C H A P T E R 2
Functions and Their Graphs

CHAPTER 2
Functions and Their Graphs

Section 2.1 Linear Equations in Two Variables

1. linear

2. slope

3. point-slope

4. parallel

5. perpendicular

6. rate or rate of change

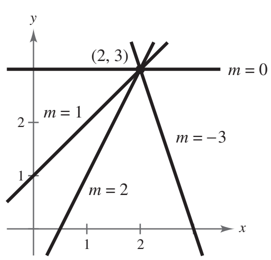
7. linear extrapolation

8. general

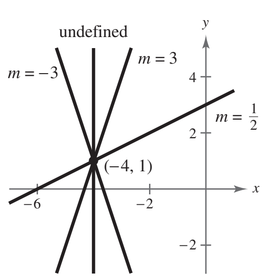
9. (a) $m = \frac{2}{3}$. Because the slope is positive, the line rises. Matches L_2.
   (b) m is undefined. The line is vertical. Matches L_3.
   (c) $m = -2$. The line falls. Matches L_1.

10. (a) $m = 0$. The line is horizontal. Matches L_2.
    (b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.
    (c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.

11.



12.



13. Two points on the line: $(0, 0)$ and $(4, 6)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6}{4} = \frac{3}{2}$$

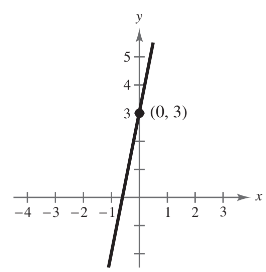
14. The line appears to go through $(0, 7)$ and $(7, 0)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{7 - 0} = -1$$

15. $y = 5x + 3$

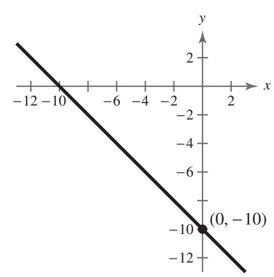
Slope: $m = 5$

y-intercept: $(0, 3)$



16. Slope: $m = -1$

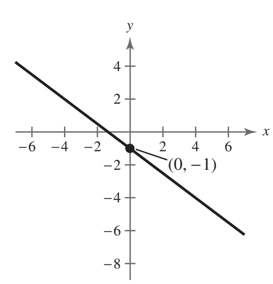
y-intercept: $(0, -10)$



17. $y = -\frac{3}{4}x - 1$

Slope: $m = -\frac{3}{4}$

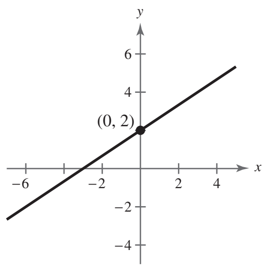
y-intercept: $(0, -1)$



18. $y = \frac{2}{3}x + 2$

Slope: $m = \frac{2}{3}$

y-intercept: $(0, 2)$

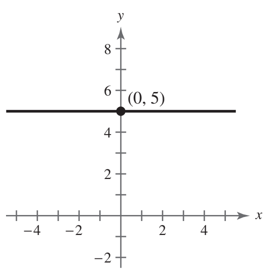


19. $y - 5 = 0$

$y = 5$

Slope: $m = 0$

y-intercept: $(0, 5)$

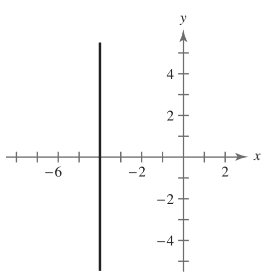


20. $x + 4 = 0$

$x = -4$

Slope: undefined (vertical line)

y-intercept: none

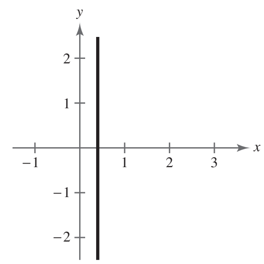


21. $5x - 2 = 0$

$x = \frac{2}{5}$, vertical line

Slope: undefined

y-intercept: none



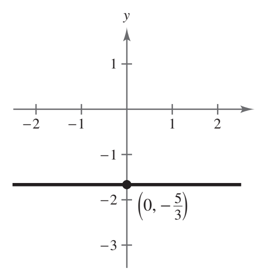
22. $3y + 5 = 0$

$3y = -5$

$y = -\frac{5}{3}$

Slope: $m = 0$

y-intercept: $\left(0, -\frac{5}{3}\right)$



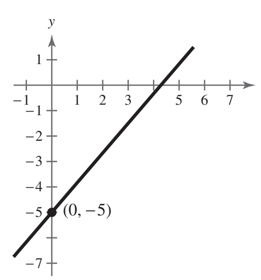
23. $7x - 6y = 30$

$-6y = -7x + 30$

$y = \frac{7}{6}x - 5$

Slope: $m = \frac{7}{6}$

y-intercept: $(0, -5)$



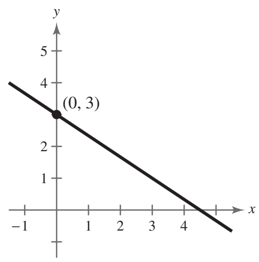
24. $2x + 3y = 9$

$$3y = -2x + 9$$

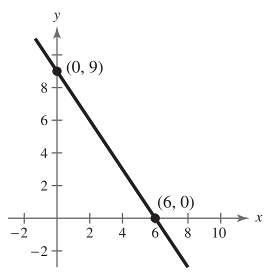
$$y = -\frac{2}{3}x + 3$$

Slope: $m = -\frac{2}{3}$

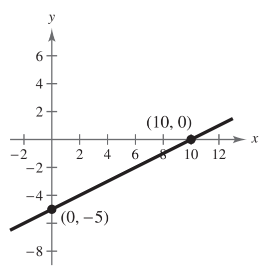
y-intercept: $(0, 3)$



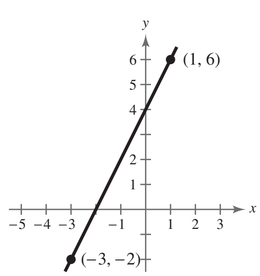
25. $m = \dfrac{0 - 9}{6 - 0} = \dfrac{-9}{6} = -\dfrac{3}{2}$



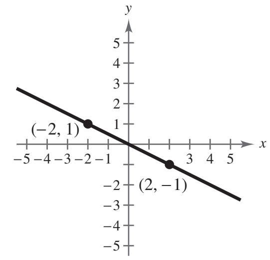
26. $m = \dfrac{-5 - 0}{0 - 10} = \dfrac{-5}{-10} = \dfrac{1}{2}$



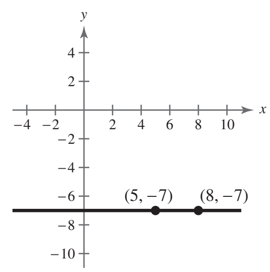
27. $m = \dfrac{6 - (-2)}{1 - (-3)} = \dfrac{8}{4} = 2$



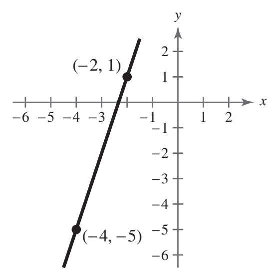
28. $m = \dfrac{1 - (-1)}{-2 - 2} = \dfrac{2}{-4} = -\dfrac{1}{2}$



29. $m = \dfrac{-7 - (-7)}{8 - 5} = \dfrac{0}{3} = 0$

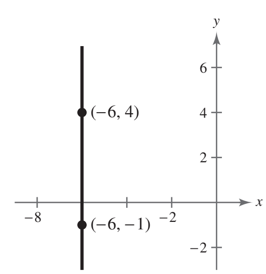


30. $m = \dfrac{-5 - 1}{-4 - (-2)} = 3$

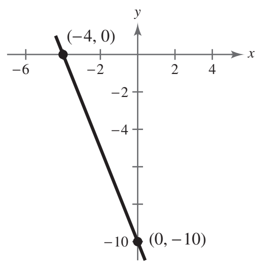


31. $m = \dfrac{4 - (-1)}{-6 - (-6)} = \dfrac{5}{0}$

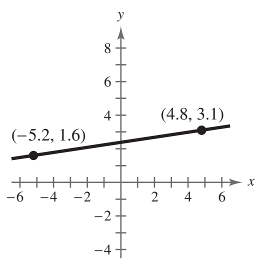
m is undefined.



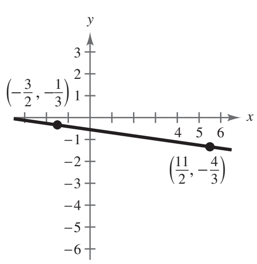
32. $m = \dfrac{0 - (-10)}{-4 - 0} = -\dfrac{5}{2}$



33. $m = \dfrac{1.6 - 3.1}{-5.2 - 4.8} = \dfrac{-1.5}{-10} = 0.15$



34. $m = \dfrac{-\frac{1}{3} - \left(-\frac{4}{3}\right)}{-\frac{3}{2} - \frac{11}{2}} = -\dfrac{1}{7}$



35. Point: $(5, 7)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three other points are $(-1, 7)$, $(0, 7)$, and $(4, 7)$.

36. Point: $(3, -2)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three other points are $(1, -2)$, $(10, -2)$, and $(-6, -2)$.

37. Point: $(-5, 4)$, Slope: $m = 2$

Because $m = 2 = \frac{2}{1}$, y increases by 2 for every one unit increase in x. Three additional points are $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

38. Point: $(0, -9)$, Slope: $m = -2$

Because $m = -2$, y decreases by 2 for every one unit increase in x. Three other points are $(-2, -5)$, $(1, -11)$, and $(3, -15)$.

39. Point: $(4, 5)$, Slope: $m = -\frac{1}{3}$

Because $m = -\frac{1}{3}$, y decreases by 1 unit for every three unit increase in x. Three additional points are $(-2, 7)$, $\left(0, -\frac{19}{4}\right)$, and $(1, 6)$.

40. Point: $(3, -4)$, Slope: $m = \frac{1}{4}$

Because $m = \frac{1}{4}$, y increases by 1 unit for every four unit increase in x. Three additional points are $(-1, -5)$, $(1, -11)$, and $(3, -15)$.

41. Point: $(-4, 3)$, Slope is undefined.

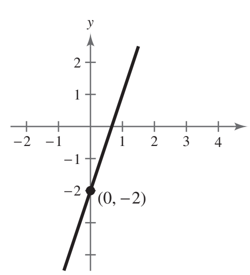
Because m is undefined, x does not change. Three points are $(-4, 0)$, $(-4, 5)$, and $(-4, 2)$.

42. Point: $(2, 14)$, Slope is undefined.

Because m is undefined, x does not change. Three other points are $(2, -3)$, $(2, 0)$, and $(2, 4)$.

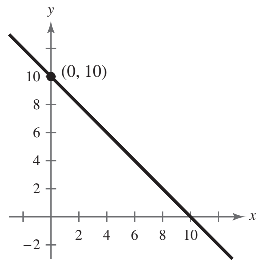
43. Point: $(0, -2)$; $m = 3$

$$y + 2 = 3(x - 0)$$
$$y = 3x - 2$$



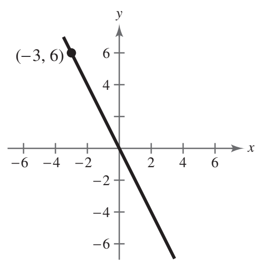
44. Point: $(0, 10)$; $m = -1$

$$y - 10 = -1(x - 0)$$
$$y - 10 = -x$$
$$y = -x + 10$$



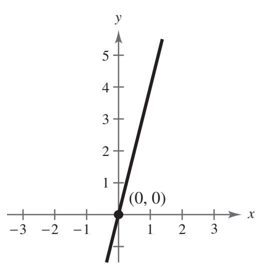
45. Point: $(-3, 6)$; $m = -2$

$$y - 6 = -2(x + 3)$$
$$y = -2x$$



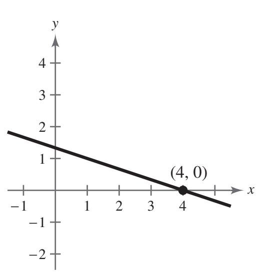
46. Point: $(0, 0)$; $m = 4$

$$y - 0 = 4(x - 0)$$
$$y = 4x$$



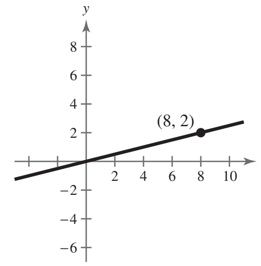
47. Point: $(4, 0)$; $m = -\frac{1}{3}$

$$y - 0 = -\tfrac{1}{3}(x - 4)$$
$$y = -\tfrac{1}{3}x + \tfrac{4}{3}$$



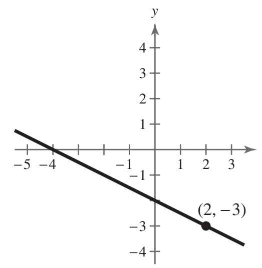
48. Point: $(8, 2)$; $m = \frac{1}{4}$

$$y - 2 = \tfrac{1}{4}(x - 8)$$
$$y - 2 = \tfrac{1}{4}x - 2$$
$$y = \tfrac{1}{4}x$$



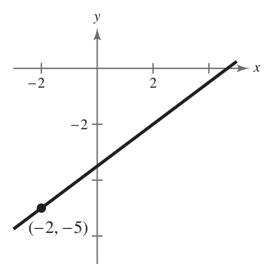
49. Point: $(2, -3)$; $m = -\dfrac{1}{2}$

$$y - (-3) = -\frac{1}{2}(x - 2)$$
$$y + 3 = -\frac{1}{2}x + 1$$
$$y = -\frac{1}{2}x - 2$$



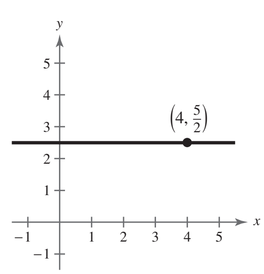
50. Point: $(-2, -5)$; $m = \frac{3}{4}$

$$y + 5 = \tfrac{3}{4}(x + 2)$$
$$4y + 20 = 3x + 6$$
$$4y = 3x - 14$$
$$y = \tfrac{3}{4}x - \tfrac{7}{2}$$



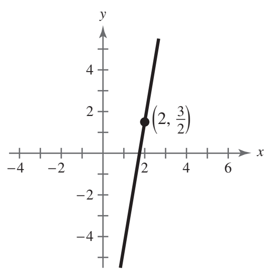
51. Point: $\left(4, \frac{5}{2}\right)$; $m = 0$

$$y - \tfrac{5}{2} = 0(x - 4)$$
$$y - \tfrac{5}{2} = 0$$
$$y = \tfrac{5}{2}$$



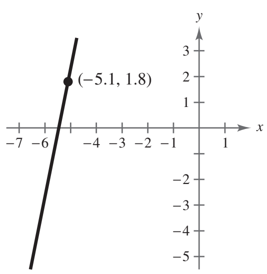
52. Point: $\left(2, \frac{3}{2}\right)$; $m = 6$

$$y - \frac{3}{2} = 6(x - 2)$$
$$y - \frac{3}{2} = 6x - 12$$
$$y = 6x - \frac{21}{2}$$



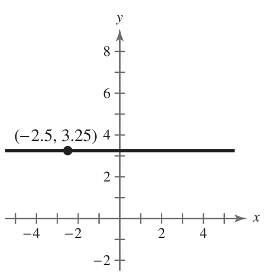
53. Point: $(-5.1, 1.8)$; $m = 5$

$$y - 1.8 = 5(x - (5.1))$$
$$y = 5x + 27.3$$



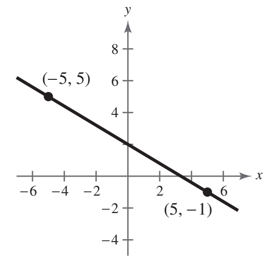
54. Point: $(-2.5, 3.25)$; $m = 0$

$$y - 3.25 = 0(x - (-2.5))$$
$$y - 3.25 = 0$$
$$y = 3.25$$



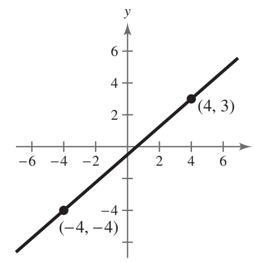
55. $(5, -1), (-5, 5)$

$$y + 1 = \frac{5 + 1}{-5 - 5}(x - 5)$$
$$y = -\frac{3}{5}(x - 5) - 1$$
$$y = -\frac{3}{5}x + 2$$



56. $(4, 3), (-4, -4)$

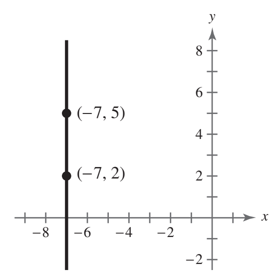
$$y - 3 = \frac{-4 - 3}{-4 - 4}(x - 4)$$
$$y - 3 = \frac{7}{8}(x - 4)$$
$$y - 3 = \frac{7}{8}x - \frac{7}{2}$$
$$y = \frac{7}{8}x - \frac{1}{2}$$



57. $(-7, 2), (-7, 5)$

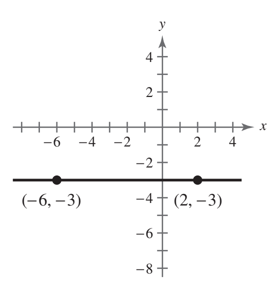
$$m = \frac{5 - 2}{-7 - (-7)} = \frac{3}{0}$$

m is undefined.



58. $(-6, -3), (2, -3)$

$$y - 4 = \frac{-3 - (-3)}{2 - (-6)}(x + 6)$$
$$y + 3 = 0(x + 6)$$
$$y + 3 = 0$$
$$y = -3$$

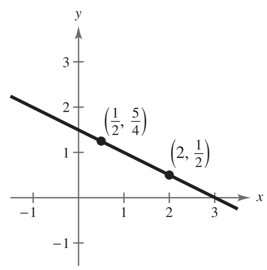


59. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$

$$y - \frac{1}{2} = \frac{\frac{5}{4} - \frac{1}{2}}{\frac{1}{2} - 2}(x - 2)$$

$$y = -\frac{1}{2}(x - 2) + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$



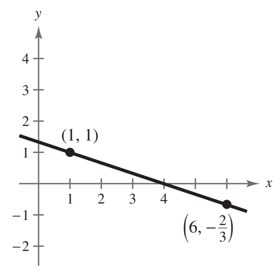
60. $(1, 1), \left(6, -\frac{2}{3}\right)$

$$y - 1 = \frac{-\frac{2}{3} - 1}{6 - 1}(x - 1)$$

$$y - 1 = -\frac{1}{3}(x - 1)$$

$$y - 1 = -\frac{1}{3}x + \frac{1}{3}$$

$$y = -\frac{1}{3}x + \frac{4}{3}$$

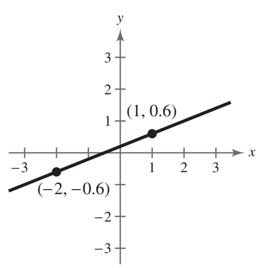


61. $(1, 0.6), (-2, -0.6)$

$$y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1)$$

$$y = 0.4(x - 1) + 0.6$$

$$y = 0.4x + 0.2$$



62. $(-8, 0.6), (2, -2.4)$

$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$

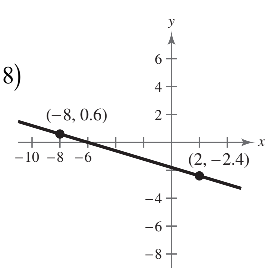
$$y - 0.6 = -\frac{3}{10}(x + 8)$$

$$10y - 6 = -3(x + 8)$$

$$10y - 6 = -3x - 24$$

$$10y = -3x - 18$$

$$y = -\frac{3}{10}x - \frac{9}{5} \text{ or } y = -0.3x - 1.8$$



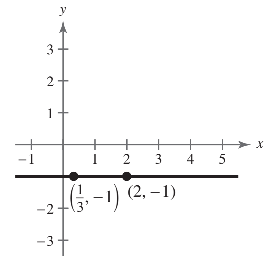
63. $(2, -1), \left(\frac{1}{3}, -1\right)$

$$y + 1 = \frac{-1 - (-1)}{\frac{1}{3} - 2}(x - 2)$$

$$y + 1 = 0$$

$$y = -1$$

The line is horizontal.

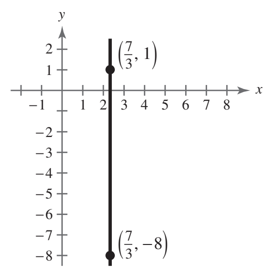


64. $\left(\frac{7}{3}, -8\right), \left(\frac{7}{3}, 1\right)$

$$m = \frac{1 - (-8)}{\frac{7}{3} - \frac{7}{3}} = \frac{9}{0} \text{ and is undefined.}$$

$$x = \frac{7}{3}$$

The line is vertical.



65. $L_1: y = -\frac{2}{3}x - 3$

$m_1 = -\frac{2}{3}$

$L_2: y = -\frac{2}{3}x - 1$

$m_2 = -\frac{2}{3}$

The slopes are equal, so the lines are parallel.

66. $L_1: y = \frac{1}{4}x - 1$

$m_1 = \frac{1}{4}$

$L_2: y = 4x + 7$

$m_2 = 4$

The lines are neither parallel nor perpendicular.

67. $L_1: y = \frac{1}{2}x - 3$

$m_1 = \frac{1}{2}$

$L_2: y = -\frac{1}{2}x + 1$

$m_2 = -\frac{1}{2}$

The lines are neither parallel nor perpendicular.

68. $L_1: y = -\frac{4}{5}x - 5$

$m_1 = -\frac{4}{5}$

$L_2: y = \frac{5}{4}x + 1$

$m_2 = \frac{5}{4}$

The slopes are negative reciprocals, so the lines are perpendicular.

69. $L_1: (0, -1), (5, 9)$

$m_1 = \frac{9 + 1}{5 - 0} = 2$

$L_2: (0, 3), (4, 1)$

$m_2 = \frac{1 - 3}{4 - 0} = -\frac{1}{2}$

The slopes are negative reciprocals, so the lines are perpendicular.

70. $L_1: (-2, -1), (1, 5)$

$m_1 = \frac{5 - (-1)}{1 - (-2)} = \frac{6}{3} = 2$

$L_2: (1, 3), (5, -5)$

$m_2 = \frac{-5 - 3}{5 - 1} = \frac{-8}{4} = -2$

The lines are neither parallel nor perpendicular.

71. $L_1: (-6, -3), (2, -3)$

$m_1 = \frac{-3 - (-3)}{2 - (-6)} = \frac{0}{8} = 0$

$L_2: \left(3, -\frac{1}{2}\right), \left(6, -\frac{1}{2}\right)$

$m_2 = \frac{-\frac{1}{2} - \left(-\frac{1}{2}\right)}{6 - 3} = \frac{0}{3} = 0$

L_1 and L_2 are both horizontal lines, so they are parallel.

72. $L_1: (4, 8), (-4, 2)$

$m_1 = \frac{2 - 8}{-4 - 4} = \frac{-6}{-8} = \frac{3}{4}$

$L_2: (3, -5), \left(-1, \frac{1}{3}\right)$

$m_2 = \frac{\frac{1}{3} - (-5)}{-1 - 3} = \frac{\frac{16}{3}}{-4} = -\frac{4}{3}$

The slopes are negative reciprocals, so the lines are perpendicular.

73. $4x - 2y = 3$

$y = 2x - \frac{3}{2}$

Slope: $m = 2$

(a) $(2, 1), m = 2$

$y - 1 = 2(x - 2)$

$y = 2x - 3$

(b) $(2, 1), m = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x - 2)$

$y = -\frac{1}{2}x + 2$

74. $x + y = 7$

$y = -x + 7$

Slope: $m = -1$

(a) $m = -1, (-3, 2)$

$y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$y = -x - 1$

(b) $m = 1, (-3, 2)$

$y - 2 = 1(x + 3)$

$y = x + 5$

75. $3x + 4y = 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

Slope: $m = -\frac{3}{4}$

(a) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = -\frac{3}{4}$

$y - \frac{7}{8} = -\frac{3}{4}\left(x - \left(-\frac{2}{3}\right)\right)$

$y = -\frac{3}{4}x + \frac{3}{8}$

(b) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = \frac{4}{3}$

$y - \frac{7}{8} = \frac{4}{3}\left(x - \left(-\frac{2}{3}\right)\right)$

$y = \frac{4}{3}x + \frac{127}{72}$

76. $5x + 3y = 0$

$$3y = -5x$$
$$y = -\tfrac{5}{3}x$$

Slope: $m = -\tfrac{5}{3}$

(a) $m = -\tfrac{5}{3}, \left(\tfrac{7}{8}, \tfrac{3}{4}\right)$

$$y - \tfrac{3}{4} = -\tfrac{5}{3}\left(x - \tfrac{7}{8}\right)$$
$$24y - 18 = -40\left(x - \tfrac{7}{8}\right)$$
$$24y - 18 = -40x + 35$$
$$24y = -40x + 53$$
$$y = -\tfrac{5}{3}x + \tfrac{53}{24}$$

(b) $m = \tfrac{3}{5}, \left(\tfrac{7}{8}, \tfrac{3}{4}\right)$

$$y - \tfrac{3}{4} = \tfrac{3}{5}\left(x - \tfrac{7}{8}\right)$$
$$40y - 30 = 24\left(x - \tfrac{7}{8}\right)$$
$$40y - 30 = 24x - 21$$
$$40y = 24x + 9$$
$$y = \tfrac{3}{5}x + \tfrac{9}{40}$$

77. $y + 5 = 0$

$$y = -5$$

Slope: $m = 0$

(a) $(-2, 4), m = 0$

$$y = 4$$

(b) $(-2, 4)$, m is undefined.

$$x = -2$$

78. $x - 4 = 0$

$$x = 4$$

Slope: m is undefined.

(a) $(3, -2)$, m is undefined.

$$x = 3$$

(b) $(3, -2), m = 0$

$$y = -2$$

79. $x - y = 4$

$$y = x - 4$$

Slope: $m = 1$

(a) $(2.5, 6.8), m = 1$

$$y - 6.8 = 1(x - 2.5)$$
$$y = x + 4.3$$

(b) $(2.5, 6.8), m = -1$

$$y - 6.8 = (-1)(x - 2.5)$$
$$y = -x + 9.3$$

80. $6x + 2y = 9$

$$2y = -6x + 9$$
$$y = -3x + \tfrac{9}{2}$$

Slope: $m = -3$

(a) $(-3.9, -1.4), m = -3$

$$y - (-1.4) = -3(x - (-3.9))$$
$$y + 1.4 = -3x - 11.7$$
$$y = -3x - 13.1$$

(b) $(-3.9, -1.4), m = \tfrac{1}{3}$

$$y - (-1.4) = \tfrac{1}{3}(x - (-3.9))$$
$$y + 1.4 = \tfrac{1}{3}x + 1.3$$
$$y = \tfrac{1}{3}x - 0.1$$

81. $\dfrac{x}{3} + \dfrac{y}{5} = 1$

$$(15)\left(\frac{x}{3} + \frac{y}{5}\right) = 1(15)$$
$$5x + 3y - 15 = 0$$

82. $(-3, 0), (0, 4)$

$$\frac{x}{-3} + \frac{y}{4} = 1$$
$$(-12)\frac{x}{-3} + (-12)\frac{y}{4} = (-12) \cdot 1$$
$$4x - 3y + 12 = 0$$

83. $\dfrac{x}{-1/6} + \dfrac{y}{-2/3} = 1$

$$6x + \frac{3}{2}y = -1$$
$$12x + 3y + 2 = 0$$

84. $\left(\dfrac{2}{3}, 0\right), (0, -2)$

$$\frac{x}{2/3} + \frac{y}{-2} = 1$$
$$\frac{3x}{2} - \frac{y}{2} = 1$$
$$3x - y - 2 = 0$$

85. $\dfrac{x}{c} + \dfrac{y}{c} = 1, c \neq 0$

$$x + y = c$$
$$1 + 2 = c$$
$$3 = c$$
$$x + y = 3$$
$$x + y - 3 = 0$$

86. $(d, 0), (0, d), (-3, 4)$

$$\frac{x}{d} + \frac{y}{d} = 1$$
$$x + y = d$$
$$-3 + 4 = d$$
$$1 = d$$
$$x + y = 1$$
$$x + y - 1 = 0$$

87. (a) $m = 135$. The sales are increasing 135 units per year.

(b) $m = 0$. There is no change in sales during the year.

(c) $m = -40$. The sales are decreasing 40 units per year.

88. (a) greatest increase = largest slope

$$(14, 182.20), (15, 233.72)$$
$$m_1 = \frac{233.72 - 182.20}{15 - 14} = 51.52$$

So, the sales increased the greatest between the years 2014 and 2015.

least increase = smallest slope

$$(13, 170.91), (14, 182.20)$$
$$m_2 = \frac{182.20 - 170.91}{14 - 13} = 11.29$$

So, the sales increased the least between the years 2013 and 2014.

(b) $(9, 42.91), (15, 233.72)$

$$m = \frac{233.72 - 42.91}{15 - 9} = \frac{190.81}{6} \approx 31.80$$

The slope of the line is about 31.8.

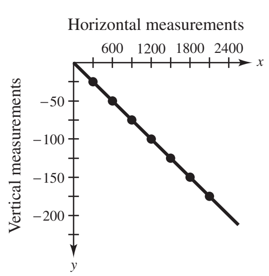
(c) The sales increased an average of about $31.8 billion each year between the years 2009 and 2015.

89. $y = \frac{6}{100}x$

$y = \frac{6}{100}(200) = 12$ feet

90. (a) and (b)

x	300	600	900	1200	1500	1800	2100
y	–25	–50	–75	–100	–125	–150	–175



(c) $m = \frac{-50 - (-25)}{600 - 300} = \frac{-25}{300} = -\frac{1}{12}$

$$y - (-50) = -\frac{1}{12}(x - 600)$$
$$y + 50 = -\frac{1}{12}x + 50$$
$$y = -\frac{1}{12}x$$

(d) Because $m = -\frac{1}{12}$, for every change in the horizontal measurement of 12 feet, the vertical measurement decreases by 1 foot.

(e) $\frac{1}{12} \approx 0.083 = 8.3\%$ grade

91. $(16, 3000), m = -150$

$$V - 3000 = -150(t - 16)$$
$$V - 3000 = -150t + 2400$$
$$V = -150t + 5400, 16 \le t \le 21$$

92. $(16, 200), m = 6.50$

$$V - 200 = 6.50(t - 16)$$
$$V - 200 = 6.50t + 104$$
$$V = 6.5t + 96, 16 \le t \le 21$$

93. The C-intercept measures the fixed costs of manufacturing when zero bags are produced.

The slope measures the cost to produce one laptop bag.

94. Monthly wages = 7% of the Sales plus the Monthly Salary

$W = 0.07S + 5000$

95. Using the points $(0, 875)$ and $(5, 0)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$m = \frac{0 - 875}{5 - 0} = -175$$

$$V = -175t + 875, \; 0 \le t \le 5.$$

96. Using the points $(0, 24{,}000)$ and $(10, 2000)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$m = \frac{2{,}000 - 24{,}000}{10 - 0} = \frac{-22{,}000}{10} = -2200.$$

Since the point $(0, 24{,}000)$ is the V-intercept, $b = 24{,}000$, the equation is

$$V = -2200t + 24{,}000, \; 0 \le t \le 10.$$

97. Using the points $(0, 32)$ and $(100, 212)$, where the first coordinate represents a temperature in degrees Celsius and the second coordinate represents a temperature in degrees Fahrenheit, you have

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}.$$

Since the point $(0, 32)$ is the F- intercept, $b = 32$, the equation is $F = 1.8C + 32$ or $C = \frac{5}{9}F - \frac{160}{9}$.

98. (a) Using the points $(1, 970)$ and $(3, 1270)$, you have

$$m = \frac{1270 - 970}{3 - 1} = \frac{300}{2} = 150.$$

Using the point-slope form with $m = 150$ and the point $(1, 970)$, you have

$$y - y_1 = m(t - t_1)$$
$$y - 970 = 150(t - 1)$$
$$y - 970 = 150t - 150$$
$$y = 150t + 820.$$

(b) The slope is $m = 150$. The slope tells you the amount of increase in the weight of an average male child's brain each year.

(c) Let $t = 2$:

$$y = 150(2) + 820$$
$$y = 300 + 820$$
$$y = 1120$$

The average brain weight at age 2 is 1120 grams.

(d) Answers will vary.

(e) Answers will vary. *Sample answer:* No. The brain stops growing after reaching a certain age.

99. (a) Total Cost = cost for fuel and maintainance + cost for operator + purchase cost

$$C = 9.5t + 11.5t + 42{,}000$$
$$C = 21t + 42{,}000$$

(b) Revenue = Rate per hour · Hours

$$R = 45t$$

(c) $P = R - C$

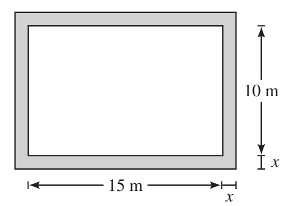
$$P = 45t - (21t + 42{,}000)$$
$$P = 24t - 42{,}000$$

(d) Let $P = 0$, and solve for t.

$$0 = 24t - 42{,}000$$
$$42{,}000 = 24t$$
$$1750 = t$$

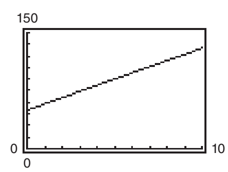
The equipment must be used 1750 hours to yield a profit of 0 dollars.

100. (a)



(b) $y = 2(15 + 2x) + 2(10 + 2x) = 8x + 50$

(c)



(d) Because $m = 8$, each 1-meter increase in x will increase y by 8 meters.

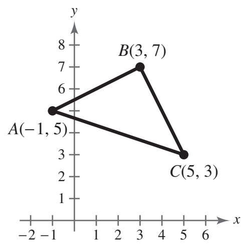
101. False. The slope with the greatest magnitude corresponds to the steepest line.

102. $(-8, 2)$ and $(-1, 4)$: $m_1 = \frac{4 - 2}{-1 - (-8)} = \frac{2}{7}$

$(0, -4)$ and $(-7, 7)$: $m_2 = \frac{7 - (-4)}{-7 - 0} = \frac{11}{-7}$

False. The lines are not parallel.

103. Find the slope of the line segments between the points A and B, and B and C.



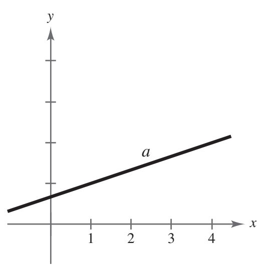
$$m_{AB} = \frac{7-5}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

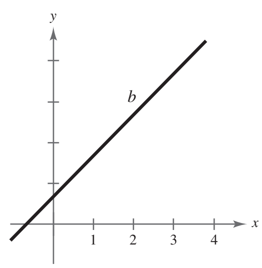
$$m_{BC} = \frac{3-7}{5-3} = \frac{-4}{2} = -2$$

Since the slopes are negative reciprocals, the line segments are perpendicular and therefore intersect to form a right angle. So, the triangle is a right triangle.

104. On a vertical line, all the points have the same x-value, so when you evaluate $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, you would have a zero in the denominator, and division by zero is undefined.

