

## 2

## MOTION ALONG A STRAIGHT LINE

## Answers to Multiple-Choice Problems

1. C, D 2. C 3. C, D 4. D 5. C 6. A, D 7. A 8. D 9. C 10. A, C, D 11. B, D 12. C

## Solutions to Problems

**\*2.1. Set Up:** Let the  $+x$  direction be to the right in the figure.**Solve:** (a) The lengths of the segments determine the distance of each point from  $O$ :

$$x_A = -5 \text{ cm}, \quad x_B = +45 \text{ cm}, \quad x_C = +15 \text{ cm}, \quad \text{and} \quad x_D = -5 \text{ cm}$$

(b) The displacement is  $\Delta x$ ; the sign of  $\Delta x$  indicates its direction. The distance is always positive.(i)  $A$  to  $B$ :  $\Delta x = x_B - x_A = +45 \text{ cm} - (-5 \text{ cm}) = +50 \text{ cm}$ . Distance is 50 cm.(ii)  $B$  to  $C$ :  $\Delta x = x_C - x_B = +15 \text{ cm} - 45 \text{ cm} = -30 \text{ cm}$ . Distance is 30 cm.(iii)  $C$  to  $D$ :  $\Delta x = x_D - x_C = -5 \text{ cm} - 15 \text{ cm} = -20 \text{ cm}$ . Distance is 20 cm.(iv)  $A$  to  $D$ :  $\Delta x = x_D - x_A = 0$ . Distance =  $2(AB) = 100 \text{ cm}$ .**Reflect:** When the motion is always in the same direction during the interval the magnitude of the displacement and the distance traveled are the same. In (iv) the ant travels to the right and then to the left and the magnitude of the displacement is less than the distance traveled.**2.2. Set Up:** From the graph the position  $x_t$  at each time  $t$  is:  $x_1 = 1.0 \text{ m}$ ,  $x_2 = 0$ ,  $x_3 = -1.0 \text{ m}$ ,  $x_4 = 0$ ,  $x_8 = 6.0 \text{ m}$ , and  $x_{10} = 6.0 \text{ m}$ .**Solve:** (a) The displacement is  $\Delta x$ . (i)  $\Delta x = x_{10} - x_1 = +5.0 \text{ m}$ ; (ii)  $\Delta x = x_{10} - x_3 = +7.0 \text{ m}$ ; (iii)  $\Delta x = x_3 - x_2 = -1.0 \text{ m}$ ; (iv)  $\Delta x = x_4 - x_2 = 0$ .(b) (i)  $3.0 \text{ m} + 1.0 \text{ m} = 4.0 \text{ m}$ ;  $90^\circ$  (ii)  $1.0 \text{ m} + 1.0 \text{ m} = 2.0 \text{ m}$ ; (iii) zero (stays at  $x = 6.0 \text{ m}$ )**2.3. Set Up:** Let the  $+x$  direction be to the right.  $x_A = 2.0 \text{ m}$ ,  $x_B = 7.0 \text{ m}$ ,  $x_C = 6.0 \text{ m}$ .**Solve:** Average velocity is

$$v_{\text{av},x} = \frac{\Delta x}{\Delta t} = \frac{x_C - x_A}{\Delta t} = \frac{+6.0 \text{ m} - 2.0 \text{ m}}{3.0 \text{ s}} = 1.3 \text{ m/s}$$

$$\text{average speed} = \frac{\text{distance}}{\text{time}} = \frac{4.0 \text{ m} + 1.0 \text{ m} + 1.0 \text{ m}}{3.0 \text{ s}} = 2.0 \text{ m/s}$$

**Reflect:** The average speed is greater than the magnitude of the average velocity.

**\*2.4. Set Up:**  $x_A = 0$ ,  $x_B = 3.0$  m,  $x_C = 9.0$  m.  $t_A = 0$ ,  $t_B = 1.0$  s,  $t_C = 5.0$  s.

**Solve: (a)**  $v_{av,x} = \frac{\Delta x}{\Delta t}$

$$A \text{ to } B: v_{av,x} = \frac{\Delta x}{\Delta t} = \frac{x_B - x_A}{t_B - t_A} = \frac{3.0 \text{ m}}{1.0 \text{ s}} = 3.0 \text{ m/s}$$

$$B \text{ to } C: v_{av,x} = \frac{x_C - x_B}{t_C - t_B} = \frac{6.0 \text{ m}}{4.0 \text{ s}} = 1.5 \text{ m/s}$$

$$A \text{ to } C: v_{av,x} = \frac{x_C - x_A}{t_C - t_A} = \frac{9.0 \text{ m}}{5.0 \text{ s}} = 1.8 \text{ m/s}$$

**(b)** The velocity is always in the same direction (+x direction), so the distance traveled is equal to the displacement in each case, and the average speed is the same as the magnitude of the average velocity.

**Reflect:** The average speed is different for different time intervals.

**2.5. Set Up:**  $t_A = 0$ ,  $t_B = 3.0$  s,  $t_C = 6.0$  s.  $x_A = 0$ ,  $x_B = 25.0$  m,  $x_C = 0$ .

**Solve: (a)**  $v_{av,x} = \frac{\Delta x}{\Delta t}$

$$A \text{ to } B: v_{av,x} = \frac{\Delta x}{\Delta t} = \frac{x_B - x_A}{t_B - t_A} = \frac{25.0 \text{ m}}{3.0 \text{ s}} = 8.3 \text{ m/s}$$

$$B \text{ to } C: v_{av,x} = \frac{x_C - x_B}{t_C - t_B} = \frac{-25.0 \text{ m}}{3.0 \text{ s}} = -8.3 \text{ m/s}$$

$$A \text{ to } C: v_{av,x} = \frac{x_C - x_A}{t_C - t_A} = 0$$

**(b)** For  $A$  to  $B$  and for  $B$  to  $C$  the distance traveled equals the magnitude of the displacement and the average speed equals the magnitude of the average velocity. For  $A$  to  $C$  the displacement is zero. Thus, the average velocity is zero but the distance traveled is not zero so the average speed is not zero. For the motion  $A$  to  $B$  and for  $B$  to  $C$  the velocity is always in the same direction but during  $A$  to  $C$  the motion changes direction.

**\*2.6. Set Up:** The positions  $x_t$  at time  $t$  are:  $x_0 = 0$ ,  $x_1 = 1.0$  m,  $x_2 = 4.0$  m,  $x_3 = 9.0$  m,  $x_4 = 16.0$  m.

**Solve: (a)** The distance is  $x_3 - x_1 = 8.0$  m.

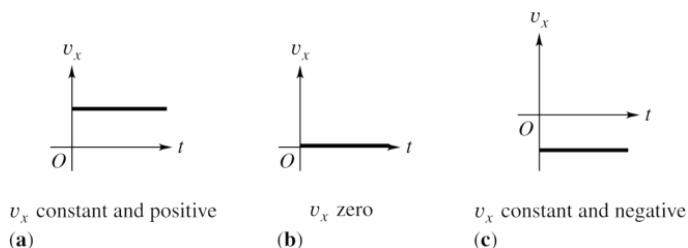
**(b)**  $v_{av,x} = \frac{\Delta x}{\Delta t}$ . (i)  $v_{av,x} = \frac{x_1 - x_0}{1.0 \text{ s}} = 1.0$  m/s; (ii)  $v_{av,x} = \frac{x_2 - x_1}{1.0 \text{ s}} = 3.0$  m/s; (iii)  $v_{av,x} = \frac{x_3 - x_2}{1.0 \text{ s}} = 5.0$  m/s;

(iv)  $v_{av,x} = \frac{x_4 - x_3}{1.0 \text{ s}} = 7.0$  m/s; (v)  $v_{av,x} = \frac{x_4 - x_0}{4.0 \text{ s}} = 4.0$  m/s

**Reflect:** In successive 1-s time intervals the boulder travels greater distances and the average velocity for the intervals increases from one interval to the next.

**2.7. Set Up:**  $v_x(t)$  is the slope of the  $x$  versus  $t$  graph. In each case this slope is constant, so  $v_x$  is constant.

**Solve:** The graphs of  $v_x$  versus  $t$  are sketched in the figure below.



**2.8. Set Up:** Average speed is the distance covered divided by the time to cover that distance. This problem involves three times: the time  $t_1$  to cover the first 60 miles, the time  $t_2 = 20$  min spent without moving in the traffic jam, and the time  $t_3$  to cover the last 40 miles. We will find these times, then divide the 100-mile trip by the total time (in hours) to find the average speed in mi/h.

**Solve:** (a) The time to cover the first 60 miles is

$$t_1 = \frac{60 \text{ mi}}{55 \text{ mi/h}} = 1.09 \text{ h}$$

The time spent waiting in the traffic jam is

$$t_2 = 20 \text{ min} = 0.33 \text{ h}$$

The time to cover the last 40 miles is

$$t_3 = \frac{40 \text{ mi}}{75 \text{ mi/h}} = 0.53 \text{ h}$$

The total time  $t$  for the trip is  $t = t_1 + t_2 + t_3 = 1.09 \text{ h} + 0.33 \text{ h} + 0.53 \text{ h} = 1.96 \text{ h}$ . The average speed for the entire trip is therefore

$$v = \frac{100 \text{ mi}}{1.96 \text{ h}} = 51 \text{ mi/h}$$

**Reflect:** This speed is less than the average of 55 and 75, as expected because you waited for 20 min in a traffic jam without moving.

**2.9. Set Up:** Assume constant speed  $v$ , so  $d = vt$ .

**Solve:** (a)  $t = \frac{d}{v} = \frac{5.0 \times 10^6 \text{ m}}{7(331 \text{ m/s})} = (2158 \text{ s}) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 36 \text{ min}$

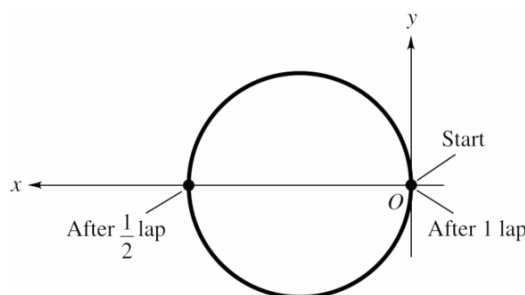
(b)  $d = vt = 7(331 \text{ m/s})(11 \text{ s}) = 2.5 \times 10^4 \text{ m} = 25 \text{ km}$

**\*2.10. Set Up:** 1.0 century = 100 years. 1 km =  $10^5$  cm.

**Solve:** (a)  $d = vt = (5.0 \text{ cm/year})(100 \text{ years}) = 500 \text{ cm} = 5.0 \text{ m}$

(b)  $t = \frac{d}{v} = \frac{550 \times 10^5 \text{ cm}}{5.0 \text{ cm/year}} = 1.1 \times 10^7 \text{ years}$

**2.11. Set Up:** The distance around the circular track is  $d = \pi(40.0 \text{ m}) = 126 \text{ m}$ . For a half lap,  $d = 63 \text{ m}$ . Use coordinates for which the origin is at her starting point and the  $x$  axis is along a diameter, as shown in the figure below.



