2}{T_b}\left[1+2\sum_{k=1}^{\infty}\cos(2\pi k f T_b)\right]^2 = \frac{A^2|F(f)|^2}{T_b}\left[\sum_{k=-\infty}^{\infty} e^{jk\omega T_b}\right]$$

$$R(k) = \sum a_n a_{n+k} P_i = A^2$$

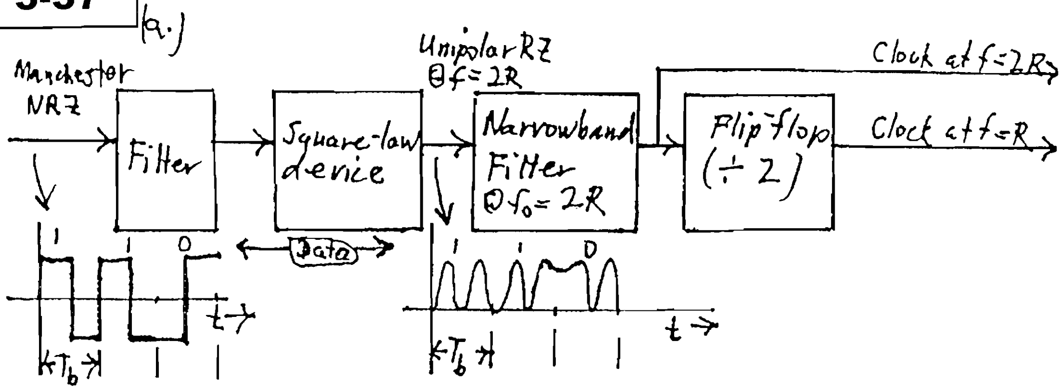
or, Using (3-38)

$$\mathcal{P}(f) = A^2 R^2 |F(f)|^2 \sum_{n=-\infty}^{\infty} \delta(f - nR)$$

where $R = 1/T_b$. The weight of the delta function at $f=R$ is $A^2R^2|F(R)|^2$. Therefore we have a periodic component in the waveform at the frequency $f=R$. Consequently, a low-pass filter (or PLL) centered on $f_0=R$ can be used to extract the bit sync signal as shown in Fig. 3-20.

## 3-37

(a.)

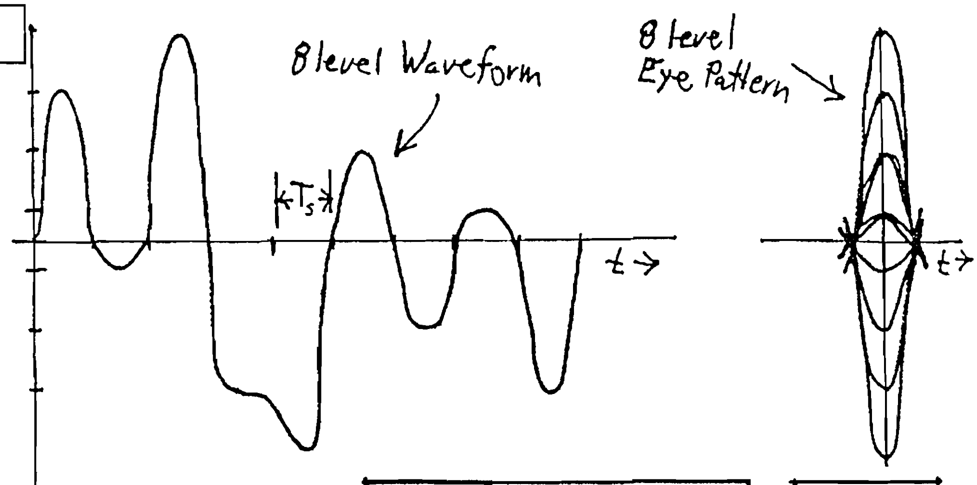



(b.) and (c.) The Manchester NRZ waveform is filtered to produce a waveform that has RZ characteristics. Consequently, the output of the square-law device will be a Unipolar RZ waveform except that it would have a bit rate of 2R instead of R where $R = 1/T_b$ is the bit rate of the input Manchester line code. Thus, using mathematics similar to the solution shown in P3-31 above, there would be delta functions at frequencies f=n2R. Consequently, a narrowband filter centered on f=2R could be used to recover bit sync at twice the clock frequency R and a flip-flop (divide by 2 frequency divider) could be used to obtain the clock signal at f=R.

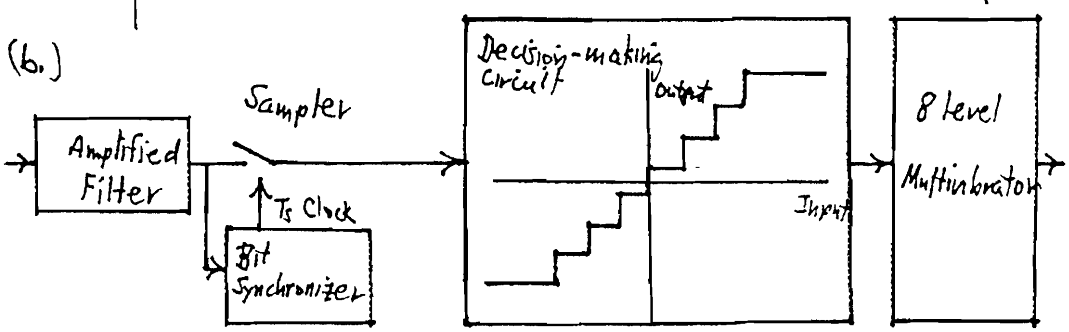
(d.) The Manchester NRZ bit synchronizer would function even when all 1s or all 0s data were received, whereas the corresponding Polar NRZ synchronizer would fail for this type of data because no pulses would appear at the square-law output for the all 1s or all 0s type of data.

## 3-38

(a.)




(b.)




Use a bit synchronizer similar to that shown in Fig. 3-20.

3-38 Cont'd.

(c) The receiver functions as described in the text associated with Fig 3-19 and Fig 3-20 except that multilevel signaling is present so that an 8 level decision circuit and 8 level multivibrator are required.

**3-39** Use the result from Prob 3-8.

(a) $n \geq 3.32 \log_{10}\left(\frac{50}{P}\right) = 3.32 \log_{10}(50) = 5.64 \Rightarrow$ Use $n = 6$ bits/word.

$f_s = 2B = 5.4\text{kHz} \Rightarrow R_{min} = nf_s = 6(5.4\text{kHz}) = \underline{\underline{32.4 \text{ kbits/sec}}}$

(b) $L = B = 2^\ell \Rightarrow \ell = 3$ bit D/A $\quad D = \frac{R}{\ell} = \frac{32.4 \text{ kbit/sec}}{3 \text{ bits/symbol}} = \underline{\underline{10.8 \text{ ksym/sec}}}$

(c) $D = \frac{2B}{1+r}$ where $r = 0$ for min BW $\Rightarrow B = \frac{D}{2} = \underline{\underline{5.4\text{kHz}}}$

**3-40** $L = B = 2^\ell \Rightarrow \ell = 3$

(a) $D = \frac{R}{\ell} = \frac{9600 \text{ bits/sec}}{3 \text{ bits/symbol}} = \underline{\underline{3.2 \text{ k symbol/sec}}}$

(b) $D = \frac{2R}{1+r} = \frac{2(2.4k)}{1+r} = 3.2k \Rightarrow \underline{\underline{r = 0.5}}$

**3-41**

(a) This is similar to the problem illustrated by Fig. 3-22 except $L = 64$ and the peak limit levels are $\pm 10$.

$$\Rightarrow R(0) = \sum_{i=1}^{64} (a_n)_i^2 \, P_i$$

$$= \underbrace{\frac{1}{64}}_{P_i = \frac{1}{64}} \left\{ 2\left[(10)^2 + (9.68)^2 + (9.37)^2 \cdots + (0.16)^2\right] \right\}$$

$\Rightarrow R(0) = 34.39155$

Using a programable calculator or a personal computer

Also $R(k) = 0$ for $k \neq 0$

3-41 Cont'd.

Using (3-36a), the PSD for $w(t)$ is

$$\mathcal{P}(f) = \frac{|F(f)|^2}{T_s}\left[34.39+0\right] = \frac{1}{T_s}\left(\frac{T_s}{2}\right)^2\left(\frac{\sin\left(\pi f \frac{T_s}{2}\right)}{\pi f \frac{T_s}{2}}\right)^2 34.39$$

$$F(f) = \mathcal{F}[f(t)] = \mathcal{F}\left[\Pi\left(\frac{2t}{T_s}\right)\right]$$

Using $T_s = 6T_b$ since $L = 2^6 = 64$, the PSD becomes

$$\mathcal{P}(f) = 17.2\left(\frac{\sin(3\pi f T_b)}{3\pi f T_b}\right)^2$$

(b.) The first-null bandwidth is $B_{null} = \frac{1}{3T_b} = \frac{1}{3}R$ Hz

(c.) The spectral efficiency is

$$\eta = \frac{R}{\frac{1}{3}R} = 3 \text{ (bits/sec)/Hz}$$

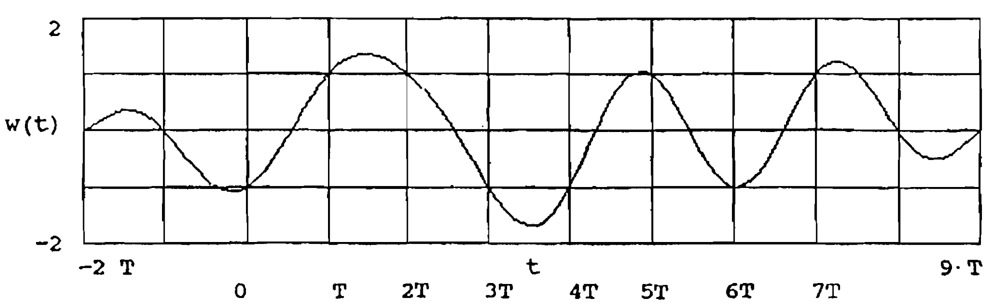
**3-42**

(a.) $B = \frac{1}{2}(1+r)D = \frac{1}{2}R = \frac{1}{2}(300) = \underline{\underline{150\ Hz}}$

$r=0,\ D=R$

(b.)