105. Since the scales for the y-axis on each graph is unknown, the slopes of the lines cannot be determined.





106.

$$\begin{aligned} d_1 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-0)^2 + (m_1 - 0)^2} \\ &= \sqrt{1 + (m_1)^2} \end{aligned}$$

$$\begin{aligned} d_2 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-0)^2 + (m_2 - 0)^2} \\ &= \sqrt{1 + (m_2)^2} \end{aligned}$$

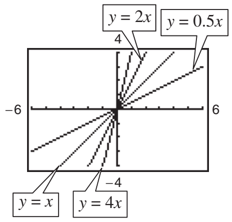
Using the Pythagorean Theorem:

$$\begin{aligned} (d_1)^2 + (d_2)^2 &= (\text{distance between } (1, m_1), \text{ and } (1, m_2))^2 \\ \left(\sqrt{1 + (m_1)^2}\right)^2 + \left(\sqrt{1 + (m_2)^2}\right)^2 &= \left(\sqrt{(1-1)^2 + (m_2 - m_1)^2}\right)^2 \\ 1 + (m_1)^2 + 1 + (m_2)^2 &= (m_2 - m_1)^2 \\ (m_1)^2 + (m_2)^2 + 2 &= (m_2)^2 - 2m_1m_2 + (m_1)^2 \\ 2 &= -2m_1m_2 \\ -\frac{1}{m_2} &= m_1 \end{aligned}$$

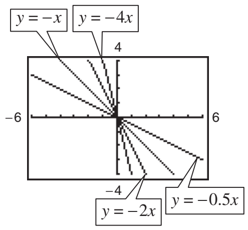
107. No, the slopes of two perpendicular lines have opposite signs. (Assume that neither line is vertical or horizontal.)

108. Because $|-4| > \left|\frac{5}{2}\right|$, the steeper line is the one with a slope of – 4. The slope with the greatest magnitude corresponds to the steepest line.

109. The line $y = 4x$ rises most quickly.



The line $y = -4x$ falls most quickly.



The greater the magnitude of the slope (the absolute value of the slope), the faster the line rises or falls.

110. (a) Matches graph (ii).

The slope is –20, which represents the decrease in the amount of the loan each week. The y-intercept is $(0, 200)$, which represents the original amount of the loan.

(b) Matches graph (iii).

The slope is 2, which represents the increase in the hourly wage for each unit produced. The y-intercept is $(0, 12.5)$, which represents the hourly rate if the employee produces no units.

(c) Matches graph (i).

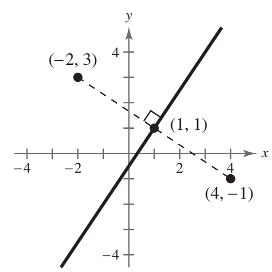
The slope is 0.32, which represents the increase in travel cost for each mile driven. The y-intercept is $(0, 32)$, which represents the fixed cost of \$30 per day for meals. This amount does not depend on the number of miles driven.

(d) Matches graph (iv).

The slope is –100, which represents the amount by which the computer depreciates each year. The y-intercept is $(0, 750)$, which represents the original purchase price.

111. Set the distance between $(4, -1)$ and (x, y) equal to the distance between $(-2, 3)$ and (x, y).

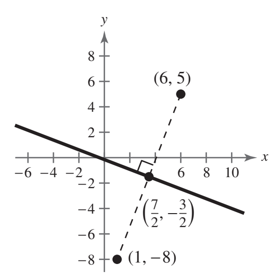
$$\sqrt{(x-4)^2 + [y-(-1)]^2} = \sqrt{[x-(-2)]^2 + (y-3)^2}$$
$$(x-4)^2 + (y+1)^2 = (x+2)^2 + (y-3)^2$$
$$x^2 - 8x + 16 + y^2 + 2y + 1 = x^2 + 4x + 4 + y^2 - 6y + 9$$
$$-8x + 2y + 17 = 4x - 6y + 13$$
$$-12x + 8y + 4 = 0$$
$$-4(3x - 2y - 1) = 0$$
$$3x - 2y - 1 = 0$$



This line is the perpendicular bisector of the line segment connecting $(4, -1)$ and $(-2, 3)$.

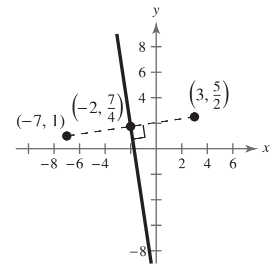
112. Set the distance between $(6, 5)$ and (x, y) equal to the distance between $(1, -8)$ and (x, y).

$$\sqrt{(x-6)^2 + (y-5)^2} = \sqrt{(x-1)^2 + (y-(-8))^2}$$
$$(x-6)^2 + (y-5)^2 = (x-1)^2 + (y+8)^2$$
$$x^2 - 12x + 36 + y^2 - 10y + 25 = x^2 - 2x + 1 + y^2 + 16y + 64$$
$$x^2 + y^2 - 12x - 10y + 61 = x^2 + y^2 - 2x + 16y + 65$$
$$-12x - 10y + 61 = -2x + 16y + 65$$
$$-10x - 26y - 4 = 0$$
$$-2(5x + 13y + 2) = 0$$
$$5x + 13y + 2 = 0$$



113. Set the distance between $\left(3, \frac{5}{2}\right)$ and (x, y) equal to the distance between $(-7, 1)$ and (x, y).

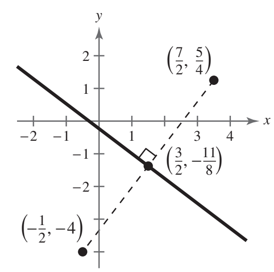
$$\sqrt{(x-3)^2 + \left(y - \frac{5}{2}\right)^2} = \sqrt{[x-(-7)]^2 + (y-1)^2}$$
$$(x-3)^2 + \left(y - \frac{5}{2}\right)^2 = (x+7)^2 + (y-1)^2$$
$$x^2 - 6x + 9 + y^2 - 5y + \frac{25}{4} = x^2 + 14x + 49 + y^2 - 2y + 1$$
$$-6x - 5y + \frac{61}{4} = 14x - 2y + 50$$
$$-24x - 20y + 61 = 56x - 8y + 200$$
$$80x + 12y + 139 = 0$$



This line is the perpendicular bisector of the line segment connecting $\left(3, \frac{5}{2}\right)$ and $(-7, 1)$.

114. Set the distance between $\left(-\frac{1}{2}, -4\right)$ and (x, y) equal to the distance between $\left(\frac{7}{2}, \frac{5}{4}\right)$ and (x, y).

$$\sqrt{\left(x - \left(-\frac{1}{2}\right)\right)^2 + (y-(-4))^2} = \sqrt{\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{5}{4}\right)^2}$$
$$\left(x + \frac{1}{2}\right)^2 + (y+4)^2 = \left(x - \frac{7}{2}\right)^2 + \left(y - \frac{5}{4}\right)^2$$
$$x^2 + x + \frac{1}{4} + y^2 + 8y + 16 = x^2 - 7x + \frac{49}{4} + y^2 - \frac{5}{2}y + \frac{25}{16}$$
$$x^2 + y^2 + x + 8y + \frac{65}{4} = x^2 + y^2 - 7x - \frac{5}{2}y + \frac{221}{16}$$
$$x + 8y + \frac{65}{4} = -7x - \frac{5}{2}y + \frac{221}{16}$$
$$8x + \frac{21}{2}y + \frac{39}{16} = 0$$
$$128x + 168y + 39 = 0$$



Section 2.2 Functions

1. domain; range; function
2. independent; dependent
3. implied domain
4. difference quotient
5. Yes, the relationship is a function. Each domain value is matched with exactly one range value.
6. No, the relationship is not a function. The domain value of −1 is matched with two output values.
7. No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.
8. Yes, the table does represent a function. Each input value is matched with exactly one output value.

Input, x	−2	0	2	4	6
Output, y	1	1	1	1	1

9. (a) Each element of A is matched with exactly one element of B, so it does represent a function.
   (b) The element 1 in A is matched with two elements, −2 and 1 of B, so it does not represent a function.
   (c) Each element of A is matched with exactly one element of B, so it does represent a function.
   (d) The element 2 in A is not matched with an element of B, so the relation does not represent a function.

10. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.
   (b) Each element of A is matched with exactly one element of B, so it does represent a function.
   (c) This is not a function from A to B (it represents a function from B to A instead).
   (d) Each element of A is matched with exactly one element of B, so it does represent a function.

11. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

    No, *y is not* a function of *x*.

12. $x^2 - y = 9 \Rightarrow y = x^2 - 9$

    Yes, *y is* a function of *x*.

13. $y = \sqrt{16 - x^2}$

    Yes, *y is* a function of *x*.

14. $y = \sqrt{x + 5}$

Yes, y *is* a function of x.

15. $y = 4 - |x|$

Yes, y *is* a function of x.

16. $|y| = 4 - x \Rightarrow y = 4 - x$ or $y = -(4 - x)$

No, y *is not* a function of x.

17. $y = -75$ or $y = -75 + 0x$

Yes, y *is* a function of x.

18. $x - 1 = 0$

$x = 1$

No, this *is not* a function of x.

19. $f(x) = 3x - 5$

(a) $f(1) = 3(1) - 5 = -2$

(b) $f(-3) = 3(-3) - 5 = -14$

(c) $f(x + 2) = 3(x + 2) - 5$
$= 3x + 6 - 5$
$= 3x + 1$

20. $V(r) = \frac{4}{3}\pi r^3$

(a) $V(3) = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$

(b) $V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{27}{8}\right) = \frac{9}{2}\pi$

(c) $V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r^3) = \frac{32}{3}\pi r^3$

21. $g(t) = 4t^2 - 3t + 5$

(a) $g(2) = 4(2)^2 - 3(2) + 5$
$= 15$

(b) $g(t - 2) = 4(t - 2)^2 - 3(t - 2) + 5$
$= 4t^2 - 19t + 27$

(c) $g(t) - g(2) = 4t^2 - 3t + 5 - 15$
$= 4t^2 - 3t - 10$

22. $h(t) = -t^2 + t + 1$

(a) $h(2) = -(2)^2 + (2) + 1 = -4 + 2 + 1 = -1$

(b) $h(-1) = -(-1)^2 + (-1) + 1 = -1 - 1 + 1 = -1$

(c) $h(x + 2) = -(x + 1)^2 + (x + 1) + 1$
$= -(x^2 + 2x + 1) + x + 1 + 1$
$= -x^2 - x + 1$

23. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

24. $f(x) = \sqrt{x + 8} + 2$

(a) $f(-8) = \sqrt{(-8) + 8} + 2 = 2$

(b) $f(1) = \sqrt{(1) + 8} + 2 = 5$

(c) $f(x - 8) = \sqrt{(x - 8) + 8} + 2 = \sqrt{x} + 2$

25. $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0) = \dfrac{1}{0^2 - 9} = -\dfrac{1}{9}$

(b) $q(3) = \dfrac{1}{3^2 - 9}$ is undefined.

(c) $q(y + 3) = \dfrac{1}{(y + 3)^2 - 9} = \dfrac{1}{y^2 + 6y}$

26. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2) = \dfrac{2(2)^2 + 3}{(2)^2} = \dfrac{8 + 3}{4} = \dfrac{11}{4}$

(b) $q(0) = \dfrac{2(0)^2 + 3}{(0)^2}$

Division by zero is undefined.

(c) $q(-x) = \dfrac{2(-x)^2 + 3}{(-x)^2} = \dfrac{2x^2 + 3}{x^2}$

27. $f(x) = \dfrac{|x|}{x}$

(a) $f(2) = \dfrac{|2|}{2} = 1$

(b) $f(-2) = \dfrac{|-2|}{-2} = -1$

(c) $f(x - 1) = \dfrac{|x - 1|}{x - 1} = \begin{cases} -1, & \text{if } x < 1 \\ 1, & \text{if } x > 1 \end{cases}$

28. $f(x) = |x| + 4$

(a) $f(2) = |2| + 4 = 6$

(b) $f(-2) = |-2| + 4 = 6$

(c) $f(x^2) = |x^2| + 4 = x^2 + 4$

29. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

30. $f(x) = \begin{cases} -3x - 3, & x < -1 \\ x^2 + 2x - 1, & x \ge -1 \end{cases}$

(a) $f(-2) = -3(-2) - 3 = 3$

(b) $f(-1) = (-1)^2 + 2(-1) - 1 = -2$

(c) $f(1) = (1)^2 + 2(1) - 1 = 2$

31.

$f(x) = -x^2 + 5$

$f(-2) = -(-2)^2 + 5 = 1$

$f(-1) = -(-1)^2 + 5 = 4$

$f(0) = -(0)^2 + 5 = 5$

$f(1) = -(1)^2 + 5 = 4$

$f(2) = -(2)^2 + 5 = 1$

x	–2	–1	0	1	2
$f(x)$	1	4	5	–4	1

32. $h(t) = \frac{1}{2}\lvert t + 3\rvert$

$h(-5) = \frac{1}{2}\lvert -5 + 3\rvert = 1$

$h(-4) = \frac{1}{2}\lvert -4 + 3\rvert = \frac{1}{2}$

$h(-3) = \frac{1}{2}\lvert -3 + 3\rvert = 0$

$h(-2) = \frac{1}{2}\lvert -2 + 3\rvert = \frac{1}{2}$

$h(-1) = \frac{1}{2}\lvert -1 + 3\rvert = 1$

t	–5	–4	–3	–2	–1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

33. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \le 0 \\ (x - 2)^2, & x > 0 \end{cases}$

$f(-2) = -\frac{1}{2}(-2) + 4 = 5$

$f(-1) = -\frac{1}{2}(-1) + 4 = 4\frac{1}{2} = \frac{9}{2}$

$f(0) = -\frac{1}{2}(0) + 4 = 4$

$f(1) = (1 - 2)^2 = 1$

$f(2) = (2 - 2)^2 = 0$

x	–2	–1	0	1	2
$f(x)$	5	$\frac{9}{2}$	4	1	0

34. $f(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \ge 3 \end{cases}$

$f(1) = 9 - (1)^2 = 8$

$f(2) = 9 - (2)^2 = 5$

$f(3) = (3) - 3 = 0$

$f(4) = (4) - 3 = 1$

$f(5) = (5) - 3 = 2$

x	1	2	3	4	5
$f(x)$	8	5	0	1	2

35. $15 - 3x = 0$

$3x = 15$

$x = 5$

36.

$f(x) = 4x + 6$

$4x + 6 = 0$

$4x = -6$

$x = -\frac{3}{2}$

37. $\dfrac{3x - 4}{5} = 0$

$3x - 4 = 0$

$x = \dfrac{4}{3}$

38.

$f(x) = \dfrac{12 - x^2}{8}$

$\dfrac{12 - x^2}{8} = 0$

$x^2 = 12$

$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

39.
$$f(x) = x^2 - 81$$
$$x^2 - 81 = 0$$
$$x^2 = 81$$
$$x = \pm 9$$

40.
$$f(x) = x^2 - 6x - 16$$
$$x^2 - 6x - 16 = 0$$
$$(x - 8)(x + 2) = 0$$
$$x - 8 = 0 \Rightarrow x = 8$$
$$x + 2 = 0 \Rightarrow x = -2$$

41.
$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x(x + 1)(x - 1) = 0$$
$x = 0, x = -1$, or $x = 1$

42.
$$f(x) = x^3 - x^2 - 3x + 3$$
$$x^3 - x^2 - 3x + 3 = 0$$
$$x^2(x - 1) - 3(x - 1) = 0$$
$$(x - 1)(x^2 - 3) = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$$

43.
$$f(x) = g(x)$$
$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x - 2)(x + 1) = 0$$
$$x - 2 = 0 \quad x + 1 = 0$$
$$x = 2 \quad x = -1$$

44.
$$f(x) = g(x)$$
$$x^2 + 2x + 1 = 5x + 19$$
$$x^2 - 3x - 18 = 0$$
$$(x - 6)(x + 3) = 0$$
$$x - 6 = 0 \quad x + 3 = 0$$
$$x = 6 \quad x = -3$$

45.
$$f(x) = g(x)$$
$$x^4 - 2x^2 = 2x^2$$
$$x^4 - 4x^2 = 0$$
$$x^2(x^2 - 4) = 0$$
$$x^2(x + 2)(x - 2) = 0$$
$$x^2 = 0 \Rightarrow x = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x - 2 = 0 \Rightarrow x = 2$$

46.
$$f(x) = g(x)$$
$$\sqrt{x} - 4 = 2 - x$$
$$x + \sqrt{x} - 6 = 0$$
$$(\sqrt{x} + 3)(\sqrt{x} - 2) = 0$$
$\sqrt{x} + 3 = 0 \Rightarrow \sqrt{x} = -3$, which is a contradiction, since $\sqrt{x}$ represents the principal square root.
$\sqrt{x} - 2 = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

47. $f(x) = 5x^2 + 2x - 1$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

48. $f(x) = 1 - 2x^2$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

49. $g(y) = \sqrt{y + 6}$
$$y + 6 \ge 0$$
Domain: $y \ge -6$

The domain is all real numbers y such that $y \ge -6$.

50. $f(t) = \sqrt[3]{t + 4}$

Because $f(t)$ is a cube root, the domain is all real numbers t.

51. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$

The domain is all real numbers x except $x = 0, x = -2$.

52.
$$h(x) = \frac{6}{x^2 - 4x}$$
$$x^2 - 4x \ne 0$$
$$x(x - 4) \ne 0$$
$$x \ne 0$$
$$x - 4 \ne 0 \Rightarrow x \ne 4$$

The domain is all real numbers x except $x = 0, x = 4$.

53. $f(s) = \dfrac{\sqrt{s-1}}{s-4}$

Domain: $s - 1 \geq 0 \Rightarrow s \geq 1$ and $s \neq 4$

The domain consists of all real numbers s, such that $s \geq 1$ and $s \neq 4$.

54. $f(x) = \dfrac{\sqrt{x+6}}{6+x}$

Domain: $x + 6 \geq 0 \Rightarrow x \geq -6$ and $x \neq -6$

The domain is all real numbers x such that $x > -6$ or $(-6, \infty)$.

55. $f(x) = \dfrac{x-4}{\sqrt{x}}$

The domain is all real numbers x such that $x > 0$ or $(0, \infty)$.

56. $f(x) = \dfrac{x+2}{\sqrt{x-10}}$

$x - 10 > 0$

$x > 10$

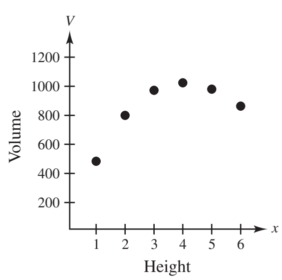
The domain is all real numbers x such that $x > 10$.

57. (a)

Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

The volume is maximum when $x = 4$ and $V = 1024$ cubic centimeters.

(b)



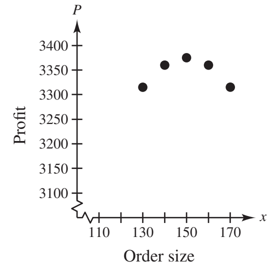
V is a function of x.

(c) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

58. (a) The maximum profit is \$3375.

(b)



Yes, P is a function of x.

(c) Profit = Revenue − Cost

$$= \begin{pmatrix}\text{price}\\ \text{per unit}\end{pmatrix}\begin{pmatrix}\text{number}\\ \text{of units}\end{pmatrix} - (\text{cost})\begin{pmatrix}\text{number}\\ \text{of units}\end{pmatrix}$$

$$= \left[90 - (x - 100)(0.15)\right]x - 60x, x > 100$$

$$= (90 - 0.15x + 15)x - 60x$$

$$= (105 - 0.15x)x - 60x$$

$$= 105x - 0.15x^2 - 60x$$

$$= 45x - 0.15x^2, x > 100$$

59. $A = s^2$ and $P = 4s \Rightarrow \dfrac{P}{4} = s$

$$A = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$$

60. $A = \pi r^2, C = 2\pi r$

$$r = \frac{C}{2\pi}$$

$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$

61. $y = -\frac{1}{10}x^2 + 3x + 6$

$y(25) = -\frac{1}{10}(25)^2 + 3(25) + 6 = 18.5$ feet

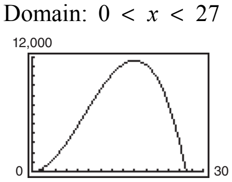
If the child holds a glove at a height of 5 feet, then the ball *will* be over the child's head because it will be at a height of 18.5 feet.

62. (a) $V = l \cdot w \cdot h = x \cdot y \cdot x = x^2 y$ where $4x + y = 108$. So, $y = 108 - 4x$ and

$V = x^2(108 - 4x) = 108x^2 - 4x^3$.

Domain: $0 < x < 27$

(b)



(c) The dimensions that will maximize the volume of the package are $18 \times 18 \times 36$. From the graph, the maximum volume occurs when $x = 18$. To find the dimension for y, use the equation $y = 108 - 4x$.

$$y = 108 - 4x = 108 - 4(18) = 108 - 72 = 36$$

63. $A = \frac{1}{2}bh = \frac{1}{2}xy$

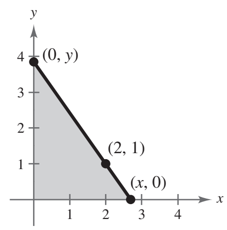
Because $(0, y)$, $(2, 1)$, and $(x, 0)$ all lie on the same line, the slopes between any pair are equal.

$$\frac{1 - y}{2 - 0} = \frac{0 - 1}{x - 2}$$

$$\frac{1 - y}{2} = \frac{-1}{x - 2}$$

$$y = \frac{2}{x - 2} + 1$$

$$y = \frac{x}{x - 2}$$



So, $A = \frac{1}{2}x\left(\frac{x}{x - 2}\right) = \frac{x^2}{2(x - 2)}$.

The domain of A includes x-values such that $x^2/\left[2(x - 2)\right] > 0$. By solving this inequality, the domain is $x > 2$.

64. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$. The domain is $0 < x < 6$.

65. For 2008 through 2011, use

$$p(t) = 2.77t + 45.2.$$

2008: $p(8) = 2.77(8) + 45.2 = 67.36\%$

2009: $p(9) = 2.77(9) + 45.2 = 70.13\%$

2010: $p(10) = 2.77(10) + 45.2 = 72.90\%$

2011: $p(11) = 2.77(11) + 45.2 = 75.67\%$

For 2011 through 2014, use

$$p(t) = 1.95t + 55.9.$$

2012: $p(12) = 1.95(12) + 55.9 = 79.30\%$

2013: $p(13) = 1.95(13) + 55.9 = 81.25\%$

2014: $p(14) = 1.95(14) + 55.9 = 83.20\%$

66. For 2000 through 2006, use

$$p(t) = -0.757t^2 + 20.80t + 127.2.$$

2002: $p(2) = -0.757(2)^2 + 20.80(2) + 127.2 = \$165{,}722$

2003: $p(3) = -0.757(3)^2 + 20.80(3) + 127.2 = \$182{,}787$

2004: $p(4) = -0.757(4)^2 + 20.80(4) + 127.2 = \$198{,}288$

2005: $p(5) = -0.757(5)^2 + 20.80(5) + 127.2 = \$212{,}275$

2006: $p(6) = -0.757(6)^2 + 20.80(6) + 127.2 = \$224{,}748$

For 2007 through 2011, use

$$p(t) = 3.879t^2 - 82.50t + 605.8.$$

2007: $p(7) = 3.879(7)^2 - 82.50(7) + 605.8 = \$218{,}371$

2008: $p(8) = 3.879(8)^2 - 82.50(8) + 605.8 = \$194{,}056$

2009: $p(9) = 3.879(9)^2 - 82.50(9) + 605.8 = \$177{,}499$

2010: $p(10) = 3.879(10)^2 - 82.50(10) + 605.8 = \$168{,}700$

2011: $p(11) = 3.879(11)^2 - 82.50(11) + 605.8 = \$167{,}659$

For 2012 through 2014, use

$$p(t) = -4.171t^2 + 124.34t - 714.2$$

2012: $p(12) = -4.171(12)^2 + 124.34(12) - 714.2 = \$177{,}256$

2013: $p(13) = -4.171(13)^2 + 124.34(13) - 714.2 = \$197{,}321$

2014: $p(14) = -4.171(14)^2 + 124.34(14) - 714.2 = \$209{,}044$

67. (a) Cost = variable costs + fixed costs

$C = 12.30x + 98{,}000$

(b) Revenue = price per unit × number of units

$R = 17.98x$

(c) Profit = Revenue − Cost

$P = 17.98x - (12.30x + 98{,}000)$

$P = 5.68x - 98{,}000$

68. (a) *Model:*

(Total cost) = (Fixed costs) + (Variable costs)

Labels: Total cost $= C$

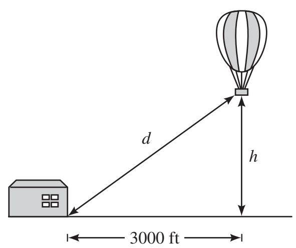
Fixed cost $= 6000$

Variable costs $= 0.95x$

Equation: $C = 6000 + 0.95x$

(b) $\bar{C} = \dfrac{C}{x} = \dfrac{6000 + 0.95x}{x} = \dfrac{6000}{x} + 0.95$

69. (a)



(b) $(3000)^2 + h^2 = d^2$

$h = \sqrt{d^2 - (3000)^2}$

Domain: $d \ge 3000$ (because both $d \ge 0$ and $d^2 - (3000)^2 \ge 0$)

70. $F(y) = 149.76\sqrt{10}y^{5/2}$

(a)

y	5	10	20	30	40
$F(y)$	26,474.08	149,760.00	847,170.49	2,334,527.36	4,792,320

The force, in tons, of the water against the dam increases with the depth of the water.

(b) It appears that approximately 21 feet of water would produce 1,000,000 tons of force.

(c) $1{,}000{,}000 = 149.76\sqrt{10}y^{5/2}$

$\dfrac{1{,}000{,}000}{149.76\sqrt{10}} = y^{5/2}$

$2111.56 \approx y^{5/2}$

$21.37 \text{ feet} \approx y$

71. (a) $R = n(\text{rate}) = n[8.00 - 0.05(n - 80)],\ n \ge 80$

$R = 12.00n - 0.05n^2 = 12n - \dfrac{n^2}{20} = \dfrac{240n - n^2}{20},\ n \ge 80$

(b)

n	90	100	110	120	130	140	150
$R(n)$	\$675	\$700	\$715	\$720	\$715	\$700	\$675

The revenue is maximum when 120 people take the trip.

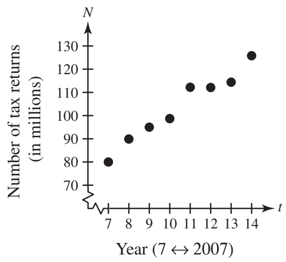
72. (a) $\dfrac{f(2014) - f(2007)}{2014 - 2007} = \dfrac{125.8 - 80.0}{7}$

$= \dfrac{45.8}{7}$

≈ 6.54

Approximately 6.54 million more tax returns were made through e-file each year from 2007 to 2014.

(b)



(c) $N = 6.54t + 34.2$

(d)

t	7	8	9	10
N	80.0	86.5	93.1	99.6

t	11	12	13	14
N	106.1	112.7	119.2	125.8

(e) The algebraic model is a good fit to the actual data.

(f) $y = 6.05x + 40.0$; The models are similar.

73.

$$\begin{aligned} f(x) &= x^2 - 2x + 4 \\ f(2+h) &= (2+h)^2 - 2(2+h) + 4 \\ &= 4 + 4h + h^2 - 4 - 2h + 4 \\ &= h^2 + 2h + 4 \\ f(2) &= (2)^2 - 2(2) + 4 = 4 \\ f(2+h) - f(2) &= h^2 + 2h \\ \frac{f(2+h) - f(2)}{h} &= \frac{h^2 + 2h}{h} = h + 2, h \neq 0 \end{aligned}$$

74.

$$\begin{aligned} f(x) &= 5x - x^2 \\ f(5+h) &= 5(5+h) - (5+h)^2 \\ &= 25 + 5h - (25 + 10h + h^2) \\ &= 25 + 5h - 25 - 10h - h^2 \\ &= -h^2 - 5h \\ f(5) &= 5(5) - (5)^2 \\ &= 25 - 25 = 0 \\ \frac{f(5+h) - f(5)}{h} &= \frac{-h^2 - 5h}{h} \\ &= \frac{-h(h+5)}{h} = -(h+5), h \neq 0 \end{aligned}$$

75.

$$\begin{aligned} f(x) &= x^3 + 3x \\ f(x+h) &= (x+h)^3 + 3(x+h) \\ &= x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h \\ \frac{f(x+h) - f(x)}{h} &= \frac{(x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h) - (x^3 + 3x)}{h} \\ &= \frac{h(3x^2 + 3xh + h^2 + 3)}{h} \\ &= 3x^2 + 3xh + h^2 + 3, h \neq 0 \end{aligned}$$

76.

$$\begin{aligned} f(x) &= 4x^3 - 2x \\ f(x+h) &= 4(x+h)^3 - 2(x+h) \\ &= 4(x^3 + 3x^2h + 3xh^2 + h^3) - 2x - 2h \\ &= 4x^3 + 12x^2h + 12xh^2 + 4h^3 - 2x - 2h \\ \frac{f(x+h) - f(x)}{h} &= \frac{(4x^3 + 12x^2h + 12xh^2 + 4h^3 - 2x - 2h) - (4x^3 - 2x)}{h} \\ &= \frac{12x^2h + 12xh^2 + 4h^3 - 2h}{h} \\ &= \frac{h(12x^2 + 12xh + 4h^2 - 2)}{h} \\ &= 12x^2 + 12xh + 4h^2 - 2, h \neq 0 \end{aligned}$$

77.
$$g(x) = \frac{1}{x^2}$$
$$\frac{g(x) - g(3)}{x - 3} = \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3}$$
$$= \frac{9 - x^2}{9x^2(x - 3)}$$
$$= \frac{-(x + 3)(x - 3)}{9x^2(x - 3)}$$
$$= -\frac{x + 3}{9x^2}, x \neq 3$$

78.
$$f(t) = \frac{1}{t - 2}$$
$$f(1) = \frac{1}{1 - 2} = -1$$
$$\frac{f(t) - f(1)}{t - 1} = \frac{\frac{1}{t - 2} - (-1)}{t - 1}$$
$$= \frac{1 + (t - 2)}{(t - 2)(t - 1)}$$
$$= \frac{(t - 1)}{(t - 2)(t - 1)}$$
$$= \frac{1}{t - 2}, t \neq 1$$

79. $f(x) = \sqrt{5x}$
$$\frac{f(x) - f(5)}{x - 5} = \frac{\sqrt{5x} - 5}{x - 5}, x \neq 5$$

80.
$$f(x) = x^{2/3} + 1$$
$$f(8) = 8^{2/3} + 1 = 5$$
$$\frac{f(x) - f(8)}{x - 8} = \frac{x^{2/3} + 1 - 5}{x - 8} = \frac{x^{2/3} - 4}{x - 8}, x \neq 8$$

81. By plotting the points, we have a parabola, so $g(x) = cx^2$. Because $(-4, -32)$ is on the graph, you have $-32 = c(-4)^2 \Rightarrow c = -2$. So, $g(x) = -2x^2$.

82. By plotting the data, you can see that they represent a line, or $f(x) = cx$. Because $(0, 0)$ and $\left(1, \frac{1}{4}\right)$ are on the line, the slope is $\frac{1}{4}$. So, $f(x) = \frac{1}{4}x$.

83. Because the function is undefined at 0, we have $r(x) = c/x$. Because $(-4, -8)$ is on the graph, you have $-8 = c/-4 \Rightarrow c = 32$. So, $r(x) = 32/x$.

84. By plotting the data, you can see that they represent $h(x) = c\sqrt{|x|}$. Because $\sqrt{|-4|} = 2$ and $\sqrt{|-1|} = 1$, and the corresponding y-values are 6 and 3, $c = 3$ and $h(x) = 3\sqrt{|x|}$.

85. False. The equation $y^2 = x^2 + 4$ is a relation between x and y. However, $y = \pm\sqrt{x^2 + 4}$ does not represent a function.

86. True. A function is a relation by definition.

87. False. The range is $[-1, \infty)$.

88. True. The set represents a function. Each x-value is mapped to exactly one y-value.

89. The domain of $f(x) = \sqrt{x - 1}$ includes $x = 1$, $x \geq 1$ and the domain of $g(x) = \frac{1}{\sqrt{x - 1}}$ does not include $x = 1$ because you cannot divide by 0. The domain of $g(x) = \frac{1}{\sqrt{x - 1}}$ is $x > 1$. So, the functions do not have the same domain.

90. Because $f(x)$ is a function of an even root, the radicand cannot be negative. $g(x)$ is an odd root, therefore the radicand can be any real number. So, the domain of g is all real numbers x and the domain of f is all real numbers x such that $x \geq 2$.

91. No; x is the independent variable, f is the name of the function.

92. (a) The height h is a function of t because for each value of t there is a corresponding value of h for $0 \leq t \leq 2.6$.

(b) Using the graph when $t = 0.5$, $h \approx 20$ feet and when $t = 1.25$, $h \approx 28$ feet.

(c) The domain of h is approximately $0 \leq t \leq 2.6$.

(d) No, the time t is not a function of the height h because some values of h correspond to more than one value of t.

93. (a) Yes. The amount that you pay in sales tax will increase as the price of the item purchased increases.

(b) No. The length of time that you study the night before an exam does not necessarily determine your score on the exam.

94. (a) No. During the course of a year, for example, your salary may remain constant while your savings account balance may vary. That is, there may be two or more outputs (savings account balances) for one input (salary).

(b) Yes. The greater the height from which the ball is dropped, the greater the speed with which the ball will strike the ground.

Section 2.3 Analyzing Graphs of Functions

1. Vertical Line Test

2. zeros

3. decreasing

4. maximum

5. average rate of change; secant

6. odd

7. Domain: $(-2, 2]$; Range: $[-1, 8]$

(a) $f(-1) = -1$

(b) $f(0) = 0$

(c) $f(1) = -1$

(d) $f(2) = 8$

8. Domain: $[-1, \infty)$; Range: $(-\infty, 7]$

(a) $f(-1) = 4$

(b) $f(0) = 3$

(c) $f(1) = 6$

(d) $f(3) = 0$

9. Domain: $(-\infty, \infty)$; Range: $(-2, \infty)$

(a) $f(2) = 0$

(b) $f(1) = 1$

(c) $f(3) = 2$

(d) $f(-1) = 3$

10. Domain: $(-\infty, \infty)$; Range: $(-\infty, 1]$

(a) $f(-2) = -3$

(b) $f(1) = 0$

(c) $f(0) = 1$

(d) $f(2) = -3$

11. A vertical line intersects the graph at most once, so *y is* a function of *x*.

12. *y is not* a function of *x*. Some vertical lines intersect the graph twice.