**Solve:** (a) After one lap she has returned to her starting point. Thus,  $\Delta x = 0$  and  $v_{\text{av},x} = 0$ .

$$\text{average speed} = \frac{d}{t} = \frac{126 \text{ m}}{62.5 \text{ s}} = 2.01 \text{ m/s}$$

(b)  $\Delta x = 40.0 \text{ m}$  and  $v_{\text{av},x} = \frac{\Delta x}{\Delta t} = \frac{40.0 \text{ m}}{28.7 \text{ s}} = 1.39 \text{ m/s}$ ; average speed  $= \frac{d}{t} = \frac{63 \text{ m}}{28.7 \text{ s}} = 2.20 \text{ m/s}$

**2.12. Set Up:** Since sound travels at a constant speed,  $\Delta x = v_x \Delta t$ ; also, from the appendix we find that 1 mile is 1.609 km.

**Solve:**  $\Delta x = (344 \text{ m/s})(7.5 \text{ s}) \left( \frac{1 \text{ mi}}{1.609 \times 10^3 \text{ m}} \right) = 1.6 \text{ mi}$

**Reflect:** The speed of sound is  $(344 \text{ m/s}) \left( \frac{1 \text{ mi}}{1.609 \times 10^3 \text{ m}} \right) \approx \frac{1}{5} \text{ mi/s}$

**2.13. Solve:** (a)  $t = \frac{d}{v}$ . touch:  $t = \frac{1.85 \text{ m}}{76.2 \text{ m/s}} = 0.0243 \text{ s}$ ; pain:  $t = \frac{1.85 \text{ m}}{0.610 \text{ m/s}} = 3.03 \text{ s}$

(b) The difference between the two times in (a) is 3.01 s.

**\*2.14. Set Up:** Use the definition of speed for this problem, being sure to convert times and distances to the requested units.

**Solve:** (a) The time between mile markers is  $2 \text{ min} = 1/30 \text{ h}$ , so the speed  $v$  in mi/h is

$$v = \frac{d}{t} = \frac{1 \text{ mi}}{1/30 \text{ h}} = 30 \text{ mi/h}$$

(b) A football field is 100 yards or 300 ft long, and  $1 \text{ mi} = 5280 \text{ ft}$ . So to travel the length of one football field at 30 mi/h takes

$$t = \frac{d}{v} = \frac{300 \text{ ft}}{30 \text{ mi/h}} \left( \frac{1 \text{ mi}}{5280 \text{ ft}} \right) = 0.0019 \text{ h}$$

**Reflect:** To better understand how long it takes to travel the length of the football field, convert the answer to part (b) to seconds:

$$t = (0.0019 \text{ h}) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 6.8 \text{ s}$$

This seems like a reasonable result for this question.

**2.15. Set Up:** Since we know the position of the mouse as a function of time, we can compute its average velocity from  $v_{\text{av},x} = \frac{\Delta x}{\Delta t}$ .

**Solve:** Calculate the position of the mouse at  $t = 0 \text{ s}$ ;  $t = 1.0 \text{ s}$ ; and  $t = 4.0 \text{ s}$ :

$$x(0 \text{ s}) = 0$$

$$x(1.0 \text{ s}) = (8.5 \text{ cm} \cdot \text{s}^{-1})(1.0 \text{ s}) - (2.5 \text{ cm} \cdot \text{s}^{-2})(1.0 \text{ s})^2 = 6.0 \text{ cm}$$

$$x(4.0 \text{ s}) = (8.5 \text{ cm} \cdot \text{s}^{-1})(4.0 \text{ s}) - (2.5 \text{ cm} \cdot \text{s}^{-2})(4.0 \text{ s})^2 = -6.0 \text{ cm}$$

The average velocities of the mouse from 0 to 1 s and from 0 to 4 s are (respectively)

$$v_{\text{av},x} = \frac{\Delta x}{\Delta t} = \frac{6.0 \text{ cm} - 0}{1.0 \text{ s}} = 6.0 \text{ cm/s}; \quad v_{\text{av},x} = \frac{\Delta x}{\Delta t} = \frac{-6.0 \text{ cm} - 0}{4.0 \text{ s}} = -1.5 \text{ cm/s}$$

**Reflect:** Since the average velocity of the mouse changes sign, the mouse must have turned around. The  $x$  versus  $t$  graph for the mouse, which is an inverted parabola, also shows that the mouse reverses direction.

**\*2.16. Set Up:** Use the normal driving time to find the distance. Use this distance to find the time on Friday.

**Solve:**  $\Delta x = v_{\text{av},x} \Delta t = (105 \text{ km/h})(1.33 \text{ h}) = 140 \text{ km}$ . Then on Friday  $\Delta t = \frac{\Delta x}{v_{\text{av},x}} = \frac{140 \text{ km}}{70 \text{ km/h}} = 2.00 \text{ h}$ . The increase in time is  $2.00 \text{ h} - 1.33 \text{ h} = 0.67 \text{ h} = 40 \text{ min}$ .

**Reflect:** A smaller average speed corresponds to a longer travel time when the distance is the same.

**2.17. Set Up:** Let  $d$  be the distance  $A$  runs in time  $t$ . Then  $B$  runs a distance  $200.0 \text{ m} - d$  in the same time  $t$ .

**Solve:**  $d = v_A t$  and  $200.0 \text{ m} - d = v_B t$ . Combine these two equations to eliminate  $d$ .  $200.0 \text{ m} - v_A t = v_B t$  and  $t = \frac{200.0 \text{ m}}{8.0 \text{ m/s} + 7.0 \text{ m/s}} = 13.3 \text{ s}$ . Then  $d = (8.0 \text{ m/s})(13.3 \text{ s}) = 106 \text{ m}$ ; they will meet 106 m from where  $A$  starts.

**\*2.18. Set Up:** The instantaneous velocity is the slope of the tangent to the  $x$  versus  $t$  graph.

**Solve:** (a) The velocity is zero where the graph is horizontal; point IV.

(b) The velocity is constant and positive where the graph is a straight line with positive slope; point I.

(c) The velocity is constant and negative where the graph is a straight line with negative slope; point V.

(d) The slope is positive and increasing at point II.

(e) The slope is positive and decreasing at point III.

**2.19. Set Up:** The instantaneous velocity at any point is the slope of the  $x$  versus  $t$  graph at that point. Estimate the slope from the graph.

**Solve:**  $A: v_x = 6.7 \text{ m/s}$ ;  $B: v_x = 6.7 \text{ m/s}$ ;  $C: v_x = 0$ ;  $D: v_x = -40.0 \text{ m/s}$ ;  $E: v_x = -40.0 \text{ m/s}$ ;  $F: v_x = -40.0 \text{ m/s}$ ;  $G: v_x = 0$ .

**Reflect:** The sign of  $v_x$  shows the direction the car is moving.  $v_x$  is constant when  $x$  versus  $t$  is a straight line.

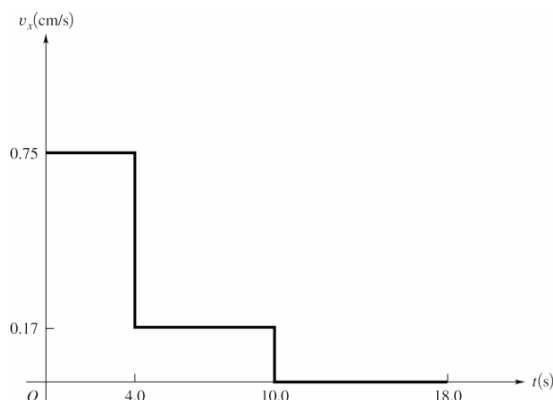
**2.20. Set Up:** Values of  $x_t$  at time  $t$  can be read from the graph:  $x_0 = 0$ ,  $x_4 = 3.0 \text{ cm}$ ,  $x_{10} = 4.0 \text{ cm}$ , and  $x_{18} = 4.0 \text{ cm}$ .  $v_x$  is constant when  $x$  versus  $t$  is a straight line.

**Solve:** The motion consists of constant velocity segments.

$$t = 0 \text{ to } 4.0 \text{ s: } v_x = \frac{3.0 \text{ cm} - 0}{4.0 \text{ s}} = 0.75 \text{ cm/s};$$

$$t = 4.0 \text{ s to } 10.0 \text{ s: } v_x = \frac{4.0 \text{ cm} - 3.0 \text{ cm}}{6.0 \text{ s}} = 0.17 \text{ cm/s}; \quad t = 10.0 \text{ s to } 18.0 \text{ s: } v_x = 0$$

The graph of  $v_x$  versus  $t$  is shown in the figure below.



**Reflect:**  $v_x$  is the slope of  $x$  versus  $t$ .

**2.21. Set Up:** The instantaneous acceleration is the slope of the  $v_x$  versus  $t$  graph.

**Solve:**  $t = 3 \text{ s}$ : The graph is horizontal, so  $a_x = 0$ .

$t = 7 \text{ s}$ : The graph is a straight line with slope  $\frac{44 \text{ m/s} - 20 \text{ m/s}}{4 \text{ s}} = 6.0 \text{ m/s}^2$ ;  $a_x = 6.0 \text{ m/s}^2$ .

$t = 11 \text{ s}$ : The graph is a straight line with slope  $\frac{0 - 44 \text{ m/s}}{4 \text{ s}} = -11 \text{ m/s}^2$ ;  $a_x = -11 \text{ m/s}^2$ .

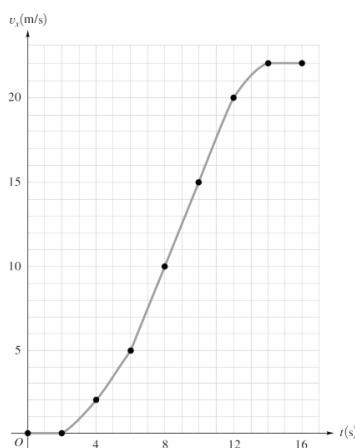
**2.22. Set Up:**  $a_{\text{av},x} = \frac{\Delta v_x}{\Delta t}$

**Solve:** (a) 0 s to 2 s:  $a_{\text{av},x} = 0$ ; 2 s to 4 s:  $a_{\text{av},x} = 1.0 \text{ m/s}^2$ ; 4 s to 6 s:  $a_{\text{av},x} = 1.5 \text{ m/s}^2$ ; 6 s to 8 s:  $a_{\text{av},x} = 2.5 \text{ m/s}^2$ ;

8 s to 10 s:  $a_{\text{av},x} = 2.5 \text{ m/s}^2$ ; 10 s to 12 s:  $a_{\text{av},x} = 2.5 \text{ m/s}^2$ ; 12 s to 14 s:  $a_{\text{av},x} = 1.0 \text{ m/s}^2$ ; 14 s to 16 s:  $a_{\text{av},x} = 0$ .

The acceleration is not constant over the entire 16-s time interval. The acceleration is constant between 6 s and 12 s.