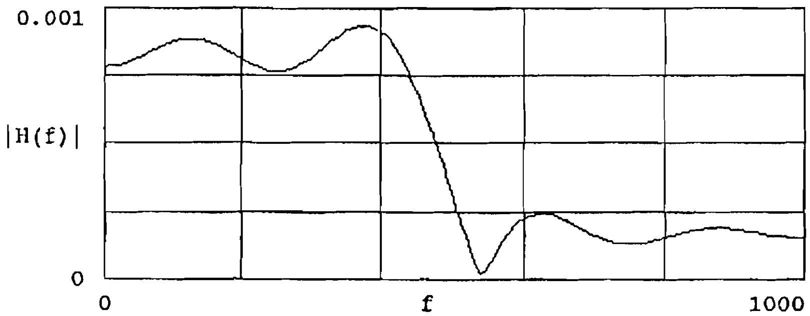
```
fs := 300            T := 1/fs            T = 0.003

t := -2·T,-1.9·T ..9 T            k := 0,1 ..7

u_k(t) := π·(1/T)·(t - k T)       ø_k(t) := sin[u_k(t)] / u_k(t)

a_0 := -1   a_1 := 1   a_2 := 1   a_3 := -1   a_4 := -1   a_5 := 1

a_6 := -1   a_7 := 1              w(t) := Σ_k a_k·ø_k(t)
```



**3-43** (a.)

f := 0,10 ..1000 fs := 1000

$$H(f) := \int_0^{0.008} \frac{\sin(\pi \cdot fs \cdot (t - 0.004))}{\pi\ fs\ (t - 0.004)} e^{-2j \cdot \pi\ f\ t}\, dt$$




(b.) From the figure above, the bandwidth for the causal approximation is $\underline{\underline{B = 540\text{ Hz}}}$

The bandwidth for the noncausal filter is $B = \frac{1}{2} f_s = \underline{\underline{500\text{ Hz}}}$

**3-44**

$$h_e(t) = \overbrace{\int_{-f_1}^{f_1} e^{j2\pi f t} df}^{①} + \overbrace{\int_{-B}^{-f_1} \frac{1}{2}\left\{1 + \cos\left[\frac{\pi}{2f_\Delta}(-f-f_1)\right]\right\} e^{j2\pi f t} df}^{②}$$

$$+ \underbrace{\int_{f_1}^{B} \frac{1}{2}\left\{1 + \cos\left[\frac{\pi}{2f_\Delta}(f-f_1)\right]\right\} e^{j2\pi f t} df}_{③}$$

$$① = \frac{e^{j2\pi f_1 t} - e^{-j2\pi f_1 t}}{j2\pi t} = \frac{\sin(2\pi f_1 t)}{\pi t}$$

$$② = \int_{-B}^{-f_1} \frac{1}{2}\left\{1 + \cos\left[\frac{\pi}{2f_\Delta}(-f-f_1)\right]\right\} e^{j2\pi f t} df$$

$$\underset{\text{Let } y = -f}{=} \int_{f_1}^{B} \frac{1}{2}\left\{1 + \cos\left[\frac{\pi}{2f_\Delta}(y-f_1)\right]\right\} e^{-j2\pi y t} dy = ③^*$$

$h_e(t) = ① + ② + ③$ and using $c + c^* = 2\,\mathrm{Re}\{c\}$

$$\Rightarrow h_e(t) = \frac{\sin(2\pi f_1 t)}{\pi t}$$

$$+ \mathrm{Re}\left\{ \int_{f}^{B} \left(1 + \cos\left[\frac{\pi}{2f_\Delta}(f-f_1)\right]\right) e^{j2\pi f t} df \right.$$

3-44. Cont'd ; $H_1(f) = \cos\left[\frac{\pi}{2f_\Delta}(f-f_1)\right]$

$$h_e(t) = \frac{\sin(2\pi f_1 t)}{\pi t} + \int_{f_1}^{B} \cos[2\pi f t]\,df + \int_{f_1}^{B} H_1(f)\cos[2\pi f t]\,df$$

$$= ① + ② + ③$$

$$① + ② = \frac{\sin(2\pi f_1 t)}{\pi t} + \frac{\sin(2\pi B t) - \sin(2\pi f_1 t)}{2\pi t}$$

$$= \frac{\sin(2\pi f_1 t) + \sin(2\pi B t)}{2\pi t} \quad ; \quad f_1 = f_0 - f_\Delta, \quad B = f_0 + f_\Delta$$

$$= \frac{\sin(2\pi f_0 t)\cos(2\pi f_\Delta t)}{\pi t}$$

(A-13)

$$\sin(x-y) + \sin(x+y) = 2\sin x \cos y$$

$$③ = \int_{f_1}^{B} \cos\left[\frac{\pi}{2f_\Delta}(f-f_1)\right]\cos[2\pi f t]\,df$$

$$= \frac{1}{2}\int_{f_1}^{B} \cos\left[\frac{\pi}{2f_\Delta}(f-f_1) - 2\pi f t\right]df + \frac{1}{2}\int_{f_1}^{B} \cos\left[\frac{\pi}{2f_\Delta}(f-f_1) + 2\pi f t\right]df$$

Using:

$B - f_1 = 2f_\Delta$ ; $\sin(\pi - x) = \sin(x) = -\sin(-x)$;

3-44 Cont'd.

$$③ = \frac{1}{2}\,\frac{\sin(2\pi B t) + \sin(2\pi f_1 t)}{\pi/2f_\Delta - 2\pi t} + \frac{1}{2}\left(\frac{-\sin(2\pi B t) - \sin(2\pi f_1 t)}{\pi/2f_\Delta + 2\pi t}\right)$$

Using Sec. A-1 with $x = f_0$ and $y = f_\Delta$

$$③ = \left[\sin(2\pi f_0 t)\cos(2\pi f_\Delta t)\right] \cdot \left[\frac{1}{\pi/2f_\Delta - 2\pi t} - \frac{1}{\pi/2f_\Delta + 2\pi t}\right]$$

$$= \left[\sin(2\pi f_0 t)\cos(2\pi f_\Delta t)\right]\left[\frac{16 f_\Delta^2 t}{\pi\left(1 - (4f_\Delta t)^2\right)}\right]$$

$$\therefore\ h_e(t) = 2f_0\,\frac{\sin(2\pi f_0 t)}{2\pi f_0 t}\left[\cos(2\pi f_\Delta t) + \frac{\cos(2\pi f_\Delta t)}{1 - (4f_\Delta t)^2} \cdot (4f_\Delta t)^2\right]$$

$$\Rightarrow\ h_e(t) = 2f_0\,\frac{\sin(2\pi f_0 t)}{2\pi f_0 t}\left[\frac{\cos[2\pi f_\Delta t]}{1 - (4f_\Delta t)^2}\right]$$

## 3-45

(a.) Select r
r := 0.75 fo := 1 f1 := (1 - r) fo fd := r fo

f := 0,0.2 ..2 fo B := (1 + r) fo

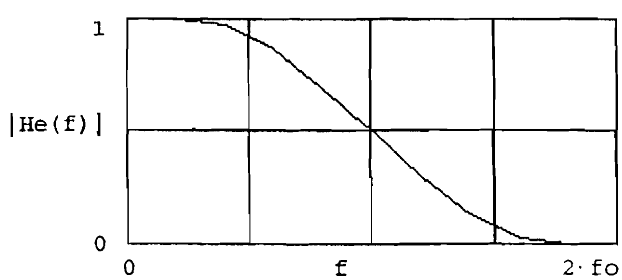
$$He(f) := (1 - \Phi(f - f1)) + 0.5\left[1 + \cos\left[\pi\,\frac{f - f1}{2\ fd}\right]\right](\Phi(f - f1) - \Phi(f - B))$$

r = 0.75

f1 = 0.25

fo = 1

B = 1.75



3-45 Cont'd.