13. A vertical line intersects the graph more than once, so *y is not* a function of *x*.

14. A vertical line intersects the graph at most once, so *y is* a function of *x*.

15.
$$f(x) = 3x + 18$$
$$3x + 18 = 0$$
$$3x = -18$$
$$x = -6$$

16.
$$f(x) = 15 - 2x$$
$$15 - 2x = 0$$
$$-2x = -15$$
$$x = \frac{15}{2}$$

17.
$$f(x) = 2x^2 - 7x - 30$$
$$2x^2 - 7x - 30 = 0$$
$$(2x + 5)(x - 6) = 0$$
$$2x + 5 = 0 \quad \text{or} \quad x - 6 = 0$$
$$x = -\tfrac{5}{2} \qquad x = 6$$

18.
$$f(x) = 3x^2 + 22x - 16$$
$$3x^2 + 22x - 16 = 0$$
$$(3x - 2)(x + 8) = 0$$
$$3x - 2 = 0 \Rightarrow x = \tfrac{2}{3}$$
$$x + 8 = 0 \Rightarrow x = -8$$

19.
$$f(x) = \frac{x + 3}{2x^2 - 6}$$
$$\frac{x + 3}{2x^2 - 6} = 0$$
$$x + 3 = 0$$
$$x = -3$$

20.
$$f(x) = \frac{x^2 - 9x + 14}{4x}$$
$$\frac{x^2 - 9x + 14}{4x} = 0$$
$$(x - 7)(x - 2) = 0$$
$$x - 7 = 0 \Rightarrow x = 7$$
$$x - 2 = 0 \Rightarrow x = 2$$

21.
$$\begin{aligned} f(x) &= \tfrac{1}{3}x^3 - 2x \\ \tfrac{1}{3}x^3 - 2x &= 0 \\ (3)\left(\tfrac{1}{3}x^3 - 2x\right) &= 0(3) \\ x^3 - 6x &= 0 \\ x\left(x^2 - 6\right) &= 0 \end{aligned}$$
$$x = 0 \quad \text{or} \quad x^2 - 6 = 0$$
$$x^2 = 6$$
$$x = \pm\sqrt{6}$$

22.
$$f(x) = -25x^4 + 9x^2$$
$$-x^2\left(25x^2 - 9\right) = 0$$
$$-x^2 = 0 \quad \text{or} \quad 25x^2 - 9 = 0$$
$$x = 0 \qquad\qquad 25x^2 = 9$$
$$x^2 = \tfrac{9}{25}$$
$$x = \pm\tfrac{3}{5}$$

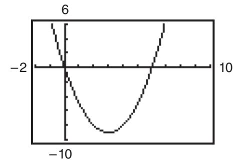
23.
$$\begin{aligned} f(x) &= x^3 - 4x^2 - 9x + 36 \\ x^3 - 4x^2 - 9x + 36 &= 0 \\ x^2(x - 4) - 9(x - 4) &= 0 \\ (x - 4)\left(x^2 - 9\right) &= 0 \end{aligned}$$
$$x - 4 = 0 \Rightarrow x = 4$$
$$x^2 - 9 = 0 \Rightarrow x = \pm 3$$

24.
$$\begin{aligned} f(x) &= 4x^3 - 24x^2 - x + 6 \\ 4x^3 - 24x^2 - x + 6 &= 0 \\ 4x^2(x - 6) - 1(x - 6) &= 0 \\ (x - 6)\left(4x^2 - 1\right) &= 0 \\ (x - 6)(2x + 1)(2x - 1) &= 0 \end{aligned}$$
$$x - 6 = 0 \quad \text{or} \quad 2x + 1 = 0 \quad \text{or} \quad 2x - 1 = 0$$
$$x = 6 \qquad\qquad x = -\tfrac{1}{2} \qquad\qquad x = \tfrac{1}{2}$$

25.
$$\begin{aligned} f(x) &= \sqrt{2x} - 1 \\ \sqrt{2x} - 1 &= 0 \\ \sqrt{2x} &= 1 \\ 2x &= 1 \\ x &= \tfrac{1}{2} \end{aligned}$$

26.
$$\begin{aligned} f(x) &= \sqrt{3x + 2} \\ \sqrt{3x + 2} &= 0 \\ 3x + 2 &= 0 \\ -\tfrac{2}{3} &= x \end{aligned}$$

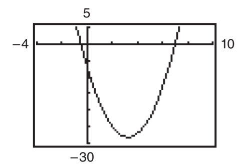
27. (a)



Zeros: $x = 0, 6$

(b)
$$\begin{aligned} f(x) &= x^2 - 6x \\ x^2 - 6x &= 0 \\ x(x - 6) &= 0 \end{aligned}$$
$$x = 0 \Rightarrow x = 0$$
$$x - 6 = 0 \Rightarrow x = 6$$

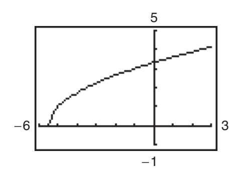
28. (a)



Zeros: $x = -0.5, 7$

(b)
$$\begin{aligned} f(x) &= 2x^2 - 13x - 7 \\ 2x^2 - 13x - 7 &= 0 \\ (2x + 1)(x - 7) &= 0 \end{aligned}$$
$$2x + 1 = 0 \Rightarrow x = -\tfrac{1}{2}$$
$$x - 7 = 0 \Rightarrow x = 7$$

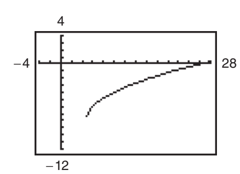
29. (a)



Zero: $x = -5.5$

(b)
$$\begin{aligned} f(x) &= \sqrt{2x + 11} \\ \sqrt{2x + 11} &= 0 \\ 2x + 11 &= 0 \\ x &= -\tfrac{11}{2} \end{aligned}$$

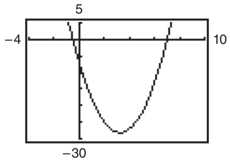
30. (a)



Zero: $x = 26$

(b)
$$\begin{aligned} f(x) &= \sqrt{3x - 14} - 8 \\ \sqrt{3x - 14} - 8 &= 0 \\ \sqrt{3x - 14} &= 8 \\ 3x - 14 &= 64 \\ x &= 26 \end{aligned}$$

31. (a)



Zero: $x = 0.3333$

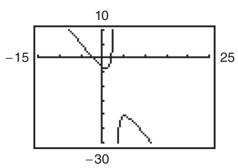
(b) $f(x) = \dfrac{3x - 1}{x - 6}$

$$\frac{3x - 1}{x - 6} = 0$$

$$3x - 1 = 0$$

$$x = \frac{1}{3}$$

32. (a)



Zeros: $x = \pm 2.1213$

(b) $f(x) = \dfrac{2x^2 - 9}{3 - x}$

$$\frac{2x^2 - 9}{3 - x} = 0$$

$$2x^2 - 9 = 0 \Rightarrow x = \pm\frac{3\sqrt{2}}{2} = \pm 2.1213$$

33. $f(x) = -\frac{1}{2}x^3$

The function is decreasing on $(-\infty, \infty)$.

34. $f(x) = x^2 - 4x$

The function is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

35. $f(x) = \sqrt{x^2 - 1}$

The function is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

36. $f(x) = x^3 - 3x^2 + 2$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 2)$.

37. $f(x) = |x + 1| + |x - 1|$

The function is increasing on $(1, \infty)$.

The function is constant on $(-1, 1)$.

The function is decreasing on $(-\infty, -1)$.

38. The function is decreasing on $(-2, -1)$ and $(-1, 0)$ and increasing on $(-\infty, -2)$ and $(0, \infty)$.

39. $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

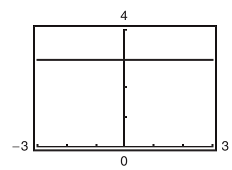
The function is decreasing on $(-1, 0)$ and increasing on $(-\infty, -1)$ and $(0, \infty)$.

40. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x + 1, & x > 2 \end{cases}$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$.

The function is constant on $(0, 2)$.

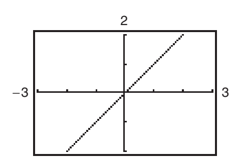
41. $f(x) = 3$



Constant on $(-\infty, \infty)$

x	−2	−1	0	1	2
$f(x)$	3	3	3	3	3

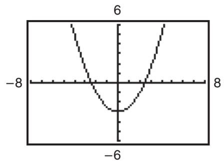
42. $g(x) = x$



Increasing on $(-\infty, \infty)$

x	−2	−1	0	1	2
$g(x)$	−2	−1	0	1	2

43. $g(x) = \frac{1}{2}x^2 - 3$

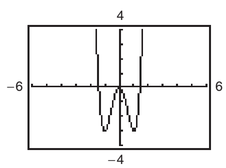


Decreasing on $(-\infty, 0)$.

Increasing on $(0, \infty)$.

x	-2	-1	0	1	2
$g(x)$	-1	$-\frac{5}{2}$	-3	$-\frac{5}{2}$	-1

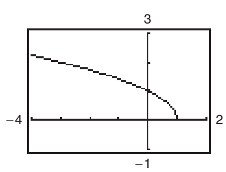
44. $f(x) = 3x^4 - 6x^2$



Increasing on $(-1, 0), (1, \infty)$; Decreasing on $(-\infty, -1), (0, 1)$

x	-2	-1	0	1	2
$f(x)$	24	-3	0	-3	24

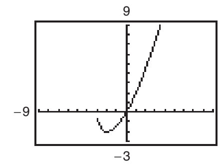
45. $f(x) = \sqrt{1 - x}$



Decreasing on $(-\infty, 1)$

x	-3	-2	-1	0	1
$f(x)$	2	$\sqrt{3}$	$\sqrt{2}$	1	0

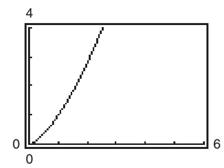
46. $f(x) = x\sqrt{x + 3}$



Increasing on $(-2, \infty)$; Decreasing on $(-3, -2)$

x	-3	-2	-1	0	1
$f(x)$	0	-2	-1.414	0	2

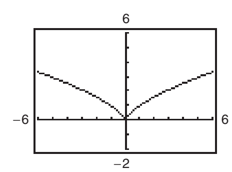
47. $f(x) = x^{3/2}$



Increasing on $(0, \infty)$

x	0	1	2	3	4
$f(x)$	0	1	2.8	5.2	8

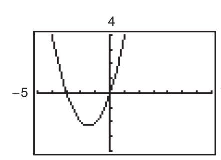
48. $f(x) = x^{2/3}$



Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

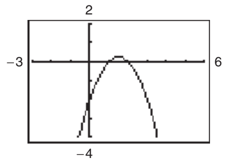
x	-2	-1	0	1	2
$f(x)$	1.59	1	0	1	1.59

49. $f(x) = x(x + 3)$



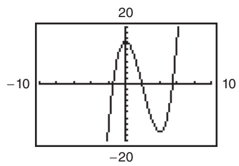
Relative minimum: $(-1.5, -2.25)$

50. $f(x) = -x^2 + 3x - 2$



Relative maximum: $(1.5, 0.25)$

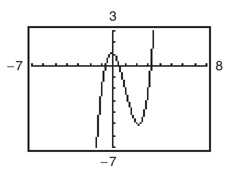
51. $h(x) = x^3 - 6x^2 + 15$



Relative minimum: $(4, -17)$

Relative maximum: $(0, 15)$

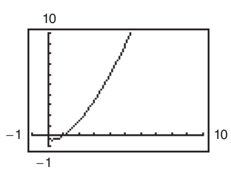
52. $f(x) = x^3 - 3x^2 - x + 1$



Relative maximum: $(-0.15, 1.08)$

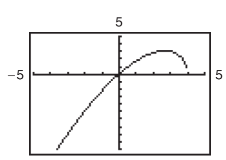
Relative minimum: $(2.15, -5.08)$

53. $h(x) = (x - 1)\sqrt{x}$



Relative minimum: $(0.33, -0.38)$

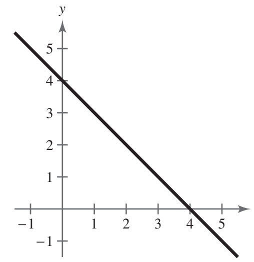
54. $g(x) = x\sqrt{4 - x}$



Relative maximum: $(2.67, 3.08)$

55. $f(x) = 4 - x$

$f(x) \geq 0$ on $(-\infty, 4]$



56. $f(x) = 4x + 2$

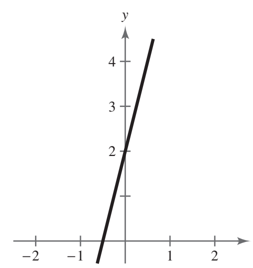
$f(x) \geq 0$ on $\left[-\frac{1}{2}, \infty\right)$

$4x + 2 \geq 0$

$4x \geq -2$

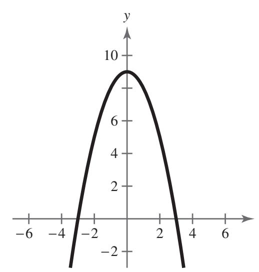
$x \geq -\frac{1}{2}$

$\left[-\frac{1}{2}, \infty\right)$



57. $f(x) = 9 - x^2$

$f(x) \geq 0$ on $[-3, 3]$



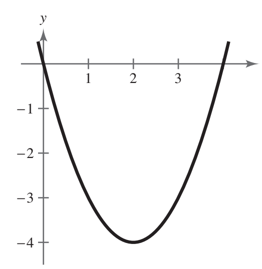
58. $f(x) = x^2 - 4x$

$f(x) \geq 0$ on $(-\infty, 0]$ and $[4, \infty)$

$x^2 - 4x \geq 0$

$x(x - 4) \geq 0$

$(-\infty, 0], [4, \infty)$



59. $f(x) = \sqrt{x-1}$

$f(x) \geq 0$ on $[1, \infty)$

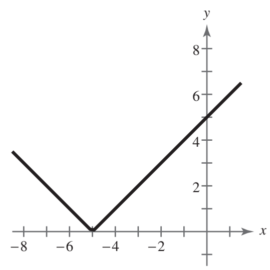
$\sqrt{x-1} \geq 0$

$x - 1 \geq 0$

$x \geq 1$

$[1, \infty)$

60. $f(x) = |x + 5|$



$f(x)$ is always greater than or equal to 0. $f(x) \geq 0$ for all x.

$(-\infty, \infty)$

61. $f(x) = -2x + 15$

$$\frac{f(3) - f(0)}{3 - 0} = \frac{9 - 15}{3} = -2$$

The average rate of change from $x_1 = 0$ to $x_2 = 3$ is -2.

62. $f(x) = x^2 - 2x + 8$

$$\frac{f(5) - f(1)}{5 - 1} = \frac{23 - 7}{4} = \frac{16}{4} = 4$$

The average rate of change from $x_1 = 1$ to $x_2 = 5$ is 4.

63. $f(x) = x^3 - 3x^2 - x$

$$\frac{f(2) - f(-1)}{2 - (-1)} = \frac{-6 - (-3)}{3} = \frac{-3}{3} = -1$$

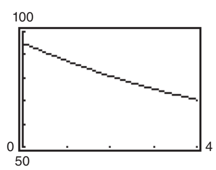
The average rate of change from $x_1 = -1$ to $x_2 = 2$ is -1.

64. $f(x) = -x^3 + 6x^2 + x$

$$\frac{f(6) - f(1)}{6 - 1} = \frac{6 - 6}{5} = 0$$

The average rate of change from $x_1 = 1$ to $x_2 = 6$ is 0.

65. (a)



(b) To find the average rate of change of the amount the U.S. Department of Energy spent for research and development from 2010 to 2014, find the average rate of change from $(0, f(0))$ to $(4, f(4))$.

$$\frac{f(4) - f(0)}{4 - 0} = \frac{70.5344 - 95.08}{4} = \frac{-24.5456}{4} = -6.1364$$

The amount the U.S. Department of Energy spent on research and development for defense decreased by about \$6.14 billion each year from 2010 to 2014.

66. Average rate of change $= \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$

$$= \frac{s(9) - s(0)}{9 - 0}$$

$$= \frac{540 - 0}{9 - 0}$$

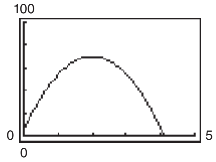
$= 60$ feet per second.

As the time traveled increases, the distance increases rapidly, causing the average speed to increase with each time increment. From $t = 0$ to $t = 4$, the average speed is less than from $t = 4$ to $t = 9$. Therefore, the overall average from $t = 0$ to $t = 9$ falls below the average found in part (b).

67. $s_0 = 6, v_0 = 64$

(a) $s = -16t^2 + 64t + 6$

(b)



(c) $\frac{s(3) - s(0)}{3 - 0} = \frac{54 - 6}{3} = 16$

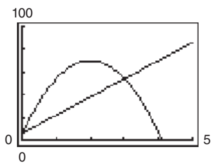
(d) The slope of the secant line is positive.

(e) $s(0) = 6, m = 16$

Secant line: $y - 6 = 16(t - 0)$

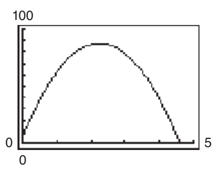
$y = 16t + 6$

(f)



68. (a) $s = -16t^2 + 72t + 6.5$

(b)



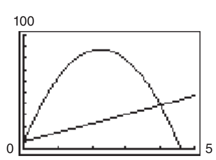
(c) The average rate of change from $t = 0$ to $t = 4$:

$\frac{s(4) - s(0)}{4 - 0} = \frac{38.5 - 6.5}{4} = \frac{32}{4} = 8$ feet per second

(d) The slope of the secant line through $(0, s(0))$ and $(4, s(4))$ is positive.

(e) The equation of the secant line:
$m = 8, y = 8t + 6.5$

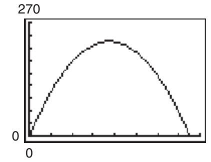
(f)



69. $v_0 = 120, s_0 = 0$

(a) $s = -16t^2 + 120t$

(b)



(c) The average rate of change from $t = 3$ to $t = 5$:

$\frac{s(5) - s(3)}{5 - 3} = \frac{200 - 216}{2} = -\frac{16}{2} = -8$ feet per second

(d) The slope of the secant line through $(3, s(3))$ and $(5, s(5))$ is negative.

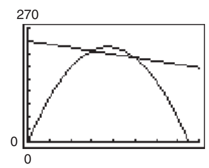
(e) The equation of the secant line: $m = -8$

Using $(5, s(5)) = (5, 200)$ we have

$y - 200 = -8(t - 5)$

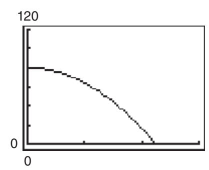
$y = -8t + 240.$

(f)



70. (a) $s = -16t^2 + 80$

(b)



(c) The average rate of change from $t = 1$ to $t = 2$:

$\frac{s(2) - s(1)}{2 - 1} = \frac{16 - 64}{1} = -\frac{48}{1} = -48$ feet per second

(d) The slope of the secant line through $(1, s(1))$ and $(2, s(2))$ is negative.

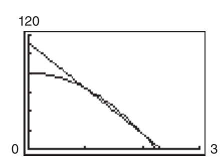
(e) The equation of the secant line: $m = -48$

Using $(1, s(1)) = (1, 64)$ we have

$y - 64 = -48(t - 1)$

$y = -48t + 112.$

(f)



71. $f(x) = x^6 - 2x^2 + 3$

$$\begin{aligned} f(-x) &= (-x)^6 - 2(-x)^2 + 3 \\ &= x^6 - 2x^2 + 3 \\ &= f(x) \end{aligned}$$

The function is even. y-axis symmetry.

72. $g(x) = x^3 - 5x$

$$\begin{aligned} g(-x) &= (-x)^3 - 5(-x) \\ &= -x^3 + 5x \\ &= -g(x) \end{aligned}$$

The function is odd. Origin symmetry.

73. $h(x) = x\sqrt{x + 5}$

$$\begin{aligned} h(-x) &= (-x)\sqrt{-x + 5} \\ &= -x\sqrt{5 - x} \\ &\neq h(x) \\ &\neq -h(x) \end{aligned}$$

The function is neither odd nor even. No symmetry.

74. $f(x) = x\sqrt{1 - x^2}$

$$\begin{aligned} f(-x) &= -x\sqrt{1 - (-x)^2} \\ &= -x\sqrt{1 - x^2} \\ &= -f(x) \end{aligned}$$

The function is odd. Origin symmetry.

75. $f(s) = 4s^{3/2}$

$$\begin{aligned} &= 4(-s)^{3/2} \\ &\neq f(s) \\ &\neq -f(s) \end{aligned}$$

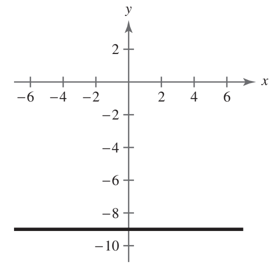
The function is neither odd nor even. No symmetry.

76. $g(s) = 4s^{2/3}$

$$\begin{aligned} g(-s) &= 4(-s)^{2/3} \\ &= 4s^{2/3} \\ &= g(s) \end{aligned}$$

The function is even. y-axis symmetry.

77.

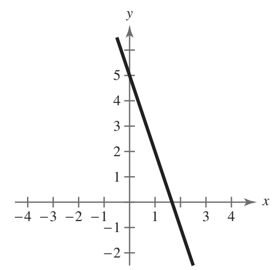


The graph of $f(x) = -9$ is symmetric to the y-axis, which implies $f(x)$ is even.

$$\begin{aligned} f(-x) &= -9 \\ &= f(x) \end{aligned}$$

The function is even.

78. $f(x) = 5 - 3x$

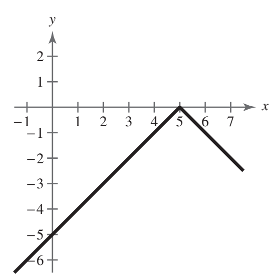


The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$\begin{aligned} f(-x) &= 5 - 3(-x) \\ &= 5 + 3x \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

The function is neither even nor odd.

79. $f(x) = -|x - 5|$

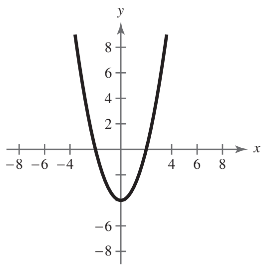


The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$\begin{aligned} f(x) &= -|(-x) - 5| \\ &= -|-x - 5| \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

The function is neither even nor odd.

80. $h(x) = x^2 - 4$

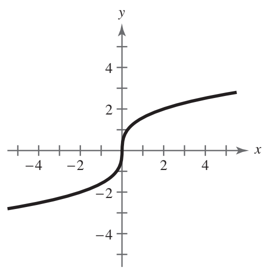


The graph displays y-axis symmetry, which implies $h(x)$ is even.

$h(-x) = (-x)^2 - 4 = x^2 - 4 = h(x)$

The function is even.

81. $f(x) = \sqrt[3]{4x}$

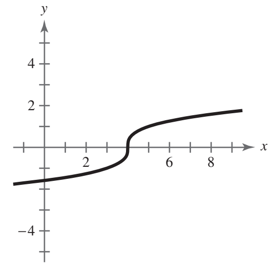


The graph displays origin symmetry, which implies $f(x)$ is odd.

$$\begin{aligned} f(-x) &= \sqrt[3]{4(-x)} \\ &= \sqrt[3]{-4x} \\ &= -\sqrt[3]{4x} \\ &= -f(x) \end{aligned}$$

The function is odd.

82. $f(x) = \sqrt[3]{x - 4}$



The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$\begin{aligned} f(-x) &= \sqrt[3]{(-x) - 4} \\ &= \sqrt[3]{-x - 4} \\ &= \sqrt[3]{-(x + 4)} \\ &= -\sqrt[3]{x + 4} \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

The function is neither even nor odd.

83. $$\begin{aligned} h &= \text{top} - \text{bottom} \\ &= 3 - (4x - x^2) \\ &= 3 - 4x + x^2 \end{aligned}$$

84. $$\begin{aligned} h &= \text{top} - \text{bottom} \\ &= (4x - x^2) - 2x \\ &= 2x - x^2 \end{aligned}$$

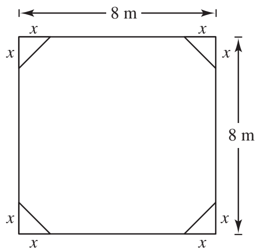
85. $$\begin{aligned} L &= \text{right} - \text{left} \\ &= 2 - \sqrt[3]{2y} \end{aligned}$$

86. $$\begin{aligned} L &= \text{right} - \text{left} \\ &= \frac{2}{y} - 0 \\ &= \frac{2}{y} \end{aligned}$$

87. The error is that $-2x^3 - 5 \neq -(2x^3 - 5)$. The correct process is as follows.

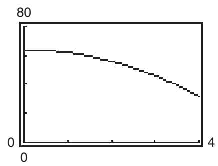
$$\begin{aligned} f(x) &= 2x^3 - 5 \\ f(-x) &= 2(-x)^3 - 5 \\ &= -2x^3 - 5 \\ &= -(2x^3 + 5) \end{aligned}$$

$f(-x) \neq -f(x)$ and $f(-x) \neq f(x)$, so the function $f(x) = 2x^3 - 5$ is neither odd nor even.

88. 

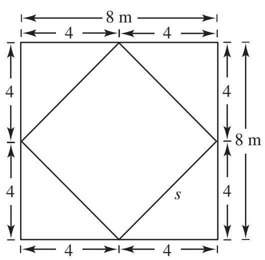
(a) $A = (8)(8) - 4\left(\frac{1}{2}\right)(x)(x) = 64 - 2x^2$

Domain: $0 \le x \le 4$

(b) 

Range: $32 \le A \le 64$

(c) When $x = 4$, the resulting figure is a square.



By the Pythagorean Theorem,
$4^2 + 4^2 = s^2 \Rightarrow s = \sqrt{32} = 4\sqrt{2}$ meters.

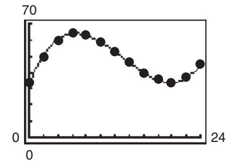
89. (a) For the average salary of college professors, a scale of $10,000 would be appropriate.

(b) For the population of the United States, use a scale of 10,000,000.

(c) For the percent of the civilian workforce that is unemployed, use a scale of 10%.

(d) For the number of games a college football team wins in a single season, single digits would be appropriate.

For each of the graphs, using the suggested scale would show yearly changes in the data clearly.

90. (a) 

(b) The model is an excellent fit.

(c) The temperature was increasing from 6 A.M. until noon $(x = 0 \text{ to } x = 6)$. Then it decreases until 2 A.M. $(x = 6 \text{ to } x = 20)$. Then the temperature increases until 6 A.M. $(x = 20 \text{ to } x = 24)$.

(d) The maximum temperature according to the model is about 63.93°F. According to the data, it is 64°F. The minimum temperature according to the model is about 33.98°F. According to the data, it is 34°F.

(e) Answers may vary. Temperatures will depend upon the weather patterns, which usually change from day to day.

91. False. The function $f(x) = \sqrt{x^2 + 1}$ has a domain of all real numbers.

92. False. An odd function is symmetric with respect to the origin, so its domain must include negative values.

93. True. A graph that is symmetric with respect to the y-axis cannot be increasing on its entire domain.

94. (a) Domain: $[-4, 5)$; Range: $[0, 9]$

(b) $(3, 0)$

(c) Increasing: $(-4, 0) \cup (3, 5)$; Decreasing: $(0, 3)$

(d) Relative minimum: $(3, 0)$

Relative maximum: $(0, 9)$

(e) Neither

95. $\left(-\frac{5}{3}, -7\right)$

(a) If f is even, another point is $\left(\frac{5}{3}, -7\right)$.

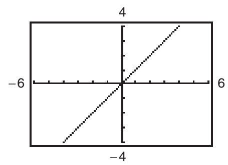
(b) If f is odd, another point is $\left(\frac{5}{3}, 7\right)$.

96. $(2a, 2c)$

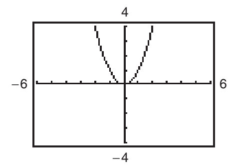
(a) $(-2a, 2c)$

(b) $(-2a, -2c)$

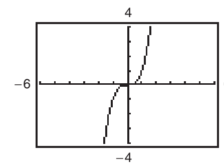
97. (a) $y = x$



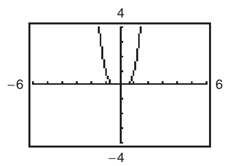
(b) $y = x^2$



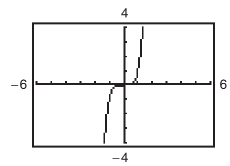
(c) $y = x^3$



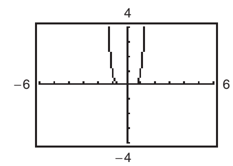
(d) $y = x^4$



(e) $y = x^5$

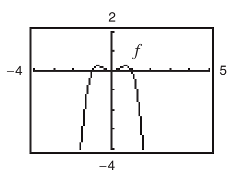


(f) $y = x^6$

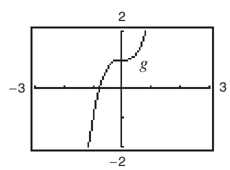


All the graphs pass through the origin. The graphs of the odd powers of x are symmetric with respect to the origin and the graphs of the even powers are symmetric with respect to the y-axis. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

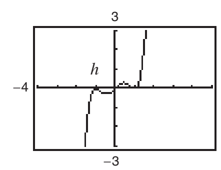
98.



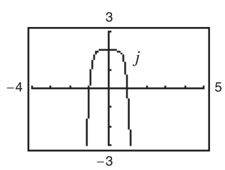
$f(x) = x^2 - x^4$ is even.



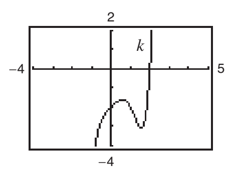
$g(x) = 2x^3 + 1$ is neither.



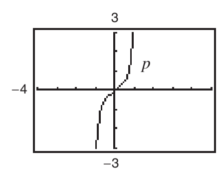
$h(x) = x^5 - 2x^3 + x$ is odd.



$j(x) = 2 - x^6 - x^8$ is even.



$k(x) = x^5 - 2x^4 + x - 2$ is neither.



$p(x) = x^9 + 3x^5 - x^3 + x$ is odd.

Equations of odd functions contain only odd powers of x. Equations of even functions contain only even powers of x. Odd functions have all variables raised to odd powers and even functions have all variables raised to even powers. A function that has variables raised to even and odd powers is neither odd nor even.

99. (a) Even. The graph is a reflection in the x-axis.
(b) Even. The graph is a reflection in the y-axis.
(c) Even. The graph is a vertical translation of f.
(d) Neither. The graph is a horizontal translation of f.

Section 2.4 A Library of Parent Functions

1. Greatest integer function

2. Identity function

3. Reciprocal function

4. Squaring function

5. Square root function

6. Constant function

7. Absolute value function

8. Cubic function

9. Linear function

10. linear

11. (a) $f(1) = 4, f(0) = 6$

$(1, 4), (0, 6)$

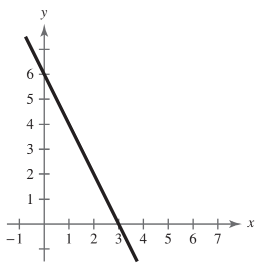
$$m = \frac{6 - 4}{0 - 1} = -2$$

$$y - 6 = -2(x - 0)$$

$$y = -2x + 6$$

$$f(x) = -2x + 6$$

(b)



12. (a) $f(-3) = -8, f(1) = 2$

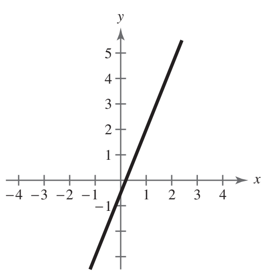
$(-3, -8), (1, 2)$

$$m = \frac{2 - (-8)}{1 - (-3)} = \frac{10}{4} = \frac{5}{2}$$

$$f(x) - 2 = \frac{5}{2}(x - 1)$$

$$f(x) = \frac{5}{2}x - \frac{1}{2}$$

(b)



13. (a) $f\left(\frac{1}{2}\right) = -\frac{5}{3}, f(6) = 2$

$\left(\frac{1}{2}, -\frac{5}{3}\right), (6, 2)$

$$m = \frac{2 - \left(-\frac{5}{3}\right)}{6 - \left(\frac{1}{2}\right)}$$

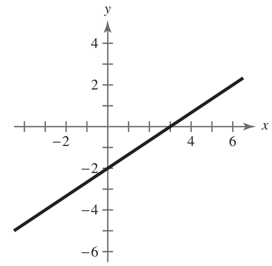
$$= \frac{\frac{11}{3}}{\frac{11}{2}} = \left(\frac{11}{3}\right) \cdot \left(\frac{2}{11}\right) = \frac{2}{3}$$

$$f(x) - 2 = \frac{2}{3}(x - 6)$$

$$f(x) - 2 = \frac{2}{3}x - 4$$

$$f(x) = \frac{2}{3}x - 2$$

(b)



14. (a) $f\left(\frac{3}{5}\right) = \frac{1}{2}, f(4) = 9$

$\left(\frac{3}{5}, \frac{1}{2}\right), (4, 9)$

$$m = \frac{9 - \left(\frac{1}{2}\right)}{4 - \left(\frac{3}{5}\right)}$$

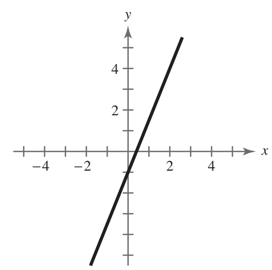
$$= \frac{\frac{17}{2}}{\frac{17}{5}} = \left(\frac{17}{2}\right) \cdot \left(\frac{5}{17}\right) = \frac{5}{2}$$

$$f(x) - 9 = \frac{5}{2}(x - 4)$$

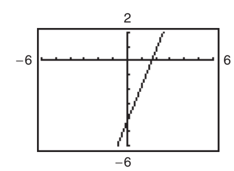
$$f(x) - 9 = \frac{5}{2}x - 10$$

$$f(x) = \frac{5}{2}x - 1$$

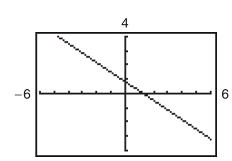
(b)



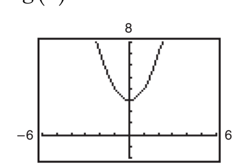
15. $f(x) = 2.5x - 4.25$



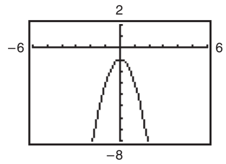
16. $f(x) = \frac{5}{6} - \frac{2}{3}x$



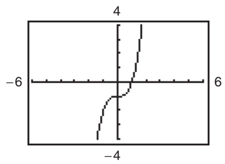
17. $g(x) = x^2 + 3$



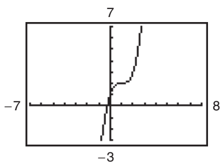
18. $g(x) = -2x^2 - 1$



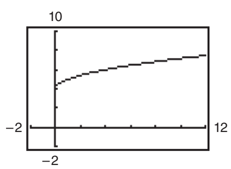
19. $f(x) = x^3 - 1$



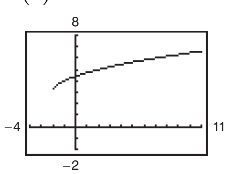
20. $f(x) = (x - 1)^3 + 2$



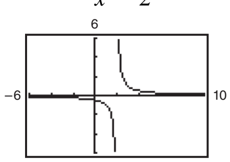
21. $f(x) = \sqrt{x} + 4$



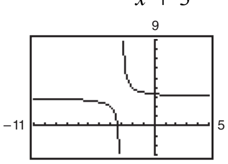
22. $h(x) = \sqrt{x + 2} + 3$



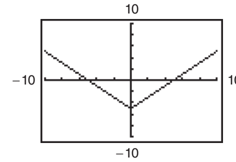
23. $f(x) = \dfrac{1}{x - 2}$



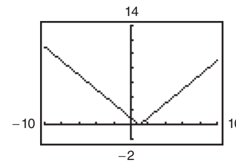
24. $k(x) = 3t + \dfrac{1}{x + 3}$



25. $g(x) = |x| - 5$



26. $f(x) = |x - 1|$



27. $f(x) = [\![x]\!]$

(a) $f(2.1) = 2$

(b) $f(2.9) = 2$

(c) $f(-3.1) = -4$

(d) $f\left(\frac{7}{2}\right) = 3$

28. $h(x) = [\![x + 3]\!]$

(a) $h(-2) = [\![1]\!] = 1$

(b) $h\left(\frac{1}{2}\right) = [\![3.5]\!] = 3$

(c) $h(4.2) = [\![7.2]\!] = 7$

(d) $h(-21.6) = [\![-18.6]\!] = -19$

29. $k(x) = [\![2x + 1]\!]$

(a) $k\left(\frac{1}{3}\right) = \left[\!\!\left[2\left(\frac{1}{3}\right) + 1\right]\!\!\right] = \left[\!\!\left[\frac{5}{3}\right]\!\!\right] = 1$

(b) $k(-2.1) = [\![2(-2.1) + 1]\!] = [\![-3.1]\!] = -4$

(c) $k(1.1) = [\![2(1.1) + 1]\!] = [\![3.2]\!] = 3$

(d) $k\left(\frac{2}{3}\right) = \left[\!\!\left[2\left(\frac{2}{3}\right) + 1\right]\!\!\right] = \left[\!\!\left[\frac{7}{3}\right]\!\!\right] = 2$