(b) The graph of  $v_x$  versus  $t$  is given in the figure below.  $t = 9 \text{ s}$ :  $a_x = 2.5 \text{ m/s}^2$ ;  $t = 13 \text{ s}$ :  $a_x = 1.0 \text{ m/s}^2$ ;  $t = 15 \text{ s}$ :  $a_x = 0$ .



**Reflect:** The acceleration is constant when the velocity changes at a constant rate. When the velocity is constant, the acceleration is zero.

**2.23. Set Up:**  $1 \text{ ft} = 0.3048 \text{ m}$ ,  $g = 9.8 \text{ m/s}^2$

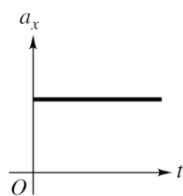
**Solve:** (a)  $5g = 49 \text{ m/s}^2$  and  $5g = (49 \text{ m/s}^2) \left( \frac{1 \text{ ft}}{0.3048 \text{ m}} \right) = 160 \text{ ft/s}^2$

(b)  $60g = 590 \text{ m/s}^2$  and  $60g = (590 \text{ m/s}^2) \left( \frac{1 \text{ ft}}{0.3048 \text{ m}} \right) = 1900 \text{ ft/s}^2$

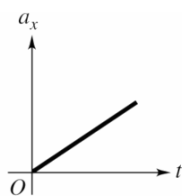
(c)  $(1.67 \text{ m/s}^2) \left( \frac{1g}{9.8 \text{ m/s}^2} \right) = 0.17g$  (d)  $(24.3 \text{ m/s}^2) \left( \frac{1g}{9.8 \text{ m/s}^2} \right) = 2.5g$

**\*2.24. Set Up:** The acceleration  $a_x$  equals the slope of the  $v_x$  versus  $t$  curve.

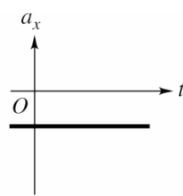
**Solve:** The qualitative graphs of acceleration as a function of time are given in the figure below.



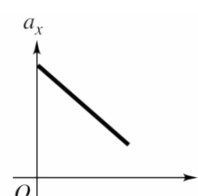
(a)



(b)



(c)



(d)

The acceleration can be described as follows: (a) positive and constant, (b) positive and increasing, (c) negative and constant, (d) positive and decreasing.

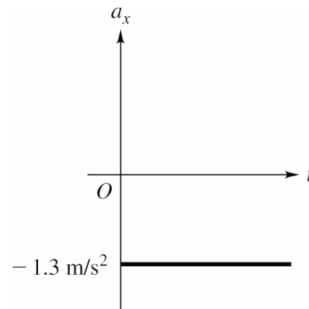
**Reflect:** When  $v_x$  and  $a_x$  have the same sign then the speed is increasing. In (c) the velocity and acceleration have opposite signs and the speed is decreasing.

**2.25. Set Up:** The acceleration  $a_x$  is the slope of the graph of  $v_x$  versus  $t$ .

**Solve:** (a) Reading from the graph, at  $t = 4.0$  s,  $v_x = 2.7$  cm/s, to the right and at  $t = 7.0$  s,  $v_x = 1.3$  cm/s, to the left.

(b)  $v_x$  versus  $t$  is a straight line with slope  $-\frac{8.0 \text{ cm/s}}{6.0 \text{ s}} = -1.3 \text{ cm/s}^2$ . The acceleration is constant and equal to  $1.3 \text{ cm/s}^2$ , to the left.

(c) The graph of  $a_x$  versus  $t$  is given in the figure below.



**\*2.26. Set Up:** Use  $a_x = \frac{\Delta v_x}{\Delta t}$  for part (a) and  $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$  for part (b). Recall that a magnitude is always positive.

**Solve:** (a) The initial speed is  $v_{0x} = 60$  mi/h and the final speed is  $v_x = 40$  mi/h so the change in speed is  $\Delta v_x = v_x - v_{0x} = 40 \text{ mi/h} - 60 \text{ mi/h} = -20 \text{ mi/h}$ . The time is  $\Delta t = 3$  s, which we will convert to hours. Thus, the magnitude of the acceleration is

$$|a_x| = \left| \frac{\Delta v_x}{\Delta t} \right| = \left| \frac{-20 \text{ mi/h}}{3 \text{ s}} \right| \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 2 \times 10^4 \text{ mi/h}^2$$

(b) The distance  $x - x_0$  traveled while braking is

$$\begin{aligned} v_x^2 &= v_{0x}^2 + 2a_x(x - x_0) \\ (x - x_0) &= \frac{v_x^2 - v_{0x}^2}{2a_x} \\ &= \frac{(40 \text{ mi/h})^2 - (60 \text{ mi/h})^2}{2(-2.4 \times 10^4 \text{ mi/h}^2)} \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \\ &= 2 \times 10^2 \text{ ft} \end{aligned}$$

where we have retained a single significant digit because we are given the time of acceleration to a single significant digit.

**Reflect:** Note that we retained two significant digits for the acceleration in the intermediate step for part (b) and only rounded down to a single significant digit at the end of the calculation.

**2.27. Set Up:**  $1 \text{ mph} = 0.4470 \text{ m/s}$  and  $1 \text{ m} = 3.281 \text{ ft}$ . Let  $x_0 = 0$ ,  $v_{0x} = 0$ ,  $t = 2.0$  s, and  $v_x = 45 \text{ mph} = 20.1 \text{ m/s}$ .

(a)  $v_x = v_{0x} + a_x t$  and  $a_x = \frac{v_x - v_{0x}}{t} = \frac{20.1 \text{ m/s} - 0}{2.0 \text{ s}} = 10 \text{ m/s}^2$

$$a_x = (10 \text{ m/s}^2) \left( \frac{3.281 \text{ ft}}{1 \text{ m}} \right) = 33 \text{ ft/s}^2$$

(b)  $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(10 \text{ m/s}^2)(2.0 \text{ s})^2 = 20 \text{ m}$ , or  $x = \frac{1}{2}(33 \text{ ft/s}^2)(2.0 \text{ s})^2 = 66 \text{ ft}$

**\*2.28. Set Up:** Take the  $+y$  direction to be upward. For part (a) we assume that the cat is in free fall with  $a_y = -g$ .

Since the cat falls a known distance, we can find its final velocity using  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ . For parts (b) and (c) we assume that the cat has a constant (but unknown) acceleration due to its interaction with the floor. We may use the equations for constant acceleration.

**Solve:** (a) Solving for  $v_y$ , we obtain

$$v_y = \pm \sqrt{v_{0y}^2 + 2a_y(y - y_0)}$$

Here we set  $\Delta y = (-4.0 \text{ ft})(1 \text{ m}/3.28 \text{ ft}) = -1.22 \text{ m}$ ,  $a_y = -g$ , and  $v_y = 0$ .

$$\begin{aligned} v_y &= -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} \\ &= -\sqrt{-2g(-1.22 \text{ m})} = -4.89 \text{ m/s} = -4.9 \text{ m/s} \end{aligned}$$

Where we choose the negative root since the cat is falling. The *speed* of the cat just before impact is the magnitude of its velocity, which is 4.9 m/s.

(b) During its impact with the floor, the cat is brought to rest over a distance of 12 cm. Thus, we have

$v_{0y} = -4.89 \text{ m/s}$ ,  $v_y = 0$ , and  $\Delta y = -0.12 \text{ m}$ . Solving  $\Delta y = \frac{1}{2}(v_y + v_{0y})t$  for time, we obtain

$$t = \frac{2\Delta y}{(v_y + v_{0y})} = \frac{2(-0.12 \text{ m})}{0 + (-4.89 \text{ m/s})} = 0.049 \text{ s}$$

(c) Solving  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  for  $a_y$ , we obtain  $a_y = \frac{(v_y^2 - v_{0y}^2)}{2\Delta y} = \frac{0^2 - (-4.89 \text{ m/s})^2}{2(-0.12 \text{ m})} = 99.6 \text{ m/s}^2$ . Since this

answer is only accurate to two significant figures, we can write it as  $1.0 \times 10^2 \text{ m/s}^2$  or approximately  $10 \text{ g}$ 's.

**Reflect:** During free fall the cat has a negative velocity and a negative acceleration—so it is speeding up. In contrast, during impact with the ground the cat has a negative velocity and a positive acceleration—so it is slowing down.

**2.29. Set Up:** Let  $+x$  be in his direction of motion. Assume constant acceleration. (a)  $v_x = 3(331 \text{ m/s}) = 993 \text{ m/s}$ ,

$v_{0x} = 0$ , and  $a_x = 5g = 49.0 \text{ m/s}^2$ ; (b)  $t = 5.0 \text{ s}$

**Solve:** (a)  $v_x = v_{0x} + a_x t$  and  $t = \frac{v_x - v_{0x}}{a_x} = \frac{993 \text{ m/s} - 0}{49.0 \text{ m/s}^2} = 20.3 \text{ s}$

Yes, the time required is larger than 5.0 s.

(b)  $v_x = v_{0x} + a_x t = 0 + (49.0 \text{ m/s}^2)(5.0 \text{ s}) = 245 \text{ m/s}$

**2.30. Set Up:** Use  $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ , where the distance is  $x - x_0 = 200 \text{ ft}$  and the initial speed is

$v_{0x} = 60 \text{ mi/h}$  for part (a) and  $v_{0x} = 100 \text{ mi/h}$  for part (b). Recall that a magnitude is always positive.

**Solve:** (a) Solving for the magnitude of the acceleration, inserting the given quantities, and converting miles to feet, we obtain

$$\begin{aligned} v_x^2 &= v_{0x}^2 + 2a_x(x - x_0) \\ |a_x| &= \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} \\ &= \left| \frac{(0 \text{ mi/h})^2 - (60 \text{ mi/h})^2}{2(200 \text{ ft})} \right| \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \\ &= 4.8 \times 10^3 \text{ mi/h}^2 \end{aligned}$$



(b) If the initial speed is 100 mi/h, the required acceleration is

$$\begin{aligned} |a_x| &= \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} \\ &= \left| \frac{(0 \text{ mi/h})^2 - (100 \text{ mi/h})^2}{2(200 \text{ ft})} \right| \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \\ &= 1.3 \times 10^5 \text{ mi/h}^2 \end{aligned}$$

**Reflect:** Note that the acceleration is negative because the speed of the car is decreasing. However, we are only interested for this problem in the magnitude of the acceleration.

**2.31. Set Up:** Let  $+x$  be the direction the jet travels and take  $x_0 = 0$ .  $a_x = 4g = 39.2 \text{ m/s}^2$ ,  $v_x = 4(331 \text{ m/s}) = 1324 \text{ m/s}$ , and  $v_{0x} = 0$ .

**Solve:** (a)  $v_x = v_{0x} + a_x t$  and  $t = \frac{v_x - v_{0x}}{a_x} = \frac{1324 \text{ m/s} - 0}{39.2 \text{ m/s}^2} = 33.8 \text{ s}$

(b)  $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(39.2 \text{ m/s}^2)(33.8 \text{ s})^2 = 2.24 \times 10^4 \text{ m} = 22.4 \text{ km}$

**2.32. Set Up:** Let  $+x$  be the direction the person travels.  $v_x = 0$  (stops),  $t = 36 \text{ ms} = 3.6 \times 10^{-2} \text{ s}$ ,  $a_x = -60g = -588 \text{ m/s}^2$ .  $a_x$  is negative since it is opposite to the direction of the motion.