(b.) $T := \frac{1}{2\ fo}$ $\qquad t := 0 + 0.001, 0.05 \,..\, 4 \cdot T$

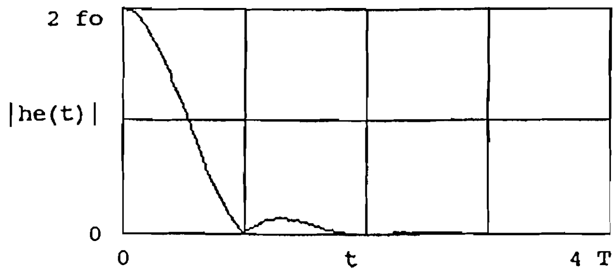
$$he(t) := 2\ fo \cdot \left[\frac{\sin(2\ \pi\ fo\ t)}{2 \cdot \pi\ fo\ t}\right] \left[\frac{\cos(2\ \pi \cdot fd \cdot t)}{1 - (4\ fd \cdot t)^2}\right]$$

r = 0.75

T = 0.5

fo = 1

fd = 0.75




**3-46** The pulse shape (impulse response) for the raised cosine-rolloff channel is given by (3-73) and the PSD for the output polar signal is given by (3-40) in (3-36a). Thus, for A=1,

$$\mathcal{P}_{out}(f) = \frac{1}{T_b} |He(f)|^2$$

where, using (3-69)

$$He(f) = \begin{cases} 1 & , |f| < f_1 \\ \frac{1}{2}\left\{1 + \cos\left[\frac{\pi(|f| - f_1)}{2 f_\Delta}\right]\right\} & , f_1 < |f| < B \\ 0 & , |f| > B \end{cases}$$

For a maximum bit rate without ISI, from (3-74),

$$R = \frac{2B}{1+r} \underset{\uparrow}{=} \frac{2B}{\frac{3}{2}} = \frac{4}{3} B \Rightarrow \underline{B = \frac{3}{4} R}$$

r = 0.5

$$f_\Delta = B - f_0 \underset{\uparrow}{=} B - \frac{f_\Delta}{r} \Rightarrow r f_\Delta = r B - f_\Delta$$

$r = \frac{f_\Delta}{f_0}$

$$\Rightarrow f_\Delta = \frac{rB}{r+1} = \frac{\frac{1}{2}B}{\frac{1}{2}+1} = \frac{\frac{1}{2}B}{\frac{3}{2}} = \frac{1}{3}B$$

$$\underset{\uparrow}{=} \frac{1}{3}\left(\frac{3}{4}R\right) = \frac{1}{4}R \qquad \Rightarrow \underline{f_\Delta = \frac{1}{4}R}$$

$B = \frac{3}{4}R$

3-46. Cont'd

From Fig. 3-25

$$f_1 = B - 2f_\Delta = \frac{3}{4}R - 2\left(\frac{1}{4}R\right) = \frac{1}{4}R$$

$$\Rightarrow \underline{f_1 = \frac{1}{4}R}$$

Thus,

$$H_e(f) = \begin{cases} 1 & , |f| < \frac{1}{4}R \\ \frac{1}{2}\left\{1 + \cos\left[\frac{\pi(|f| - \frac{1}{4}R)}{2(\frac{1}{4}R)}\right]\right\} & , \frac{1}{4}R < |f| < \frac{3}{4}R \\ 0 & , f > \frac{3}{4}R \end{cases}$$

Or

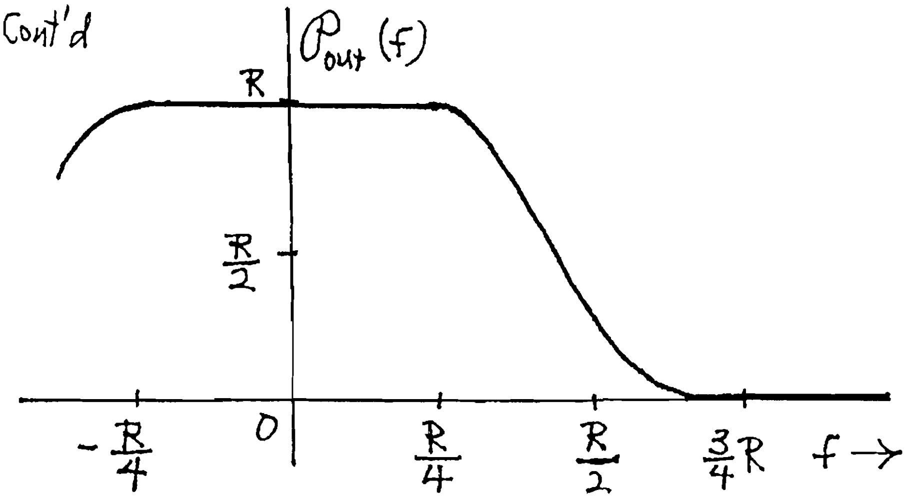
$$H_e(f) = \begin{cases} 1 & , |f| < \frac{1}{4}R \\ \frac{1}{2}\left\{1 + \cos\left[2\pi\left(|\frac{f}{R}| - \frac{1}{4}\right)\right]\right\} & , \frac{1}{4}R < |f| < \frac{3}{4}R \\ 0 & , |f| > \frac{3}{4}R \end{cases}$$

Then, the PSD at the output is

$$\mathcal{P}_{out}(f) = \begin{cases} R & , |f| < \frac{1}{4}R \\ \frac{R}{4}\left\{1 + \cos\left[2\pi\left(|\frac{f}{R}| - \frac{1}{4}\right)\right]\right\}^2 & , \frac{1}{4}R < |f| < \frac{3}{4}R \\ 0 & , |f| > \frac{3}{4}R \end{cases}$$

A sketch of this result is shown at the top of the next page.

3-46. Cont'd




**3-47** $h_e(t) = \int_{-\infty}^{\infty} H_e(f)\, e^{j2\pi f t}\, df$

For $t = nT_s \ni h_e(nT_s) = \int_{-\infty}^{\infty} H_e(f)\, e^{j2\pi n T_s f}\, df$

Break into multiple integrals, each with a $\frac{1}{T_s}$ wide interval.

$$\ni h_e(nT_s) = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s}+\frac{k}{T_s}}^{\frac{1}{2T_s}+\frac{k}{T_s}} H_e(f)\, e^{j2\pi n T_s f}\, df$$

Let $f_1 = f - \frac{k}{T_s}$

Then $$h_e(nT_s) = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} H_e\left(f_1 + \frac{k}{T_s}\right) e^{j2\pi n T_s \left(f_1 + \frac{k}{T_s}\right)}\, df_1$$

Or
$$h_e(nT_s) = \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} \sum_{k=-\infty}^{\infty} H_e\left(f_1 + \frac{k}{T_s}\right) e^{j2\pi n T_s f_1}\, df_1$$

Assume $\sum_{-\infty}^{\infty} H_e\left(f_1 + \frac{k}{T_s}\right) = T_s, \quad |f_1| < \frac{1}{2T_s}$

Then
$$h(nT_s) = \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} T_s\, e^{j2\pi n T_s f_1}\, df_1 = \frac{\cancel{T_s}\, e^{j2\pi n T_s f_1}}{j2\pi n \cancel{T_s}}\Bigg|_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}}$$

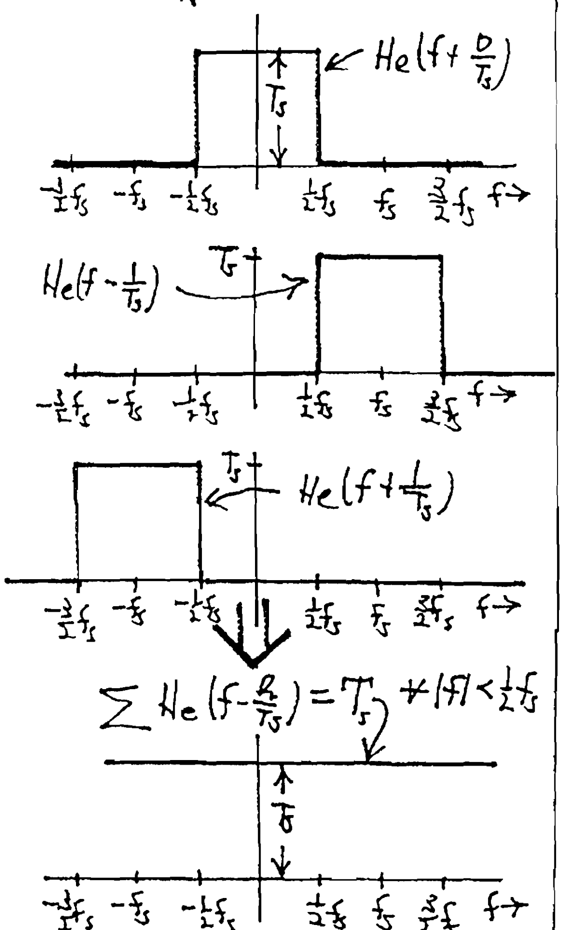
or
$$h(nT_s) = \frac{e^{j\cancel{2}\pi n \cancel{T_s}\frac{1}{\cancel{2}\cancel{T_s}}} - e^{-j\cancel{2}\pi n \cancel{T_s}\frac{1}{\cancel{2}\cancel{T_s}}}}{j2\pi n} = \frac{\sin(n\pi)}{n\pi} = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Q.E.D.

**3-48** See if $\sum_{k=-\infty}^{\infty} H_e\left(f+\frac{k}{T_s}\right) = T_s$ (1)

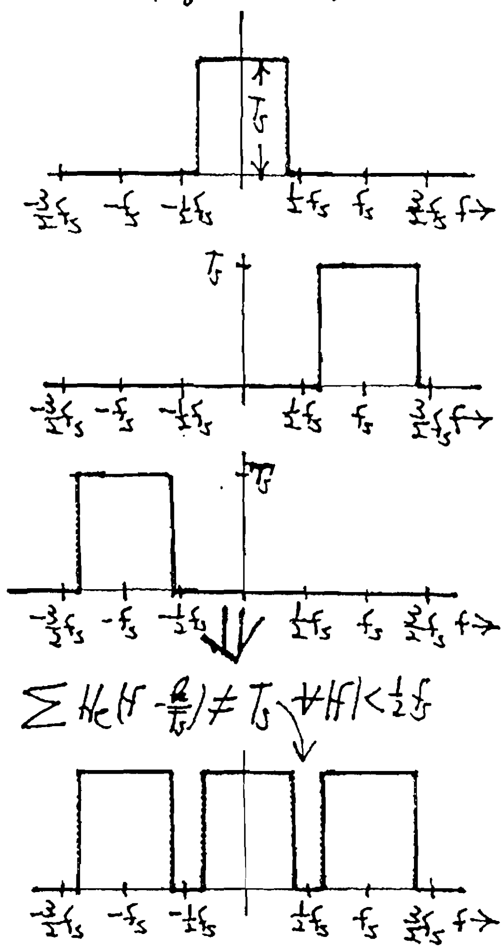
is satisfied for $|f| < \frac{1}{2}f_s$ where $\frac{1}{T_s} = \frac{2}{T_0}$

(a) $H_e(f) = \frac{T_0}{2}\Pi\left(\frac{f}{\frac{2}{T_0}}\right) = T_s\Pi\left(\frac{f}{\frac{1}{T_s}}\right) = T_s\Pi\left(\frac{f}{f_s}\right)$




<u>Yes</u>, Nyquist criterion is satisfied

(b) $H_e(f) = \frac{T_0}{2}\Pi\left(\frac{f}{\frac{3}{4}\left(\frac{2}{T_0}\right)}\right) = T_s\Pi\left(\frac{f}{\frac{3}{4}\frac{1}{T_s}}\right) = T_s\Pi\left(\frac{f}{\frac{3}{4}f_s}\right)$




<u>No</u>, Nyquist criterion is not satisfied.