30. $g(x) = -7[\![x + 4]\!] + 6$

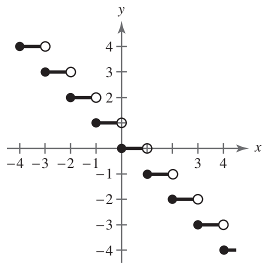
(a) $g\left(\frac{1}{8}\right) = -7\left[\!\!\left[\frac{1}{8} + 4\right]\!\!\right] + 6$
$= -7\left[\!\!\left[4\frac{1}{8}\right]\!\!\right] + 6 = -7(4) + 6 = -22$

(b) $g(9) = -7[\![9 + 4]\!] + 6$
$= -7[\![13]\!] + 6 = -7(13) + 6 = -85$

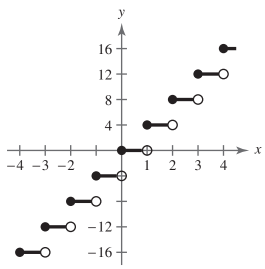
(c) $g(-4) = -7[\![-4 + 4]\!] + 6$
$= -7[\![0]\!] + 6 = -7(0) + 6 = 6$

(d) $g\left(\frac{3}{2}\right) = -7\left[\!\!\left[\frac{3}{2} + 4\right]\!\!\right] + 6$
$= -7\left[\!\!\left[5\frac{1}{2}\right]\!\!\right] + 6 = -7(5) + 6 = -29$

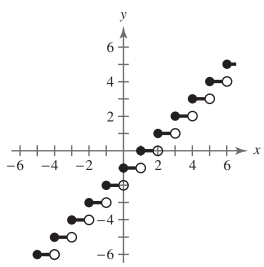
31. $g(x) = -[\![x]\!]$



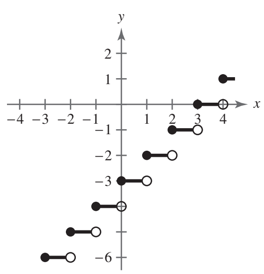
32. $g(x) = 4[\![x]\!]$



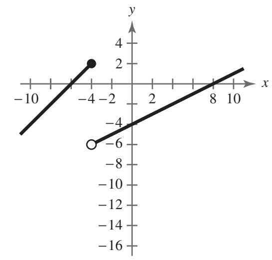
33. $g(x) = [\![x]\!] - 1$



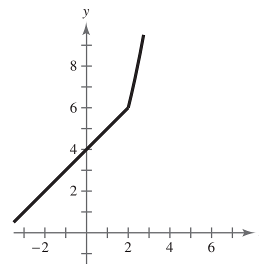
34. $g(x) = [\![x - 3]\!]$



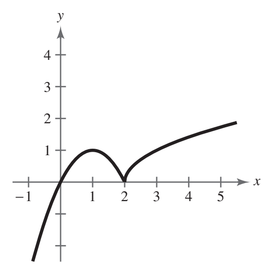
35. $g(x) = \begin{cases} x + 6, & x \le -4 \\ \frac{1}{2}x - 4, & x > -4 \end{cases}$



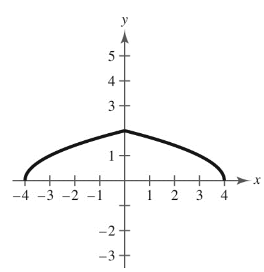
36. $f(x) = \begin{cases} 4 + x, & x \le 2 \\ x^2 + 2, & x > 2 \end{cases}$



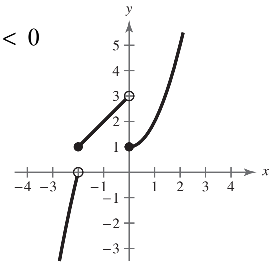
37. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \le 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$



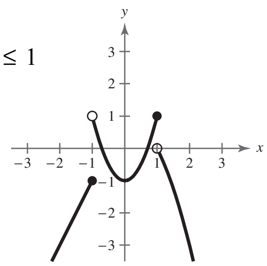
38. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \ge 0 \end{cases}$



39. $h(x) = \begin{cases} 4 - x^2, & x < -2 \\ 3 + x, & -2 \le x < 0 \\ x^2 + 1, & x \ge 0 \end{cases}$

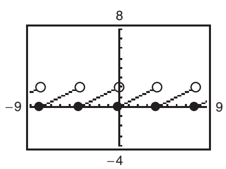


40. $k(x) = \begin{cases} 2x + 1, & x \le -1 \\ 2x^2 - 1, & -1 < x \le 1 \\ 1 - x^2, & x > 1 \end{cases}$



41. $s(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)$

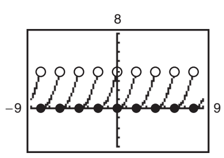
(a)



(b) Domain: $(-\infty, \infty)$; Range: $[0, 2)$

42. $k(x) = 4\left(\frac{1}{2}x - [\![\frac{1}{2}x]\!]\right)^2$

(a)



(b) Domain: $(-\infty, \infty)$; Range: $[0, 4)$

43. (a) $W(30) = 14(30) = 420$

$W(40) = 14(40) = 560$

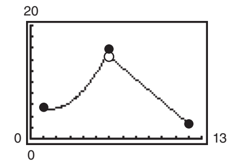
$W(45) = 21(45 - 40) + 560 = 665$

$W(50) = 21(50 - 40) + 560 = 770$

(b) $W(h) = \begin{cases} 14h, & 0 < h \le 36 \\ 21(h - 36) + 504, & h > 36 \end{cases}$

(c) $W(h) = \begin{cases} 16h, & 0 < h \le 40 \\ 24(h - 40) + 640, & h > 40 \end{cases}$

44. (a)



The domain of $f(x) = -1.97x + 26.3$ is $6 < x \le 12$. One way to see this is to notice that this is the equation of a line with negative slope, so the function values are decreasing as x increases, which matches the data for the corresponding part of the table. The domain of $f(x) = 0.505x^2 - 1.47x + 6.3$ is then $1 \le x \le 6$.

(b) $f(5) = 0.505(5)^2 - 1.47(5) + 6.3$

$= 0.505(25) - 7.35 + 6.3 = 11.575$

$f(11) = -1.97(11) + 26.3 = 4.63$

These values represent the revenue in thousands of dollars for the months of May and November, respectively.

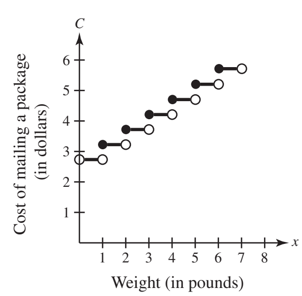
(c) These values are quite close to the actual data values.

45. Answers will vary. *Sample answer:*

Interval	*Input Pipe*	*Drain Pipe 1*	*Drain Pipe 2*
[0, 5]	Open	Closed	Closed
[5, 10]	Open	Open	Closed
[10, 20]	Closed	Closed	Closed
[20, 30]	Closed	Closed	Open
[30, 40]	Open	Open	Open
[40, 45]	Open	Closed	Open
[45, 50]	Open	Open	Open
[50, 60]	Open	Open	Closed

46. (a) $C = 0.5[\![x]\!] + 2.72$

(b)



47. For the first two hours, the slope is 1. For the next six hours, the slope is 2. For the final hour, the slope is $\frac{1}{2}$.

$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ 2t - 2, & 2 < t \le 8 \\ \frac{1}{2}t + 10, & 8 < t \le 9 \end{cases}$$

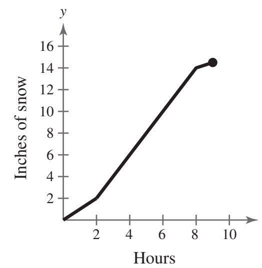
To find $f(t) = 2t - 2$, use $m = 2$ and $(2, 2)$.

$y - 2 = 2(t - 2) \Rightarrow y = 2t - 2$

To find $f(t) = \frac{1}{2}t + 10$, use $m = \frac{1}{2}$ and $(8, 14)$.

$y - 14 = \frac{1}{2}(t - 8) \Rightarrow y = \frac{1}{2}t + 10$

Total accumulation $= 14.5$ inches



48. $f(x) = x^2$

(a) Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

(b) x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

(c) Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$

(d) Even; the graph has y-axis symmetry.

$f(x) = x^3$

(a) Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

(b) x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

(c) Increasing: $(-\infty, \infty)$

(d) Odd; the graph has origin symmetry.

49. False. A piecewise-defined function is a function that is defined by two or more equations over a specified domain. That domain may or may not include x- and y-intercepts.

50. False. The vertical line $x = 2$ has an x-intercept at the point $(2, 0)$ but does not have a y-intercept. The horizontal line $y = 3$ has a y-intercept at the point $(0, 3)$ but does not have an x-intercept.

Section 2.5 Transformations of Functions

1. rigid

2. $-f(x)$; $f(-x)$

3. vertical stretch; vertical shrink

4. (a) iv

(b) ii

(c) iii

(d) i

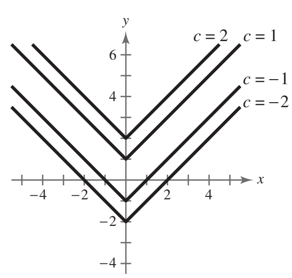
5. (a) $f(x) = |x| + c$ Vertical shifts

$c = -2$: $f(x) = |x| - 2$ 2 units down

$c = -1$: $f(x) = |x| - 1$ 1 unit down

$c = 1$: $f(x) = |x| + 1$ 1 unit up

$c = 2$: $f(x) = |x| + 2$ 2 units up



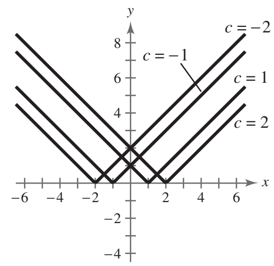
(b) $f(x) = |x - c|$ Horizontal shifts

$c = -2$: $f(x) = |x - (-2)| = |x + 2|$ 2 units left

$c = -1$: $f(x) = |x - (-1)| = |x + 1|$ 1 unit left

$c = 1$: $f(x) = |x - (1)| = |x - 1|$ 1 unit right

$c = 2$: $f(x) = |x - (2)| = |x - 2|$ 2 units right



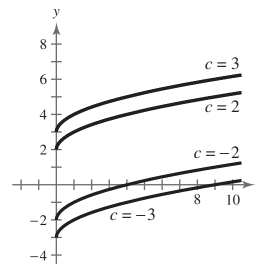
6. (a) $f(x) = \sqrt{x} + c$ Vertical shifts

$c = -3$: $f(x) = \sqrt{x} - 3$ 3 units down

$c = -2$: $f(x) = \sqrt{x} - 2$ 2 units down

$c = 2$: $f(x) = \sqrt{x} + 2$ 2 units up

$c = 3$: $f(x) = \sqrt{x} + 3$ 3 units up



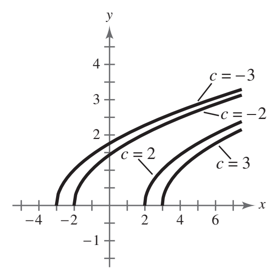
(b) $f(x) = \sqrt{x - c}$ Horizontal shifts

$c = -3$: $f(x) = \sqrt{x - (-3)} = \sqrt{x + 3}$ 3 units left

$c = -2$: $f(x) = \sqrt{x - (-2)} = \sqrt{x + 2}$ 2 units left

$c = 2$: $f(x) = \sqrt{x - 2}$ 2 units right

$c = 3$: $f(x) = \sqrt{x - 3}$ 3 units right



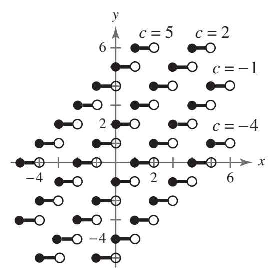
7. (a) $f(x) = [\![x]\!] + c$ Vertical shifts

$c = -4$: $f(x) = [\![x]\!] - 4$ 4 units down

$c = -1$: $f(x) = [\![x]\!] - 1$ 1 unit down

$c = 2$: $f(x) = [\![x]\!] + 2$ 2 units up

$c = 5$: $f(x) = [\![x]\!] + 5$ 5 units up



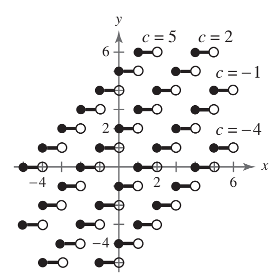
(b) $f(x) = [\![x + c]\!]$ Horizontal shifts

$c = -4$: $f(x) = [\![x - (-4)]\!] = [\![x + 4]\!]$ 4 units left

$c = -1$: $f(x) = [\![x - (-1)]\!] = [\![x + 1]\!]$ 1 unit left

$c = 2$: $f(x) = [\![x - (2)]\!] = [\![x - 2]\!]$ 2 units right

$c = 5$: $f(x) = [\![x - (5)]\!] = [\![x - 5]\!]$ 5 units right



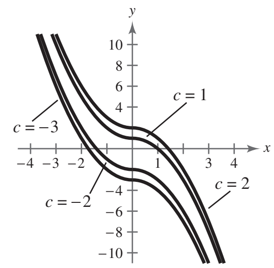
8. (a) $f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \geq 0 \end{cases}$ Vertical shifts

$c = -3$: $f(x) = \begin{cases} x^2 - 3, & x < 0 \\ -x^2 - 3, & x \geq 0 \end{cases}$ 3 units down

$c = -2$: $f(x) = \begin{cases} x^2 - 2, & x < 0 \\ -x^2 - 2, & x \geq 0 \end{cases}$ 2 units down

$c = 1$: $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ -x^2 + 1, & x \geq 0 \end{cases}$ 1 unit up

$c = 2$: $f(x) = \begin{cases} x^2 + 2, & x < 0 \\ -x^2 + 2, & x \geq 0 \end{cases}$ 2 units up



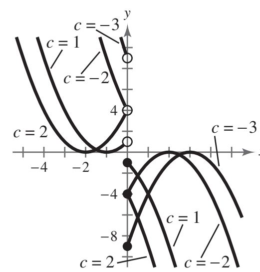
(b) $f(x) = \begin{cases} (x + c)^2, & x < 0 \\ -(x + c)^2, & x \geq 0 \end{cases}$ Horizontal shifts

$c = -3$: $f(x) = \begin{cases} (x - 3)^2, & x < 0 \\ -(x - 3)^2, & x \geq 0 \end{cases}$ 3 units right

$c = -2$: $f(x)\begin{cases} (x - 2)^2, & x < 0 \\ -(x - 2)^2, & x \geq 0 \end{cases}$ 2 units right

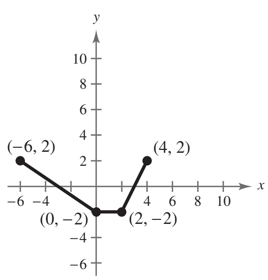
$c = 1$: $f(x) = \begin{cases} (x + 1)^2, & x < 0 \\ -(x + 1)^2, & x \geq 0 \end{cases}$ 1 unit left

$c = 2$: $f(x) = \begin{cases} (x + 2)^2, & x < 0 \\ -(x + 2)^2, & x \geq 0 \end{cases}$ 2 units left



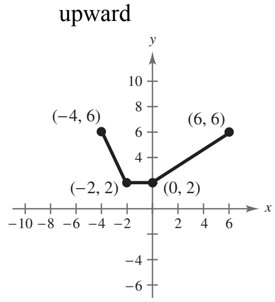
9. (a) $y = f(-x)$

Reflection in the y-axis



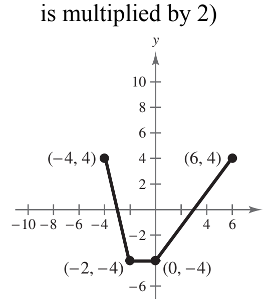
(b) $y = f(x) + 4$

Vertical shift 4 units upward



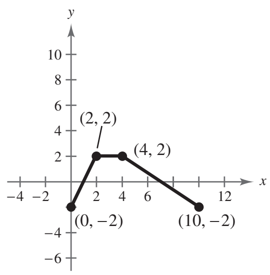
(c) $y = 2f(x)$

Vertical stretch (each y-value is multiplied by 2)



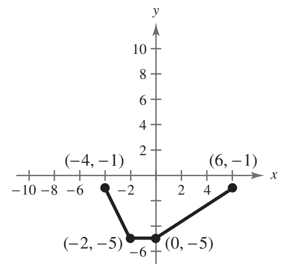
(d) $y = -f(x - 4)$

Reflection in the x-axis and a horizontal shift 4 units to the right



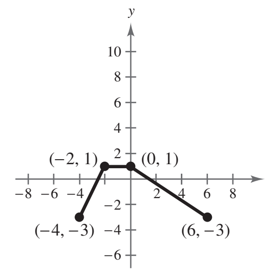
(e) $y = f(x) - 3$

Vertical shift 3 units downward



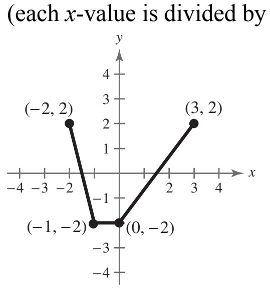
(f) $y = -f(x) - 1$

Reflection in the x-axis and a vertical shift 1 unit downward



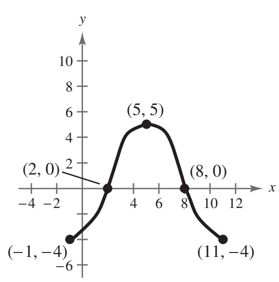
(g) $y = f(2x)$

Horizontal shrink (each x-value is divided by 2)



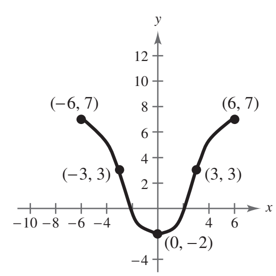
10. (a) $y = f(x - 5)$

Horizontal shift 5 units to the right



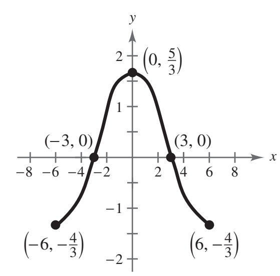
(b) $y = -f(x) + 3$

Reflection in the x-axis and a vertical shift 3 units upward



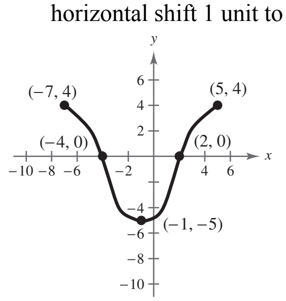
(c) $y = \frac{1}{3}f(x)$

Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$



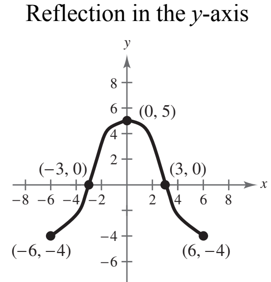
(d) $y = -f(x + 1)$

Reflection in the x-axis and a horizontal shift 1 unit to the left



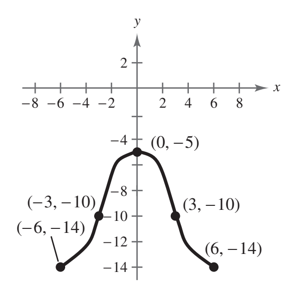
(e) $y = f(-x)$

Reflection in the y-axis



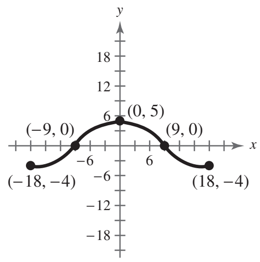
(f) $y = f(x) - 10$

Vertical shift 10 units downward



(g) $y = f\left(\frac{1}{3}x\right)$

Horizontal stretch
$\left(\text{each } x\text{-value is multiplied by } 3\right)$



11. Parent function: $f(x) = x^2$

(a) Vertical shift 1 unit downward

$g(x) = x^2 - 1$

(b) Reflection in the x-axis, horizontal shift 1 unit to the left, and a vertical shift 1 unit upward

$g(x) = -(x + 1)^2 + 1$

12. Parent function: $f(x) = x^3$

(a) Shifted upward 1 unit

$g(x) = x^3 + 1$

(b) Reflection in the x-axis, shifted to the left 3 units and down 1 unit

$g(x) = -(x + 3)^3 - 1$

13. Parent function: $f(x) = |x|$

(a) Reflection in the x-axis and a horizontal shift 3 units to the left

$g(x) = -|x + 3|$

(b) Horizontal shift 2 units to the right and a vertical shift 4 units downward

$g(x) = |x - 2| - 4$

14. Parent function: $f(x) = \sqrt{x}$

(a) A vertical shift 7 units downward and a horizontal shift 1 unit to the left

$g(x) = \sqrt{x + 1} - 7$

(d) Reflection in the x- and y-axis and shifted to the right 3 units and downward 4 units

$g(x) = -\sqrt{-x + 3} - 4$

15. Parent function: $f(x) = x^3$

Horizontal shift 2 units to the right

$y = (x - 2)^3$

16. Parent function: $y = x$

Vertical shrink $\left(\text{each } y \text{ value is multiplied by } \frac{1}{2}\right)$

$y = \frac{1}{2}x$

17. Parent function: $f(x) = x^2$

Reflection in the x-axis

$y = -x^2$

18. Parent function: $y = [\![x]\!]$

Vertical shift 4 units upward

$y = [\![x]\!] + 4$

19. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 1 unit upward

$y = -\sqrt{x} + 1$

20. Parent function: $y = |x|$

Horizontal shift 2 units to the left

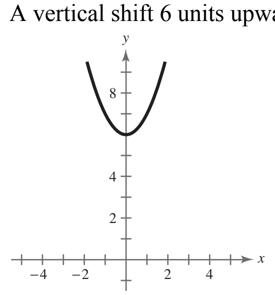
$y = |x + 2|$

21. $g(x) = x^2 + 6$

(a) Parent function: $f(x) = x^2$

(b) A vertical shift 6 units upward

(c)



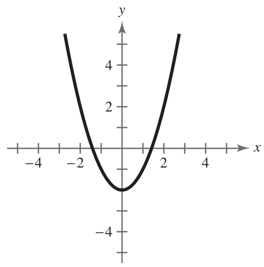
(d) $g(x) = f(x) + 6$

22. $g(x) = x^2 - 2$

(a) Parent function: $f(x) = x^2$

(b) A vertical shift 2 units downward

(c)



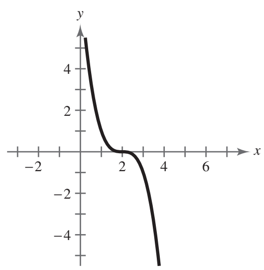
(d) $g(x) = f(x) - 2$

23. $g(x) = -(x - 2)^3$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift of 2 units to the right and a reflection in the x-axis

(c)



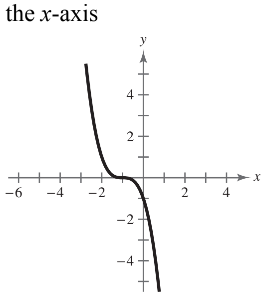
(d) $g(x) = -f(x - 2)$

24. $g(x) = -(x + 1)^3$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift 1 unit to the left and a reflection in the x-axis

(c)



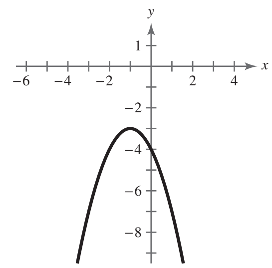
(d) $g(x) = -f(x + 1)$

25. $g(x) = -3 - (x + 1)^2$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis, a vertical shift 3 units downward and a horizontal shift 1 unit left

(c)



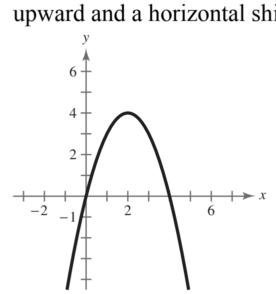
(d) $g(x) = -f(x + 1) - 3$

26. $g(x) = 4 - (x - 2)^2$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis, a vertical shift 4 units upward and a horizontal shift 2 units right

(c)



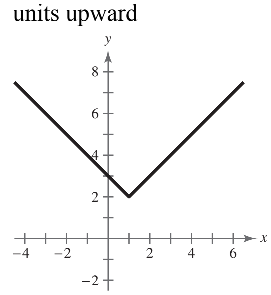
(d) $g(x) = -f(x - 2) + 4$

27. $g(x) = |x - 1| + 2$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shift 1 unit right and a vertical shift 2 units upward

(c)



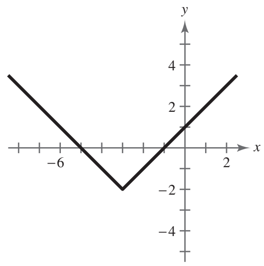
(d) $g(x) = f(x - 1) + 2$

28. $g(x) = |x + 3| - 2$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shift 3 units left and a vertical shift 2 units downward

(c)



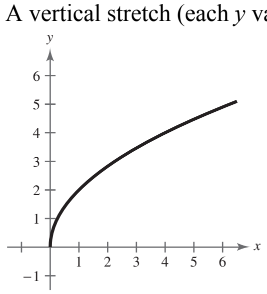
(d) $g(x) = f(x + 3) - 2$

29. $g(x) = 2\sqrt{x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) A vertical stretch (each y value is multiplied by 2)

(c)



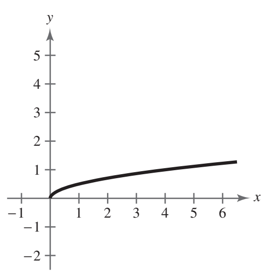
(d) $g(x) = 2f(x)$

30. $g(x) = \frac{1}{2}\sqrt{x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) A vertical shrink $\left(\text{each } y \text{ value is multiplied by } \frac{1}{2}\right)$

(c)



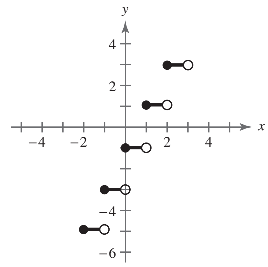
(d) $g(x) = \frac{1}{2}f(x)$

31. $g(x) = 2[\![x]\!] - 1$

(a) Parent function: $f(x) = [\![x]\!]$

(b) A vertical shift of 1 unit downward and a vertical stretch (each y value is multiplied by 2)

(c)



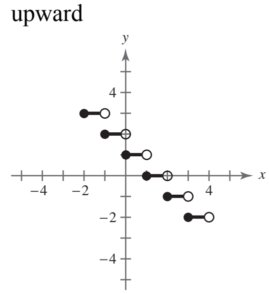
(d) $g(x) = 2f(x) - 1$

32. $g(x) = -[\![x]\!] + 1$

(a) Parent function: $f(x) = [\![x]\!]$

(b) Reflection in the x-axis and a vertical shift 1 unit upward

(c)



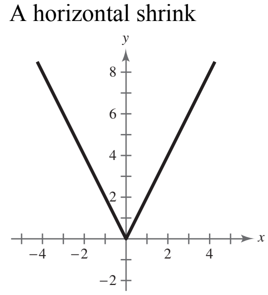
(d) $g(x) = -f(x) + 1$

33. $g(x) = |2x|$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shrink

(c)



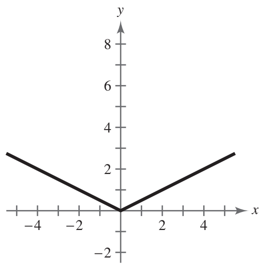
(d) $g(x) = f(2x)$

34. $g(x) = \left|\frac{1}{2}x\right|$

(a) Parent function: $f(x) = |x|$

(b) A horizontal stretch

(c)



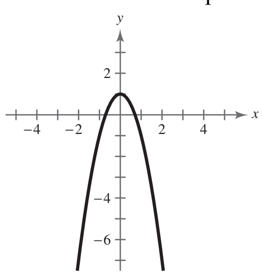
(d) $g(x) = f\left(\frac{1}{2}x\right)$

35. $g(x) = -2x^2 + 1$

(a) Parent function: $f(x) = x^2$

(b) A vertical stretch, reflection in the x-axis and a vertical shift 1 unit upward

(c)



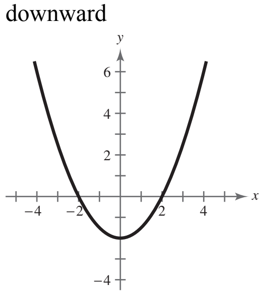
(d) $g(x) = -2f(x) + 1$

36. $g(x) = \frac{1}{2}x^2 - 2$

(a) Parent function: $f(x) = x^2$

(b) A vertical shrink and a vertical shift 2 units downward

(c)



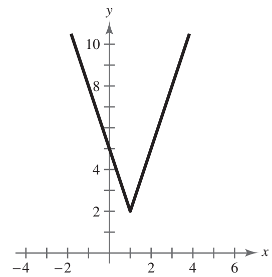
(d) $g(x) = \frac{1}{2}f(x) - 2$

37. $g(x) = 3|x - 1| + 2$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shift of 1 unit to the right, a vertical stretch, and a vertical shift 2 units upward

(c)



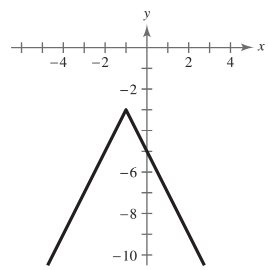
(d) $g(x) = 3f(x - 1) + 2$

38. $g(x) = -2|x + 1| - 3$

(a) Parent function: $f(x) = |x|$

(b) A reflection in the x-axis, a vertical stretch, a horizontal shift 1 unit to the left, and a vertical shift 3 units downward

(c)



(d) $g(x) = -2f(x + 1) - 3$

39. $g(x) = (x - 3)^2 - 7$

40. $g(x) = -(x + 2)^2 + 9$

41. $f(x) = x^3$ moved 13 units to the right

$g(x) = (x - 13)^3$

42. $f(x) = x^3$ moved 6 units to the left, 6 units downward, and reflected in the y-axis (in that order)

$g(x) = (-x + 6)^3 - 6$

43. $g(x) = -|x| + 12$

44. $g(x) = |x + 4| - 8$

45. $f(x) = \sqrt{x}$ moved 6 units to the left and reflected in both the x- and y-axes

$g(x) = -\sqrt{-x + 6}$

46. $f(x) = \sqrt{x}$ moved 9 units downward and reflected in both the x-axis and the y-axis

$$g(x) = -\left(\sqrt{-x} - 9\right)$$
$$= -\sqrt{-x} + 9$$

47. $f(x) = x^2$

(a) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 3)

$g(x) = -3x^2$

(b) Vertical shift 3 units upward and a vertical stretch (each y-value is multiplied by 4)

$g(x) = 4x^2 + 3$

48. $f(x) = x^3$

(a) Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = \frac{1}{4}x^3$

(b) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 2)

$g(x) = -2x^3$

49. $f(x) = |x|$

(a) Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}|x|$

(b) Vertical stretch (each y-value is multiplied by 3) and a vertical shift 3 units downward

$g(x) = 3|x| - 3$

50. $f(x) = \sqrt{x}$

(a) Vertical stretch (each y-value is multiplied by 8)

$g(x) = 8\sqrt{x}$

(b) Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = -\frac{1}{4}\sqrt{x}$

51. Parent function: $f(x) = x^3$

Vertical stretch (each y-value is multiplied by 2)

$g(x) = 2x^3$

52. Parent function: $f(x) = |x|$

Vertical stretch (each y-value is multiplied by 6)

$g(x) = 6|x|$

53. Parent function: $f(x) = x^2$

Reflection in the x-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}x^2$

54. Parent function: $y = [\![x]\!]$

Horizontal stretch (each x-value is multiplied by 2)

$g(x) = [\![\frac{1}{2}x]\!]$

55. Parent function: $f(x) = \sqrt{x}$

Reflection in the y-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = \frac{1}{2}\sqrt{-x}$

56. Parent function: $f(x) = |x|$

Reflection in the x-axis, vertical shift of 2 units downward, vertical stretch (each y-value is multiplied by 2)

$g(x) = -2|x| - 2$

57. Parent function: $f(x) = x^3$

Reflection in the x-axis, horizontal shift 2 units to the right and a vertical shift 2 units upward

$g(x) = -(x - 2)^3 + 2$

58. Parent function: $f(x) = |x|$

Horizontal shift of 4 units to the left and a vertical shift of 2 units downward

$g(x) = |x + 4| - 2$

59. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 3 units downward

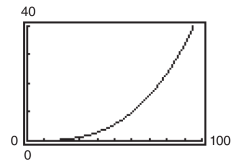
$g(x) = -\sqrt{x} - 3$

60. Parent function: $f(x) = x^2$

Horizontal shift of 2 units to the right and a vertical shift of 4 units upward

$g(x) = (x - 2)^2 + 4$

61. (a)

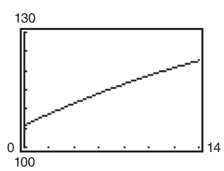


(b) $H(x) = 0.00004636x^3$

$$H\left(\frac{x}{1.6}\right) = 0.00004636\left(\frac{x}{1.6}\right)^3$$
$$= 0.00004636\left(\frac{x^3}{4.096}\right)$$
$$= 0.0000113184x^3 = 0.00001132x^3$$

The graph of $H\left(\frac{x}{1.6}\right)$ is a horizontal stretch of the graph of $H(x)$.

62. (a) The graph of $N(x) = -0.023(x - 33.12)^2 + 131$ is a reflection in the x-axis, a vertical shrink, a horizontal shift 33.12 units to the right and a vertical shift 131 units upward of the parent graph $f(x) = x^2$.



(b) The average rate of change from $t = 0$ to $t = 14$ is given by the following.

$$\frac{N(14) - N(0)}{10 - 3} \approx \frac{122.591 - 105.770}{7}$$
$$= \frac{16.821}{14}$$
$$\approx 1.202$$

The number of households in the United States increased by an average of 1.202 million or 1,202,000 households each year from 2000 to 2014.

(c) Let $t = 22$:

$$N(22) = -0.023(22 - 33.12)^2 + 131$$
$$\approx 128.156$$

In 2022, the number of households in the United States will be about 128.2 million households. Answers will vary. *Sample answer:* Yes, because the number of households has been increasing on average.

63. False. $y = f(-x)$ is a reflection in the y-axis.

64. False. $y = -f(x)$ is a reflection in the x-axis.

65. True. Because $|x| = |-x|$, the graphs of $f(x) = |x| + 6$ and $f(x) = |-x| + 6$ are identical.

66. False. The point $(-2, -61)$ lies on the transformation.

67. $y = f(x + 2) - 1$

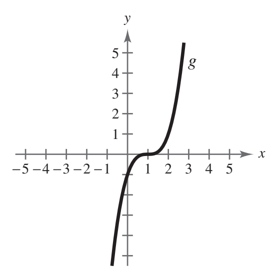
Horizontal shift 2 units to the left and a vertical shift 1 unit downward

$$(0, 1) \rightarrow (0 - 2, 1 - 1) = (-2, 0)$$
$$(1, 2) \rightarrow (1 - 2, 2 - 1) = (-1, 1)$$
$$(2, 3) \rightarrow (2 - 2, 3 - 1) = (0, 2)$$

68. (a) Answers will vary. *Sample answer*: To graph $f(x) = 3x^2 - 4x + 1$ use the point-plotting method since it is not written in a form that is easily identified by a sequence of translations of the parent function $y = x^2$.

(b) Answers will vary. *Sample answer*: To graph $f(x) = 2(x - 1)^2 - 6$ use the method of translating the parent function $y = x^2$ since it is written in a form such that a sequence of translations is easily identified.

69. Since the graph of $g(x)$ is a horizontal shift one unit to the right of $f(x) = x^3$, the equation should be $g(x) = (x - 1)^3$ and not $g(x) = (x + 1)^3$.