**Solve:**  $v_x = v_{0x} + a_x t$  so  $v_{0x} = -a_x t$ . Then  $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$  gives  $x = -\frac{1}{2}a_x t^2$ .

$$x = -\frac{1}{2}(-588 \text{ m/s}^2)(3.6 \times 10^{-2} \text{ s})^2 = 38 \text{ cm}$$

**Reflect:** We could also find the initial speed:  $v_{0x} = -a_x t = -(-588 \text{ m/s}^2)(3.6 \times 10^{-2} \text{ s}) = 21 \text{ m/s} = 47 \text{ mi/h}$

**2.33. Set Up:** Take the  $+x$  direction to be the direction of motion of the boulder.

**Solve:** (a) Use the motion during the first second to find the acceleration.  $v_{0x} = 0$ ,  $x_0 = 0$ ,  $x = 2.00 \text{ m}$ , and  $t = 1.00 \text{ s}$ .

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \text{ and } a_x = \frac{2x}{t^2} = \frac{2(2.00 \text{ m})}{(1.00 \text{ s})^2} = 4.00 \text{ m/s}^2$$

$$v_x = v_{0x} + a_x t = (4.00 \text{ m/s}^2)(1.00 \text{ s}) = 4.00 \text{ m/s}$$

For the second second,  $v_{0x} = 4.00 \text{ m/s}$ ,  $a_x = 4.00 \text{ m/s}^2$ , and  $t = 1.00 \text{ s}$ .

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = (4.00 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}(4.00 \text{ m/s}^2)(1.00 \text{ s})^2 = 6.00 \text{ m}$$

We can also solve for the location at  $t = 2.00 \text{ s}$ , starting at  $t = 0$ :

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(4.00 \text{ m/s}^2)(2.00 \text{ s})^2 = 8.00 \text{ m},$$

which agrees with 2.00 m in the first second and 6.00 m in the second second. The boulder speeds up so it travels farther in each successive second.

(b) We have already found  $v_x = 4.00 \text{ m/s}$  after the first second. After the second second,

$$v_x = v_{0x} + a_x t = 4.00 \text{ m/s} + (4.00 \text{ m/s}^2)(1.00 \text{ s}) = 8.00 \text{ m/s}$$

**\*2.34. Set Up:** Let  $+x$  be in the direction of motion of the bullet.  $v_{0x} = 0$ ,  $x_0 = 0$ ,  $v_x = 335$  m/s, and  $x = 0.127$  m.

**Solve:** (a)  $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$  and

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(335 \text{ m/s})^2 - 0}{2(0.127 \text{ m})} = 4.42 \times 10^5 \text{ m/s}^2 = 4.51 \times 10^4 g$$

(b)  $v_x = v_{0x} + a_x t$  so  $t = \frac{v_x - v_{0x}}{a_x} = \frac{335 \text{ m/s} - 0}{4.42 \times 10^5 \text{ m/s}^2} = 0.758 \text{ ms}$

**Reflect:** The acceleration is very large compared to  $g$ . In (b) we could also use  $(x - x_0) = \left(\frac{v_{0x} + v_x}{2}\right)t$  to calculate

$$t = \frac{2(x - x_0)}{v_x} = \frac{2(0.127 \text{ m})}{335 \text{ m/s}} = 0.758 \text{ ms.}$$

**2.35. Set Up:** Take  $+x$  in the direction to be in the direction in which the drag racer moves. Use

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$  to find the acceleration with  $v_{0x} = 0$ ,  $(x - x_0) = 0.25$  mi, and  $t = 10$  s.

**Solve:** (a)  $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$  gives

(b) The acceleration of gravity is  $9.8 \text{ m/s}^2$ , which is about 20% greater than the acceleration of the drag racer.

(c) To get the final speed, use  $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$  and insert the known quantities. This gives

$$\begin{aligned} v_x^2 &= v_{0x}^2 + 2a_x(x - x_0) \\ v_x &= \sqrt{v_{0x}^2 + 2a_x(x - x_0)} \\ &= \sqrt{0 + 2(8.047)(0.25 \text{ mi})\left(\frac{1609.34 \text{ m}}{1 \text{ mi}}\right)} \\ &= 80 \text{ m/s} \end{aligned}$$

or about 180 mi/h.

**Solve:** Note that we retained the positive sign in part (c) because the drag racer moves in the  $+x$  direction. The result for part (c) seems like a reasonable speed for a drag racer.

**2.36. Set Up:**  $1 \text{ mi/h} = 1.466 \text{ ft/s}$ . The car travels at constant speed during the reaction time. Let  $+x$  be the direction the car is traveling, so  $a_x = -12.0 \text{ ft/s}^2$  after the brakes are applied.

**Solve:** (a)  $v_{0x} = (15.0 \text{ mi/h})\left(\frac{1.466 \text{ ft/s}}{1 \text{ mi/h}}\right) = 22.0 \text{ ft/s}$ . During the reaction time the car travels a distance of  $(22.0 \text{ ft/s})(0.7 \text{ s}) = 15.4 \text{ ft}$ .

For the motion after the brakes are applied,  $v_{0x} = 22.0 \text{ ft/s}$ ,  $a_x = -12.0 \text{ ft/s}^2$ , and  $v_x = 0$ .  $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

gives  $(x - x_0) = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (22.0 \text{ ft/s})^2}{2(-12.0 \text{ ft/s}^2)} = 20.2 \text{ ft}$ .

The total distance is  $15.4 \text{ ft} + 20.2 \text{ ft} = 35.6 \text{ ft}$ .

(b)  $v_{0x} = (55.0 \text{ mi/h})\left(\frac{1.466 \text{ ft/s}}{1 \text{ mi/h}}\right) = 80.6 \text{ ft/s}$ . A calculation similar to that of part (a) gives a total stopping distance of  $(x - x_0) = 56.4 \text{ ft} + 270.7 \text{ ft} = 327 \text{ ft}$ .

**\*2.37. Set Up:**  $0.250 \text{ mi} = 1320 \text{ ft}$ .  $60.0 \text{ mph} = 88.0 \text{ ft/s}$ . Let  $+x$  be the direction the car is traveling.

**Solve:** (a) Braking:  $v_{0x} = 88.0 \text{ ft/s}$ ,  $x - x_0 = 146 \text{ ft}$ ,  $v_x = 0$ .  $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$  gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{0 - (88.0 \text{ ft/s})^2}{2(146 \text{ ft})} = -26.5 \text{ ft/s}^2$$

Speeding up:  $v_{0x} = 0$ ,  $x - x_0 = 1320 \text{ ft}$ ,  $t = 19.9 \text{ s}$ .  $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$  gives

$$a_x = \frac{2(x - x_0)}{t^2} = \frac{2(1320 \text{ ft})}{(19.9 \text{ s})^2} = 6.67 \text{ ft/s}^2$$

(b)  $v_x = v_{0x} + a_x t = 0 + (6.67 \text{ ft/s}^2)(19.9 \text{ s}) = 133 \text{ ft/s} = 90.5 \text{ mph}$

(c)  $t = \frac{v_x - v_{0x}}{a_x} = \frac{0 - 88.0 \text{ ft/s}}{-26.5 \text{ ft/s}^2} = 3.32 \text{ s}$

**Reflect:** The magnitude of the acceleration while braking is much larger than when speeding up. That is why it takes much longer to go from 0 to 60 mph than to go from 60 to 0 mph.

**2.38. Set Up:** Break this problem into two parts: In the first part, the car accelerates at  $a_x = 2 \text{ m/s}^2$  for an unknown distance until it reaches at speed of  $v_x = 20 \text{ m/s}$ . It accelerates over a time we'll call  $t_1$ , which we do not know initially. In the second part of the problem, the car continues at this speed for an unknown time  $t_2$ . We know that the total time  $t = t_1 + t_2 = 30 \text{ s}$ .

**Solve:** During the acceleration phase, use  $v_x^2 = v_{0x} + 2a_x d_1$  to find the distance  $d_1$  that the car travels. This gives

$$\begin{aligned} v_x^2 &= v_{0x} + 2a_x d_1 \\ d_1 &= \frac{v_x^2}{2a_x} \\ &= \frac{(20 \text{ m/s})^2}{2(2 \text{ m/s}^2)} \\ &= 100 \text{ m} \end{aligned}$$

where we have used  $v_{0x} = 0$ . The time it takes to travel this distance may be found from

$$\begin{aligned} d_1 &= v_{0x}t_1 + \frac{1}{2}a_x t_1^2 = \frac{1}{2}a_x t_1^2 \\ t_1 &= \sqrt{\frac{2d_1}{a_x}} = \sqrt{\frac{2\left[v_x^2/(2a_x)\right]}{a_x}} = \frac{v_x}{a_x} = \frac{20 \text{ m/s}}{2 \text{ m/s}^2} = 10 \text{ s} \end{aligned}$$

Thus, the car travels for another 20 s at 20 m/s, covering an additional distance of  $20 \text{ s} \times 20 \text{ m/s} = 400 \text{ m}$ . The total distance covered in the initial 30 s is thus  $100 \text{ m} + 400 \text{ m} = 500 \text{ m}$ .

**2.39. Set Up:**  $A = \pi r^2$  and  $C = 2\pi r$ , where  $r$  is the radius.

**Solve:**  $\frac{A_1}{r_1^2} = \frac{A_2}{r_2^2}$  and  $A_2 = \left(\frac{r_2}{r_1}\right)^2 A_1 = \left(\frac{2r_1}{r_1}\right)^2 A = 4A$

$$\frac{C_1}{r_1} = \frac{C_2}{r_2} \text{ and } C_2 = \left(\frac{r_2}{r_1}\right) C_1 = \left(\frac{2r_1}{r_1}\right) C = 2C$$

**2.40. Set Up:** Let  $L$  be the length of each side of the cube. The cube has 6 faces of area  $L^2$ , so  $A = 6L^2$ .  $V = L^3$ .

**Solve:**  $\frac{A_1}{L_1^2} = \frac{A_2}{L_2^2}$  and  $A_2 = \left(\frac{L_2}{L_1}\right)^2 A_1 = \left(\frac{3L_1}{L_1}\right)^2 A_1 = 9A_1$ ; surface area increases by a factor of 9.

$\frac{V_1}{L_1^3} = \frac{V_2}{L_2^3}$  and  $V_2 = \left(\frac{L_2}{L_1}\right)^3 V_1 = \left(\frac{3L_1}{L_1}\right)^3 V_1 = 27V_1$ ; volume increases by a factor of 27.

**\*2.41. Set Up:** The volume of a cylinder of radius  $R$  and height  $H$  is given by  $V = \pi R^2 H$ . We know the ratio of the heights of the two tanks and their volumes. From this information we can determine the ratio of their radii.