**3-49** (a.) Substitute (3-69) into $H_e(f) = \begin{cases} 1+Y(f), & |f| < f_0 \\ Y(f), & f_0 < |f| < 2f_0 \end{cases}$

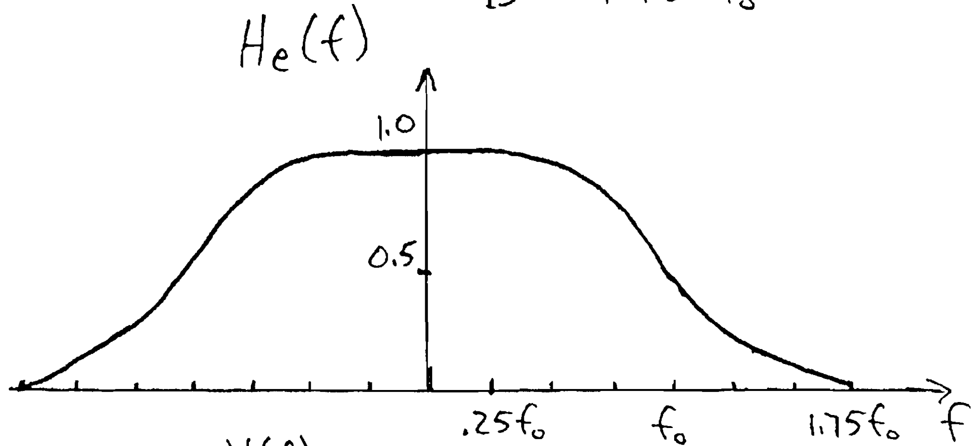
$$\Rightarrow Y(f) = \begin{cases} 0, & |f| < f_1 \\ \frac{1}{2}\left\{-1+\cos\left[\frac{\pi(|f|-f_1)}{2f_\Delta}\right]\right\}, & f_1 < |f| < f_0 \\ \frac{1}{2}\left\{1+\cos\left[\frac{\pi(|f|-f_1)}{2f_\Delta}\right]\right\}, & f_0 < |f| < B \\ 0, & |f| > B \end{cases}$$

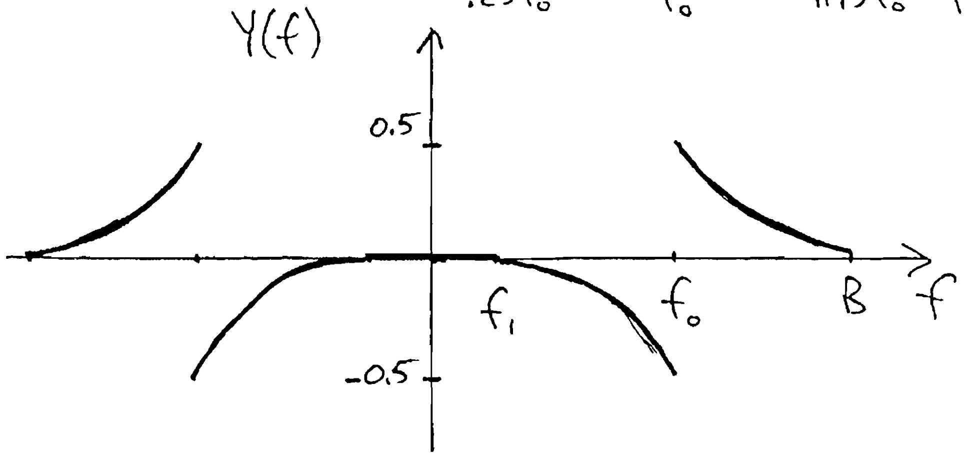
3-49 Cont'd (b.)

$$r = 0.75 = \frac{f_\Delta}{f_0} \Longrightarrow f_1 = f_0(1-0.75)$$

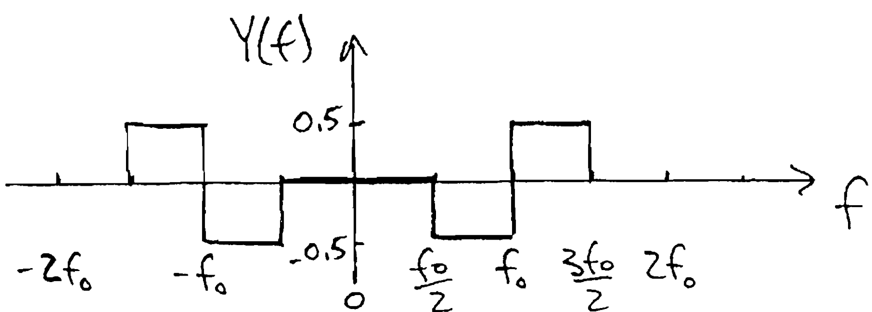
$$f_1 = 0.25 f_0$$

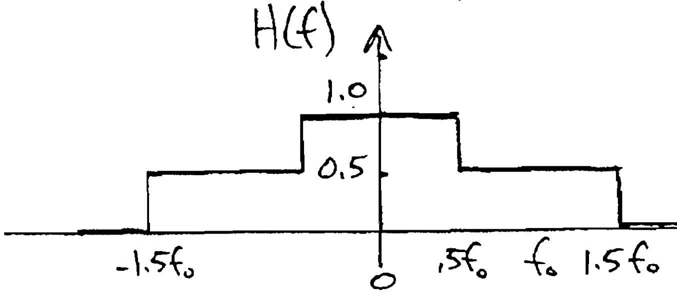
$$B = 1.75 f_0$$




(c.)




$$\underline{\underline{B = 1.5 f_0}}$$

**3-50** $M = 16 = 2^4 \Rightarrow n = 4$

(a) Binary PCM $\Rightarrow \ell = 1$, $R = nf_s = 4f_s = D$

$$D = \frac{2B}{1+r} = \frac{2(4\text{ kHz})}{1+0.5} = \underline{\underline{5.33 \text{ kbits/sec}}}$$

(b) From (a) $f_s = \frac{D}{4} = \frac{5.33k}{4} = 1.33\text{ kHz}$

$$B_{\substack{analog\\max}} = \frac{f_s}{2} = \frac{1.33k}{2} = \underline{\underline{667 \text{ Hz}}}$$

**3-51** $L = 2^\ell = 4 \Rightarrow \ell = 2$

(a) $D = \frac{R}{\ell} = \frac{R}{2}$

$$\Rightarrow R = 2D = 2\left(\frac{2B}{1+r}\right) = \underline{\underline{10.67 \text{ kbit/sec}}}$$

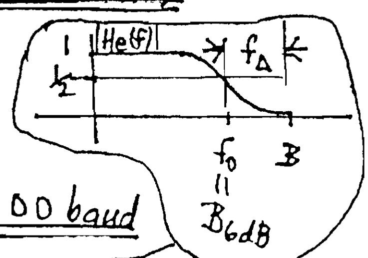
(b) $M = 2^n = L^N = (2^\ell)^{n/\ell} = 16 \Rightarrow n = 4$

$$R = nf_s \Rightarrow f_s = \frac{R}{n} = \frac{10.67\text{ kb/s}}{4\text{ bits/samp}} = \underline{\underline{2.66 \text{ k samples/sec}}}$$

$$B_{\substack{analog\\max}} = \frac{f_s}{2} = \frac{2.66k}{2} = \underline{\underline{1.33 \text{ kHz}}}$$

**3-52** (a) $L = 2^\ell = 4 \Rightarrow \ell = 2$

$$D = R/\ell = \frac{2400}{2} = \underline{\underline{1200 \text{ baud}}}$$




(b) $B = \frac{1}{2}(1+r)D$ where $r = \frac{f_\Delta}{f_0} = 0 \Rightarrow B = f_0 = B_{6dB}$

$$\Rightarrow B_{6dB} = \frac{1}{2}(1+0)D = \frac{1}{2}(1200) = \underline{\underline{600 \text{ Hz}}}$$

(c)

$$B_{absolute} = \frac{1}{2}(1+r)D = \frac{1}{2}(1+0.5)(1200) = \frac{3}{4}(1200)$$

$r = 0.5$

$$\Rightarrow \underline{\underline{B_{absolute} = 900 \text{ Hz}}}$$

**3-53**

PCM with $\mu = 255$ companding.