70. (a) Increasing on the interval $(-2, 1)$ and decreasing on the intervals $(-\infty, -2)$ and $(1, \infty)$

(b) Increasing on the interval $(-1, 2)$ and decreasing on the intervals $(-\infty, -1)$ and $(2, \infty)$

(c) Increasing on the intervals $(-\infty, -1)$ and $(2, \infty)$ and decreasing on the interval $(-1, 2)$

(d) Increasing on the interval $(0, 3)$ and decreasing on the intervals $(-\infty, 0)$ and $(3, \infty)$

(e) Increasing on the intervals $(-\infty, 1)$ and $(4, \infty)$ and decreasing on the interval $(1, 4)$

71. (a) The profits were only $\frac{3}{4}$ as large as expected:

$g(t) = \frac{3}{4}f(t)$

(b) The profits were \$10,000 greater than predicted:

$g(t) = f(t) + 10{,}000$

(c) There was a two-year delay: $g(t) = f(t - 2)$

72. No. $g(x) = -x^4 - 2$. Yes. $h(x) = -(x - 3)^4$

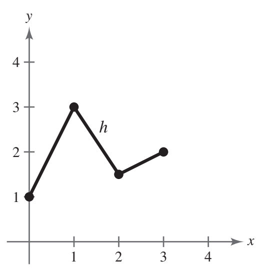
Section 2.6 Combinations of Functions: Composite Functions

1. addition; subtraction; multiplication; division

2. composition

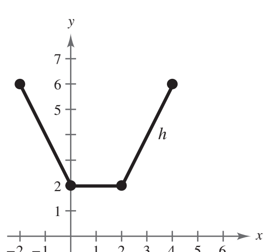
3.

x	0	1	2	3
f	2	3	1	2
g	−1	0	$\frac{1}{2}$	0
$f + g$	1	3	$\frac{3}{2}$	2



4.

x	−2	0	1	2	4
f	2	0	1	2	4
g	4	2	1	0	2
$f + g$	6	2	2	2	6



5. $f(x) = x + 2, g(x) = x - 2$

(a) $(f + g)(x) = f(x) + g(x)$
$= (x + 2) + (x - 2)$
$= 2x$

(b) $(f - g)(x) = f(x) - g(x)$
$= (x + 2) - (x - 2)$
$= 4$

(c) $(fg)(x) = f(x) \cdot g(x)$
$= (x + 2)(x - 2)$
$= x^2 - 4$

(d) $\left(\frac{f}{x}\right)(x) = \frac{f(x)}{g(x)} = \frac{x + 2}{x - 2}$

Domain: all real numbers x except $x = 2$

6. $f(x) = 2x - 5, g(x) = 2 - x$

(a) $(f + g)(x) = 2x - 5 + 2 - x = x - 3$

(b) $(f - g)(x) = 2x - 5 - (2 - x)$
$= 2x - 5 - 2 + x$
$= 3x - 7$

(c) $(fg)(x) = (2x - 5)(2 - x)$
$= 4x - 2x^2 - 10 + 5x$
$= -2x^2 + 9x - 10$

(d) $\left(\frac{f}{g}\right)(x) = \frac{2x - 5}{2 - x}$

Domain: all real numbers x except $x = 2$

7. $f(x) = x^2, g(x) = 4x - 5$

(a) $(f + g)(x) = f(x) + g(x)$
$= x^2 + (4x - 5)$
$= x^2 + 4x - 5$

(b) $(f - g)(x) = f(x) - g(x)$
$= x^2 - (4x - 5)$
$= x^2 - 4x + 5$

(c) $(fg)(x) = f(x) \cdot g(x)$
$= x^2(4x - 5)$
$= 4x^3 - 5x^2$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

$= \dfrac{x^2}{4x - 5}$

Domain: all real numbers x except $x = \dfrac{5}{4}$

8. $f(x) = 3x + 1, g(x) = x^2 - 16$

(a) $(f + g)(x) = f(x) + g(x)$

$= 3x + 1 + x^2 - 16$

$= x^2 + 3x - 15$

(b) $(f - g)(x) = f(x) - g(x)$

$= 3x + 1 - (x^2 - 16)$

$= -x^2 + 3x + 17$

(c) $(fg)(x) = f(x) \cdot g(x)$

$= (3x + 1)(x^2 - 16)$

$= 3x^3 + x^2 - 48x - 16$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x + 1}{x^2 - 16}$

Domain: all real numbers x, except $x \neq \pm 4$

9. $f(x) = x^2 + 6, g(x) = \sqrt{1 - x}$

(a) $(f + g)(x) = f(x) + g(x) = x^2 + 6 + \sqrt{1 - x}$

(b) $(f - g)(x) = f(x) - g(x) = x^2 + 6 - \sqrt{1 - x}$

(c) $(fg)(x) = f(x) \cdot g(x) = (x^2 + 6)\sqrt{1 - x}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 + 6}{\sqrt{1 - x}} = \dfrac{(x^2 + 6)\sqrt{1 - x}}{1 - x}$

Domain: $x < 1$

10. $f(x) = \sqrt{x^2 - 4}, g(x) = \dfrac{x^2}{x^2 + 1}$

(a) $(f + g)(x) = \sqrt{x^2 - 4} + \dfrac{x^2}{x^2 + 1}$

(b) $(f - g)(x) = \sqrt{x^2 - 4} - \dfrac{x^2}{x^2 + 1}$

(c) $(fg)(x) = \sqrt{x^2 - 4}\left(\dfrac{x^2}{x^2 + 1}\right) = \dfrac{x^2\sqrt{x^2 - 4}}{x^2 + 1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \sqrt{x^2 - 4} \div \dfrac{x^2}{x^2 + 1}$

$= \dfrac{(x^2 + 1)\sqrt{x^2 - 4}}{x^2}$

Domain: $x^2 - 4 \geq 0$

$x^2 \geq 4 \Rightarrow x \geq 2$ or $x \leq -2$

$|x| \geq 2$

11. $f(x) = \dfrac{x}{x + 1}, g(x) = x^3$

(a) $(f + g)(x) = \dfrac{x}{x + 1} + x^3 = \dfrac{x + x^4 + x^3}{x + 1}$

(b) $(f - g)(x) = \dfrac{x}{x + 1} - x^3 = \dfrac{x - x^4 - x^3}{x + 1}$

(c) $(fg)(x) = \dfrac{x}{x + 1} \cdot x^3 = \dfrac{x^4}{x + 1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{x + 1} \div x^3 = \dfrac{x}{x + 1} \cdot \dfrac{1}{x^3} = \dfrac{1}{x^2(x + 1)}$

Domain: all real numbers x except $x = 0$ and $x = -1$

12. $f(x) = \dfrac{2}{x}, g(x) = \dfrac{1}{x^2 - 1}$

(a) $(f + g)(x) = \dfrac{2}{x} + \dfrac{1}{x^2 - 1} = \dfrac{2(x^2 - 1) + x}{x(x^2 - 1)} = \dfrac{2x^2 + x - 2}{x(x^2 - 1)}$

(b) $(f - g)(x) = \dfrac{2}{x} - \dfrac{1}{x^2 - 1} = \dfrac{2(x^2 - 1) - x}{x(x^2 - 1)} = \dfrac{2x^2 - x - 2}{x(x^2 - 1)}$

(c) $(fg)(x) = \left(\dfrac{2}{x}\right)\left(\dfrac{1}{x^2 - 1}\right) = \dfrac{2}{x(x^2 - 1)}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{2}{x} \div \frac{1}{x^2 - 1}$

$$= \frac{\frac{2}{x}}{\frac{1}{x^2 - 1}}$$

$$= \left(\frac{2}{x}\right)(x^2 - 1)$$

$$= \frac{2(x^2 - 1)}{x}$$

Domain: all real numbers x, $x \neq 0, \pm 1$.

For Exercises 13–24, $f(x) = x + 3$ and $g(x) = x^2 - 2$.

13. $(f + g)(2) = f(2) + g(2)$
$= (2 + 3) + (2^2 - 2)$
$= 7$

14. $(f + g)(-1) = f(-1) + g(-1)$
$= (-1 + 3) + ((-1)^2 - 2)$
$= 1$

15. $(f - g)(0) = f(0) - g(0)$
$= (0 + 3) - ((0)^2 - 2)$
$= 5$

16. $(f - g)(1) = f(1) - g(1)$
$= (1 + 3) - ((1)^2 - 2)$
$= 5$

17. $(f - g)(3t) = f(3t) - g(3t)$
$= ((3t) + 3) - ((3t)^2 - 2)$
$= 3t + 3 - (9t^2 - 2)$
$= -9t^2 + 3t + 5$

18. $(f + g)(t - 2) = f(t - 2) + g(t - 2)$
$= ((t - 2) + 3) + ((t - 2)^2 - 2)$
$= t + 1 + (t^2 - 4t + 4 - 2)$
$= t^2 - 3t + 3$

19. $(fg)(6) = f(6)g(6)$
$= ((6) + 3)((6)^2 - 2)$
$= (9)(34)$
$= 306$

20. $(fg)(-6) = f(-6)g(-6)$
$= ((-6) + 3)((-6)^2 - 2)$
$= (-3)(34)$
$= -102$

21. $(f/g)(5) = f(5) \,/\, g(5)$
$= ((5) + 3) \,/\, ((5)^2 - 2)$
$= \frac{8}{23}$

22. $(f/g)(0) = f(0) \,/\, g(0)$
$= ((0) + 3) \,/\, ((0)^2 - 2)$
$= -\frac{3}{2}$

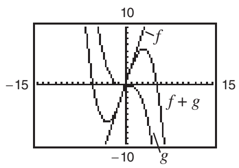
23. $(f/g)(-1) - g(3) = f(-1) \,/\, g(-1) - g(3)$
$= ((-1) + 3) \,/\, ((-1)^2 - 2) - ((3)^2 - 2)$
$= (2/-1) - 7$
$= -2 - 7 = -9$

$$\begin{aligned}(fg)(5) + f(4) &= f(5)g(5) + f(4)\\ &= ((5) + 3)((5)^2 - 2) + ((4) + 3)\\ &= (8)(23) + 7\\ &= 184 + 7 = 191\end{aligned}$$

24.

25. $f(x) = 3x,\ g(x) = -\dfrac{x^3}{10}$

$(f + g)(x) = 3x - \dfrac{x^3}{10}$

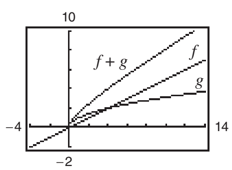


For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

26. $f(x) = \dfrac{x}{2},\ g(x) = \sqrt{x}$

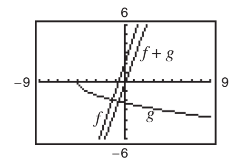
$(f + g)(x) = \dfrac{x}{2} + \sqrt{x}$



$g(x)$ contributes most to the magnitude of the sum for $0 \le x \le 2$. $f(x)$ contributes most to the magnitude of the sum for $x > 6$.

27. $f(x) = 3x + 2,\ g(x) = -\sqrt{x + 5}$

$(f + g)x = 3x - \sqrt{x + 5} + 2$

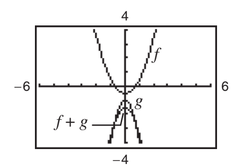


For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $f(x)$ contributes most to the magnitude.

28. $f(x) = x^2 - \frac{1}{2},\ g(x) = -3x^2 - 1$

$(f + g)(x) = -2x^2 - \frac{3}{2}$



For $0 \le x \le 2$, $g(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

29. $f(x) = x + 8,\ g(x) = x - 3$

(a) $(f \circ g)(x) = f(g(x)) = f(x - 3) = (x - 3) + 8 = x + 5$

(b) $(g \circ f)(x) = g(f(x)) = g(x + 8) = (x + 8) - 3 = x + 5$

(c) $(g \circ g)(x) = g(g(x)) = g(x - 3) = (x - 3) - 3 = x - 6$

30. $f(x) = -4x,\ g(x) = x + 7$

(a) $(f \circ g)(x) = f(g(x)) = f(x + 7) = -4(x + 7) = -4x - 28$

(b) $(g \circ f)(x) = g(f(x)) = g(-4x) = (-4x) + 7 = -4x + 7$

(c) $(g \circ g)(x) = g(g(x)) = g(x + 7) = (x + 7) + 7 = x + 14$

31. $f(x) = x^2, g(x) = x - 1$

(a) $(f \circ g)(x) = f(g(x)) = f(x - 1) = (x - 1)^2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 1$

(c) $(g \circ g)(x) = g(g(x)) = g(x - 1) = x - 2$

32. $f(x) = 3x, g(x) = x^4$

(a) $(f \circ g)(x) = f(g(x)) = f(x^4) = 3(x^4) = 3x^4$

(b) $(g \circ f)(x) = g(f(x)) = g(3x) = (3x)^4 = 81x^4$

(c) $(g \circ g)(x) = g(g(x)) = g(x^4) = (x^4)^4 = x^{16}$

33. $f(x) = \sqrt[3]{x - 1}, g(x) = x^3 + 1$

(a) $(f \circ g)(x) = f(g(x))$
$= f(x^3 + 1)$
$= \sqrt[3]{(x^3 + 1) - 1}$
$= \sqrt[3]{x^3} = x$

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt[3]{x - 1})$
$= (\sqrt[3]{x - 1})^3 + 1$
$= (x - 1) + 1 = x$

(c) $(g \circ g)(x) = g(g(x))$
$= g(x^3 + 1)$
$= (x^3 + 1)^3 + 1$
$= x^9 + 3x^6 + 3x^3 + 2$

34. $f(x) = x^3, g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^3 = \dfrac{1}{x^3}$

(b) $(g \circ f)(x) = g(f(x)) = g(x^3) = \dfrac{1}{x^3}$

(c) $(g \circ g)(x) = g(g(x)) = g\left(\dfrac{1}{x}\right) = x$

35. $f(x) = \sqrt{x + 4}$ Domain: $x \geq -4$

$g(x) = x^2$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 + 4}$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt{x + 4}) = (\sqrt{x + 4})^2 = x + 4$

Domain: $x \geq -4$

36. $f(x) = \sqrt[3]{x - 5}$ Domain: all real numbers x

$g(x) = x^3 + 1$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x))$
$= f(x^3 + 1)$
$= \sqrt[3]{x^3 + 1 - 5}$
$= \sqrt[3]{x^3 - 4}$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt[3]{x - 5})$
$= (\sqrt[3]{x - 5})^3 + 1$
$= x - 5 + 1 = x - 4$

Domain: all real numbers x

37. $f(x) = x^3$ Domain: all real numbers x

$g(x) = x^{2/3}$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x^{2/3}) = (x^{2/3})^3 = x^2$

Domain: all real numbers x.

(b) $(g \circ f)(x) = g(f(x)) = g(x^3) = (x^3)^{2/3} = x^2$

Domain: all real numbers x.

38. $f(x) = x^5$ Domain: all real numbers x

$g(x) = \sqrt[4]{x} = x^{1/4}$ Domain: all real numbers $x \geq 0$

(a) $(f \circ g)(x) = f(g(x)) = f(x^{1/4}) = (x^{1/4})^5 = x^{5/4}$

Domain: all real numbers $x \geq 0$.

(b) $(g \circ f)(x) = g(f(x)) = g(x^5) = (x^5)^{1/4} = x^{5/4}$

Domain: all real numbers $x \geq 0$.

39. $f(x) = |x|$ Domain: all real numbers x

$g(x) = x + 6$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 6) = |x + 6|$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(|x|) = |x| + 6$

Domain: all real numbers x

40. $f(x) = |x - 4|$ Domain: all real numbers x

$g(x) = 3 - x$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(3 - x) = |(3 - x) - 4| = |-x - 1|$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(|x - 4|) = 3 - (|x - 4|) = 3 - |x - 4|$

Domain: all real numbers x

41. $f(x) = \frac{1}{x}$ Domain: all real numbers x except $x = 0$

$g(x) = x + 3$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 3) = \frac{1}{x + 3}$

Domain: all real numbers x except $x = -3$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{x} + 3$

Domain: all real numbers x except $x = 0$

42. $f(x) = \frac{3}{x^2 - 1}$ Domain: all real numbers x except $x = \pm 1$

$g(x) = x + 1$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 1) = \frac{3}{(x + 1)^2 - 1} = \frac{3}{x^2 + 2x + 1 - 1} = \frac{3}{x^2 + 2x}$

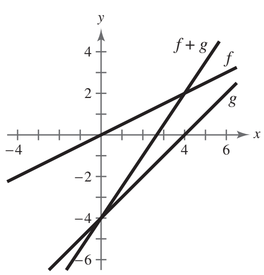
Domain: all real numbers x except $x = 0$ and $x = -2$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{3}{x^2 - 1}\right) = \frac{3}{x^2 - 1} + 1 = \frac{3 + x^2 - 1}{x^2 - 1} = \frac{x^2 + 2}{x^2 - 1}$

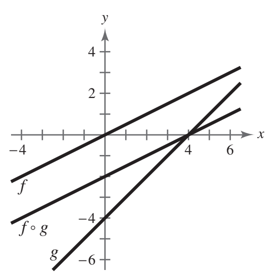
Domain: all real numbers x except $x = \pm 1$

43. $f(x) = \frac{1}{2}x, g(x) = x - 4$

(a)

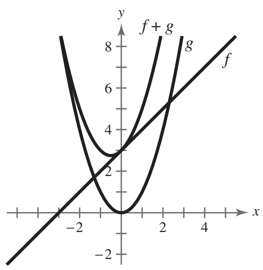


(b)

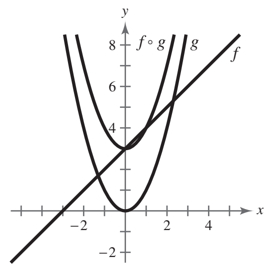


44. $f(x) = x + 3, g(x) = x^2$

(a)



(b)



45. (a) $(f + g)(3) = f(3) + g(3) = 2 + 1 = 3$

(b) $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{0}{2} = 0$

46. (a) $(f - g)(1) = f(1) - g(1) = 2 - 3 = -1$

(b) $(fg)(4) = f(4) \cdot g(4) = 4 \cdot 0 = 0$

47. (a) $(f \circ g)(2) = f(g(2)) = f(2) = 0$

(b) $(g \circ f)(2) = g(f(2)) = g(0) = 4$

48. (a) $(f \circ g)(1) = f(g(1)) = f(3) = 2$

(b) $(g \circ f)(3) = g(f(3)) = g(2) = 2$

49. $h(x) = (2x^2 + 1)^2$

One possibility: Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$.

50. $h(x) = (1 - x)^3$

One possibility: Let $g(x) = 1 - x$ and $f(x) = x^3$, then $(f \circ g)(x) = h(x)$.

51. $h(x) = \sqrt[3]{x^2 - 4}$

One possibility: Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$.

52. $h(x) = \sqrt{9 - x}$

One possibility: Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$, then $(f \circ g)(x) = h(x)$.

53. $h(x) = \dfrac{1}{x + 2}$

One possibility: Let $f(x) = 1/x$ and $g(x) = x + 2$, then $(f \circ g)(x) = h(x)$.

54. $h(x) = \dfrac{4}{(5x + 2)^2}$

One possibility: Let $g(x) = 5x + 2$ and $f(x) = \dfrac{4}{x^2}$, then $(f \circ g)(x) = h(x)$.

55. $h(x) = \dfrac{-x^2 + 3}{4 - x^2}$

One possibility: Let $f(x) = \dfrac{x + 3}{4 + x}$ and $g(x) = -x^2$, then $(f \circ g)(x) = h(x)$.

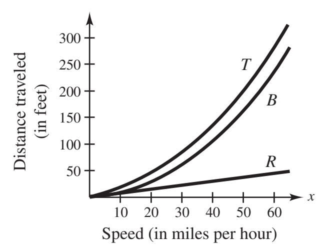
56. $h(x) = \dfrac{27x^3 + 6x}{10 - 27x^3}$

One possibility: Let $g(x) = x^3$ and

$f(x) = \dfrac{27x + 6\sqrt[3]{x}}{10 - 27x}$, then $(f \circ g)(x) = h(x)$.

57. (a) $T(x) = R(x) + B(x) = \frac{3}{4}x + \frac{1}{15}x^2$

(b)



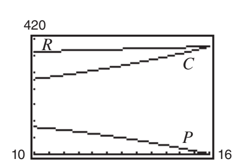
(c) $B(x)$; As x increases, $B(x)$ increases at a faster rate.

58. (a) $P = R - C$

$= (341 + 3.2t) - (254 - 9t + 1.1t^2)$

$= -1.1t^2 + 12.2t + 87$

(b)



59. (a) $c(t) = \dfrac{b(t) - d(t)}{p(t)} \times 100$

(b) $c(16)$ represents the percent change in the population due to births and deaths in the year 2016.

60. (a) $p(t) = d(t) + c(t)$

(b) $p(16)$ represents the number of dogs and cats in 2016.

(c) $h(t) = \dfrac{p(t)}{n(t)} = \dfrac{d(t) + c(t)}{n(t)}$

$h(t)$ represents the number of dogs and cats per capita.

61. (a) $r(x) = \dfrac{x}{2}$

(b) $A(r) = \pi r^2$

(c) $(A \circ r)(x) = A(r(x)) = A\left(\dfrac{x}{2}\right) = \pi\left(\dfrac{x}{2}\right)^2$

$(A \circ r)(x)$ represents the area of the circular base of the tank on the square foundation with side length x.

62. (a)
$$\begin{aligned} N(T(t)) &= N(3t + 2) \\ &= 10(3t + 2)^2 - 20(3t + 2) + 600 \\ &= 10(9t^2 + 12t + 4) - 60t - 40 + 600 \\ &= 90t^2 + 60t + 600 \\ &= 30(3t^2 + 2t + 20), \quad 0 \le t \le 6 \end{aligned}$$

This represents the number of bacteria in the food as a function of time.

(b) Use $t = 0.5$.

$N(T(0.5)) = 30\left(3(0.5)^2 + 2(0.5) + 20\right) = 652.5$

After half an hour, there will be about 653 bacteria.

(c)
$$\begin{aligned} 30(3t^2 + 2t + 20) &= 1500 \\ 3t^2 + 2t + 20 &= 50 \\ 3t^2 + 2t - 30 &= 0 \end{aligned}$$

By the Quadratic Formula, $t \approx -3.513$ or 2.846.

Choosing the positive value for t, you have $t \approx 2.846$ hours.

63. (a) $f(g(x)) = f(0.03x) = 0.03x - 500{,}000$

(b) $g(f(x)) = g(x - 500{,}000) = 0.03(x - 500{,}000)$

$g(f(x))$ represents your bonus of 3% of an amount over \$500,000.

64. (a) $R(p) = p - 2000$ the cost of the car after the factory rebate.

(b) $S(p) = 0.9p$ the cost of the car with the dealership discount.

(c)
$$\begin{aligned} (R \circ S)(p) &= R(0.9p) = 0.9p - 2000 \\ (S \circ R)(p) &= S(p - 2000) \\ &= 0.9(p - 2000) = 0.9p - 1800 \end{aligned}$$

$(R \circ S)(p)$ represents the factory rebate *after* the dealership discount.

$(S \circ R)(p)$ represents the dealership discount after the factory rebate.

(d)
$$\begin{aligned} (R \circ S)(p) &= (R \circ S)(20{,}500) \\ &= 0.9(20{,}500) - 2000 = \$16{,}450 \\ (S \circ R)(p) &= (S \circ R)(20{,}500) \\ &= 0.9(20{,}500) - 1800 = \$16{,}650 \end{aligned}$$

$(R \circ S)(20{,}500)$ yields the lower cost because 10% of the price of the car is more than \$2000.

65. False. $(f \circ g)(x) = 6x + 1$ and $(g \circ f)(x) = 6x + 6$

66. True. $(f \circ g)(c)$ is defined only when $g(c)$ is in the domain of f.

67. Let O = oldest sibling, M = middle sibling, Y = youngest sibling.

Then the ages of each sibling can be found using the equations:

$O = 2M$

$M = \frac{1}{2}Y + 6$

(a) $O(M(Y)) = 2\left(\frac{1}{2}(Y) + 6\right) = 12 + Y$; Answers will vary.

(b) Oldest sibling is 16: $O = 16$

Middle sibling:
$$\begin{aligned} O &= 2M \\ 16 &= 2M \\ M &= 8 \text{ years old} \end{aligned}$$

Youngest sibling:
$$\begin{aligned} M &= \tfrac{1}{2}Y + 6 \\ 8 &= \tfrac{1}{2}Y + 6 \\ 2 &= \tfrac{1}{2}Y \\ Y &= 4 \text{ years old} \end{aligned}$$

68. (a) $Y(M(O)) = 2\left(\frac{1}{2}O\right) - 12 = O - 12$; Answers will vary.

(b) Youngest sibling is $2 \to Y = 2$

Middle sibling: $M = \frac{1}{2}Y + 6$

$M = \frac{1}{2}(2) + 6$

$M = 7$ years old

Oldest sibling: $O = 2M$

$O = 2(7)$

$O = 14$ years old

69. Let $f(x)$ and $g(x)$ be two odd functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)][-g(x)] \quad \text{because } f \text{ and } g \text{ are odd} \\ &= f(x)g(x) \\ &= h(x). \end{aligned}$$

So, $h(x)$ is even.

Let $f(x)$ and $g(x)$ be two even functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= f(x)g(x) \quad \text{because } f \text{ and } g \text{ are even} \\ &= h(x). \end{aligned}$$

So, $h(x)$ is even.

70. Let $f(x)$ be an odd function, $g(x)$ be an even function, and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)]g(x) \quad \text{because } f \text{ is odd and } g \text{ is even} \\ &= -f(x)g(x) \\ &= -h(x). \end{aligned}$$

So, *h* is odd and the product of an odd function and an even function is odd.

71. (a) Answer not unique. *Sample answer*: $f(x) = x + 3,\ g(x) = x + 2$

$(f \circ g)(x) = f(g(x)) = (x + 2) + 3 = x + 5$

$(g \circ f)(x) = g(f(x)) = (x + 3) + 2 = x + 5$

(b) Answer not unique. *Sample answer*: $f(x) = x^2$, $g(x) = x^3$

$(f \circ g)(x) = f(g(x)) = (x^3)^2 = x^6$

$(g \circ f)(x) = g(f(x)) = (x^2)^3 = x^6$

72. (a) $f(p)$: matches L_2; For example, an original price of $p = \$15.00$ corresponds to a sale price of $S = \$7.50$.

(b) $g(p)$: matches L_1; For example an original price of $p = \$20.00$ corresponds to a sale price of $S = \$15.00$.

(c) $(g \circ f)(p)$: matches L_4; This function represents applying a 50% discount to the original price *p*, then subtracting a \$5 discount.

(d) $(f \circ g)(p)$ matches L_3; This function represents subtracting a \$5 discount from the original price *p*, then applying a 50% discount.

73. (a) $g(x) = \frac{1}{2}[f(x) + f(-x)]$

To determine if $g(x)$ is even, show $g(-x) = g(x)$.

$$g(-x) = \frac{1}{2}[f(-x) + f(-(-x))] = \frac{1}{2}[f(-x) + f(x)] = \frac{1}{2}[f(x) + f(-x)] = g(x) \checkmark$$

$h(x) = \frac{1}{2}[f(x) - f(-x)]$

To determine if $h(x)$ is odd show $h(-x) = -h(x)$.

$$h(-x) = \frac{1}{2}[f(-x) - f(-(-x))] = \frac{1}{2}[f(-x) - f(x)] = -\frac{1}{2}[f(x) - f(-x)] = -h(x) \checkmark$$

(b) Let $f(x) = a$ function

$f(x) =$ even function + odd function.

Using the result from part (a) $g(x)$ is an even function and $h(x)$ is an odd function.

$$f(x) = g(x) + h(x) = \frac{1}{2}\left[f(x) + f(-x)\right] + \frac{1}{2}\left[f(x) - f(-x)\right] = \frac{1}{2}f(x) + \frac{1}{2}f(-x) + \frac{1}{2}f(x) - \frac{1}{2}f(-x) = f(x) \checkmark$$

(c) $f(x) = x^2 - 2x + 1$

$$f(x) = g(x) + h(x)$$

$$\begin{aligned} g(x) &= \frac{1}{2}\left[f(x) + f(-x)\right] = \frac{1}{2}\left[x^2 - 2x + 1 + (-x)^2 - 2(-x) + 1\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 + x^2 + 2x + 1\right] = \frac{1}{2}\left[2x^2 + 2\right] = x^2 + 1 \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{2}\left[f(x) - f(-x)\right] = \frac{1}{2}\left[x^2 - 2x + 1 - \left((-x)^2 - 2(-x) + 1\right)\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 - x^2 - 2x - 1\right] = \frac{1}{2}[-4x] = -2x \end{aligned}$$

$$f(x) = \left(x^2 + 1\right) + (-2x)$$

$$k(x) = \frac{1}{x + 1}$$

$$k(x) = g(x) + h(x)$$

$$\begin{aligned} g(x) &= \frac{1}{2}\left[k(x) + k(-x)\right] = \frac{1}{2}\left[\frac{1}{x + 1} + \frac{1}{-x + 1}\right] \\ &= \frac{1}{2}\left[\frac{1 - x + x + 1}{(x + 1)(1 - x)}\right] = \frac{1}{2}\left[\frac{2}{(x + 1)(1 - x)}\right] \\ &= \frac{1}{(x + 1)(1 - x)} = \frac{-1}{(x + 1)(x - 1)} \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{2}\left[k(x) - k(-x)\right] = \frac{1}{2}\left[\frac{1}{x + 1} - \frac{1}{1 - x}\right] \\ &= \frac{1}{2}\left[\frac{1 - x - (x + 1)}{(x + 1)(1 - x)}\right] = \frac{1}{2}\left[\frac{-2x}{(x + 1)(1 - x)}\right] \\ &= \frac{-x}{(x + 1)(1 - x)} = \frac{x}{(x + 1)(x - 1)} \end{aligned}$$

$$k(x) = \left(\frac{-1}{(x + 1)(x - 1)}\right) + \left(\frac{x}{(x + 1)(x - 1)}\right)$$

Section 2.7 Inverse Functions

1. inverse

2. f^{-1}

3. range; domain

4. $y = x$

5. one-to-one

6. Horizontal

7. $f(x) = 6x$

$f^{-1}(x) = \frac{x}{6} = \frac{1}{6}x$

$f(f^{-1}(x)) = f\left(\frac{x}{6}\right) = 6\left(\frac{x}{6}\right) = x$

$f^{-1}(f(x)) = f^{-1}(6x) = \frac{6x}{6} = x$

8. $f(x) = \frac{1}{3}x$

$f^{-1}(x) = 3x$

$f(f^{-1}(x)) = f(3x) = \frac{1}{3}(3x) = x$

$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$

9. $f(x) = 3x + 1$

$f^{-1}(x) = \frac{x - 1}{3}$

$f(f^{-1}(x)) = f\left(\frac{x-1}{3}\right) = 3\left(\frac{x-1}{3}\right) + 1 = x$

$f^{-1}(f(x)) = f^{-1}(3x + 1) = \frac{(3x + 1) - 1}{3} = x$

10. $f(x) = \frac{x - 3}{2}$

$f^{-1}(x) = 2x + 3$

$f(f^{-1}(x)) = f(2x + 3) = \frac{(2x + 3) - 3}{3} = \frac{2x}{2} = x$

$f^{-1}(f(x)) = f^{-1}\left(\frac{x-3}{2}\right) = 2\left(\frac{x-3}{2}\right) + 3$

$= (x - 3) + 3 = x$

11. $f(x) = x^2 - 4, x \geq 0$

$f^{-1}(x) = \sqrt{x + 4}$

$f(f^{-1}(x)) = f\left(\sqrt{x+4}\right) = \left(\sqrt{x+4}\right)^2 - 4 = (x + 4) - 4 = x$

$f^{-1}(f(x)) = f^{-1}(x^2 - 4) = \sqrt{(x^2 - 4) + 4} = \sqrt{x^2} = x$

12. $f(x) = x^2 + 2, x \geq 0$

$f^{-1}(x) = \sqrt{x - 2}$

$f(f^{-1}(x)) = f\left(\sqrt{x-2}\right) = \left(\sqrt{x-2}\right)^2 + 2 = (x - 2) + 2 = x$

$f^{-1}(f(x)) = f^{-1}(x^2 + 2) = \sqrt{(x^2 + 2) - 2} = \sqrt{x^2} = x$

13. $f(x) = x^3 + 1$

$f^{-1}(x) = \sqrt[3]{x - 1}$

$f(f^{-1}(x)) = f\left(\sqrt[3]{x-1}\right) = \left(\sqrt[3]{x-1}\right)^3 + 1 = (x - 1) + 1 = x$

$f^{-1}(f(x)) = f^{-1}(x^3 + 1) = \sqrt[3]{(x^3 + 1) - 1} = \sqrt[3]{x^3} = x$

14. $f(x) = \frac{x^5}{4}$

$f^{-1}(x) = \sqrt[5]{4x}$

$f(f^{-1}(x)) = f\left(\sqrt[5]{4x}\right) = \frac{\left(\sqrt[5]{4x}\right)^5}{4} = \frac{4x}{4} = x$

$f^{-1}(f(x)) = f^{-1}\left(\frac{x^5}{4}\right) = \sqrt[5]{4\left(\frac{x^5}{4}\right)} = \sqrt[5]{x^5} = x$

15. $(f \circ g)(x) = f(g(x)) = f(4x + 9) = \dfrac{4x + 9 - 9}{4} = \dfrac{4x}{4} = x$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{x - 9}{4}\right) = 4\left(\dfrac{x - 9}{4}\right) + 9 = x - 9 + 9 = x$

16. $f(g(x)) = f\left(-\dfrac{2x + 8}{3}\right) = -\dfrac{3}{2}\left(-\dfrac{2x + 8}{3}\right) - 4$

$= \dfrac{1}{2}(2(x + 4)) - 4$

$= x + 4 - 4 = x$

$g(f(x)) = g\left(-\dfrac{3}{2}x - 4\right) = -\dfrac{2\left(-\frac{3}{2}x - 4\right) + 8}{3}$

$= -\dfrac{-3x - 8 + 8}{3}$

$= -\dfrac{-3x}{3}$

$= x$

17. $f(g(x)) = f\left(\sqrt[3]{4x}\right) = \dfrac{\left(\sqrt[3]{4x}\right)^3}{4}$

$= \dfrac{4x}{4}$

$= x$

$g(f(x)) = g\left(\dfrac{x^3}{4}\right) = \sqrt[3]{4\left(\dfrac{x^3}{4}\right)}$

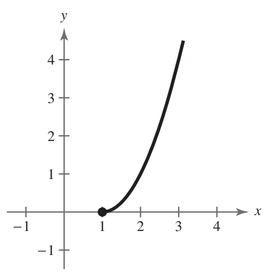
$= \sqrt[3]{x^3}$

$= x$

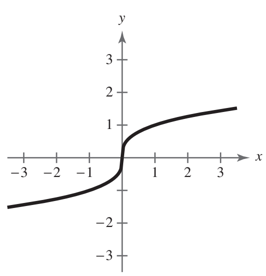
18. $(f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{x - 5}\right) = \left(\sqrt[3]{x - 5}\right)^3 + 5 = x - 5 + 5 = x$

$(g \circ f)(x) = g(f(x)) = g(x^3 + 5) = \sqrt[3]{x^3 + 5 - 5} = \sqrt[3]{x^3} = x$

19.



20.