**Solve:** We take the ratio of the volumes of the two tanks:  $\frac{V_{\text{large}}}{V_{\text{small}}} = \frac{218}{150} = \frac{\pi R_{\text{large}}^2 H_{\text{large}}}{\pi R_{\text{small}}^2 H_{\text{small}}} = 1.20 \left(\frac{R_{\text{large}}}{R_{\text{small}}}\right)^2$ , where we

have used  $\frac{H_{\text{large}}}{H_{\text{small}}} = 1.20$ . Solving for the ratio of the radii we obtain  $\frac{R_{\text{large}}}{R_{\text{small}}} = \sqrt{\left(\frac{1}{1.20}\right)\left(\frac{218}{150}\right)} = 1.10$ . Thus, the larger radius is 10% larger than the smaller radius.

**Reflect:** All of the ratios used are dimensionless, and independent of the units used for measurement.

**2.42. Set Up:** The formula for volume  $V = \frac{4}{3}\pi R^3$  shows that  $V$  is proportional to  $R^3$ .

**Solve:** If  $V \propto R^3$  then  $R \propto V^{1/3}$ . Thus, if  $V$  decreases by a factor of 8,  $R$  will decrease by a factor of  $8^{1/3} = 2$ .

**\*2.43. Set Up:**  $a_A = a_B$ ,  $x_{0A} = x_{0B} = 0$ ,  $v_{0x,A} = v_{0x,B} = 0$ , and  $t_A = 2t_B$ .

**Solve: (a)**  $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$  gives  $x_A = \frac{1}{2}a_A t_A^2$  and  $x_B = \frac{1}{2}a_B t_B^2$ .  $a_A = a_B$  gives  $\frac{x_A}{t_A^2} = \frac{x_B}{t_B^2}$  and

$$x_B = \left(\frac{t_B}{t_A}\right)^2 x_A = \left(\frac{1}{2}\right)^2 (250 \text{ km}) = 62.5 \text{ km}$$

**(b)**  $v_x = v_{0x} + a_x t$  gives  $a_A = \frac{v_A}{t_A}$  and  $a_B = \frac{v_B}{t_B}$ . Since  $a_A = a_B$ ,  $\frac{v_A}{t_A} = \frac{v_B}{t_B}$  and

$$v_B = \left(\frac{t_B}{t_A}\right) v_A = \left(\frac{1}{2}\right) (350 \text{ m/s}) = 175 \text{ m/s}$$

**Reflect:**  $v_x$  is proportional to  $t$  and for  $v_{0x} = 0$ ,  $x$  is proportional to  $t^2$ .

**2.44. Set Up:**  $a_A = 3a_B$  and  $v_{0A} = v_{0B}$ . Let  $x_{0A} = x_{0B} = 0$ . Since cars stop,  $v_A = v_B = 0$ .

**Solve: (a)**  $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$  gives  $a_A x_A = a_B x_B$ , and  $x_B = \left(\frac{a_A}{a_B}\right) x_A = 3D$

**(b)**  $v_x = v_{0x} + a_x t$  gives  $a_A t_A = a_B t_B$ , so  $t_A = \left(\frac{a_B}{a_A}\right) t_B = \frac{1}{3} T$

**2.45. Set Up:** We are given that  $a_A = 2a_B$  and  $t_B = 2t_A$ , where the subscripts refer to cyclist  $A$  and  $B$ .

**Solve:** The motion of each cyclist must satisfy the equation  $d = \frac{1}{2}at^2$  (where we have ignored their initial speed because they start at rest). By taking the ratio of this equation for cyclist  $A$  and  $B$ , we find

$$\frac{d_A}{d_B} = \frac{\frac{1}{2}a_A t_A^2}{\frac{1}{2}a_B t_B^2}$$

Using  $a_A = 2a_B$  and  $t_B = 2t_A$ , we get

$$\frac{d_A}{d_B} = \frac{\frac{1}{2}a_A t_A^2}{\frac{1}{2}a_B t_B^2} = \frac{\frac{1}{2}2a_B t_A^2}{\frac{1}{2}a_B (2t_A)^2} = \frac{1}{2}$$

We also know that  $v^2 = 2ad$  for each cyclist, from which we obtain the ratio

$$\frac{v_A^2}{v_B^2} = \frac{2a_A d_A}{2a_B d_B}$$

Using the previous result for the ratio of distances, we find

$$\frac{v_A^2}{v_B^2} = \frac{2a_A d_A}{2a_B d_B} = \frac{2(2a_B) \frac{1}{2}}{2a_B \frac{1}{2}} = 1$$

$$\frac{v_A}{v_B} = 1$$

**Reflect:** We used the positive square root for the ratio of speeds because both cyclists move in the same direction.

**2.46. Set Up:** Let  $+y$  be upward.  $a_y = -9.80 \text{ m/s}^2$ .  $v_y = 0$  at the maximum height.

**Solve:** (a)  $y - y_0 = 0.220 \text{ m}$ ,  $a_y = -9.80 \text{ m/s}^2$ ,  $v_y = 0$ .  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives

$$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80 \text{ m/s}^2)(0.220 \text{ m})} = 2.08 \text{ m/s.}$$

(b) When the flea returns to ground,  $v_y = -v_{0y}$ .  $v_y = v_{0y} + a_y t$  gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-2.08 \text{ m/s} - 2.08 \text{ m/s}}{-9.80 \text{ m/s}^2} = 0.424 \text{ s}$$

(c)  $a = 9.80 \text{ m/s}^2$ , downward, at all points in the motion.

**2.47. Set Up:** Let  $+y$  be downward.  $a_y = 9.80 \text{ m/s}^2$

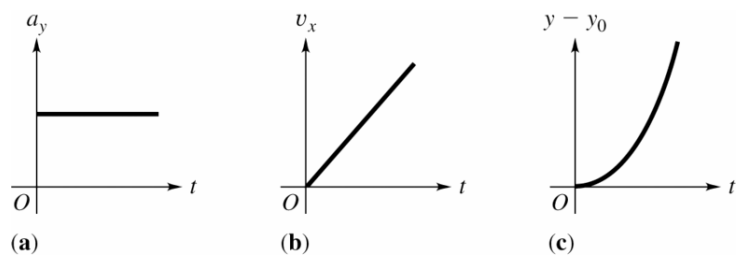
**Solve:** (a)  $v_{0y} = 0$ ,  $t = 2.50 \text{ s}$ ,  $a_y = 9.80 \text{ m/s}^2$ .

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80 \text{ m/s}^2)(2.50 \text{ s})^2 = 30.6 \text{ m}$$

The building is 30.6 m tall.

(b)  $v_y = v_{0y} + a_y t = 0 + (9.80 \text{ m/s}^2)(2.50 \text{ s}) = 24.5 \text{ m/s}$

(c) The graphs of  $a_y$ ,  $v_y$ , and  $y$  versus  $t$  are given in the figure below. Take  $y = 0$  at the ground.



**2.48. Set Up:** Take downward to be the positive direction, so that the acceleration of gravity is  $g = 9.8 \text{ m/s}^2$ . The initial speed of the marble is  $v_0 = 0$  so we can ignore terms with initial speed. The distance traveled by the marble is  $x - x_0 = 830 \text{ m}$ .

**Solve:** (a) We know  $x - x_0 = \frac{1}{2}at^2 = \frac{1}{2}gt^2$  because the acceleration is that of gravity, so  $a = g$ . Solving for the time  $t$  gives

$$t = \sqrt{\frac{2(x - x_0)}{g}} = \sqrt{\frac{2(830 \text{ m})}{9.8 \text{ m/s}^2}} = 13 \text{ s}$$

where we have retained only the positive square root as being physically relevant.

(b) To find the speed  $v$  of the marble just before it hits, we solve for  $v$  in

$$\begin{aligned} v^2 &= 2a(x - x_0) = 2g(x - x_0) \\ v &= \sqrt{2g(x - x_0)} = \sqrt{2(9.8 \text{ m/s}^2)(830 \text{ m})} = 1.3 \times 10^2 \text{ m/s} \end{aligned}$$

where we have again retained only the positive square root as being physically relevant.

**\*2.49. Set Up:** Take  $+y$  upward.  $v_y = 0$  at the maximum height.  $a_y = -0.379g = -3.71 \text{ m/s}^2$ .

**Solve:** Consider the motion from the maximum height back to the initial level. For this motion  $v_{0y} = 0$  and

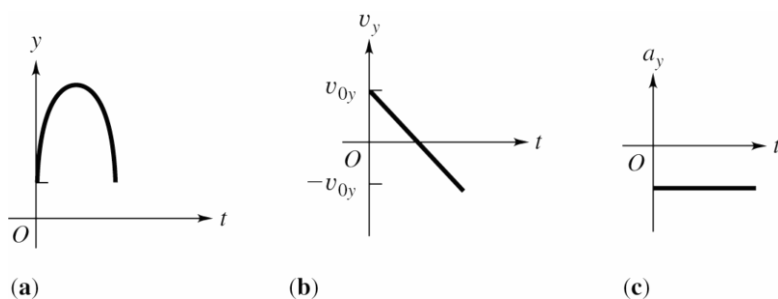
$$t = 4.25 \text{ s. } y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.71 \text{ m/s}^2)(4.25 \text{ s})^2 = -33.5 \text{ m.}$$

The ball went 33.5 m above its original position.

(b) Consider the motion from just after it was hit to the maximum height. For this motion  $v_y = 0$  and  $t = 4.25 \text{ s}$ .

$$v_y = v_{0y} + a_y t \text{ gives } v_{0y} = -a_y t = -(-3.71 \text{ m/s}^2)(4.25 \text{ s}) = 15.8 \text{ m/s.}$$

(c) The graphs are sketched in the figure below.



**Reflect:** The answers can be checked several ways. For example,  $v_y = 0$ ,  $v_{0y} = 15.8 \text{ m/s}$ , and  $a_y = -3.7 \text{ m/s}^2$  in  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (15.8 \text{ m/s})^2}{2(-3.71 \text{ m/s}^2)} = 33.6 \text{ m}$$

which agrees with the height calculated in (a).

**2.50. Set Up:** Take  $+y$  to be downward.  $v_{0y} = 0$  and let  $y_0 = 0$ .

**Solve:** (a)  $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$  gives  $a_y = \frac{2y}{t^2} = \frac{2(11.26 \text{ m})}{(3.17 \text{ s})^2} = 2.24 \text{ m/s}^2 = 0.229g$ .

(b)  $v_y = v_{0y} + a_y t = (2.24 \text{ m/s}^2)(3.17 \text{ s}) = 7.10 \text{ m/s}$

**\*2.51. Set Up:** Take  $+y$  upward.  $a_y = -9.80 \text{ m/s}^2$ . The initial velocity of the sandbag equals the velocity of the balloon, so  $v_{0y} = +5.00 \text{ m/s}$ . When the balloon reaches the ground,  $y - y_0 = -40.0 \text{ m}$ . At its maximum height the sandbag has  $v_y = 0$ .