$\Rightarrow$ Use (3-25) with (3-26b)

$$\left(\frac{S}{N}\right)_{dB} = 6.02n + \alpha = 6.02n + 4.77 - 20\log[\ln(1+\mu)] \underset{\mu=255}{=} 6.02n - 10.1$$

$$\Rightarrow n = \frac{40 + 10.1}{6.02} = \frac{50.108}{6.02} = 8.32 \text{ bits}$$

$n = 8.32$ bits $\Rightarrow$ Use 9 bits to get 40 dB $(S/N)$

$$R = f_s n = 2Bn = 2(3400)(9) = \underline{\underline{61.2 \text{ kbits/sec}}}$$

**3-54**

DPCM Use (3-80a) where, depending on the properties of the analog input signal, $-3 < \alpha < 15$. $\alpha = -3$ gives the worst case result and $\alpha = 15$ gives the best result.

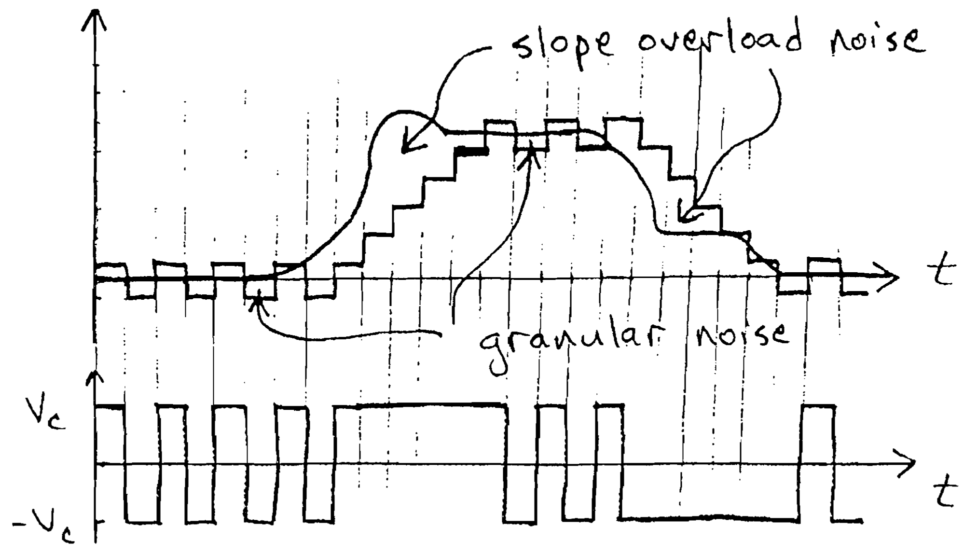
$$\Rightarrow n = \begin{Bmatrix} 7.1, \text{ worst case} \\ 4.1, \text{ best case} \end{Bmatrix} \underset{\text{Use next largest integer value}}{=} \begin{Bmatrix} 8, \text{ worst case} \\ 5, \text{ best case} \end{Bmatrix}$$

thus

$$R = 2Bn = 2(3400)n = \underline{\underline{\begin{Bmatrix} 54 \text{ kBits/sec, worst case} \\ 34 \text{ kbits/sec, best case} \end{Bmatrix}}}$$

Even for the worst case, the bit rate required for DPCM would be slightly less than that required for $\mu = 255$ PCM. Consequently, DPCM with a bit rate of about 48 kbits/sec might be selected as the best solution where it is realized that for certain types of inputs, the 40 dB S/N goal would not be achieved.

**3-55** DM




**3-56**

(a) To prevent slope overload,

$$\delta > \frac{2\pi f_a A}{f_s} = \frac{2\pi\,(10\text{ kHz})(0.5\text{ V})}{10(2)(10\text{ kHz})} = \underline{\underline{0.157\text{ V}}}$$

(b)

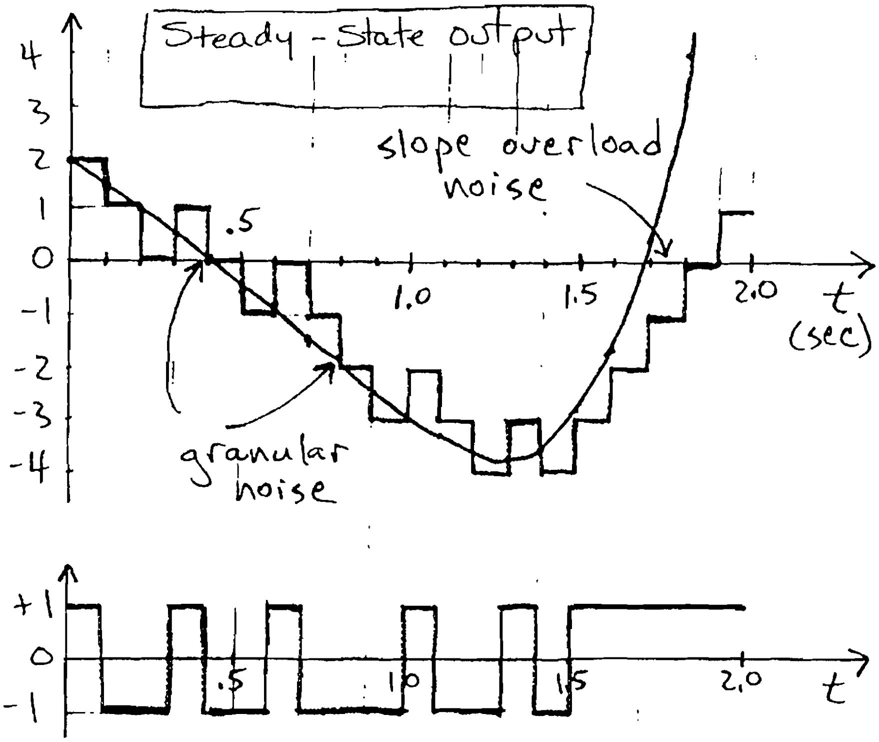
$$\mathcal{P}_n(f) = \frac{\delta^2}{6f_s} = \frac{(0.157)^2}{6(200\text{ kHz})} = \underline{\underline{2.06\times10^{-5}\ \frac{V^2}{\text{kHz}} = \frac{N_0}{2}}}$$

(c)

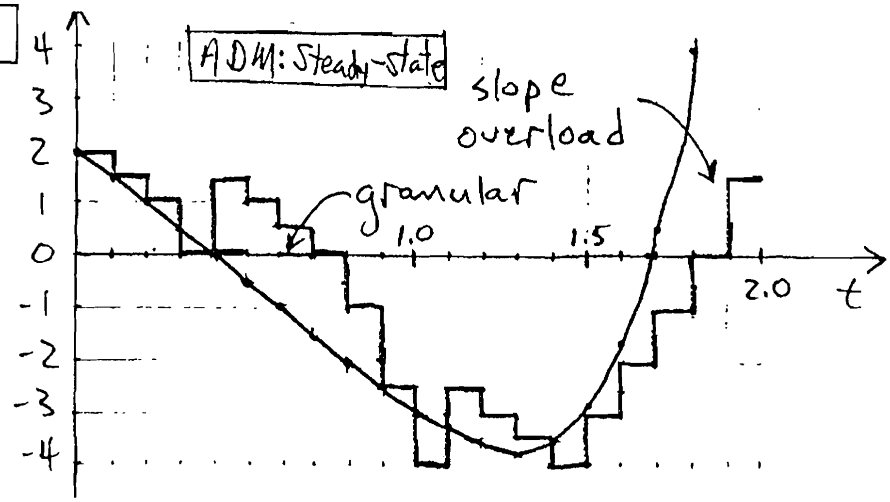
$$\left(\frac{S}{N}\right)_{\text{rcvr out}} = \frac{\frac{A^2}{2}}{2B\left(\frac{N_0}{2}\right)} = \frac{(0.5\text{ V})^2/2}{2(200\text{ kHz})\left(2.06\times10^{-5}\frac{V}{\text{kHz}}\right)} = 15.2$$

$$\Rightarrow \left(\frac{S}{N}\right)_{dB} = 10\log(15.2) = \underline{\underline{11.82\text{ dB}}}$$

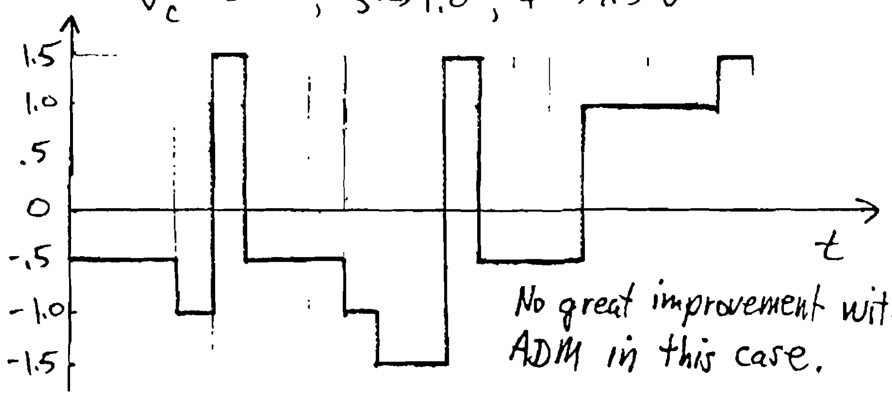
**3-57** $v_{in}(t) = 0.1t^8 - 5t + 2$, $V_c = 1$ volt, $f_s = 10$ Hz




**3-58**




$V_c = 0.5$ ; $3 \rightarrow 1.0$ ; $4 \rightarrow 1.5$ v




No great improvement with ADM in this case.