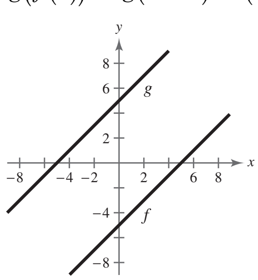


21. $f(x) = x - 5,\ g(x) = x + 5$

(a) $f(g(x)) = f(x + 5) = (x + 5) - 5 = x$

$g(f(x)) = g(x - 5) = (x - 5) + 5 = x$

(b)

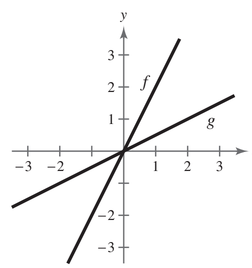


22. $f(x) = 2x,\ g(x) = \dfrac{x}{2}$

(a) $f(g(x)) = f\left(\dfrac{x}{2}\right) = 2\left(\dfrac{x}{2}\right) = x$

$g(f(x)) = g(2x) = \dfrac{2x}{2} = x$

(b)

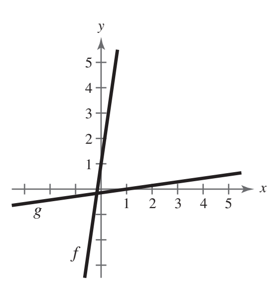


23. $f(x) = 7x + 1,\ g(x) = \dfrac{x - 1}{7}$

(a) $f(g(x)) = f\left(\dfrac{x - 1}{7}\right) = 7\left(\dfrac{x - 1}{7}\right) + 1 = x$

$g(f(x)) = g(7x + 1) = \dfrac{(7x + 1) - 1}{7} = x$

(b)



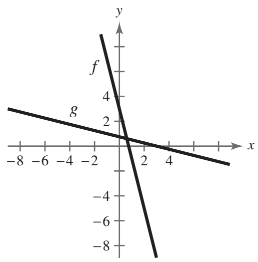
24. $f(x) = 3 - 4x, g(x) = \dfrac{3 - x}{4}$

(a) $f(g(x)) = f\left(\dfrac{3 - x}{4}\right) = 3 - 4\left(\dfrac{3 - x}{4}\right)$

$= 3 - (3 - x) = x$

$g(f(x)) = g(3 - 4x) = \dfrac{3 - (3 - 4x)}{4} = \dfrac{4x}{4} = x$

(b)

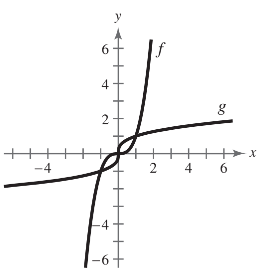


25. $f(x) = x^3, g(x) = \sqrt[3]{x}$

(a) $f(g(x)) = f(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$

$g(f(x)) = g(x^3) = \sqrt[3]{(x^3)} = x$

(b)

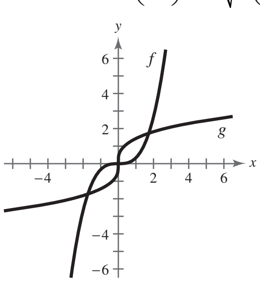


26. $f(x) = \dfrac{x^3}{3}, g(x) = \sqrt[3]{3x}$

(a) $f(g(x)) = f(\sqrt[3]{3x}) = \dfrac{(\sqrt[3]{3x})^3}{3} = \dfrac{3x}{3} = x$

$g(f(x)) = g\left(\dfrac{x^3}{3}\right) = \sqrt[3]{3\left(\dfrac{x^3}{3}\right)} = \sqrt[3]{x^3} = x$

(b)



27. $f(x) = \sqrt{x + 5}, g(x) = x^2 - 5, x \geq 0$

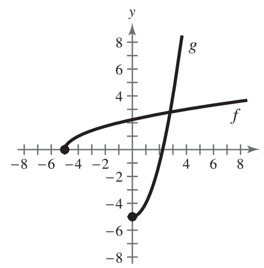
(a) $f(g(x)) = f(x^2 - 5), x \geq 0$

$= \sqrt{(x^2 - 5) + 5} = x$

$g(f(x)) = g(\sqrt{x + 5})$

$= (\sqrt{x + 5})^2 - 5 = x$

(b)



28. $f(x) = 1 - x^3, g(x) = \sqrt[3]{1 - x}$

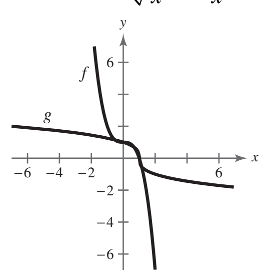
(a) $f(g(x)) = f(\sqrt[3]{1 - x}) = 1 - (\sqrt[3]{1 - x})^3$

$= 1 - (1 - x) = x$

$g(f(x)) = g(1 - x^3) = \sqrt[3]{1 - (1 - x^3)}$

$= \sqrt[3]{x^3} = x$

(b)

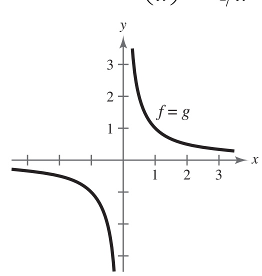


29. $f(x) = \dfrac{1}{x}, g(x) = \dfrac{1}{x}$

(a) $f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

$g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

(b)

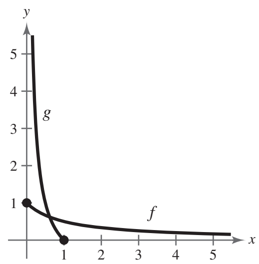


30. $f(x) = \dfrac{1}{1+x}, x \geq 0;\ g(x) = \dfrac{1-x}{x}, 0 < x \leq 1$

(a) $$f(g(x)) = f\left(\frac{1-x}{x}\right) = \frac{1}{1+\left(\frac{1-x}{x}\right)} = \frac{1}{\frac{x}{x}+\frac{1-x}{x}} = \frac{1}{\frac{1}{x}} = x$$

$$g(f(x)) = g\left(\frac{1}{1+x}\right) = \frac{1-\left(\frac{1}{1+x}\right)}{\left(\frac{1}{1+x}\right)} = \frac{\frac{1+x}{1+x}-\frac{1}{1+x}}{\frac{1}{1+x}} = \frac{\frac{x}{1+x}}{\frac{1}{1+x}} = \frac{x}{1+x}\cdot\frac{x+1}{1} = x$$

(b)

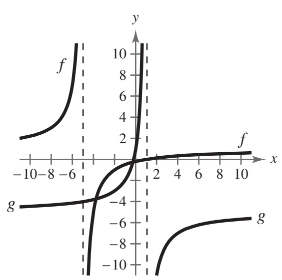


31. $f(x) = \dfrac{x-1}{x+5},\ g(x) = -\dfrac{5x+1}{x-1}$

(a) $$f(g(x)) = f\left(-\frac{5x+1}{x-1}\right) = \frac{\left(-\frac{5x+1}{x-1}-1\right)}{\left(-\frac{5x+1}{x-1}+5\right)}\cdot\frac{x-1}{x-1} = \frac{-(5x+1)-(x-1)}{-(5x+1)+5(x-1)} = \frac{-6x}{-6} = x$$

$$g(f(x)) = g\left(\frac{x-1}{x+5}\right) = -\frac{\left[5\left(\frac{x-1}{x+5}\right)+1\right]}{\left[\frac{x-1}{x+5}-1\right]}\cdot\frac{x+5}{x+5} = -\frac{5(x-1)+(x+5)}{(x-1)-(x+5)} = -\frac{6x}{-6} = x$$

(b)

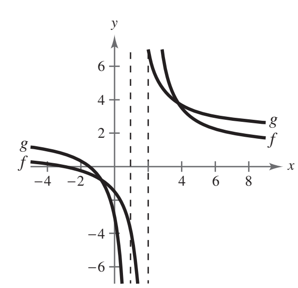


32. $f(x) = \dfrac{x+3}{x-2},\ g(x) = \dfrac{2x+3}{x-1}$

(a) $$f(g(x)) = f\left(\frac{2x+3}{x-1}\right) = \frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2} = \frac{\frac{2x+3+3x-3}{x-1}}{\frac{2x+3-2x+2}{x-1}} = \frac{5x}{5} = x$$

$$g(f(x)) = g\left(\frac{x+3}{x-2}\right) = \frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1} = \frac{\frac{2x+6+3x-6}{x-2}}{\frac{x+3-x+2}{x-2}} = \frac{5x}{5} = x$$

(b)



33. No, $\{(-2, -1), (1, 0), (2, 1), (1, 2), (-2, 3), (-6, 4)\}$ does not represent a function. -2 and 1 are paired with two different values.

34. Yes, $\{(10, -3), (6, -2), (4, -1), (1, 0), (-3, 2), (10, 2)\}$ does represent a function.

35.

x	3	5	7	9	11	13
$f^{-1}(x)$	-1	0	1	2	3	4

36.

x	10	5	0	-5	-10	-15
$f^{-1}(x)$	-3	-2	-1	0	1	2

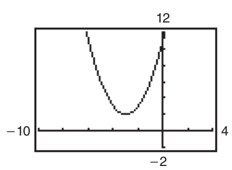
37. Yes, because no horizontal line crosses the graph of *f* at more than one point, *f has* an inverse.

38. No, because some horizontal lines intersect the graph of *f* twice, *f does not* have an inverse.

39. No, because some horizontal lines cross the graph of *f* twice, *f does not* have an inverse.

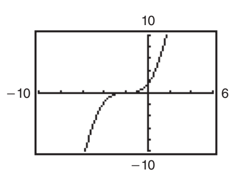
40. Yes, because no horizontal lines intersect the graph of *f* at more than one point, *f has* an inverse.

41. $g(x) = (x + 3)^2 + 2$



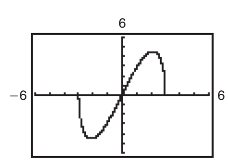
g does not pass the Horizontal Line Test, so *g does not* have an inverse.

42. $f(x) = \frac{1}{5}(x + 2)^3$



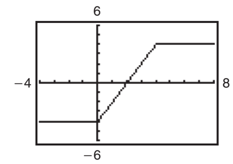
f does pass the Horizontal Line Test, so *f does* have an inverse.

43. $f(x) = x\sqrt{9 - x^2}$



f does not pass the Horizontal Line Test, so *f does not* have an inverse.

44. $h(x) = |x| - |x - 4|$

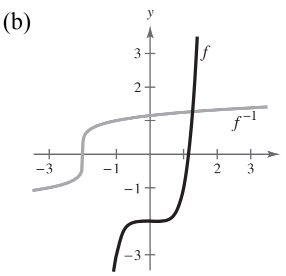


h does not pass the Horizontal Line Test, so *h does not* have an inverse.

45. (a)

$$f(x) = x^5 - 2$$
$$y = x^5 - 2$$
$$x = y^5 - 2$$
$$y = \sqrt[5]{x + 2}$$
$$f^{-1}(x) = \sqrt[5]{x + 2}$$

(b)



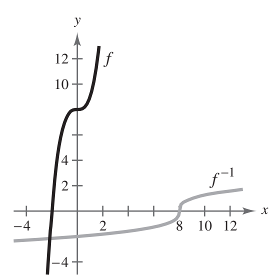
(c) The graph of f^{-1} is the reflection of the graph of *f* in the line $y = x$.

(d) The domains and ranges of *f* and f^{-1} are all real numbers.

46. (a)

$$f(x) = x^3 + 8$$
$$y = x^3 + 8$$
$$x = y^3 + 8$$
$$x - 8 = y^3$$
$$\sqrt[3]{x - 8} = y$$
$$f^{-1}(x) = \sqrt[3]{x - 8}$$

(b)

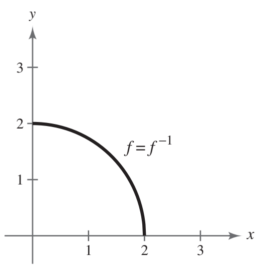


(c) The graph of f^{-1} is the reflection of *f* in the line $y = x$.

(d) The domains and ranges of *f* and f^{-1} are all real numbers.

47. (a)
$$f(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$$
$$y = \sqrt{4 - x^2}$$
$$x = \sqrt{4 - y^2}$$
$$x^2 = 4 - y^2$$
$$y^2 = 4 - x^2$$
$$y = \sqrt{4 - x^2}$$
$$f^{-1}(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$$

(b)

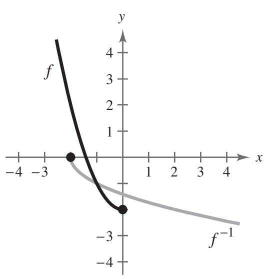


(c) The graph of f^{-1} is the same as the graph of *f*.

(d) The domains and ranges of *f* and f^{-1} are all real numbers *x* such that $0 \le x \le 2$.

48. (a)
$$f(x) = x^2 - 2,\ x \le 0$$
$$y = x^2 - 2$$
$$x = y^2 - 2$$
$$\pm\sqrt{x + 2} = y$$
$$f^{-1}(x) = -\sqrt{x + 2}$$

(b)



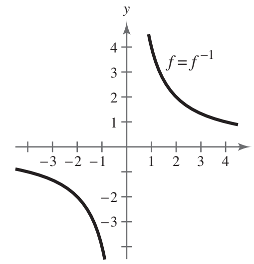
(c) The graph of f^{-1} is the reflection of *f* in the line $y = x$.

(d) $[-2, \infty)$ is the range of *f* and domain of f^{-1}.

$(-\infty, 0]$ is the domain of *f* and the range of f^{-1}.

49. (a)
$$f(x) = \frac{4}{x}$$
$$y = \frac{4}{x}$$
$$x = \frac{4}{y}$$
$$xy = 4$$
$$y = \frac{4}{x}$$
$$f^{-1}(x) = \frac{4}{x}$$

(b)

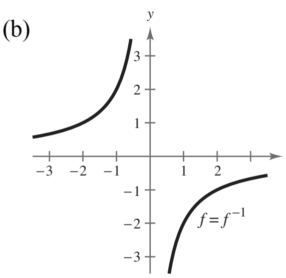


(c) The graph of f^{-1} is the same as the graph of *f*.

(d) The domains and ranges of *f* and f^{-1} are all real numbers except for 0.

50. (a)
$$f(x) = -\frac{2}{x}$$
$$y = -\frac{2}{x}$$
$$x = -\frac{2}{y}$$
$$y = -\frac{2}{x}$$
$$f^{-1}(x) = -\frac{2}{x}$$

(b)

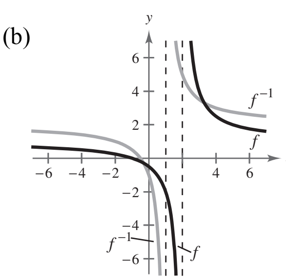


(c) The graphs are the same.

(d) The domains and ranges of *f* and f^{-1} are all real numbers except for 0.

51. (a)
$$f(x) = \frac{x + 1}{x - 2}$$
$$y = \frac{x + 1}{x - 2}$$
$$x = \frac{y + 1}{y - 2}$$
$$x(y - 2) = y + 1$$
$$xy - 2x = y + 1$$
$$xy - y = 2x + 1$$
$$y(x - 1) = 2x + 1$$
$$y = \frac{2x + 1}{x - 1}$$
$$f^{-1}(x) = \frac{2x + 1}{x - 1}$$

(b)



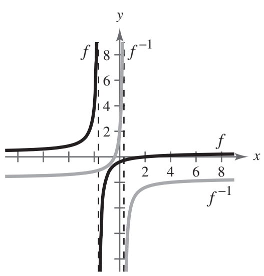
(c) The graph of f^{-1} is the reflection of graph of *f* in the line $y = x$.

(d) The domain of f and the range of f^{-1} is all real numbers except 2.

The range of f and the domain of f^{-1} is all real numbers except 1.

52. (a)
$$f(x) = \frac{x-2}{3x+5}$$
$$y = \frac{x-2}{3x+5}$$
$$x = \frac{y-2}{3y+5}$$
$$3xy + 5x - y + 2 = 0$$
$$3xy - y = -5x - 2$$
$$y(3x-1) = -5x - 2$$
$$y = \frac{-5x-2}{3x-1}$$
$$y = \frac{-5x-2}{3x-1}$$
$$f^{-1}(x) = -\frac{5x+2}{3x-1}$$

(b)



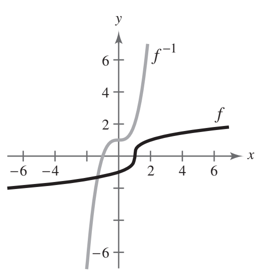
(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domain of f and the range of f^{-1} is all real numbers except $x = -\frac{5}{3}$.

The range of f and the domain of f^{-1} is all real numbers x except $x = \frac{1}{3}$.

53. (a)
$$f(x) = \sqrt[3]{x-1}$$
$$y = \sqrt[3]{x-1}$$
$$x = \sqrt[3]{y-1}$$
$$x^3 = y - 1$$
$$y = x^3 + 1$$
$$f^{-1}(x) = x^3 + 1$$

(b)

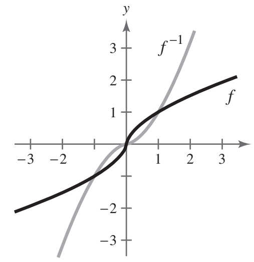


(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

54. (a)
$$f(x) = x^{3/5}$$
$$y = x^{3/5}$$
$$x = y^{3/5}$$
$$x^{5/3} = \left(y^{3/5}\right)^{5/3}$$
$$x^{5/3} = y$$
$$f^{-1}(x) = x^{5/3}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

55.
$$f(x) = x^4$$
$$y = x^4$$
$$x = y^4$$
$$y = \pm\sqrt[4]{x}$$

This does not represent y as a function of x. f does not have an inverse.

56.
$$f(x) = \frac{1}{x^2}$$
$$y = \frac{1}{x^2}$$
$$x = \frac{1}{y^2}$$
$$y^2 = \frac{1}{x}$$
$$y = \pm\sqrt{\frac{1}{x}}$$

This does not represent y as a function of x. f does not have an inverse.

57.
$$g(x) = \frac{x+1}{6}$$
$$y = \frac{x+1}{6}$$
$$x = \frac{y+1}{6}$$
$$6x = y + 1$$
$$y = 6x - 1$$

This is a function of x, so g has an inverse.

$$g^{-1}(x) = 6x - 1$$

58. $f(x) = 3x + 5$

$$y = 3x + 5$$
$$x = 3y + 5$$
$$x - 5 = 3y$$
$$\frac{x - 5}{3} = y$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{x - 5}{3}$$

59. $p(x) = -4$

$$y = -4$$

Because $y = -4$ for all x, the graph is a horizontal line and fails the Horizontal Line Test. p does not have an inverse.

60. $f(x) = 0$

$$y = 0$$

Because $y = 0$ for all x, the graph is a horizontal line and fails the Horizontal Line Test. f does not have an inverse.

61. $f(x) = (x + 3)^2, x \ge -3 \Rightarrow y \ge 0$

$$y = (x + 3)^2, x \ge -3, y \ge 0$$
$$x = (y + 3)^2, y \ge -3, x \ge 0$$
$$\sqrt{x} = y + 3, y \ge -3, x \ge 0$$
$$y = \sqrt{x} - 3, x \ge 0, y \ge -3$$

This is a function of x, so f has an inverse.

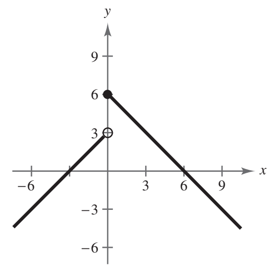
$$f^{-1}(x) = \sqrt{x} - 3, x \ge 0$$

62. $q(x) = (x - 5)^2$

$$y = (x - 5)^2$$
$$x = (y - 5)^2$$
$$\pm\sqrt{x} = y - 5$$
$$5 \pm \sqrt{x} = y$$

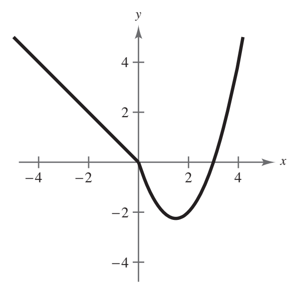
This does not represent y as a function of x, so q does not have an inverse.

63. $f(x) = \begin{cases} x + 3, & x < 0 \\ 6 - x, & x \ge 0 \end{cases}$



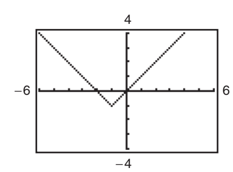
This graph fails the Horizontal Line Test, so f does not have an inverse.

64. $f(x) = \begin{cases} -x, & x \le 0 \\ x^2 - 3x, & x > 0 \end{cases}$



The graph fails the Horizontal Line Test, so f does not have an inverse.

65. $h(x) = |x + 1| - 1$



The graph fails the Horizontal Line Test, so h does not have an inverse.

66. $f(x) = |x - 2|, x \le 2 \Rightarrow y \ge 0$

$$y = |x - 2|, x \le 2, y \ge 0$$
$$x = |y - 2|, y \le 2, x \ge 0$$
$$x = y - 2 \quad \text{or} \quad -x = y - 2$$
$$2 + x = y \quad \text{or} \quad 2 - x = y$$

The portion that satisfies the conditions $y \le 2$ and $x \ge 0$ is $2 - x = y$. This is a function of x, so f has an inverse.

$$f^{-1}(x) = 2 - x, x \ge 0$$

67. $f(x) = \sqrt{2x + 3} \Rightarrow x \geq -\frac{3}{2}, y \geq 0$

$$y = \sqrt{2x + 3}, x \geq -\frac{3}{2}, y \geq 0$$
$$x = \sqrt{2y + 3}, y \geq -\frac{3}{2}, x \geq 0$$
$$x^2 = 2y + 3, x \geq 0, y \geq -\frac{3}{2}$$
$$y = \frac{x^2 - 3}{2}, x \geq 0, y \geq -\frac{3}{2}$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{x^2 - 3}{2}, x \geq 0$$

68. $f(x) = \sqrt{x - 2} \Rightarrow x \geq 2, y \geq 0$

$$y = \sqrt{x - 2}, x \geq 2, y \geq 0$$
$$x = \sqrt{y - 2}, y \geq 2, x \geq 0$$
$$x^2 = y - 2, x \geq 0, y \geq 2$$
$$x^2 + 2 = y, x \geq 0, y \geq 2$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = x^2 + 2, x \geq 0$$

69. $f(x) = \frac{6x + 4}{4x + 5}$

$$y = \frac{6x + 4}{4x + 5}$$
$$x = \frac{6y + 4}{4y + 5}$$
$$x(4y + 5) = 6y + 4$$
$$4xy + 5x = 6y + 4$$
$$4xy - 6y = -5x + 4$$
$$y(4x - 6) = -5x + 4$$
$$y = \frac{-5x + 4}{4x - 6}$$
$$= \frac{5x - 4}{6 - 4x}$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{5x - 4}{6 - 4x}$$

70. The graph of f passes the Horizontal Line Test. So, you know f is one-to-one and has an inverse function.

$$f(x) = \frac{5x - 3}{2x + 5}$$
$$y = \frac{5x - 3}{2x + 5}$$
$$x = \frac{5y - 3}{2y + 5}$$
$$x(2y + 5) = 5y - 3$$
$$2xy + 5x = 5y - 3$$
$$2xy - 5y = -5x - 3$$
$$y(2x - 5) = -(5x + 3)$$
$$y = -\frac{5x + 3}{2x - 5}$$
$$f^{-1}(x) = -\frac{5x + 3}{2x - 5}$$

71. $f(x) = |x + 2|$

domain of $f: x \geq -2$, range of $f: y \geq 0$

$$f(x) = |x + 2|$$
$$y = |x + 2|$$
$$x = y + 2$$
$$x - 2 = y$$

So, $f^{-1}(x) = x - 2$.

domain of $f^{-1}: x \geq 0$, range of $f^{-1}: y \geq -2$

72. $f(x) = |x - 5|$

domain of $f: x \geq 5$, range of $f: y \geq 0$

$$f(x) = |x - 5|$$
$$y = x - 5$$
$$x = y - 5$$
$$x + 5 = y$$

So, $f^{-1}(x) = x + 5$.

domain $f^{-1}: x \geq 0$, range of $f^{-1}: y \geq 5$

73. $f(x) = (x + 6)^2$

domain of $f: x \geq -6$, range of $f: y \geq 0$

$$\begin{aligned} f(x) &= (x + 6)^2 \\ y &= (x + 6)^2 \\ x &= (y + 6)^2 \\ \sqrt{x} &= y + 6 \\ \sqrt{x} - 6 &= y \end{aligned}$$

So, $f^{-1}(x) = \sqrt{x} - 6$.

domain of $f^{-1}: x \geq 0$, range of $f^{-1}: y \geq -6$

74. $f(x) = (x - 4)^2$

domain of $f: x \geq 4$, range of $f: y \geq 0$

$$\begin{aligned} f(x) &= (x - 4)^2 \\ y &= (x - 4)^2 \\ x &= (y - 4)^2 \\ \sqrt{x} &= y - 4 \\ \sqrt{x} + 4 &= y \end{aligned}$$

So, $f^{-1}(x) = \sqrt{x} + 4$.

domain of $f^{-1}: x \geq 0$, range of $f^{-1}: y \geq 4$

75. $f(x) = -2x^2 + 5$

domain of $f: x \geq 0$, range of $f: y \leq 5$

$$\begin{aligned} f(x) &= -2x^2 + 5 \\ y &= -2x^2 + 5 \\ x &= -2y^2 + 5 \\ x - 5 &= -2y^2 \\ 5 - x &= 2y^2 \\ \sqrt{\frac{5 - x}{2}} &= y \\ \frac{\sqrt{5 - x}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} &= y \\ \frac{\sqrt{2(5 - x)}}{2} &= y \end{aligned}$$

So, $f^{-1}(x) = \dfrac{\sqrt{-2(x - 5)}}{2}$.

domain of $f^{-1}(x): x \leq 5$, range of $f^{-1}(x): y \geq 0$

76. $f(x) = \frac{1}{2}x^2 - 1$

domain of $f: x \geq 0$, range of $f: y \geq -1$

$$\begin{aligned} f(x) &= \tfrac{1}{2}x^2 - 1 \\ y &= \tfrac{1}{2}x^2 - 1 \\ x &= \tfrac{1}{2}y^2 - 1 \\ x + 1 &= \tfrac{1}{2}y^2 \\ 2x + 2 &= y^2 \\ \sqrt{2x + 2} &= y \end{aligned}$$

So, $f^{-1}(x) = \sqrt{2x + 2}$.

domain of $f^{-1}: x \geq -1$, range of $f^{-1}: y \geq 0$

77. $f(x) = |x - 4| + 1$

domain of $f: x \geq 4$, range of $f: y \geq 1$

$$\begin{aligned} f(x) &= |x - 4| + 1 \\ y &= x - 3 \\ x &= y - 3 \\ x + 3 &= y \end{aligned}$$

So, $f^{-1}(x) = x + 3$.

domain of $f^{-1}: x \geq 1$, range of $f^{-1}: y \geq 4$

78. $f(x) = -|x - 1| - 2$

domain of $f: x \geq 1$, range of $f: y \leq -2$

$$\begin{aligned} f(x) &= -|x - 1| - 2 \\ y &= -|x - 1| - 2 \\ x &= -(y - 1) - 2 \\ x &= -y - 1 \\ -x - 1 &= y \end{aligned}$$

So, $f^{-1}(x) = -x - 1$.

domain of $f^{-1}: x \leq -2$, range of $f^{-1}: y \geq 1$

In Exercises 79–84, $f(x) = \frac{1}{8}x - 3$, $f^{-1}(x) = 8(x + 3)$, $g(x) = x^3$, $g^{-1}(x) = \sqrt[3]{x}$.

79. $$\begin{aligned} (f^{-1} \circ g^{-1})(1) &= f^{-1}(g^{-1}(1)) \\ &= f^{-1}(\sqrt[3]{1}) \\ &= 8(\sqrt[3]{1} + 3) = 32 \end{aligned}$$

80. $\left(g^{-1} \circ f^{-1}\right)(-3) = g^{-1}\left(f^{-1}(-3)\right)$
$= g^{-1}\left(8(-3 + 3)\right)$
$= g^{-1}(0) = \sqrt[3]{0} = 0$

81. $\left(f^{-1} \circ f^{-1}\right)(4) = f^{-1}\left(f^{-1}(4)\right)$
$= f^{-1}\left(8[4 + 3]\right)$
$= 8\left[8(4 + 3) + 3\right]$
$= 8\left[8(7) + 3\right]$
$= 8(59) = 472$

82. $\left(g^{-1} \circ g^{-1}\right)(-1) = g^{-1}\left(g^{-1}(-1)\right)$
$= g^{-1}\left(\sqrt[3]{-1}\right)$
$= \sqrt[3]{\sqrt[3]{-1}}$
$= \sqrt[3]{-1}$
$= -1$

83. $(f \circ g)(x) = f(g(x)) = f\left(x^3\right) = \frac{1}{8}x^3 - 3$
$y = \frac{1}{8}x^3 - 3$
$x = \frac{1}{8}y^3 - 3$
$x + 3 = \frac{1}{8}y^3$
$8(x + 3) = y^3$
$\sqrt[3]{8(x + 3)} = y$
$(f \circ g)^{-1}(x) = 2\sqrt[3]{x + 3}$

84. $g^{-1} \circ f^{-1} = g^{-1}\left(f^{-1}(x)\right)$
$= g^{-1}\left(8(x + 3)\right)$
$= \sqrt[3]{8(x + 3)}$
$= 2\sqrt[3]{x + 3}$

In Exercises 85–88, $f(x) = x + 4$, $f^{-1}(x) = x - 4$, $g(x) = 2x - 5$, $g^{-1}(x) = \dfrac{x + 5}{2}$.

85. $\left(g^{-1} \circ f^{-1}\right)(x) = g^{-1}\left(f^{-1}(x)\right)$
$= g^{-1}(x - 4)$
$= \dfrac{(x - 4) + 5}{2}$
$= \dfrac{x + 1}{2}$

86. $\left(f^{-1} \circ g^{-1}\right)(x) = f^{-1}\left(g^{-1}(x)\right)$
$= f^{-1}\left(\dfrac{x + 5}{2}\right)$
$= \dfrac{x + 5}{2} - 4$
$= \dfrac{x + 5 - 8}{2}$
$= \dfrac{x - 3}{2}$

87. $(f \circ g)(x) = f(g(x))$
$= f(2x - 5)$
$= (2x - 5) + 4$
$= 2x - 1$
$(f \circ g)^{-1}(x) = \dfrac{x + 1}{2}$

Note: Comparing Exercises 85 and 87,
$(f \circ g)^{-1}(x) = \left(g^{-1} \circ f^{-1}\right)(x).$

88. $(g \circ f)(x) = g(f(x))$
$= g(x + 4)$
$= 2(x + 4) - 5$
$= 2x + 8 - 5$
$= 2x + 3$
$y = 2x + 3$
$x = 2y + 3$
$x - 3 = 2y$
$\dfrac{x - 3}{2} = y$
$(g \circ f)^{-1}(x) = \dfrac{x - 3}{2}$

89. (a) $y = 10 + 0.75x$
$x = 10 + 0.75y$
$x - 10 = 0.75y$
$\dfrac{x - 10}{0.75} = y$

So, $f^{-1}(x) = \dfrac{x - 10}{0.75}$.

x = hourly wage, y = number of units produced

(b) $y = \dfrac{24.25 - 10}{0.75} = 19$

So, 19 units are produced.

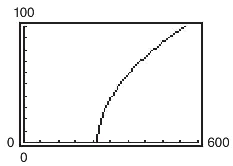
90. (a)

$$y = 0.03x^2 + 245.50,\ 0 < x < 100$$
$$\Rightarrow 245.50 < y < 545.50$$
$$x = 0.03y^2 + 245.50$$
$$x - 245.50 = 0.03y^2$$
$$\frac{x - 245.50}{0.03} = y^2$$
$$\sqrt{\frac{x - 245.50}{0.03}} = y,\ 245.50 < x < 545.50$$
$$f^{-1}(x) = \sqrt{\frac{x - 245.50}{0.03}}$$

x = temperature in degrees Fahrenheit

y = percent load for a diesel engine

(b)



(c)
$$0.03x^2 + 245.50 \le 500$$
$$0.03x^2 \le 254.50$$
$$x^2 \le 8483.33$$
$$x \le 92.10$$

Thus, $0 < x \le 92.10$.

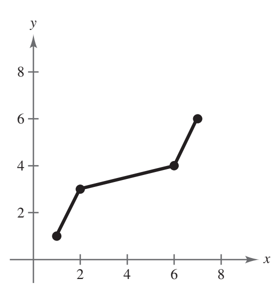
91. False. $f(x) = x^2$ is even and does not have an inverse.

92. True. If $f(x)$ has an inverse and it has a y-intercept at $(0, b)$, then the point $(b, 0)$, must be a point on the graph of $f^{-1}(x)$.

93.

x	1	3	4	6
f	1	2	6	7

x	1	2	6	7
$f^{-1}(x)$	1	3	4	6



94.

x	-4	-2	0	3
f	3	4	0	-1

The graph does not pass the Horizontal Line Test, so $f^{-1}(x)$ does not exist.

95. Let $(f \circ g)(x) = y$. Then $x = (f \circ g)^{-1}(y)$. Also,

$$(f \circ g)(x) = y \Rightarrow f(g(x)) = y$$
$$g(x) = f^{-1}(y)$$
$$x = g^{-1}(f^{-1}(y))$$
$$x = (g^{-1} \circ f^{-1})(y).$$

Because f and g are both one-to-one functions, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

96. Let $f(x)$ be a one-to-one odd function. Then $f^{-1}(x)$ exists and $f(-x) = -f(x)$. Letting (x, y) be any point on the graph of $f(x) \Rightarrow (-x, -y)$ is also on the graph of $f(x)$ and $f^{-1}(-y) = -x = -f^{-1}(y)$. So, $f^{-1}(x)$ is also an odd function.

97. If $f(x) = k(2 - x - x^3)$ has an inverse and $f^{-1}(3) = -2$, then $f(-2) = 3$. So,

$$f(-2) = k\left(2 - (-2) - (-2)^3\right) = 3$$
$$k(2 + 2 + 8) = 3$$
$$12k = 3$$
$$k = \tfrac{3}{12} = \tfrac{1}{4}.$$

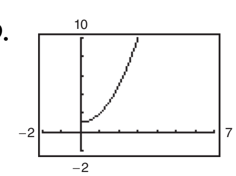
So, $k = \frac{1}{4}$.

98.

x	-10	0	7	45
$f(f^{-1}(x))$	-10	0	7	45
$f^{-1}(f(x))$	-10	0	7	45

$f(x)$ and $f^{-1}(x)$ are inverses of each other.

99.



There is an inverse function $f^{-1}(x) = \sqrt{x - 1}$ because the domain of f is equal to the range of f^{-1} and the range of f is equal to the domain of f^{-1}.

100. (a) $C(x)$ is represented by graph m and $C^{-1}(x)$ is represented by graph n.

(b) $C(x)$ represents the cost of making x units of personalized T-shirts. $C^{-1}(x)$ represents the number of personalized T-shirts that can be made for a given cost.

101. This situation could be represented by a one-to-one function if the runner does not stop to rest. The inverse function would represent the time in hours for a given number of miles completed.

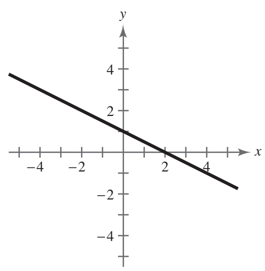
102. This situation cannot be represented by a one-to-one function because it oscillates.