**Solve:** (a)  $t = 0.250 \text{ s}$ :

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (5.00 \text{ m/s})(0.250 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.250 \text{ s})^2 = 0.94 \text{ m}$$

The sandbag is 40.9 m above the ground.

$$v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.250 \text{ s}) = 2.55 \text{ m/s}$$

$t = 1.00 \text{ s}$ :

$$y - y_0 = (5.00 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = 0.10 \text{ m}$$

The sandbag is 40.1 m above the ground.

$$v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(1.00 \text{ s}) = -4.80 \text{ m/s}$$

(b)  $y - y_0 = -40.0 \text{ m}$ ,  $v_{0y} = 5.00 \text{ m/s}$ ,  $a_y = -9.80 \text{ m/s}^2$ .  $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$  gives  $-40.0 \text{ m} = (5.00 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$ .  $(4.90 \text{ m/s}^2)t^2 - (5.00 \text{ m/s})t - 40.0 \text{ m} = 0$  and

$$t = \frac{1}{9.80} \left( 5.00 \pm \sqrt{(-5.00)^2 - 4(4.90)(-40.0)} \right) \text{ s} = (0.51 \pm 2.90) \text{ s}$$

$t$  must be positive, so  $t = 3.41 \text{ s}$ .

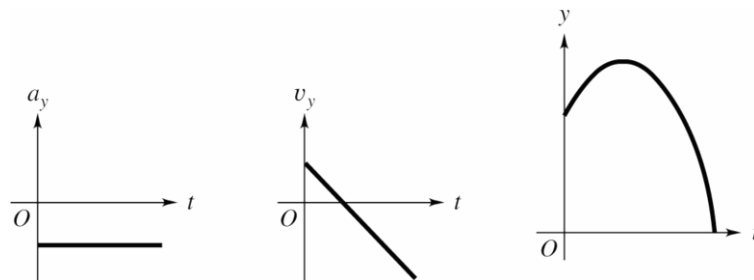
(c)  $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(3.41 \text{ s}) = -28.4 \text{ m/s}$

(d)  $v_{0y} = 5.00 \text{ m/s}$ ,  $a_y = -9.80 \text{ m/s}^2$ ,  $v_y = 0$ .  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (5.00 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 1.28 \text{ m}$$

The maximum height is 41.3 m above the ground.

(e) The graphs of  $a_y$ ,  $v_y$ , and  $y$  versus  $t$  are given in the figure below. Take  $y = 0$  at the ground.



**\*2.52. Set Up:**  $a_M = 0.170a_E$ . Take  $+y$  to be upward and  $y_0 = 0$ .

**Solve: (a)**  $v_{0E} = v_{0M}$ .  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  with  $v_y = 0$  at the maximum height gives  $2a_y y = -v_{0y}^2$ , so  $a_M y_M = a_E y_E$ .

$$y_M = \left( \frac{a_E}{a_M} \right) y_E = \left( \frac{1}{0.170} \right) (12.0 \text{ m}) = 70.6 \text{ m}$$

(b) Consider the time to the maximum height on the earth. The total travel time is twice this. First solve for  $v_{0y}$ , with  $v_y = 0$  and  $y = 12.0 \text{ m}$ .  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives

$$v_{0y} = \sqrt{-2(-a_E)y} = \sqrt{-2(-9.8 \text{ m/s}^2)(12.0 \text{ m})} = 15.3 \text{ m/s}$$

Then  $v_y = v_{0y} + a_y t$  gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 15.3 \text{ m/s}}{-9.8 \text{ m/s}^2} = 1.56 \text{ s}$$

The total time is  $2(1.56 \text{ s}) = 3.12 \text{ s}$ . Then, on the moon  $v_y = v_{0y} + a_y t$  with  $v_{0y} = 15.3 \text{ m/s}$ ,  $v_y = 0$ , and  $a = -1.666 \text{ m/s}^2$  gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 15.3 \text{ m/s}}{-1.666 \text{ m/s}^2} = 9.18 \text{ s}$$

The total time is 18.4 s. It takes 15.3 s longer on the moon.

**Reflect:** The maximum height is proportional to  $1/a$ , so the height on the moon is greater. Since the acceleration is the rate of change of the speed, the wrench loses speed at a slower rate on the moon and it takes more time for its speed to reach  $v = 0$  at the maximum height. In fact,  $t_M/t_E = a_E/a_M = 1/0.170 = 5.9$ , which agrees with our calculated times. But to find the difference in the times we had to solve for the actual times, not just their ratios.

**2.53. Set Up:** Take  $+y$  downward.  $a_y = +9.80 \text{ m/s}^2$ .

**Solve: (a)**  $v_y = v_{0y} + a_y t = 15.0 \text{ m/s} + (9.80 \text{ m/s}^2)(2.00 \text{ s}) = 34.6 \text{ m/s}$

(b)  $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 = (15.0 \text{ m/s})(2.00 \text{ s}) + \frac{1}{2} (9.80 \text{ m/s}^2)(2.00 \text{ s})^2 = 49.6 \text{ m}$

(c)  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives  $v_y = \sqrt{(15.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(10.0 \text{ m})} = 20.5 \text{ m/s}$



**2.54. Set Up:** Take  $+y$  upward.  $a_y = -9.80 \text{ m/s}^2$ . When the rock reaches the ground,  $y - y_0 = -60.0 \text{ m}$ .

**Solve:** (a)  $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$  gives  $-60.0 \text{ m} = (12.0 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$ .  $(4.90 \text{ m/s}^2)t^2 - (12.0 \text{ m/s})t - 60.0 \text{ m} = 0$  and

$$t = \frac{1}{9.80} \left( 12.0 \pm \sqrt{(-12.0)^2 - 4(4.90)(-60.0)} \right) \text{ s} = (1.22 \pm 3.71) \text{ s}$$

$t$  must be positive, so  $t = 4.93 \text{ s}$ .

(b)  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives  $v_y = -\sqrt{(12.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-60.0 \text{ m})} = -36.3 \text{ m/s}$ .

**Reflect:** We could have taken downward to be  $+y$ . Then  $y - y_0$ ,  $v_y$ , and  $a_y$  are all positive, but  $v_{0y}$  is negative. The same results are obtained with this alternative choice of coordinates.

**2.55. Set Up:** Take  $+x$  to be in the direction the sled travels.  $1610 \text{ km/h} = 447 \text{ m/s}$ .  $1020 \text{ km/h} = 283 \text{ m/s}$ . Assume the acceleration is constant.

**Solve:** (a)  $v_{0x} = 0$ .  $v_x = v_{0x} + a_x t$  gives  $a_x = \frac{v_x - v_{0x}}{t} = \frac{447 \text{ m/s} - 0}{1.80 \text{ s}} = 248 \text{ m/s}^2 = 25.3g$

(b)  $(x - x_0) = \left( \frac{v_{0x} + v_x}{2} \right) t = \left( \frac{447 \text{ m/s}}{2} \right) (1.80 \text{ s}) = 402 \text{ m}$

(c) Solve for  $a_x$  and compare to  $40g$ .  $v_x = 0$ .

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{0 - 283 \text{ m/s}}{1.40 \text{ s}} = -202 \text{ m/s}^2 = -20.6g.$$

The figures are inconsistent, if the acceleration while stopping is constant. The acceleration while stopping could reach  $40g$  if the acceleration wasn't constant.

**\*2.56. Set Up:** Use subscripts f and s to refer to the faster and slower stones, respectively. Take  $+y$  to be upward and  $y_0 = 0$  for both stones.  $v_{0f} = 3v_{0s}$ . When a stone reaches the ground,  $y = 0$ .

**Solve:** (a)  $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$  gives  $a_y = -\frac{2v_{0y}}{t}$ . Since both stones have the same  $a_y$ ,  $\frac{v_{0f}}{t_f} = \frac{v_{0s}}{t_s}$  and

$$t_s = t_f \left( \frac{v_{0s}}{v_{0f}} \right) = \left( \frac{1}{3} \right) (10 \text{ s}) = 3.3 \text{ s}$$

(b) Since  $v_y = 0$  at the maximum height, then  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives  $a_y = -\frac{v_{0y}^2}{2y}$ . Since both have the same

$$a_y, \frac{v_{0f}^2}{y_f} = \frac{v_{0s}^2}{y_s} \text{ and } y_f = y_s \left( \frac{v_{0f}}{v_{0s}} \right)^2 = 9H.$$

**Reflect:** The faster stone reaches a greater height so it travels a greater distance than the slower stone and takes more time to return to the ground.

**2.57. Set Up:** Take  $+y$  to be downward and  $y_0 = 0$ . Both coconuts have the same acceleration,  $a_y = g$ . Let  $A$  be the coconut that falls from the greater height and let  $B$  be the other coconut.  $y_A = 2y_B$ .  $v_{0A} = v_{0B} = 0$ .

**Solve:** (a)  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives  $a_y = \frac{v_y^2}{2y}$  and  $\frac{v_A^2}{y_A} = \frac{v_B^2}{y_B}$ .  $v_B = v_A \sqrt{\frac{y_B}{y_A}} = V \sqrt{\frac{1}{2}} = V/\sqrt{2}$

(b)  $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$  gives  $a_y = \frac{2y}{t^2}$  and  $\frac{y_A}{t_A^2} = \frac{y_B}{t_B^2}$ .  $t_A = t_B \sqrt{\frac{y_A}{y_B}} = \sqrt{2}T$

**\*2.58. Set Up:**  $\vec{v}_{T/E} = 65$  mph, north.  $\vec{v}_{VW/E} = 42$  mph, south. Let  $+y$  be north.

**Solve:**  $\vec{v}_{T/E} = \vec{v}_{T/VW} + \vec{v}_{VW/E}$  and  $\vec{v}_{T/VW} = \vec{v}_{T/E} - \vec{v}_{VW/E}$

(a)  $(\vec{v}_{T/VW})_y = (\vec{v}_{T/E})_y - (\vec{v}_{VW/E})_y = 65 \text{ mph} - (-42 \text{ mph}) = 107 \text{ mph}$ . Relative to the VW, the Toyota is traveling north at 107 mph.  $\vec{v}_{VW/T} = -\vec{v}_{T/VW}$ . Relative to the Toyota the VW is traveling south at 107 mph.

(b) The answers are the same as in (a).

**2.59. Set Up:** Your velocity relative to the highway is  $\vec{v}_{Y/H} = 70$  mi/h, east. Let east be the positive direction.

**Solve:** (a) When you and the truck travel in the same direction, the difference between your speed and that of the truck is 20 mi/h. Thus,

$$\begin{aligned}v_{Y/H} - v_{T/H} &= 20 \text{ mi/h} \\v_{T/H} &= v_{Y/H} - 20 \text{ mi/h} = 50 \text{ mi/h}\end{aligned}$$

(b) If the truck is traveling toward you at 50 mi/h (i.e.,  $\vec{v}_{T/H} = -50$  mi/h), then your relative velocity  $\vec{v}_{Y/T}$  would be the sum of your velocity with respect to the highway and the highway's velocity with respect to the truck, or

$$\vec{v}_{Y/T} = \vec{v}_{Y/H} + \vec{v}_{H/T}$$

Using  $\vec{v}_{H/T} = -\vec{v}_{T/H} = 50$  mi/h, this gives  $\vec{v}_{Y/T} = 70 \text{ mi/h} + 50 \text{ mi/h} = 120 \text{ mi/h}$ .