**3-59**

(a.) From (3-84)

$$\delta = \frac{2\pi f_a A}{f_s} \quad ; \quad f_a = 3.4\,\text{kHz} \;\; \& \;\; A = \frac{1}{2}$$

We need to determine the $f_s$ which the Channel can support. Assuming that a $r=0$ roll-off factor is used, then

$$f_s = D = 2B = 2(1\,\text{MHz}) = 2\times 10^6 \ \frac{\text{Samples}}{\text{sec}}$$

$$\Rightarrow \delta = \frac{2\pi(3.4k)(\frac{1}{2})}{2\times 10^6} = \underline{\underline{0.00534}}$$

(Note: The channel has to be equalized with a Nyquist filter.)

(b.)

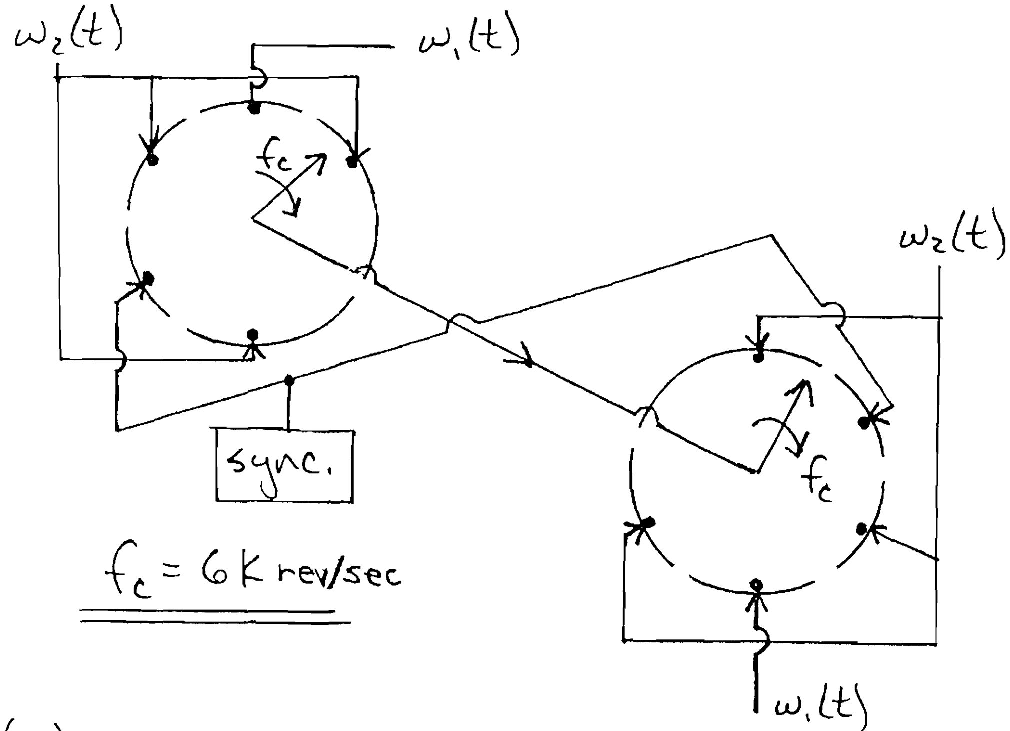
$$\delta = \frac{2\pi(3.4k)(\frac{1}{2})}{25\times 10^3} = \underline{\underline{0.427}}$$

(Note: No Channel equalization required.)

**3-60** TDM

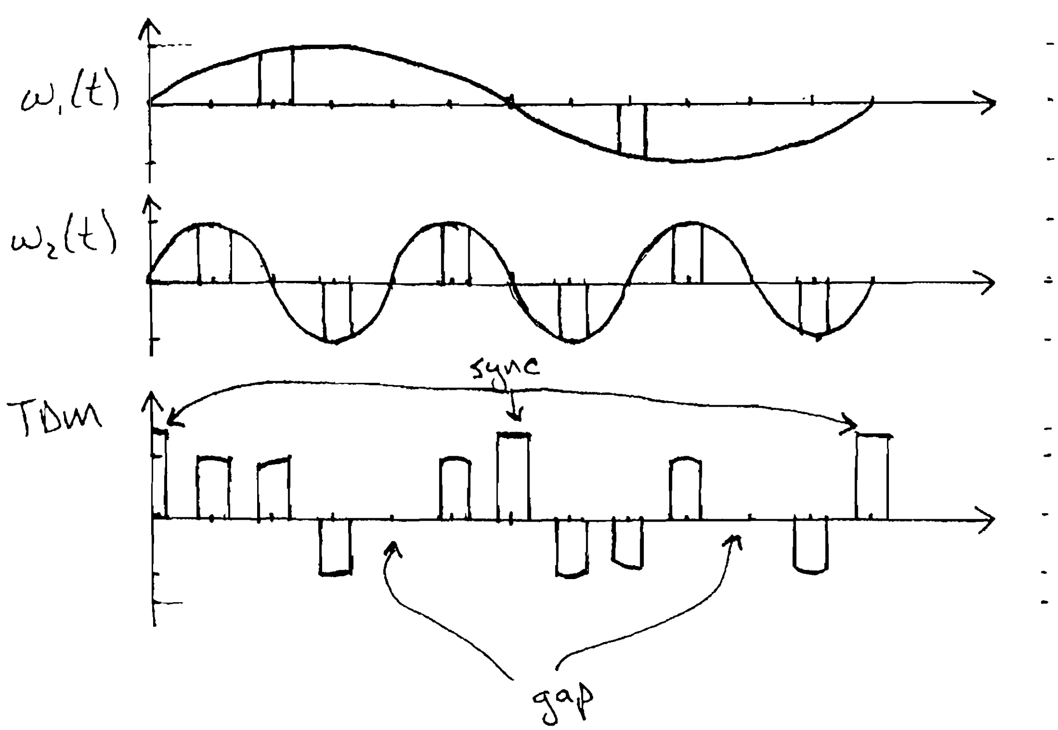
(a.) $w_1(t)$ ; $B = 3\text{KHz} \Rightarrow f_{s1} = 6\text{K samples/sec}$

$w_2(t)$ ; $B = 9\text{KHz} \Rightarrow f_{s2} = 18\text{K samples/sec}$




$f_c = 6\text{K rev/sec}$

(b.)

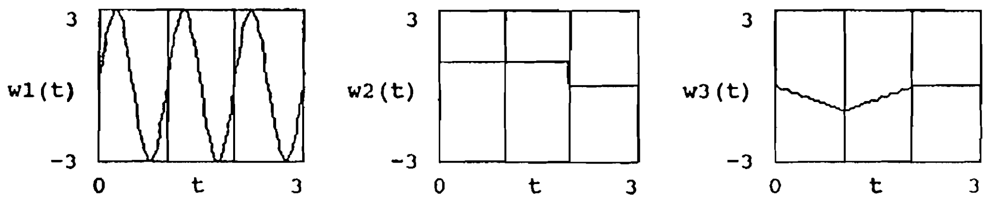


**3-61**

$T := 0.05 \qquad t := 0, 0.05 .. 3$

$w1(t) := 3\cdot\sin(2\cdot\pi\cdot t) \qquad w2(t) := \mathrm{if}((|t| - 1) \le 1, 1, 0)$

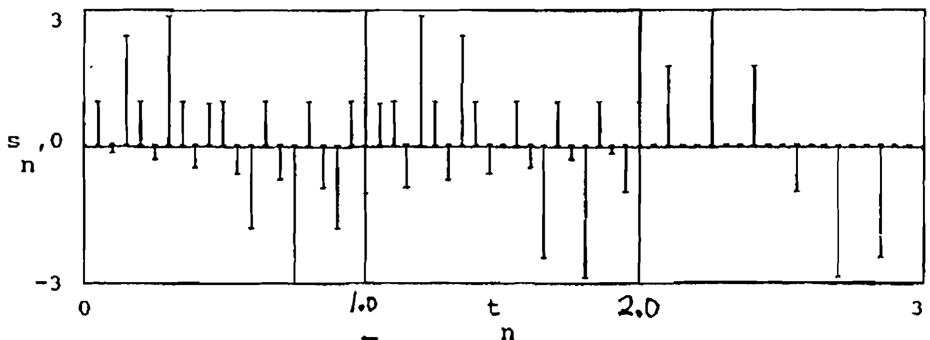
$w3(t) := \mathrm{if}(|t - 1| \le 1, |t - 1| - 1, 0)$



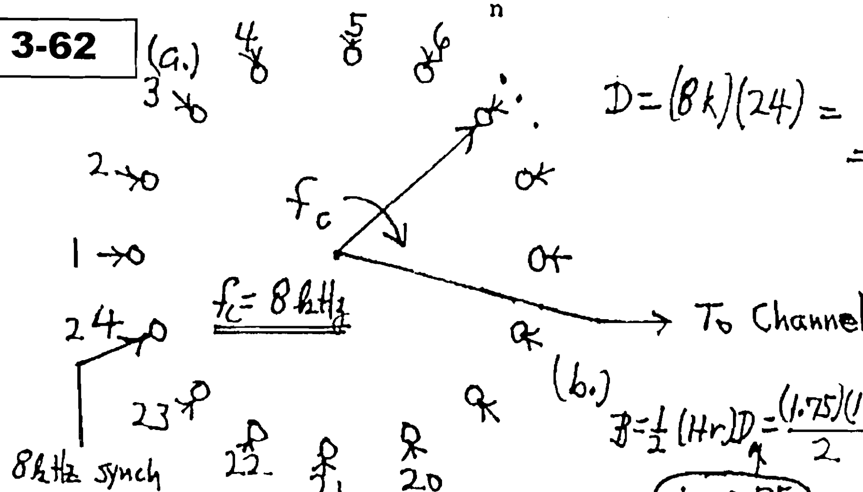

$n := 0 .. 62 \qquad t_n := n\cdot T \qquad k := 0 .. 20$

$k1(k) := 3\cdot k \qquad k2(k) := 3\cdot k + 1 \qquad k3(k) := 3\cdot k + 2$

$s_{k1(k)} := w1\left[t_{k1(k)}\right] \qquad s_{k2(k)} := w2\left[t_{k2(k)}\right] \qquad s_{k3(k)} := w3\left[t_{k3(k)}\right]$