Review Exercises for Chapter 2

1. $y = -\frac{1}{2}x + 1$

Slope: $m = -\frac{1}{2}$

y-intercept: $(0, 1)$



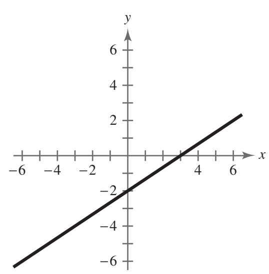
2. $2x - 3y = 6$

$$-3y = -2x + 6$$

$$y = \frac{2}{3}x - 2$$

Slope: $m = \frac{2}{3}$

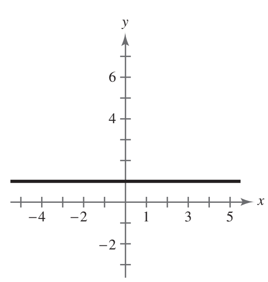
y-intercept: $(0, -2)$



3. $y = 1$

Slope: $m = 0$

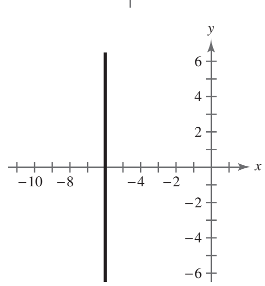
y-intercept: $(0, 1)$



4. $x = -6$

Slope: m is undefined.

y-intercept: none



5. $(5, -2), (-1, 4)$

$$m = \frac{4 - (-2)}{-1 - 5} = \frac{6}{-6} = -1$$

6. $(-1, 6), (3, -2)$

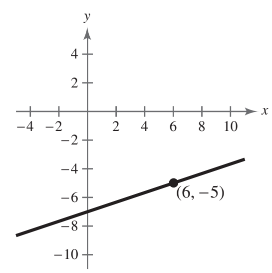
$$m = \frac{-2 - 6}{3 - (-1)} = \frac{-8}{4} = -2$$

7. $(6, -5), m = \frac{1}{3}$

$$y - (-5) = \frac{1}{3}(x - 6)$$

$$y + 5 = \frac{1}{3}x - 2$$

$$y = \frac{1}{3}x - 7$$



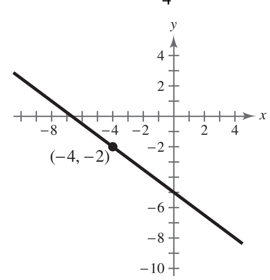
8. $(-4, -2), m = -\frac{3}{4}$

$$y - (-2) = -\frac{3}{4}(x - (-4))$$

$$y + 2 = -\frac{3}{4}(x + 4)$$

$$y + 2 = -\frac{3}{4}x - 3$$

$$y = -\frac{3}{4}x - 5$$



9. $(-6, 4), (4, 9)$

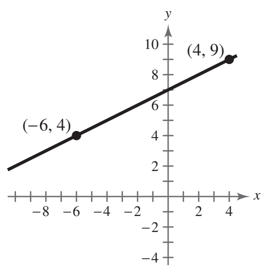
$$m = \frac{9-4}{4-(-6)} = \frac{5}{10} = \frac{1}{2}$$

$$y - 4 = \frac{1}{2}(x - (-6))$$

$$y - 4 = \frac{1}{2}(x + 6)$$

$$y - 4 = \frac{1}{2}x + 3$$

$$y = \frac{1}{2}x + 7$$



10. $(-9, -3), (-3, -5)$

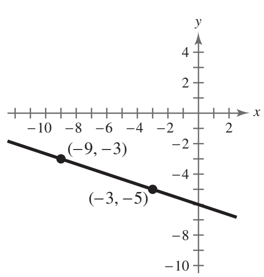
$$m = \frac{-5-(-3)}{-3-(-9)} = \frac{-2}{6} = -\frac{1}{3}$$

$$y - (-3) = -\frac{1}{3}(x - (-9))$$

$$y + 3 = -\frac{1}{3}(x + 9)$$

$$y + 3 = -\frac{1}{3}x - 3$$

$$y = -\frac{1}{3}x - 6$$



11. Point: $(3, -2)$

$$5x - 4y = 8$$

$$y = \tfrac{5}{4}x - 2$$

(a) Parallel slope: $m = \frac{5}{4}$

$$y - (-2) = \tfrac{5}{4}(x - 3)$$

$$y + 2 = \tfrac{5}{4}x - \tfrac{15}{4}$$

$$y = \tfrac{5}{4}x - \tfrac{23}{4}$$

(b) Perpendicular slope: $m = -\frac{4}{5}$

$$y - (-2) = -\tfrac{4}{5}(x - 3)$$

$$y + 2 = -\tfrac{4}{5}x + \tfrac{12}{5}$$

$$y = -\tfrac{4}{5}x + \tfrac{2}{5}$$

12. Point: $(-8, 3)$, $2x + 3y = 5$

$$3y = 5 - 2x$$

$$y = \tfrac{5}{3} - \tfrac{2}{3}x$$

(a) Parallel slope: $m = -\frac{2}{3}$

$$y - 3 = -\tfrac{2}{3}(x + 8)$$

$$3y - 9 = -2x - 16$$

$$3y = -2x - 7$$

$$y = -\tfrac{2}{3}x - \tfrac{7}{3}$$

(b) Perpendicular slope: $m = \frac{3}{2}$

$$y - 3 = \tfrac{3}{2}(x + 8)$$

$$2y - 6 = 3x + 24$$

$$2y = 3x + 30$$

$$y = \tfrac{3}{2}x + 15$$

13. *Verbal Model:* Sale price = (List price) − (Discount)

Labels: Sale price = S

List price = L

Discount = 20% of $L = 0.2L$

Equation: $S = L - 0.2L$

$S = 0.8L$

14. *Verbal Model:* Amount earned = (starting fee) + (per page rate) · (number of *pages*)

Labels: Hourly wage = A

Starting fee = 50

Per page rate = 2.50

Number of pages = p

Equation: $A = 50 + 2.5p$

15. $16x - y^4 = 0$

$y^4 = 16x$

$y = \pm 2\sqrt[4]{x}$

No, y is not a function of x. Some x-values correspond to two y-values.

16. $2x - y - 3 = 0$

$2x - 3 = y$

Yes, the equation represents y as a function of x.

17. $y = \sqrt{1 - x}$

Yes, the equation represents y as a function of x. Each x-value, $x \le 1$, corresponds to only one y-value.

18. $|y| = x + 2$ corresponds to $y = x + 2$ or $-y = x + 2$.

No, y is not a function of x. Some x-values correspond to two y-values.

19. $g(x) = x^{4/3}$

(a) $g(8) = 8^{4/3} = 2^4 = 16$

(b) $g(t + 1) = (t + 1)^{4/3}$

(c) $(-27)^{4/3} = (-3)^4 = 81$

(d) $g(-x) = (-x)^{4/3} = x^{4/3}$

20. $h(x) = |x - 2|$

(a) $h(-4) = |-4 - 2| = |-6| = 6$

(b) $h(-2) = |-2 - 2| = |-4| = 4$

(c) $h(0) = |0 - 2| = |-2| = 2$

(d) $h(-x + 2) = |-x + 2 - 2| = |-x| = |x|$

21. $f(x) = \sqrt{25 - x^2}$

Domain: $25 - x^2 \ge 0$

$(5 + x)(5 - x) \ge 0$

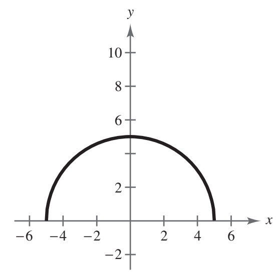
Critical numbers: $x = \pm 5$

Test intervals: $(-\infty, -5), (-5, 5), (5, \infty)$

Test: Is $25 - x^2 \ge 0$?

Solution set: $-5 \le x \le 5$

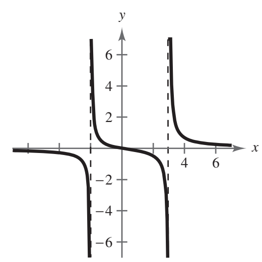
Domain: all real numbers x such that $-5 \le x \le 5$, or $[-5, 5]$



22. $h(x) = \dfrac{x}{x^2 - x - 6}$

$= \dfrac{x}{(x + 2)(x - 3)}$

Domain: All real numbers x except $x = -2, 3$



23. $v(t) = -32t + 48$

$v(1) = 16$ feet per second

24. $0 = -32t + 48$

$t = \frac{48}{32} = 1.5$ seconds

25. $f(x) = 2x^2 + 3x - 1$

$$\frac{f(x + h) - f(x)}{h} = \frac{\left[2(x + h)^2 + 3(x + h) - 1\right] - \left(2x^2 + 3x - 1\right)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 1 - 2x^2 - 3x + 1}{h}$$

$$= \frac{h(4x + 2h + 3)}{h}$$

$$= 4x + 2h + 3, \quad h \ne 0$$

26. $$\begin{aligned} f(x) &= x^3 - 5x^2 + x \\ f(x+h) &= (x+h)^3 - 5(x+h)^2 + (x+h) \\ &= x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h \\ \frac{f(x+h) - f(x)}{h} &= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h - x^3 + 5x^2 - x}{h} \\ &= \frac{3x^2h + 3xh^2 + h^3 - 10xh - 5h^2 + h}{h} \\ &= \frac{h(3x^2 + 3xh + h^2 - 10x - 5h + 1)}{h} \\ &= 3x^2 + 3xh + h^2 - 10x - 5h + 1, \; h \neq 0 \end{aligned}$$

27. $y = (x - 3)^2$

A vertical line intersects the graph no more than once, so *y is* a function of *x*.

28. $x = -|4 - y|$

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

29. $f(x) = 5x^2 + 4x - 1$

$$\begin{aligned} 5x^2 + 4x - 1 &= 0 \\ (5x - 1)(x + 1) &= 0 \\ 5x - 1 = 0 &\Rightarrow x = \tfrac{1}{5} \\ x + 1 = 0 &\Rightarrow x = -1 \end{aligned}$$

30. $f(x) = \dfrac{8x + 3}{11 - x}$

$$\begin{aligned} \frac{8x + 3}{11 - x} &= 0 \\ 8x + 3 &= 0 \\ x &= -\tfrac{3}{8} \end{aligned}$$

31. $f(x) = \sqrt{2x + 1}$

$$\begin{aligned} \sqrt{2x + 1} &= 0 \\ 2x + 1 &= 0 \\ 2x &= -1 \\ x &= -\tfrac{1}{2} \end{aligned}$$

32. $f(x) = x^3 - x^2$

$$\begin{aligned} x^3 - x^2 &= 0 \\ x^2(x - 1) &= 0 \end{aligned}$$

$x^2 = 0$ or $x - 1 = 0$

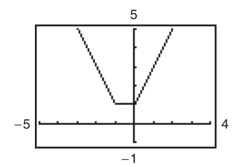
$x = 0$ $\qquad$ $x = 1$

33. $f(x) = |x| + |x + 1|$

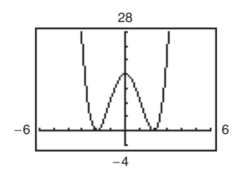
f is increasing on $(0, \infty)$.

f is decreasing on $(-\infty, -1)$.

f is constant on $(-1, 0)$.



34. $f(x) = (x^2 - 4)^2$

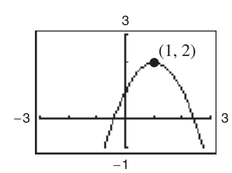


f is increasing on $(-2, 0)$ and $(2, \infty)$.

f is decreasing on $(-\infty, -2)$ and $(0, 2)$.

35. $f(x) = -x^2 + 2x + 1$

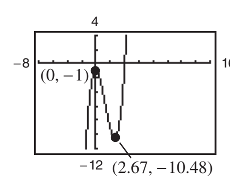
Relative maximum: $(1, 2)$



36. $f(x) = x^3 - 4x^2 - 1$

Relative minimum: $(2.67, -10.48)$

Relative maximum: $(0, -1)$



37. $f(x) = -x^2 + 8x - 4$

$$\frac{f(4) - f(0)}{4 - 0} = \frac{12 - (-4)}{4} = 4$$

The average rate of change of f from $x_1 = 0$ to $x_2 = 4$ is 4.

38. $f(x) = x^3 + 2x + 1$

$$\frac{f(3) - f(1)}{3 - 1} = \frac{34 - 4}{2} = \frac{30}{2} = 15$$

The average rate of change of f from $x_1 = 1$ to $x_2 = 3$ is 15.

39. $f(x) = x^5 + 4x - 7$

$$\begin{aligned} f(-x) &= (-x)^5 + 4(-x) - 7 \\ &= -x^5 - 4x - 7 \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

The function is neither even nor odd, so the graph has no symmetry.

40. $f(x) = x^4 - 20x^2$

$$f(-x) = (-x)^4 - 20(-x)^2 = x^4 - 20x^2 = f(x)$$

The function is even, so the graph has y-axis symmetry.

41. $f(x) = 2x\sqrt{x^2 + 3}$

$$\begin{aligned} f(-x) &= 2(-x)\sqrt{(-x)^2 + 3} \\ &= -2x\sqrt{x^2 + 3} \\ &= -f(x) \end{aligned}$$

The function is odd, so the graph has origin symmetry.

42. $f(x) = \sqrt[5]{6x^2}$

$$f(-x) = \sqrt[5]{6(-x)^2} = \sqrt[5]{6x^2} = f(x)$$

The function is even, so the graph has y-axis symmetry.

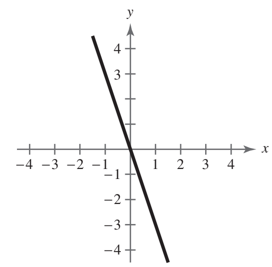
43. (a) $f(2) = -6, f(-1) = 3$

Points: $(2, -6), (-1, 3)$

$$m = \frac{3 - (-6)}{-1 - 2} = \frac{9}{-3} = -3$$

$$\begin{aligned} y - (-6) &= -3(x - 2) \\ y + 6 &= -3x + 6 \\ y &= -3x \\ f(x) &= -3x \end{aligned}$$

(b)



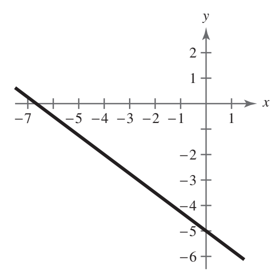
44. (a) $f(0) = -5, f(4) = -8$

$(0, -5), (4, -8)$

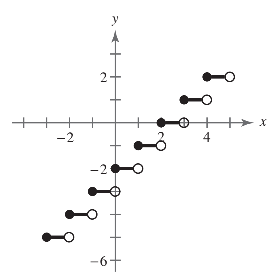
$$m = \frac{-8 - (-5)}{4 - 0} = -\frac{3}{4}$$

$$\begin{aligned} y - (-5) &= -\frac{3}{4}(x - 0) \\ y &= -\frac{3}{4}x - 5 \\ f(x) &= -\frac{3}{4}x - 5 \end{aligned}$$

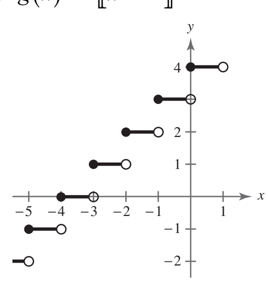
(b)



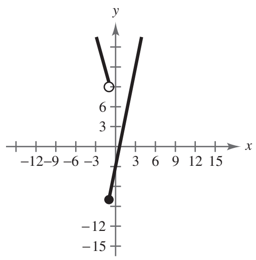
45. $g(x) = [\![x]\!] - 2$



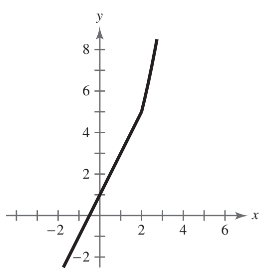
46. $g(x) = [\![x + 4]\!]$



47. $f(x) = \begin{cases} 5x - 3, & x \geq -1 \\ -4x + 5, & x < -1 \end{cases}$



48. $f(x) = \begin{cases} 2x + 1, & x \leq 2 \\ x^2 + 1, & x > 2 \end{cases}$

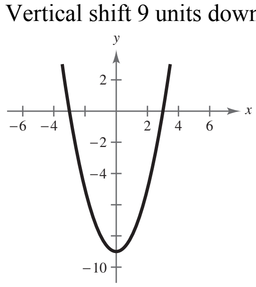


49. (a) $f(x) = x^2$

(b) $h(x) = x^2 - 9$

Vertical shift 9 units downward

(c)



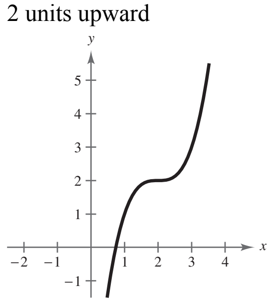
(d) $h(x) = f(x) - 9$

50. (a) $f(x) = x^3$

(b) $h(x) = (x - 2)^3 + 2$

Horizontal shift 2 units to the right; vertical shift 2 units upward

(c)



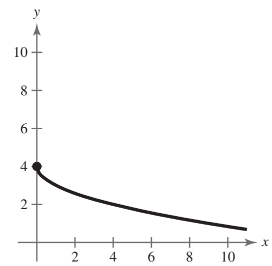
(d) $h(x) = f(x - 2) + 2$

51. (a) $f(x) = \sqrt{x}$

(b) $h(x) = -\sqrt{x} + 4$

Reflection in the x-axis and a vertical shift 4 units upward

(c)



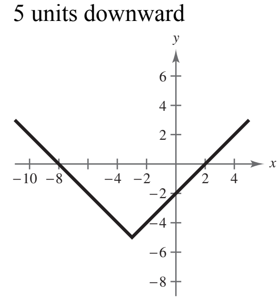
(d) $h(x) = -f(x) + 4$

52. (a) $f(x) = |x|$

(b) $h(x) = |x + 3| - 5$

Horizontal shift 3 units to the left and a vertical shift 5 units downward

(c)



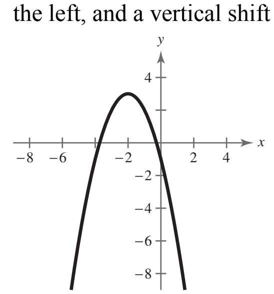
(d) $h(x) = f(x + 3) - 5$

53. (a) $f(x) = x^2$

(b) $h(x) = -(x + 2)^2 + 3$

Reflection in the x-axis, a horizontal shift 2 units to the left, and a vertical shift 3 units upward

(c)



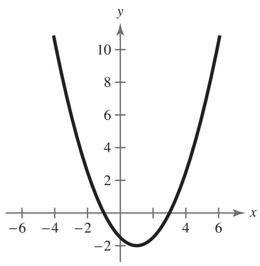
(d) $h(x) = -f(x + 2) + 3$

54. (a) $f(x) = x^2$

(b) $h(x) = \frac{1}{2}(x - 1)^2 - 2$

Horizontal shift one unit to the right, vertical shrink, and a vertical shift 2 units downward

(c)



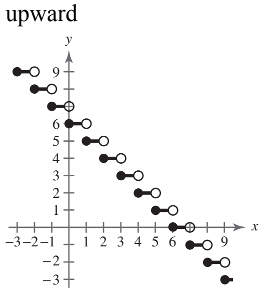
(d) $h(x) = \frac{1}{2}f(x - 1) - 2$

55. (a) $f(x) = [\![x]\!]$

(b) $h(x) = -[\![x]\!] + 6$

Reflection in the x-axis and a vertical shift 6 units upward

(c)



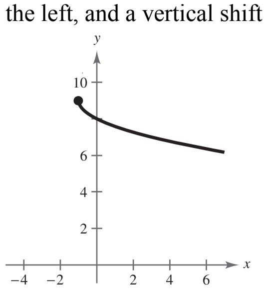
(d) $h(x) = -f(x) + 6$

56. (a) $f(x) = \sqrt{x}$

(b) $h(x) = -\sqrt{x + 1} + 9$

Reflection in the x-axis, a horizontal shift 1 unit to the left, and a vertical shift 9 units upward

(c)



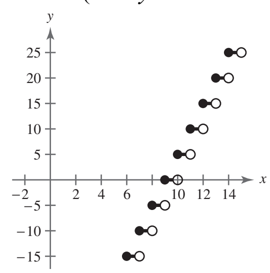
(d) $h(x) = -f(x + 1) + 9$

57. (a) $f(x) = [\![x]\!]$

(b) $h(x) = 5[\![x - 9]\!]$

Horizontal shift 9 units to the right and a vertical stretch (each y-value is multiplied by 5)

(c)



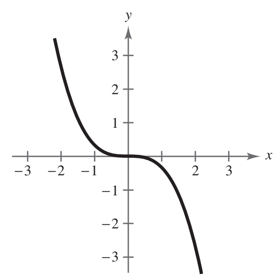
(d) $h(x) = 5f(x - 9)$

58. (a) $f(x) = x^3$

(b) $h(x) = -\frac{1}{3}x^3$

Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$

(c)



(d) $h(x) = -\frac{1}{3}f(x)$

59. $f(x) = x^2 + 3, g(x) = 2x - 1$

(a) $(f + g)(x) = (x^2 + 3) + (2x - 1) = x^2 + 2x + 2$

(b) $(f - g)(x) = (x^2 + 3) - (2x - 1) = x^2 - 2x + 4$

(c) $(fg)(x) = (x^2 + 3)(2x - 1) = 2x^3 - x^2 + 6x - 3$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + 3}{2x - 1}$, Domain: $x \neq \dfrac{1}{2}$

60. $f(x) = x^2 - 4, g(x) = \sqrt{3 - x}$

(a) $(f + g)(x) = f(x) + g(x) = x^2 - 4 + \sqrt{3 - x}$

(b) $(f - g)(x) = f(x) - g(x) = x^2 - 4 - \sqrt{3 - x}$

(c) $(fg)(x) = f(x)g(x) = (x^2 - 4)(\sqrt{3 - x})$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 - 4}{\sqrt{3 - x}}$, Domain: $x < 3$

61. $f(x) = \frac{1}{3}x - 3, g(x) = 3x + 1$

The domains of f and g are all real numbers.

(a) $(f \circ g)(x) = f(g(x))$
$= f(3x + 1)$
$= \frac{1}{3}(3x + 1) - 3$
$= x + \frac{1}{3} - 3$
$= x - \frac{8}{3}$

Domain: all real numbers

(b) $(g \circ f)(x) = g(f(x))$
$= g\left(\frac{1}{3}x - 3\right)$
$= 3\left(\frac{1}{3}x - 3\right) + 1$
$= x - 9 + 1$
$= x - 8$

Domain: all real numbers

62. $f(x) = \sqrt{x + 1}, g(x) = x^2$

The domain of f is all real numbers x such that $x \geq -1$. The domain of g is all real numbers.

(a) $(f \circ g)(x) = f(g(x))$
$= f(x^2)$
$= \sqrt{x^2 + 1}$

Domain: all real numbers

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt{x + 1})$
$= (\sqrt{x + 1})^2$
$= x + 1$

Domain: all real numbers x such that $x \geq -1$

In Exercises 63-64 use the following functions. $\boldsymbol{f(x) = x - 100, g(x) = 0.95x}$

63. $(f \circ g)(x) = f(0.95x) = 0.95x - 100$ represents the sale price if first the 5% discount is applied and then the \$100 rebate.

64. $(g \circ f)(x) = g(x - 100) = 0.95(x - 100) = 0.95x - 95$ represents the sale price if first the \$100 rebate is applied and then the 5% discount.

65. $f(x) = \dfrac{x - 4}{5}$

$y = \dfrac{x - 4}{5}$

$x = \dfrac{y - 4}{5}$

$5x = y - 4$

$y = 5x + 4$

So, $f^{-1}(x) = 5x + 4$.

$f(f^{-1}(x)) = f(5x + 4) = \dfrac{5x + 4 - 4}{5} = \dfrac{5x}{5} = x$

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{x - 4}{5}\right) = 5\left(\dfrac{x - 4}{5}\right) + 4 = x - 4 + 4 = x$

66. $f(x) = x^3 - 1$

$y = x^3 - 1$

$x = y^3 - 1$

$x + 1 = y^3$

$\sqrt[3]{x + 1} = y$

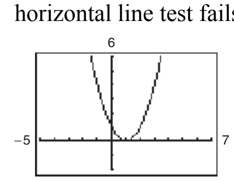
$f^{-1}(x) = \sqrt[3]{x + 1}$

$f(f^{-1}(x)) = \left(\sqrt[3]{x + 1}\right)^3 - 1 = x$

$f^{-1}(f(x)) = \sqrt[3]{(x^3 - 1) + 1} = x$

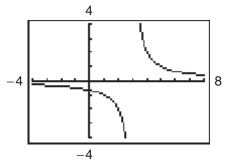
67. $f(x) = (x - 1)^2$

No, the function does not have an inverse because the horizontal line test fails.



68. $h(t) = \dfrac{2}{t - 3}$

Yes, the function has an inverse because no horizontal lines intersect the graph at more than one point. The function has an inverse.



69. (a) $f(x) = \frac{1}{2}x - 3$

$y = \frac{1}{2}x - 3$

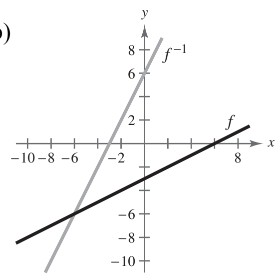
$x = \frac{1}{2}y - 3$

$x + 3 = \frac{1}{2}y$

$2(x + 3) = y$

$f^{-1}(x) = 2x + 6$

(b)



(c) The graph of f^{-1} is the reflection of the graph of *f* in the line $y = x$.

(d) The domains and ranges of *f* and f^{-1} are the set of all real numbers.

70. (a) $f(x) = \sqrt{x + 1}$

$y = \sqrt{x + 1}$

$x = \sqrt{y + 1}$

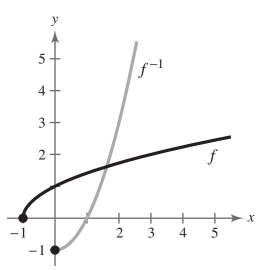
$x^2 = y + 1$

$x^2 - 1 = y$

$f^{-1}(x) = x^2 - 1, \quad x \ge 0$

Note: The inverse must have a restricted domain.

(b)



(c) The graph of f^{-1} is the reflection of the graph of *f* in the line $y = x$.

(d) The domain of *f* and the range of f^{-1} is $[-1, \infty)$.

The range of *f* and the domain of f^{-1} is $[0, \infty)$.

71. $f(x) = 2(x - 4)^2$ is increasing on $(4, \infty)$.

Let $f(x) = 2(x - 4)^2, x > 4$ and $y > 0$.

$y = 2(x - 4)^2$

$x = 2(y - 4)^2, x > 0, y > 4$

$\dfrac{x}{2} = (y - 4)^2$

$\sqrt{\dfrac{x}{2}} = y - 4$

$\sqrt{\dfrac{x}{2}} + 4 = y$

$f^{-1}(x) = \sqrt{\dfrac{x}{2}} + 4, x > 0$

72. $f(x) = |x - 2|$ is increasing on $(2, \infty)$.

Let $f(x) = x - 2, x > 2, y > 0$.

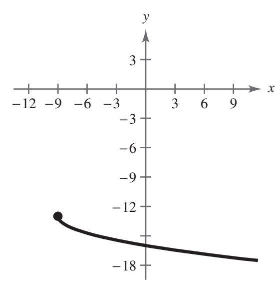
$y = x - 2$

$x = y - 2, x > 0, y > 2$

$x + 2 = y, x > 0, y > 2$

$f^{-1}(x) = x + 2, x > 0$

73. False. The graph is reflected in the *x*-axis, shifted 9 units to the left, then shifted 13 units downward.



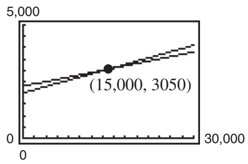
74. True. If $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$, then the domain of *g* is all real numbers, which is equal to the range of *f* and vice versa

Problem Solving for Chapter 2

1. (a) $W_1 = 0.07S + 2000$

(b) $W_2 = 0.05S + 2300$

(c)



Point of intersection: $(15{,}000, 3050)$

Both jobs pay the same, \$3050, if you sell \$15,000 per month.

(d) No. If you think you can sell \$20,000 per month, keep your current job with the higher commission rate. For sales over \$15,000 it pays more than the other job.

2. Mapping numbers onto letters is *not* a function. Each number between 2 and 9 is mapped to more than one letter.

$\{(2, A), (2, B), (2, C), (3, D), (3, E), (3, F), (4, G), (4, H), (4, I), (5, J), (5, K), (5, L),$
$(6, M), (6, N), (6, O), (7, P), (7, Q), (7, R), (7, S), (8, T), (8, U), (8, V), (9, W), (9, X), (9, Y), (9, Z)\}$

Mapping letters onto numbers *is* a function. Each letter is only mapped to one number.

$\{(A, 2), (B, 2), (C, 2), (D, 3), (E, 3), (F, 3), (G, 4), (H, 4), (I, 4), (J, 5), (K, 5), (L, 5),$
$(M, 6), (N, 6), (O, 6), (P, 7), (Q, 7), (R, 7), (S, 7), (T, 8), (U, 8), (V, 8), (W, 9), (X, 9), (Y, 9), (Z, 9)\}$

3. (a) Let $f(x)$ and $g(x)$ be two even functions.

Then define $h(x) = f(x) \pm g(x)$.

$$h(-x) = f(-x) \pm g(-x)$$
$$= f(x) \pm g(x) \text{ because } f \text{ and } g \text{ are even}$$
$$= h(x)$$

So, $h(x)$ is also even.

(b) Let $f(x)$ and $g(x)$ be two odd functions.

Then define $h(x) = f(x) \pm g(x)$.

$$h(-x) = f(-x) \pm g(-x)$$
$$= -f(x) \pm g(x) \text{ because } f \text{ and } g \text{ are odd}$$
$$= -h(x)$$

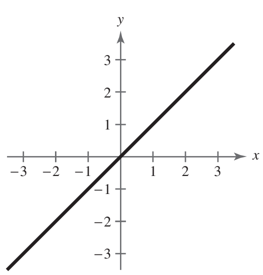
So, $h(x)$ is also odd. $(\text{If } f(x) \neq g(x))$

(c) Let $f(x)$ be odd and $g(x)$ be even. Then define $h(x) = f(x) \pm g(x)$.

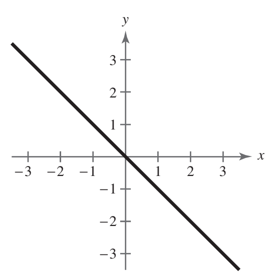
$$h(-x) = f(-x) \pm g(-x)$$
$$= -f(x) \pm g(x) \text{ because } f \text{ is odd and } g \text{ is even}$$
$$\neq h(x)$$
$$\neq -h(x)$$

So, $h(x)$ is neither odd nor even.

4. $f(x) = x$



$g(x) = -x$



$(f \circ f)(x) = x$ and $(g \circ g)(x) = x$

These are the only two linear functions that are their own inverse functions since m has to equal $1/m$ for this to be true.

General formula: $y = -x + c$

5. $f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0 = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 = f(x)$

So, $f(x)$ is even.

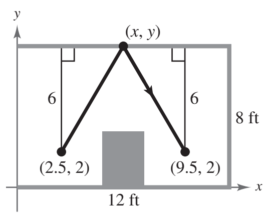
6. It appears, from the drawing, that the triangles are equal; thus $(x, y) = (6, 8)$.

The line between $(2.5, 2)$ and $(6, 8)$ is $y = \frac{12}{7}x - \frac{16}{7}$.

The line between $(9.5, 2)$ and $(6, 8)$ is $y = -\frac{12}{7}x + \frac{128}{7}$.

The path of the ball is:

$$f(x) = \begin{cases} \frac{12}{7}x - \frac{16}{7}, & 2.5 \le x \le 6 \\ -\frac{12}{7}x + \frac{128}{7}, & 6 < x \le 9.5 \end{cases}$$



7. (a) April 11: 10 hours

April 12: 24 hours

April 13: 24 hours

April 14: $23\frac{2}{3}$ hours

Total: $81\frac{2}{3}$ hours

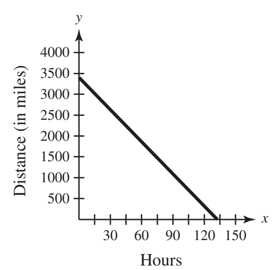
(b) $\text{Speed} = \dfrac{\text{distance}}{\text{time}} = \dfrac{2100}{81\frac{2}{3}} = \dfrac{180}{7} = 25\frac{5}{7}$ mph

(c) $D = -\dfrac{180}{7}t + 3400$

Domain: $0 \le t \le \dfrac{1190}{9}$

Range: $0 \le D \le 3400$

(d)



8. (a) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(2) - f(1)}{2 - 1} = \dfrac{1 - 0}{1} = 1$

(b) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.5) - f(1)}{1.5 - 1} = \dfrac{0.75 - 0}{0.5} = 1.5$

(c) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.25) - f(1)}{1.25 - 1} = \dfrac{0.4375 - 0}{0.25} = 1.75$

(d) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.125) - f(1)}{1.125 - 1} = \dfrac{0.234375 - 0}{0.125} = 1.875$

(e) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.0625) - f(1)}{1.0625 - 1} = \dfrac{0.12109375 - 0}{0.625} = 1.9375$

(f) Yes, the average rate of change appears to be approaching 2.

(g) a. $(1, 0), (2, 1), m = 1, y = x - 1$

b. $(1, 0), (1.5, 0.75), m = \dfrac{0.75}{0.5} = 1.5, y = 1.5x - 1.5$

c. $(1, 0), (1.25, 0.4375), m = \dfrac{0.4375}{0.25} = 1.75, y = 1.75x - 1.75$

d. $(1, 0), (1.125, 0.234375), m = \dfrac{0.234375}{0.125} = 1.875, y = 1.875x - 1.875$

e. $(1, 0), (1.0625, 0.12109375), m = \dfrac{0.12109375}{0.0625} = 1.9375, y = 1.9375x - 1.9375$

(h) $(1, f(1)) = (1, 0), m \to 2, y = 2(x - 1), y = 2x - 2$

9. (a)–(d) Use $f(x) = 4x$ and $g(x) = x + 6$.

(a) $(f \circ g)(x) = f(x + 6) = 4(x + 6) = 4x + 24$

(b) $(f \circ g)^{-1}(x) = \dfrac{x - 24}{4} = \dfrac{1}{4}x - 6$

(c) $f^{-1}(x) = \dfrac{1}{4}x$

$g^{-1}(x) = x - 6$

(d) $(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\dfrac{1}{4}x\right) = \dfrac{1}{4}x - 6$

(e) $f(x) = x^3 + 1$ and $g(x) = 2x$

$(f \circ g)(x) = f(2x) = (2x)^3 + 1 = 8x^3 + 1$

$(f \circ g)^{-1}(x) = \sqrt[3]{\dfrac{x - 1}{8}} = \dfrac{1}{2}\sqrt[3]{x - 1}$

$f^{-1}(x) = \sqrt[3]{x - 1}$

$g^{-1}(x) = \dfrac{1}{2}x$

$(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\sqrt[3]{x - 1}\right) = \dfrac{1}{2}\sqrt[3]{x - 1}$

(f) Answers will vary.

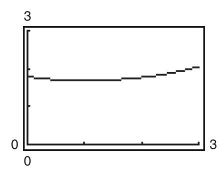
(g) Conjecture: $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$

10. (a) The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. The total time is

$$T(x) = \frac{\sqrt{4 + x^2}}{2} + \frac{\sqrt{1 + (3 - x)^2}}{4}$$
$$= \frac{1}{2}\sqrt{4 + x^2} + \frac{1}{4}\sqrt{x^2 - 6x + 10}.$$

(b) Domain of $T(x)$: $0 \le x \le 3$

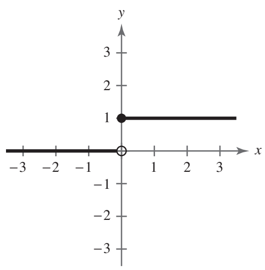
(c)



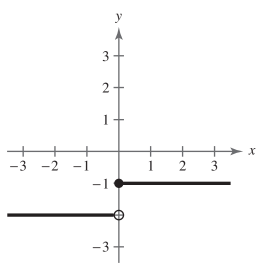
(d) $T(x)$ is a minimum when $x = 1$.

(e) Answers will vary. *Sample answer:* To reach point Q in the shortest amount of time, you should row to a point one mile down the coast, and then walk the rest of the way. The distance $x = 1$ yields a time of 1.68 hours.