**2.60. Set Up:** Use coordinates with  $+y$  downward. Relative to the earth the package has  $v_{0y} = +3.50$  m/s and

$$a_y = 9.80 \text{ m/s}^2.$$

**Solve:** The velocity of the package relative to the ground just before it hits is

$$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(3.50 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(8.50 \text{ m})} = 13.4 \text{ m/s}$$

(a)  $\vec{v}_{P/G} = 13.4$  m/s, downward.  $\vec{v}_{H/G} = 3.50$  m/s, downward.  $\vec{v}_{P/G} = \vec{v}_{P/H} + \vec{v}_{H/G}$  and  $\vec{v}_{P/H} = \vec{v}_{P/G} - \vec{v}_{H/G}$ .  
 $\vec{v}_{P/H} = 9.9$  m/s, downward.

(b)  $\vec{v}_{H/P} = -\vec{v}_{P/H}$ , so  $\vec{v}_{H/P} = 9.9$  m/s, upward.

**Reflect:** Since the helicopter is traveling downward, the package is moving slower relative to the helicopter than its speed relative to the ground.

**2.61. Set Up:**  $\vec{v}_{P/A} = 600$  mph and is east for the first 200 mi and west for the return 200 mi. The time is the distance relative to the ground divided by the speed relative to the ground.

**Solve:**  $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$

(a)  $v_{A/E} = 0$  and  $v_{P/E} = 600$  mph.  $t = \frac{4000 \text{ mi}}{600 \text{ mi/h}} = 6.67 \text{ h}$

(b) San Francisco to Chicago:  $\vec{v}_{A/E} = 150$  mph, east.  $\vec{v}_{P/A} = 600$  mph, east.

$$v_{P/E} = v_{P/A} + v_{A/E} = 750 \text{ mph. } t = \frac{2000 \text{ mi}}{750 \text{ mi/h}} = 2.67 \text{ h}$$

Chicago to San Francisco:  $\vec{v}_{A/E} = 150$  mph, east.  $\vec{v}_{P/A} = 600$  mph, west.

$$v_{P/E} = v_{P/A} - v_{A/E} = 450 \text{ mph. } t = \frac{2000 \text{ mi}}{450 \text{ mi/h}} = 4.44 \text{ h}$$

The total time is  $2.67 \text{ h} + 4.44 \text{ h} = 7.11 \text{ h}$ .

**2.62. Set Up:** At  $t=0$  the auto and truck are at the same position. The auto overtakes the truck when after time  $T$  they have both traveled a distance  $d$ .

**Solve: (a)** Apply  $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$  to the motion of each vehicle. The auto has  $v_{0x} = 0$  and  $a_x = 2.50 \text{ m/s}^2$ , so  $d = \frac{1}{2}(2.50 \text{ m/s}^2)T^2$ . The truck has  $v_{0x} = 15.0 \text{ m/s}$  and  $a_x = 0$ , so  $d = (15.0 \text{ m/s})T$ . Combining these two equations gives  $(1.25 \text{ m/s}^2)T^2 = (15.0 \text{ m/s})T$  and  $T = 12.0 \text{ s}$ . Then  $d = (15.0 \text{ m/s})(12.0 \text{ s}) = 180 \text{ m}$ .

**(b)**  $v_x = v_{0x} + a_x t = 0 + (2.50 \text{ m/s}^2)(12.0 \text{ s}) = 30.0 \text{ m/s}$

**\*2.63. Set Up:** The trooper's initial speed is  $v_{0,T} = 30 \text{ m/s}$  and his acceleration is  $a_T = 2.5 \text{ m/s}^2$ . The speeder's speed is constant at  $v_s = 50 \text{ m/s}$ .

**Solve: (a)** From the point at which the speeder passes the trooper to the point at which the trooper catches up with the speeder, both will have traveled the same distance  $d$ . Thus,

$$v_{0,T}t + \frac{1}{2}a_T t^2 = v_s t$$

The left-hand side is the distance traveled by the trooper in time  $t$ , and the right-hand side is the same distance traveled by the speeder during the same time  $t$ . Solving for the time  $t$ , we find

$$\begin{aligned} v_{0,T} + \frac{1}{2}a_T t &= v_s \\ t &= 2 \frac{v_s - v_{0,T}}{a_T} \\ &= 2 \frac{50 \text{ m/s} - 30 \text{ m/s}}{2.5 \text{ m/s}^2} \\ &= 16 \text{ s} \end{aligned}$$

**(b)** The distance covered by the speeder (and the trooper) when the trooper catches the speeder is  $d = v_s t = (50 \text{ m/s})(16 \text{ s}) = 800 \text{ m}$

**Reflect:** The answer for part (b) may also be found by calculating the distance covered by the trooper during the chase. This gives  $d = v_{0,T}t + \frac{1}{2}a_T t^2 = (30 \text{ m/s})(16 \text{ s}) + \frac{1}{2}(2.5 \text{ m/s}^2)(16 \text{ s})^2 = 800 \text{ m}$ .

**\*2.64. Set Up:** Let  $+y$  to be upward and  $y_0 = 0$ .  $a_M = a_E/6$ . At the maximum height  $v_y = 0$ .

**Solve: (a)**  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ . Since  $v_y = 0$  and  $v_{0y}$  is the same for both rocks,  $a_M y_M = a_E y_E$  and

$$y_E = \left( \frac{a_M}{a_E} \right) y_M = H/6$$

**(b)**  $v_y = v_{0y} + a_y t$ .  $a_M t_M = a_E t_E$  and  $t_M = \left( \frac{a_E}{a_M} \right) t_E = 6(4.0 \text{ s}) = 24.0 \text{ s}$

**Reflect:** On the moon, where the acceleration is less, the rock reaches a greater height and takes more time to reach that maximum height.

**2.65. Set Up:** Let +y to be downward.  $v_{0y} = 2.0$  m/s,  $v_y = 1.3$  m/s, and  $y - y_0 = 0.020$  m.

**Solve:** (a)  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{(1.3 \text{ m/s})^2 - (2.0 \text{ m/s})^2}{2(0.020 \text{ m})} = -58 \text{ m/s}^2 = -5.9g$$

(b)  $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$  gives  $t = \frac{2(y - y_0)}{v_{0y} + v_y} = \frac{2(0.020 \text{ m})}{2.0 \text{ m/s} + 1.3 \text{ m/s}} = 12 \text{ ms}$

**\*2.66. Set Up:** Let +y be downward. The egg has  $v_{0y} = 0$  and  $a_y = 9.80$  m/s<sup>2</sup>. Find the distance the professor walks during the time  $t$  it takes the egg to fall to the height of his head. At this height, the egg has  $y - y_0 = 44.2$  m.

**Solve:**  $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$  gives

$$t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(44.2 \text{ m})}{9.80 \text{ m/s}^2}} = 3.00 \text{ s}$$

The professor walks a distance  $x - x_0 = v_{0x}t = (1.20 \text{ m/s})(3.00 \text{ s}) = 3.60$  m. Release the egg when your professor is 3.60 m from the point directly below you.

**Reflect:** Just before the egg lands its speed is  $(9.80 \text{ m/s}^2)(3.00 \text{ s}) = 29.4$  m/s. It is traveling much faster than the professor.

**2.67. Set Up:** Use the two kinematics equations  $x_n - x_{n-1} = v_n t + \frac{1}{2}at^2$  and  $v_n^2 = v_{n-1}^2 + 2a(x_n - x_{n-1})$ , where the subscript  $n$  gives the number of second since the marble was released. Note that  $x_0 = 0$  and  $v_0 = 0$ .

**Solve:** In the first second ( $n = 1$ ), the distance moved is

$$x = \frac{1}{2}at^2 = \frac{1}{2}a$$

where we used  $t = 1$  s,  $x_0 = 0$ , and  $v_0 = 0$ . The speed at the end of the first second is

$$v_1^2 = v_0^2 + 2a(x - x_0) = 2ax = a^2 \Rightarrow v_1 = a$$

(a) Between one and two seconds ( $n = 2$ ), the marble moves a distance

$$x_2 - x_1 = v_1 t + \frac{1}{2}at^2 = v_1 + \frac{1}{2}a = \frac{3}{2}a = 3x$$

The speed at the end of the two seconds is

$$v_2^2 = v_1^2 + 2a(x_2 - x_1) = a^2 + 2a(3x) = 4a^2 \Rightarrow v_2 = 2a$$

(b) Between two and three seconds ( $n = 3$ ), the marble moves a distance

$$x_3 - x_2 = v_2 t + \frac{1}{2}at^2 = v_2 + \frac{1}{2}a = 2a + \frac{1}{2}a = \frac{5}{2}a = 5x$$

(c) In the  $n$ th second of motion, the marble moves a distance  $(2n - 1)x$ .

**\*2.68. Set Up:** Since air resistance is ignored, the boulder is in free-fall and has a constant downward acceleration of magnitude  $9.80 \text{ m/s}^2$ . Apply the acceleration equations to the motion of the boulder. Take  $+y$  to be upward.

**Solve:** (a)  $v_{0y} = 40.0 \text{ m/s}$ ,  $v_y = +20.0 \text{ m/s}$ ,  $a_y = -9.80 \text{ m/s}^2$ .  $v_y = v_{0y} + a_y t$  gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{20.0 \text{ m/s} - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = +2.04 \text{ s}$$

(b)  $v_y = -20.0 \text{ m/s}$ .  $t = \frac{v_y - v_{0y}}{a_y} = \frac{-20.0 \text{ m/s} - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = +6.12 \text{ s}$

(c)  $y - y_0 = 0$ ,  $v_{0y} = +40.0 \text{ m/s}$ ,  $a_y = -9.80 \text{ m/s}^2$ .  $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$  gives  $t = 0$  and

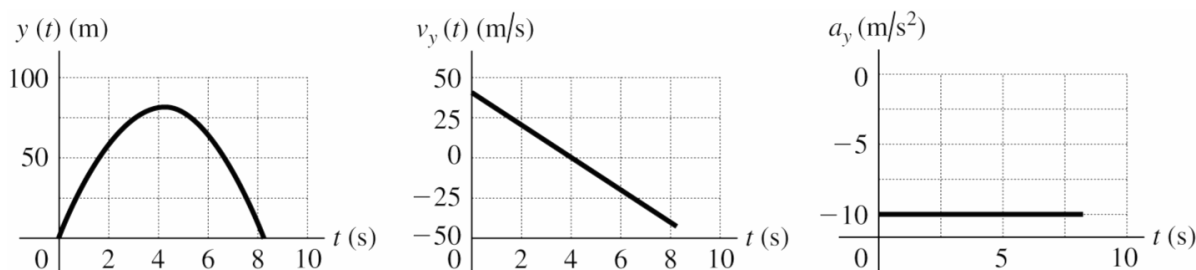
$$t = -\frac{2v_{0y}}{a_y} = -\frac{2(40.0 \text{ m/s})}{-9.80 \text{ m/s}^2} = +8.16 \text{ s}$$

(d)  $v_y = 0$ ,  $v_{0y} = +40.0 \text{ m/s}$ ,  $a_y = -9.80 \text{ m/s}^2$ .  $v_y = v_{0y} + a_y t$  gives  $t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 4.08 \text{ s}$ .