**3-62** (a.)




$f_c = 8\ \text{kHz}$

$D = (8k)(24) = 192\ \frac{\text{kpulses}}{\text{sec}}$

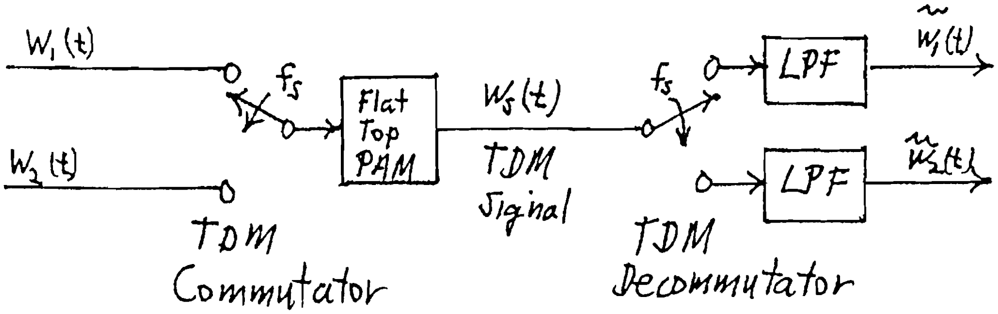
(b.) $B = \frac{1}{2}(1+r)D = \frac{(1.75)(192k)}{2} = 168\ \text{kHz}$

$r = 0.75$

**3-63** (a.) Each analog signal has a highest frequency of $B = 3\ \text{kHz}$

$\Rightarrow$ The minimum sampling frequency for each analog signal is $f_s = 2B = 6\ \text{kHz}$

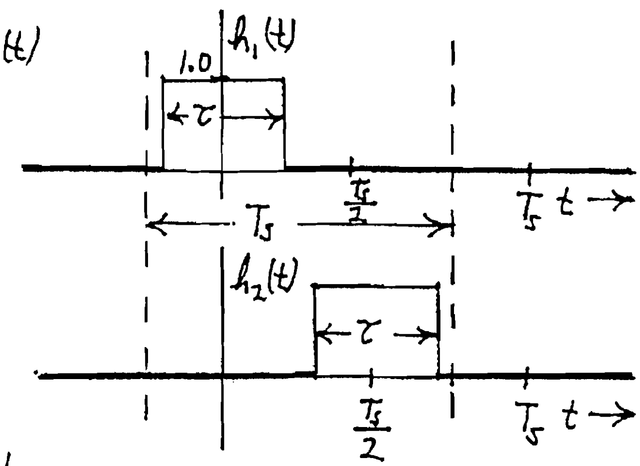
3-63.(a.) Cont'd




(b.) Referring to (3-8), the sampled TDM signal is

$$w_s(t) = \sum_{k=-\infty}^{\infty} w_1(kT_s)\, h_1(t-kT_s) + \sum_{k=-\infty}^{\infty} w_2(kT_s)\, h_2(t-kT_s)$$

where $h_1(t)$ and $h_2(t)$ are shown in the figure and $\tau \le \frac{T_s}{2}$ and $f_s \ge 2B$.




Following the same procedure as described in (3-8) thru (3-13), the spectrum of the TDM instantaneously sampled (flat-topped) PAM signal is

$$W_s(f) = \frac{1}{T_s} H_1(f) \sum_{k=-\infty}^{\infty} W_1(f-kf_s) + \frac{1}{T_s} H_2(f) \sum_{k=-\infty}^{\infty} W_2(f-kf_s)$$

where $H_1(f) = \tau\left(\frac{\sin(\pi f \tau)}{\pi f \tau}\right)$ and $H_2(f) = \tau\left(\frac{\sin(\pi f \tau)}{\pi f \tau}\right) e^{-j2\pi f \frac{T_s}{2}}$

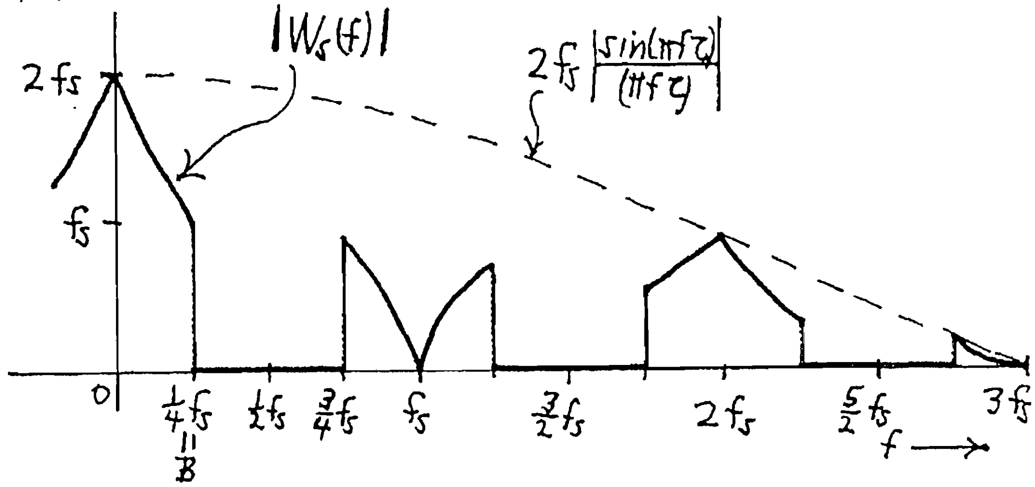
$$\Rightarrow W_s(f) = f_s\, \tau \frac{\sin(\pi f \tau)}{\pi f \tau} \sum_{k=-\infty}^{\infty} \Pi\left(\frac{f-kf_s}{2B}\right) + 2B\tau\left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right)$$

$$+ 2B\tau\left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right) e^{-j\pi T_s f} \sum_{k=-\infty}^{\infty} \Lambda\left(\frac{f-kf_s}{B}\right)$$

3-63. (b.) Cont'd

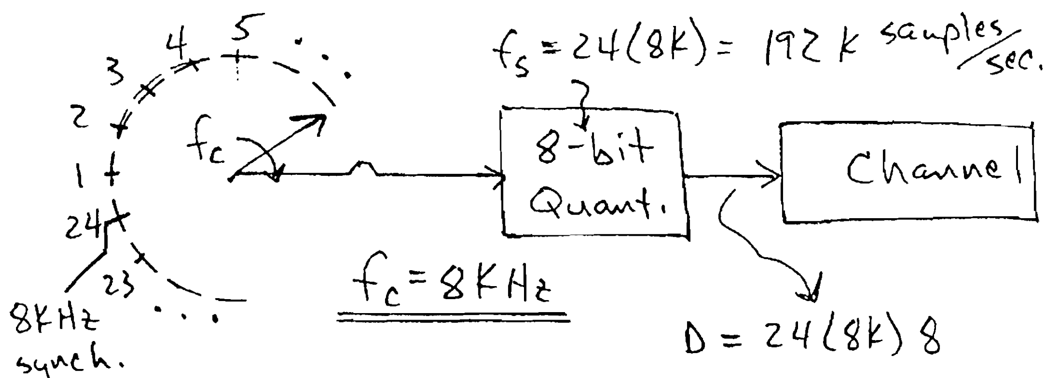
Thus,
$$\underline{\underline{|W_s(f)| = f_s\left|\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right| \sum_{k=-\infty}^{\infty} \left|\Pi\left(\frac{f-kf_s}{2B}\right) + e^{j\pi T_s f}\Lambda\left(\frac{f-kf_s}{B}\right)\right|}}$$

For the sketch, let the parameters be the same as those shown in Fig. 3-6. Let $\tau/T_s = 1/3$, $f_s = 4B$. Using a programable calculator, the following sketch is obtained.




**3-64** (a.) TDM - PCM

$f_s = 24(8k) = 192k$ samples/sec.




$\underline{\underline{f_c = 8\text{ KHz}}}$

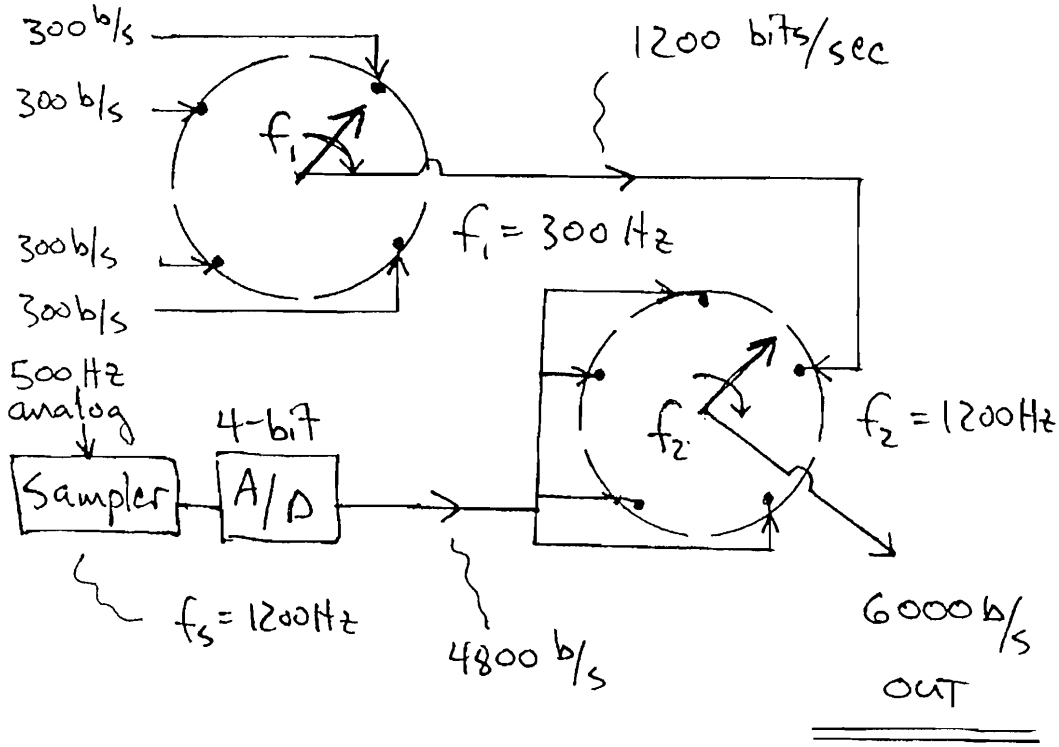
$D = 24(8k)\,8$

$\underline{\underline{= 1.536 \text{ Mpukes/sec}}}$

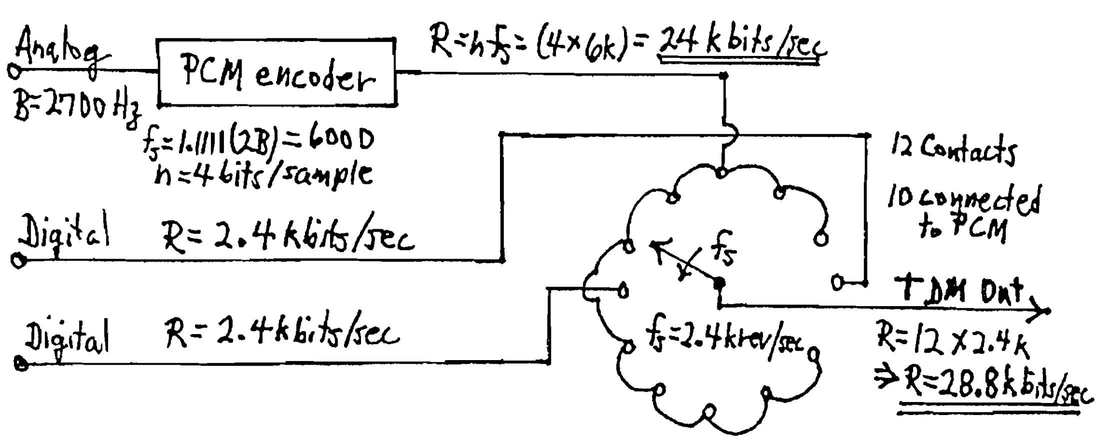
(b.) $r = 0.75$

$$B = \frac{D(1+r)}{2} = \frac{1.536(10^6)(1.75)}{2} = \underline{\underline{1.344 \text{ MHz}}}$$

**3-65**


300 b/s
300 b/s
300 b/s
300 b/s
1200 bits/sec
$f_1$
$f_1$ = 300 Hz
500 Hz analog
4-bit
Sampler
A/D
$f_s$ = 1200 Hz
4800 b/s
$f_2$
$f_2$ = 1200 Hz
6000 b/s OUT

**3-66**

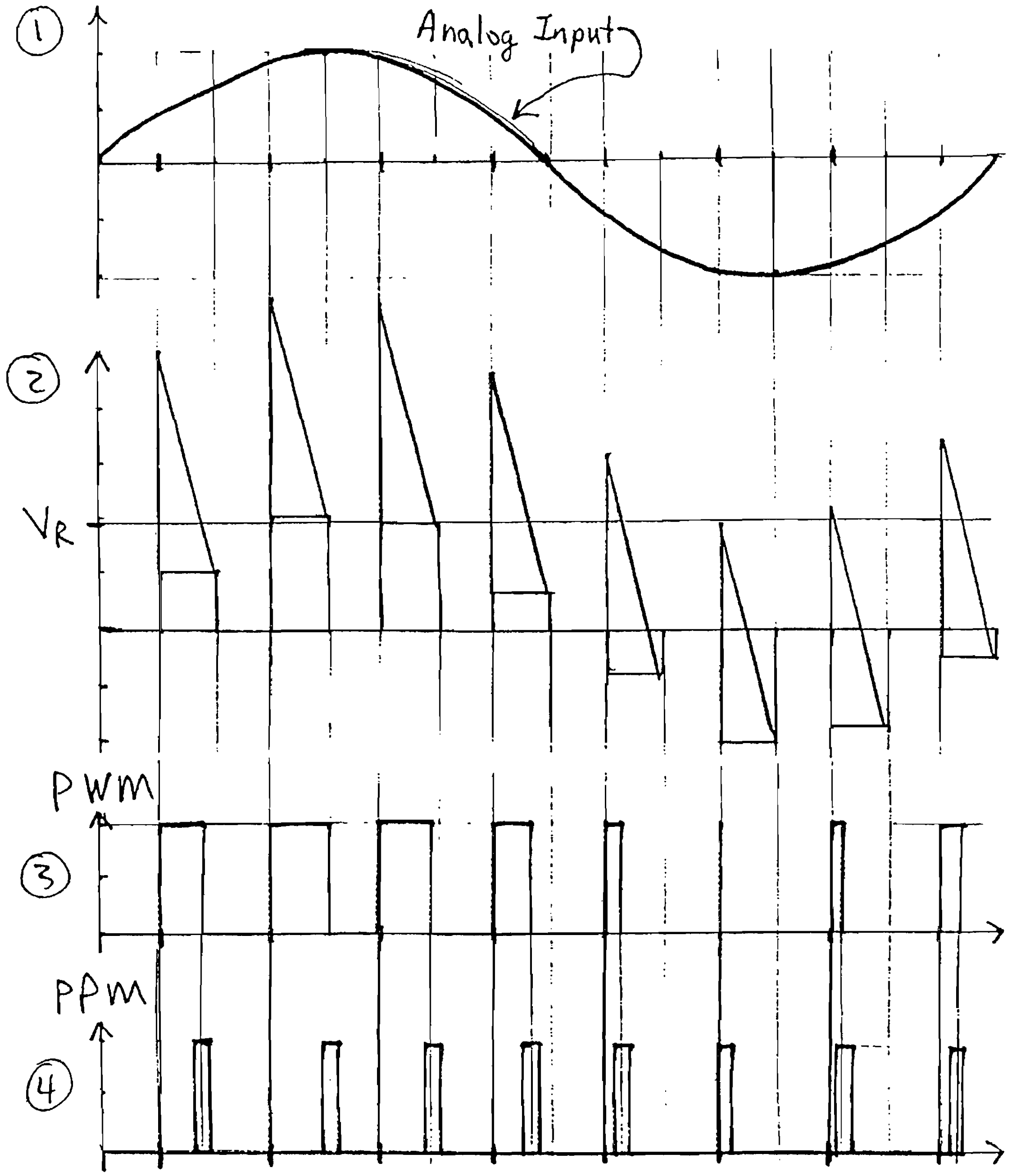

Analog
B = 2700 Hz
PCM encoder
$f_s = 1.1111(2B) = 6000$
$n = 4$ bits/sample
$R = nf_s = (4 \times 6k) = 24$ k bits/sec
Digital R = 2.4 kbits/sec
Digital R = 2.4 kbits/sec
12 Contacts
10 connected to PCM
$f_s$
$f_s$ = 2.4 krev/sec
TDM Out
$R = 12 \times 2.4k$
$\Rightarrow R = 28.8$ kbits/sec

**3-67** Available capacity $\triangleq AC = (1.544 \text{ Mbits/sec})(0.99) = 1.53 \text{ Mbits/sec}$

(a) $\frac{AC}{110} = 13{,}896$, (b) $\frac{AC}{8000} = 191$, (c) $\frac{AC}{9600} = 159$, (d) $\frac{AC}{64\times10^3} = 23$

(e) $\frac{AC}{144\times10^3} = 10$. If sources are active only 10% of the time, a statistical multiplexer could support 10 times the number of terminals.

**3-68**



**3-69**

The information in a PAM signal is contained in the amplitude of each pulse. The information in a PWM signal is contained in the width of each pulse. However, the information in a PPM signal is contained in the position of each pulse relative to each clocking time. A clocking signal must be available at the receiver in order to determine the relative position of each pulse and the clocking signal must be synchronized with the clocking signal used at the transmitter.

**3-70**

(a) For PCM a $N=8$ dimensional system is used since any of the 256 messages can be represented by

$$s_i(t) = \sum_{j=1}^{8} s_{ij}\,\varphi_j(t)$$

where $s_{ij} = \pm 1$ for binary PCM.

$$T_0 = \frac{1}{10 \text{ mess/sec}} = \underline{\underline{0.1 \text{ sec/message}}}$$

$$B = \frac{1}{2}\left(\frac{N}{T_0}\right) = \frac{1}{2}\left(\frac{8}{0.1}\right) = \underline{\underline{40 \text{ Hz}}}$$

(b) For PPM a $N = 256$ dimensional system is used:

$$s_i(t) = \sum_{j=1}^{256} s_{ij}\,\varphi_j(t) \quad \text{where } s_{ij} = \delta_{ij}$$

$$B = \frac{1}{2}\left(\frac{N}{T_0}\right) = \frac{1}{2}\left(\frac{256}{0.1}\right) = \underline{\underline{1{,}280 \text{ Hz}}}$$