(e) The acceleration is  $9.80 \text{ m/s}^2$ , downward, at all points in the motion.

(f) The graphs are sketched in the following figure.

**Reflect:** We have  $v_y = 0$  at the maximum height. The time to reach the maximum height is half the total time in the air, so the answer in part (d) is half the answer in part (c). Also note that  $2.04 \text{ s} < 4.08 \text{ s} < 6.12 \text{ s}$ . The boulder is going upward until it reaches its maximum height, and after the maximum height it is traveling downward.



**2.69. Set Up:** We are given that, in  $t = 5 \text{ s}$ , the velocity changes from  $v_{0,x} = -25 \text{ m/s}$  to  $v_x = -50 \text{ m/s}$ .

**Solve:** (a) The magnitude of the average acceleration is  $|a_{av}| = \left| \frac{v_x - v_{0,x}}{t} \right| = \left| \frac{-50 \text{ m/s} - (-25 \text{ m/s})}{5 \text{ s}} \right| = 5 \text{ m/s}^2$ .

(b) The average acceleration is in the same direction as the initial velocity: the negative  $x$  direction.

(c) During this period, the car travels a distance  $d$  given by

$$d = v_{0,x}t + \frac{1}{2}at^2 = (-25 \text{ m/s})(5 \text{ s}) + \frac{1}{2}(-5 \text{ m/s}^2)(5 \text{ s})^2 = -2 \times 10^2 \text{ m}$$

Thus the car travels a distance of 200 m in the negative  $x$  direction.

**2.70. Set Up:** The average acceleration is given by  $a_{av,x} = \frac{\Delta v_x}{\Delta t}$ . The time for each beat is  $\frac{1}{72} \text{ min}$ , or  $\frac{60}{72} \text{ s}$ .

**Solve:**  $a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{425 \text{ cm/s} - 0}{60/72 \text{ s}} = 510 \text{ cm/s}^2$

**Reflect:** This is about  $\frac{1}{2}g$ .

**2.71. Set Up:** Define north to be the positive direction. The acceleration of the northbound car is  $a_N = 2 \text{ m/s}^2$  and the constant velocity of the southbound car is  $v_S = -25 \text{ m/s}$ . The both move for the same amount of time, and the distances they cover must sum to 200 m, or  $x_N + |x_S| = 200 \text{ m}$ , where we used the absolute value of the southbound car's displacement because it moves in the negative direction.

**Solve: (a)** When they pass each other, we have  $x_N = \frac{1}{2}a_N t^2$  and  $x_S = -(200 \text{ m} - x_N) = v_S t$ . Solving for the time  $t$  gives

$$t = -\frac{200 \text{ m} - x_N}{v_S} = -\frac{200 \text{ m} - \frac{1}{2}a_N t^2}{v_S}$$

$$t = \frac{v_S \pm \sqrt{(400 \text{ m})a_N + v_S^2}}{a_N} = \frac{(-25 \text{ m/s}) \pm \sqrt{(400 \text{ m})(2 \text{ m/s}^2) + (-25 \text{ m/s})^2}}{2 \text{ m/s}^2} = 6.4 \text{ s}, -31 \text{ s}$$

The cars pass each other 6.4 s after the light changes.

**(b)** The distance  $x_N$  from the red light is

$$x_N = \frac{1}{2}a_N t^2 = \frac{1}{2}(2 \text{ m/s}^2)(6.375 \text{ s})^2 = 41 \text{ m}$$

**Reflect:** The distance from the red light may be checked by using the formula for distance traveled by the southbound car. This gives  $x_S = -(200 - x_N) = v_S t$ , or  $x_N = v_S t + 200 \text{ m} = (-25 \text{ m/s})(6.375 \text{ s}) + 200 \text{ m} = 41 \text{ m}$ .

**\*2.72. Set Up:** Take  $+y$  to be upward. There are two periods of constant acceleration:  $a_y = +2.50 \text{ m/s}^2$  while the engines fire and  $a_y = -9.8 \text{ m/s}^2$  after they shut off. Constant acceleration equations can be applied within each period of constant acceleration.

**Solve: (a)** Find the speed and height at the end of the first 20.0 s.  $a_y = +2.50 \text{ m/s}^2$ ,  $v_{0y} = 0$ , and  $y_0 = 0$ .

$$v_y = v_{0y} + a_y t = (2.50 \text{ m/s}^2)(20.0 \text{ s}) = 50.0 \text{ m/s} \quad \text{and} \quad y = y_0 + v_{0y} t + \frac{1}{2}a_y t^2 = \frac{1}{2}(2.50 \text{ m/s}^2)(20.0 \text{ s})^2 = 500 \text{ m}.$$

Next consider the motion from this point to the maximum height.  $y_0 = 500 \text{ m}$ ,  $v_y = 0$ ,  $v_{0y} = 50.0 \text{ m/s}$ , and  $a_y = -9.8 \text{ m/s}^2$ .  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$  gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (50.0 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = +128 \text{ m}$$

so  $y = 628 \text{ m}$ . The duration of this part of the motion is obtained from  $v_y = v_{0y} + a_y t$ :

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-50 \text{ m/s}}{-9.8 \text{ m/s}^2} = 5.10 \text{ s}$$

**(b)** At the highest point,  $v_y = 0$  and  $a_y = 9.8 \text{ m/s}^2$ , downward.

**(c)** Consider the motion from the maximum height back to the ground.  $a_y = -9.8 \text{ m/s}^2$ ,  $v_{0y} = 0$ ,  $y = 0$ , and

$y_0 = 628 \text{ m}$ .  $y = y_0 + v_{0y} t + \frac{1}{2}a_y t^2$  gives

$$t = \sqrt{\frac{2(y - y_0)}{a_y}} = 11.3 \text{ s}$$

The total time the rocket is in the air is  $20.0 \text{ s} + 5.10 \text{ s} + 11.3 \text{ s} = 36.4 \text{ s}$ .  $v_y = v_{0y} + a_y t = (-9.8 \text{ m/s}^2)(11.3 \text{ s}) = -111 \text{ m/s}$ . Just before it hits the ground the rocket will have speed 111 m/s.

**Reflect:** We could calculate the time of free fall directly by considering the motion from the point of engine shutoff to the ground:  $v_{0y} = 50.0 \text{ m/s}$ ,  $y - y_0 = 500 \text{ m}$  and  $a_y = -9.8 \text{ m/s}^2$ .  $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$  gives  $t = 16.4 \text{ s}$ , which agrees with a total time of 36.4 s.

**2.73. Set Up:** Assume straight line motion along the  $+x$  axis. For the starting phase we may use the kinematic equations for constant acceleration:  $v_{0x} = 0$ ,  $v_x = 8.00 \text{ m/s}$ , and  $\Delta t = 1.40 \text{ s}$ ;  $v_x = \frac{\Delta v_x}{\Delta t}$ , and  $\Delta x = \frac{v_x + v_{0x}}{2} \Delta t$ .

**Solve:** (a)  $v_x = \frac{\Delta v_x}{\Delta t} = \frac{8.00 \text{ m/s} - 0}{1.40 \text{ s}} = 5.71 \text{ m/s}^2$

(b) His acceleration to a top speed of 11.8 m/s occurs over a time of 7.02 s. Thus,  $a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{11.8 \text{ m/s} - 0}{7.02 \text{ s}} = 1.68 \text{ m/s}^2$ .

(c) The distance traveled during the starting phase is  $\Delta x = \left(\frac{8.00 \text{ m/s} + 0}{2}\right)(1.40 \text{ s}) = 5.60 \text{ m}$ .

**Reflect:** We cannot calculate the distance that the sprinter travels during the second phase of the race, since we do not know that his acceleration is constant during this phase.

**\*2.74. Set Up:** Let  $t_{\text{fall}}$  be the time for the rock to fall to the ground and let  $t_s$  be the time it takes the sound to travel from the impact point back to you.  $t_{\text{fall}} + t_s = 10.0 \text{ s}$ . Both the rock and sound travel a distance  $d$  that is equal to the height of the cliff. Take  $+y$  downward for the motion of the rock. The rock has  $v_{0y} = 0$  and  $a_y = 9.80 \text{ m/s}^2$ .

**Solve:** (a) For the rock,  $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$  gives  $t_{\text{fall}} = \sqrt{\frac{2d}{9.80 \text{ m/s}^2}}$ .

For the sound,  $t_s = \frac{d}{330 \text{ m/s}} = 10.0 \text{ s}$ . Let  $\alpha^2 = d$ .  $0.00303\alpha^2 + 0.4518\alpha - 10.0 = 0$ .  $\alpha = 19.6$  and  $d = 384 \text{ m}$ .

(b) You would have calculated  $d = \frac{1}{2}(9.80 \text{ m/s}^2)(10.0 \text{ s})^2 = 490 \text{ m}$ . You would have overestimated the height of the cliff. It actually takes the rock less time than 10.0 s to fall to the ground.

**Reflect:** Once we know  $d$  we can calculate that  $t_{\text{fall}} = 8.8 \text{ s}$  and  $t_s = 1.2 \text{ s}$ . The time for the sound of impact to travel back to you is 12% of the total time and cannot be neglected. The rock has speed 86 m/s just before it strikes the ground.

### Solutions to Passage Problems

**2.75. Set Up:** Since the blood momentarily comes to rest,  $v_{0x} = 0$ . Also, we know that  $v_x = 0.8 \text{ m/s}$  and  $\Delta t = 250 \text{ ms}$ .

**Solve:**  $a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{0.8 \text{ m/s} - 0}{0.250 \text{ s}} = 3.2 \text{ m/s}^2$ . Thus, the correct answer is C.

**\*2.76. Set Up:** Assuming that the aorta and arteries are circular in cross-sectional area, we can use

$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2$ . Let  $d_a$  be the diameter of the aorta and  $d_b$  be the diameter of each branch.

**Solve:** Since the combined area of the two arteries is equal to that of the aorta we have  $\pi \left(\frac{d_b}{2}\right)^2 + \pi \left(\frac{d_b}{2}\right)^2 = \pi \left(\frac{d_a}{2}\right)^2$ ,

which reduces to  $2d_b^2 = d_a^2$ . Thus, we have  $d_b = d_a/\sqrt{2}$ . The correct answer is B.

**2.77. Set Up:** The graph shows the blood velocity during a single heartbeat. Note that the blood velocity does not change direction (i.e., it is always positive or zero).

**Solve:** At 0.25 s the blood flow does not change direction; rather the velocity stops increasing and begins to decrease. The blood flow increases monotonically over the initial 0.25 s, so it does not decrease at 0.10 s. The acceleration in blood velocity is given by the slope of the curve. The magnitude of the slope is greatest near 0.10 s, which means that the magnitude of the blood acceleration is greatest at 0.10 s. The correct answer is D.