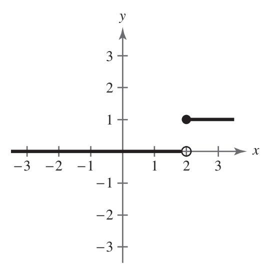
11. $H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$



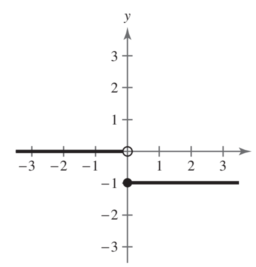
(a) $H(x) - 2$



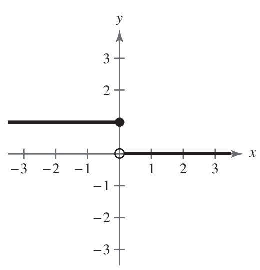
(b) $H(x - 2)$



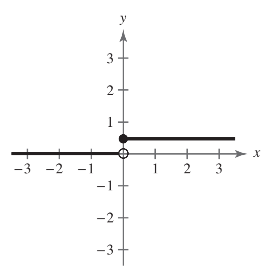
(c) $-H(x)$



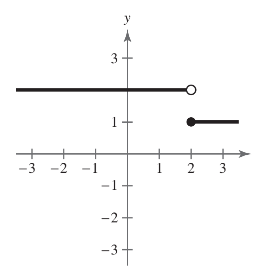
(d) $H(-x)$



(e) $\frac{1}{2}H(x)$



(f) $-H(x - 2) + 2$



12. $f(x) = y = \dfrac{1}{1 - x}$

(a) Domain: all real numbers x except $x = 1$

Range: all real numbers y except $y = 0$

(b) $f(f(x)) = f\left(\dfrac{1}{1 - x}\right)$

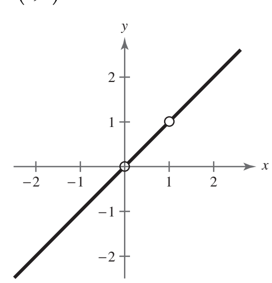
$$= \frac{1}{1 - \left(\dfrac{1}{1 - x}\right)} = \frac{1}{\dfrac{1 - x - 1}{1 - x}}$$

$$= \frac{1 - x}{-x} = \frac{x - 1}{x}$$

Domain: all real numbers x except $x = 0$ and $x = 1$

(c) $f(f(f(x))) = f\left(\dfrac{x - 1}{x}\right) = \dfrac{1}{1 - \left(\dfrac{x - 1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

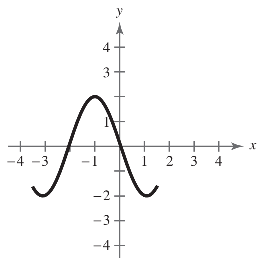
The graph is not a line. It has holes at $(0, 0)$ and $(1, 1)$.



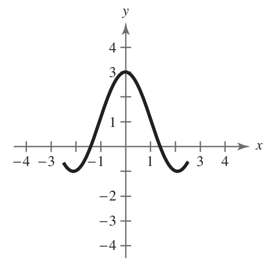
13. $(f \circ (g \circ h))(x) = f((g \circ h)(x)) = f(g(h(x))) = (f \circ g \circ h)(x)$

$((f \circ g) \circ h)(x) = (f \circ g)(h(x)) = f(g(h(x))) = (f \circ g \circ h)(x)$

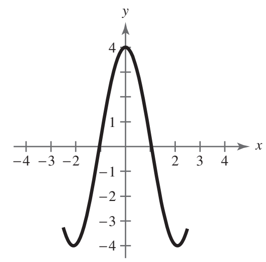
14. (a) $f(x + 1)$



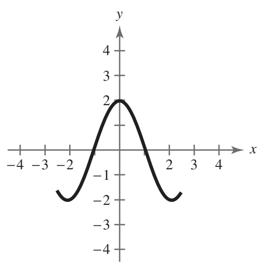
(b) $f(x) + 1$



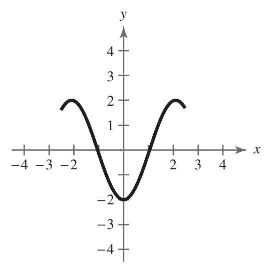
(c) $2f(x)$



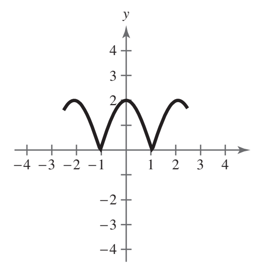
(d) $f(-x)$



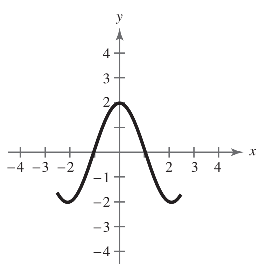
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$



15.

x	$f(x)$	$f^{-1}(x)$
-4	—	2
-3	4	1
-2	1	0
-1	0	—
0	-2	-1
1	-3	-2
2	-4	—
3	—	—
4	—	-3

(a)

x	$f(f^{-1}(x))$
-4	$f(f^{-1}(-4)) = f(2) = -4$
-2	$f(f^{-1}(-2)) = f(0) = -2$
0	$f(f^{-1}(0)) = f(-1) = 0$
4	$f(f^{-1}(4)) = f(-3) = 4$

(b)

x	$(f + f^{-1})(x)$
-3	$f(-3) + f^{-1}(-3) = 4 + 1 = 5$
-2	$f(-2) + f^{-1}(-2) = 1 + 0 = 1$
0	$f(0) + f^{-1}(0) = -2 + (-1) = -3$
1	$f(1) + f^{-1}(1) = -3 + (-2) = -5$

(c)

x	$(f \cdot f^{-1})(x)$
-3	$f(-3)f^{-1}(-3) = (4)(1) = 4$
-2	$f(-2)f^{-1}(-2) = (1)(0) = 0$
0	$f(0)f^{-1}(0) = (-2)(-1) = 2$
1	$f(1)f^{-1}(1) = (-3)(-2) = 6$

(d)

x	$\lvert f^{-1}(x)\rvert$
-4	$\lvert f^{-1}(-4)\rvert = \lvert 2\rvert = 2$
-3	$\lvert f^{-1}(-3)\rvert = \lvert 1\rvert = 1$
0	$\lvert f^{-1}(0)\rvert = \lvert -1\rvert = 1$
4	$\lvert f^{-1}(4)\rvert = \lvert -3\rvert = 3$

Practice Test for Chapter 2

1. Find the equation of the line through $(2, 4)$ and $(3, -1)$.

2. Find the equation of the line with slope $m = 4/3$ and y-intercept $b = -3$.

3. Find the equation of the line through $(4, 1)$ perpendicular to the line $2x + 3y = 0$.

4. If it costs a company \$32 to produce 5 units of a product and \$44 to produce 9 units, how much does it cost to produce 20 units? (Assume that the cost function is linear.)

5. Given $f(x) = x^2 - 2x + 1$, find $f(x - 3)$.

6. Given $f(x) = 4x - 11$, find $\dfrac{f(x) - f(3)}{x - 3}$.

7. Find the domain and range of $f(x) = \sqrt{36 - x^2}$.

8. Which equations determine y as a function of x?
   (a) $6x - 5y + 4 = 0$
   (b) $x^2 + y^2 = 9$
   (c) $y^3 = x^2 + 6$

9. Sketch the graph of $f(x) = x^2 - 5$.

10. Sketch the graph of $f(x) = |x + 3|$.

11. Sketch the graph of $f(x) = \begin{cases} 2x + 1, & \text{if } x \geq 0, \\ x^2 - x, & \text{if } x < 0. \end{cases}$

12. Use the graph of $f(x) = |x|$ to graph the following:
    (a) $f(x + 2)$
    (b) $-f(x) + 2$

13. Given $f(x) = 3x + 7$ and $g(x) = 2x^2 - 5$, find the following:
    (a) $(g - f)(x)$
    (b) $(fg)(x)$

14. Given $f(x) = x^2 - 2x + 16$ and $g(x) = 2x + 3$, find $f(g(x))$.

15. Given $f(x) = x^3 + 7$, find $f^{-1}(x)$.

16. Which of the following functions have inverses?
    (a) $f(x) = |x - 6|$
    (b) $f(x) = ax + b, a \neq 0$
    (c) $f(x) = x^3 - 19$

17. Given $f(x) = \sqrt{\frac{3 - x}{x}}, 0 < x \leq 3,$ find $f^{-1}(x)$.

Exercises 18–20, true or false?

18. $y = 3x + 7$ and $y = \frac{1}{3}x - 4$ are perpendicular.

19. $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

20. If a function has an inverse, then it must pass both the Vertical Line Test and the Horizontal Line Test.

2 Functions and Their Graphs

2.1 Linear Equations in Two Variables

Objectives

- Use slope to graph linear equations in two variables.
- Find the slope of a line given two points on the line.
- Write linear equations in two variables.
- Use slope to identify parallel and perpendicular lines.
- Use slope and linear equations in two variables to model and solve real-life problems.

Using Slope

Using Slope

The simplest mathematical model for relating two variables is the **linear equation in two variables** $y = mx + b$.

The equation is called *linear* because its graph is a line. (In mathematics, the term *line* means *straight line*.)

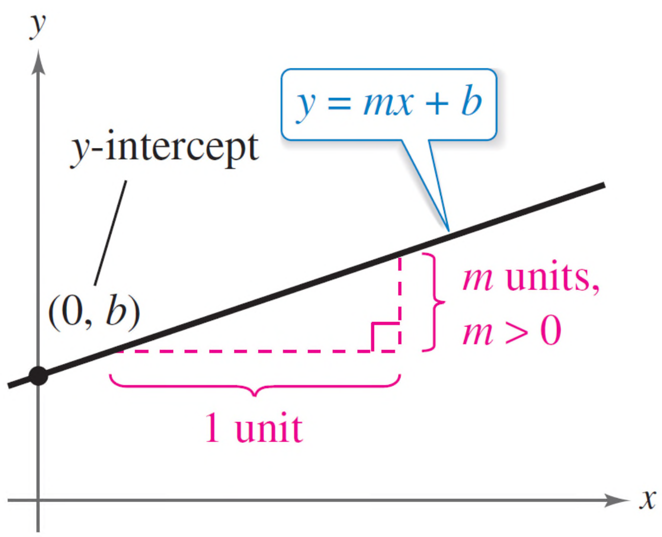
By letting $x = 0$, you obtain

$$y = m(0) + b$$

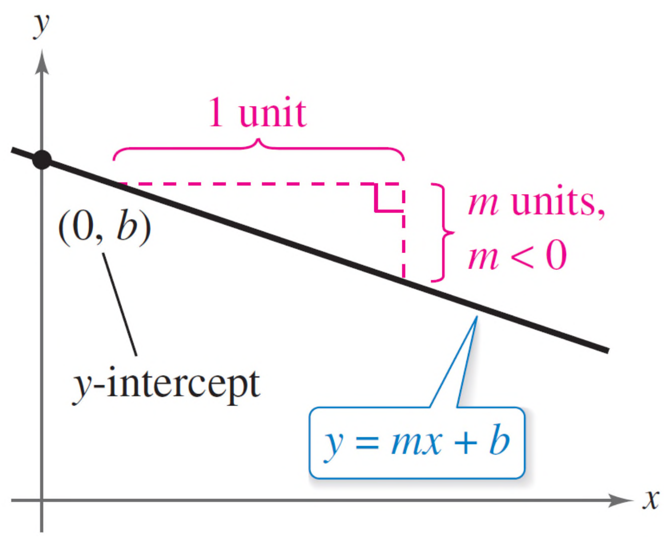
$$= b.$$

Using Slope

So, the line crosses the y-axis at $y = b$, as shown in the figures below.



Positive slope, line rises.



Negative slope, line falls.

Using Slope

In other words, the *y*-intercept is (0, *b*).

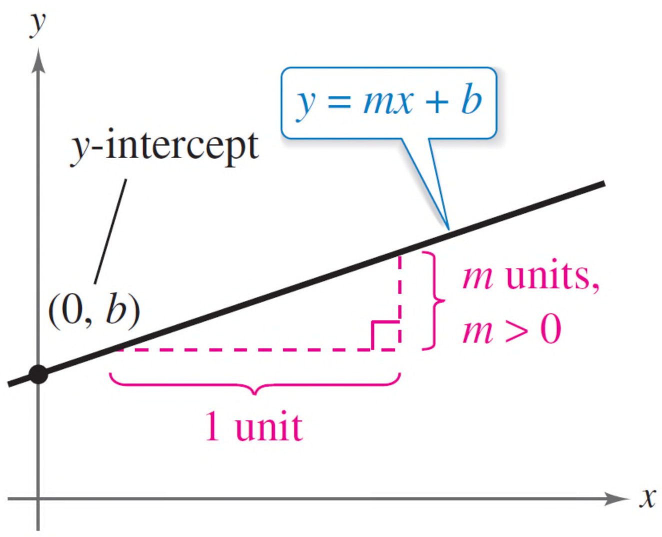
The steepness or slope of the line is *m*.

$$y = mx + b$$

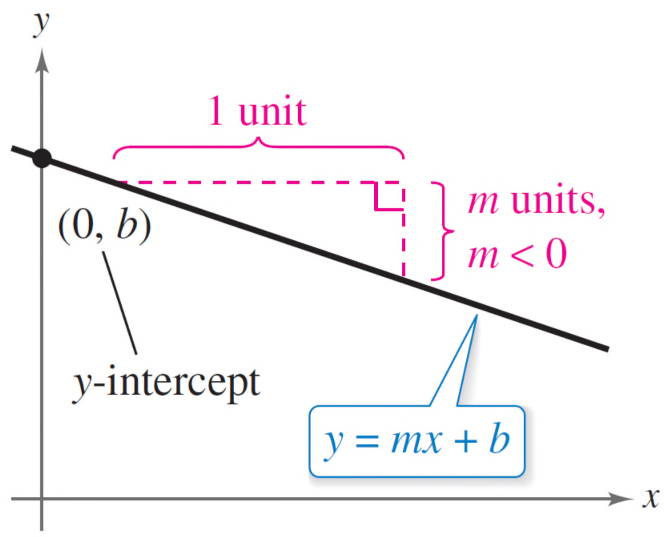
Slope → m; *y*-Intercept → b

Using Slope

The **slope** of a nonvertical line is the number of units the line rises (or falls) vertically for each unit of horizontal change from left to right, as shown below.



Positive slope, line rises.



Negative slope, line falls.

Using Slope

A linear equation written in **slope-intercept form** has the form $y = mx + b$.

The Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

Once you have determined the slope and the y-intercept of a line, it is a relatively simple matter to sketch its graph.

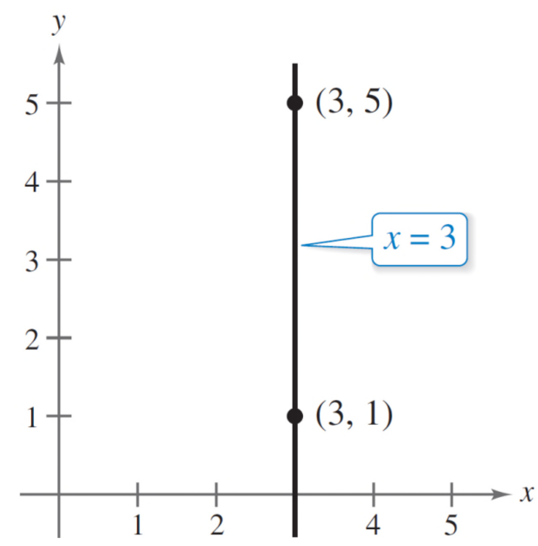
Using Slope

In the next example, note that none of the lines is vertical.

A vertical line has an equation of the form

$$x = a.$$ Vertical line

The equation of a vertical line cannot be written in the form $y = mx + b$ because the slope of a vertical line is undefined, as indicated in Figure 2.1.



Slope is undefined.
Figure 2.1

Example 1 – *Graphing a Linear Equation*

Sketch the graph of each linear equation.

a. $y = 2x + 1$

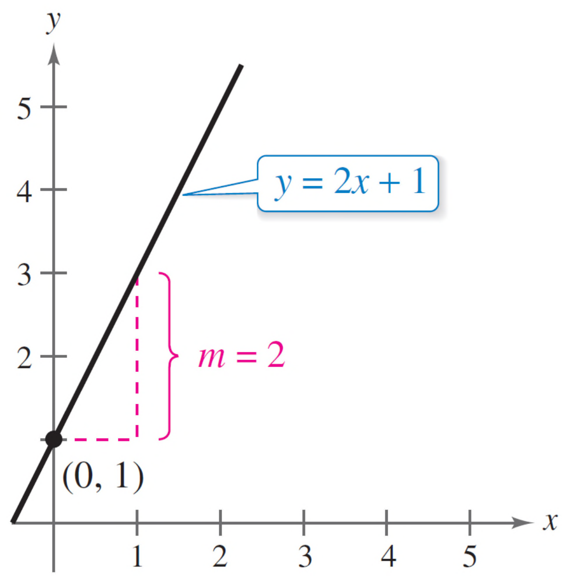
b. $y = 2$

c. $x + y = 2$

Example 1(a) – *Solution*

Because $b = 1$, the y-intercept is (0, 1).

Moreover, because the slope is $m = 2$, the line *rises* two units for each unit the line moves to the right.



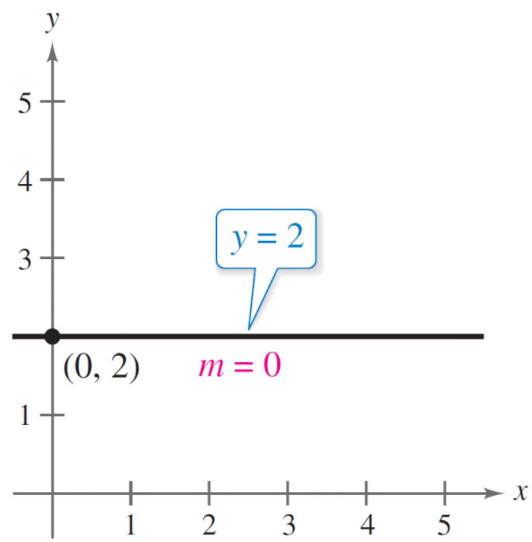
When m is positive, the line rises.

Example 1(b) – *Solution*

cont'd

By writing this equation in the form $y = (0)x + 2$, you can see that the y-intercept is (0, 2) and the slope is zero.

A zero slope implies that the line is horizontal—that is, it does not rise *or* fall.



When *m* is 0, the line is horizontal.

Example 1(c) – *Solution*

cont'd

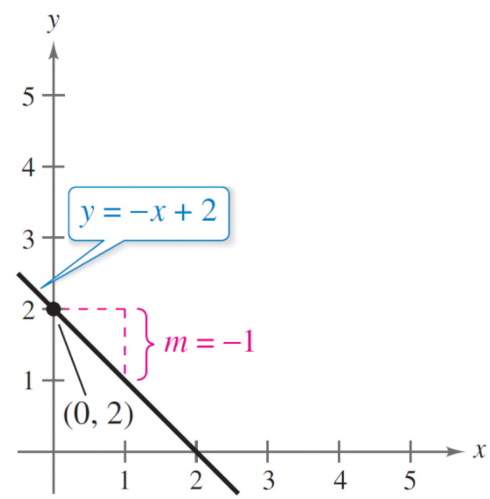
By writing this equation in slope-intercept form

$$x + y = 2$$ Write original equation.

$$y = -x + 2$$ Subtract *x* from each side.

$$y = (-1)x + 2$$ Write in slope-intercept form.

you can see that the *y*-intercept is (0, 2). Moreover, because the slope is $m = -1$, the line *falls* one unit for each unit the line moves to the right.



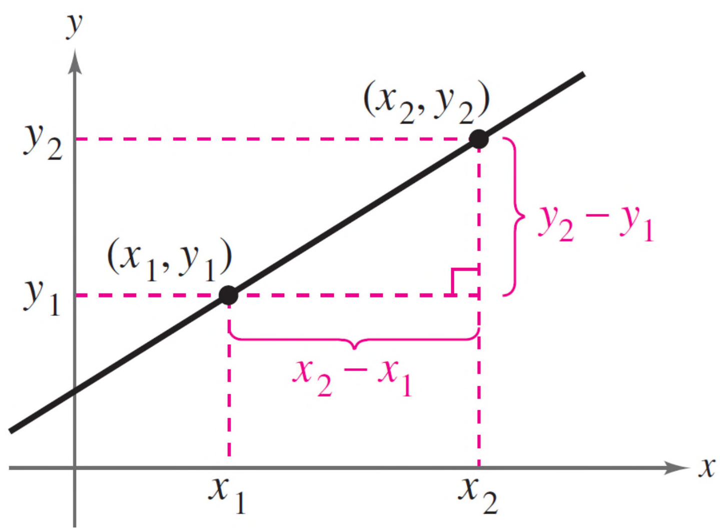
When *m* is negative, the line falls.

Finding the Slope of a Line

Finding the Slope of a Line

Given an equation of a line, you can find its slope by writing the equation in slope-intercept form. If you are not given an equation, then you can still find the slope of a line.

For instance, suppose you want to find the slope of the line passing through the points (x_1, y_1) and (x_2, y_2), as shown below.



Finding the Slope of a Line

As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction.

$$y_2 - y_1 = \text{the change in } y = \text{rise}$$

and

$$x_2 - x_1 = \text{the change in } x = \text{run}$$

The ratio of $(y_2 - y_1)$ to $(x_2 - x_1)$ represents the slope of the line that passes through the points (x_1, y_1) and (x_2, y_2).

Finding the Slope of a Line

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

The Slope of a Line Passing Through Two Points

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where $x_1 \neq x_2$.

Finding the Slope of a Line

When using the formula for slope, the *order of subtraction* is important. Given two points on a line, you are free to label either one of them as (x_1, y_1) and the other as (x_2, y_2).

However, once you have done this, you must form the numerator and denominator using the same order of subtraction.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Correct

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Correct

$$m = \frac{y_2 - y_1}{x_1 - x_2}$$

Incorrect

Finding the Slope of a Line

For instance, the slope of the line passing through the points (3, 4) and (5, 7) can be calculated as

$$m = \frac{7 - 4}{5 - 3} = \frac{3}{2}$$

or, reversing the subtraction order in both the numerator and denominator, as

$$m = \frac{4 - 7}{3 - 5} = \frac{-3}{-2} = \frac{3}{2}.$$

Example 2 – *Finding the Slope of a Line Through Two Points*

Find the slope of the line passing through each pair of points.

a. (–2, 0) and (3, 1)

b. (–1, 2) and (2, 2)

c. (0, 4) and (1, –1)

d. (3, 4) and (3, 1)

Example 2(a) – *Solution*

Letting $(x_1, y_1) = (–2, 0)$ and $(x_2, y_2) = (3, 1)$, you obtain a slope of

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - 0}{3 - (-2)}$$

$$= \frac{1}{5}.$$

See Figure 2.2.

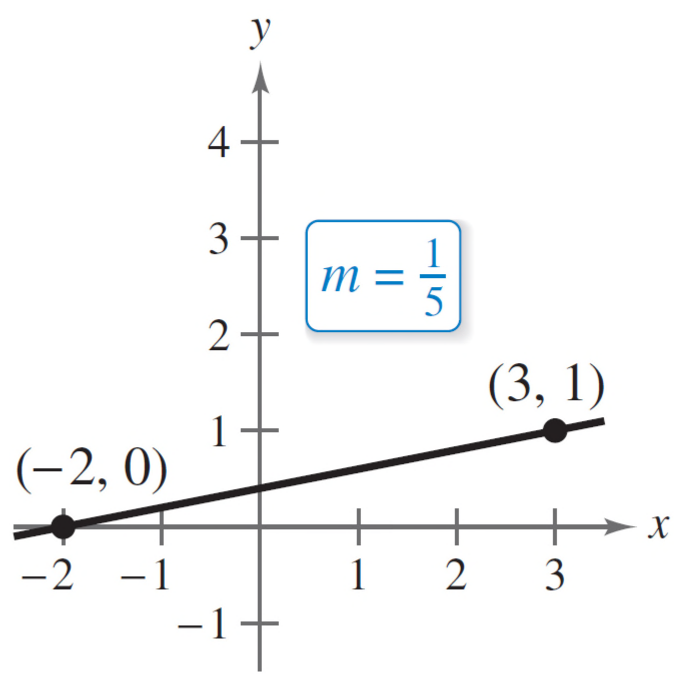


Figure 2.2

Example 2(b) – *Solution* cont'd

The slope of the line passing through (–1, 2) and (2, 2) is

$$m = \frac{2 - 2}{2 - (-1)}$$

$$= \frac{0}{3}$$

$$= 0.$$

See Figure 2.3.

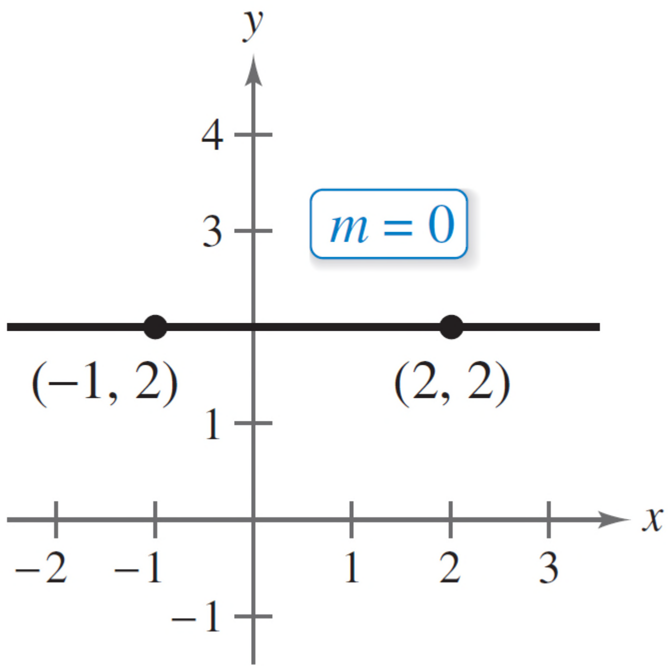


Figure 2.3

Example 2(c) – *Solution*

cont'd

The slope of the line passing through (0, 4) and (1, –1) is

$$m = \frac{-1 - 4}{1 - 0}$$

$$= \frac{-5}{1}$$

$$= -5.$$

See Figure 2.4.

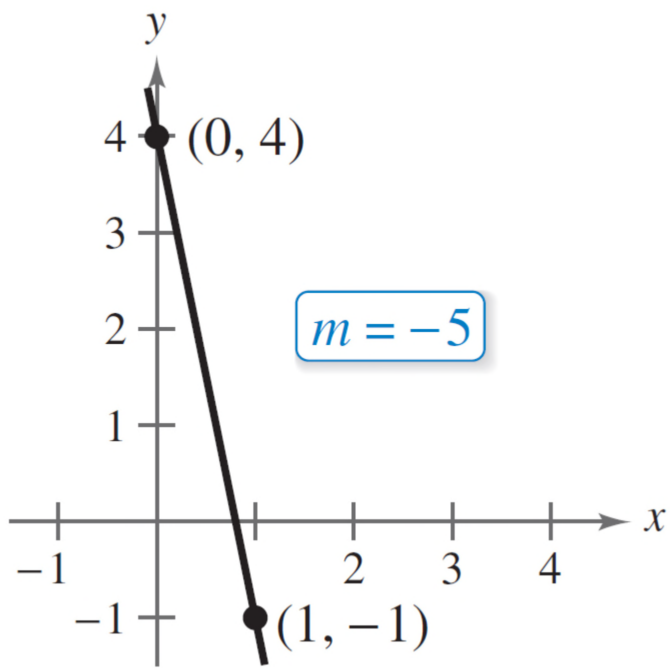


Figure 2.4

Example 2(d) – *Solution*

cont'd

The slope of the line passing through (3, 4) and (3, 1) is

$$m = \frac{1 - 4}{3 - 3}$$

$$= \frac{-3}{0}.$$

See Figure 2.5.

Because division by 0 is undefined, the slope is undefined and the line is vertical.

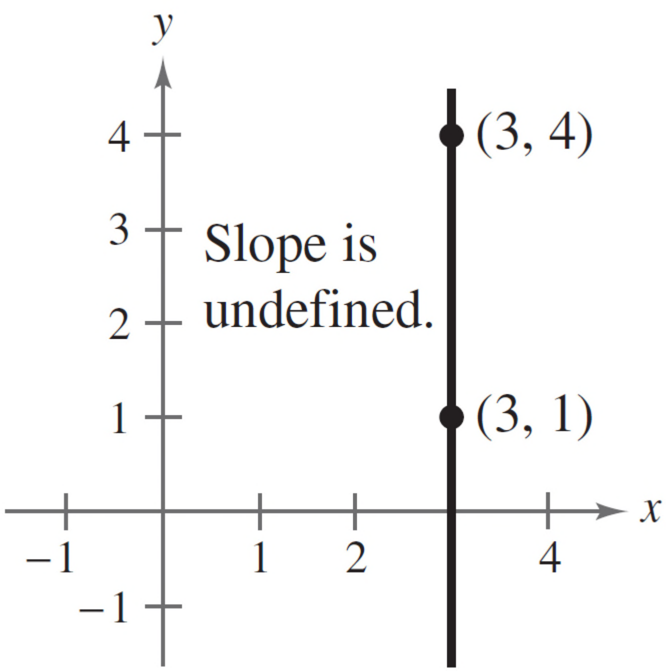


Figure 2.5

Writing Linear Equations in Two Variables

Writing Linear Equations in Two Variables

If (x_1, y_1) is a point on a line of slope m and (x, y) is *any other* point on the line, then

$$\frac{y - y_1}{x - x_1} = m.$$

This equation involving the variables x and y, rewritten in the form

$$y - y_1 = m(x - x_1)$$

is the **point-slope form** of the equation of a line.

Writing Linear Equations in Two Variables

Point-Slope Form of the Equation of a Line

The equation of the line with slope m passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

The point-slope form is most useful for *finding* the equation of a line.

Example 3 – *Using the Point-Slope Form*

Find the slope-intercept form of the equation of the line that has a slope of 3 and passes through the point (1, –2).

Solution:

Use the point-slope form with $m = 3$ and $(x_1, y_1) = (1, -2)$.

$$y - y_1 = m(x - x_1)$$ Point-slope form

$$y - (-2) = 3(x - 1)$$ Substitute for m, x_1 and y_1.

Example 3 – *Solution*

cont'd

$$y + 2 = 3x - 3 \qquad \text{Simplify.}$$

$$y = 3x - 5 \qquad \text{Write in slope-intercept form.}$$

The slope-intercept form of the equation of the line is $y = 3x - 5$.

Figure 2.6 shows the graph of this equation.

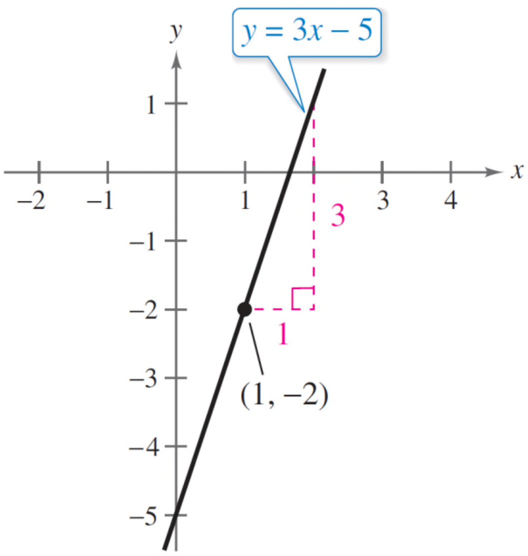


Figure 2.6

Writing Linear Equations in Two Variables

The point-slope form can be used to find an equation of the line passing through two points (x_1, y_1) and (x_2, y_2).

To do this, first find the slope of the line

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

and then use the point-slope form to obtain the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

Two-point form

This is sometimes called the **two-point form** of the equation of a line.

Parallel and Perpendicular Lines

Parallel and Perpendicular Lines

Slope can tell you whether two nonvertical lines in a plane are parallel, perpendicular, or neither.

Parallel and Perpendicular Lines

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is,

$$m_1 = m_2.$$

2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = \frac{-1}{m_2}.$$

Example 4 – *Finding Parallel and Perpendicular Lines*

Find the slope-intercept form of the equations of the lines that pass through the point (2, –1) and are

(a) parallel to and

(b) perpendicular to the line $2x - 3y = 5$.

Solution:

By writing the equation of the given line in slope-intercept form

$2x - 3y = 5$ Write original equation.

$-3y = -2x + 5$ Subtract $2x$ from each side.

Example 4 – *Solution*

cont'd

$$y = \frac{2}{3}x - \frac{5}{3}$$ Write in slope-intercept form.

you can see that it has a slope of $m = \frac{2}{3}$, as shown in Figure 2.7.

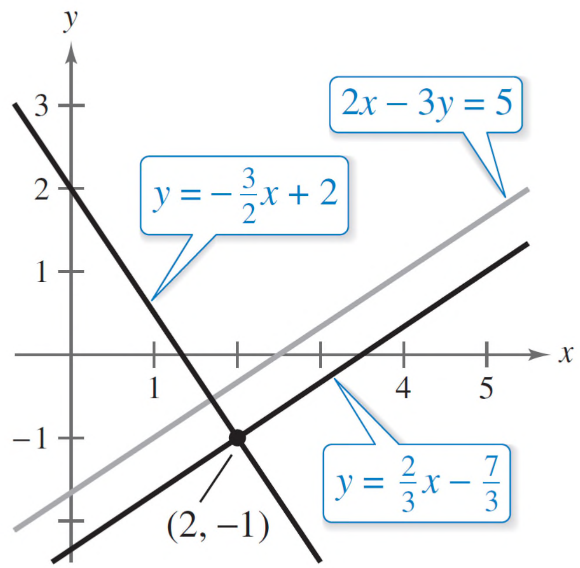


Figure 2.7

Example 4(a) – *Solution*

cont'd

Any line parallel to the given line must also have a slope of $\frac{2}{3}$.

So, the line through (2, –1) that is parallel to the given line has the following equation.

$y - (-1) = \frac{2}{3}(x - 2)$ Write in point-slope form.

$3(y + 1) = 2(x - 2)$ Multiply each side by 3.

$3y + 3 = 2x - 4$ Distributive Property

$y = \frac{2}{3}x - \frac{7}{3}$ Write in slope-intercept form.

Example 4(b) – *Solution*

cont'd

Any line perpendicular to the given line must have a slope of $-\frac{3}{2}$ (because $-\frac{3}{2}$ is the negative reciprocal of $\frac{2}{3}$).

So, the line through (2, –1) that is perpendicular to the given line has the following equation.

$y - (-1) = -\frac{3}{2}(x - 2)$	Write in point-slope form.
$2(y + 1) = -3(x - 2)$	Multiply each side by 2.
$2y + 2 = -3x + 6$	Distributive Property
$y = -\frac{3}{2}x + 2$	Write in slope-intercept form.

Parallel and Perpendicular Lines

Notice in Example 4 how the slope-intercept form is used to obtain information about the graph of a line, whereas the point-slope form is used to write the equation of a line.

Applications

Applications

In real-life problems, the slope of a line can be interpreted as either a *ratio* or a *rate*.

If the x-axis and y-axis have the same unit of measure, then the slope has no units and is a **ratio.**

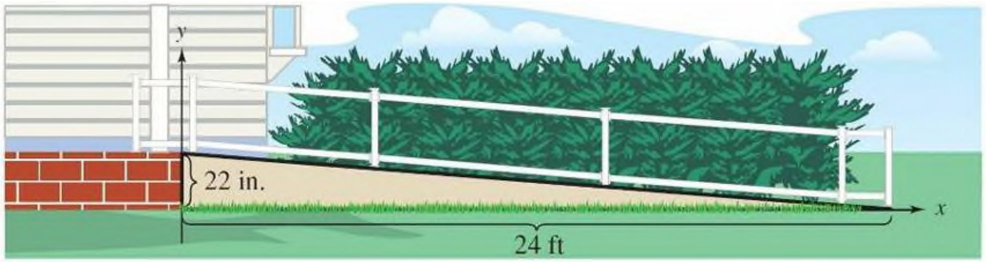
If the x-axis and y-axis have different units of measure, then the slope is a **rate** or **rate of change.**

Example 5 – *Using Slope as a Ratio*

The maximum recommended slope of a wheelchair ramp is $\frac{1}{12}$. A business is installing a wheelchair ramp that rises 22 inches over a horizontal length of 24 feet. Is the ramp steeper than recommended?

Example 5 – *Solution*

The horizontal length of the ramp is 24 feet or 12(24) = 288 inches, as shown below.



So, the slope of the ramp is

$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

Example 5 – *Solution*

cont'd

$$= \frac{22 \text{ in.}}{288 \text{ in.}}$$

$$\approx 0.076.$$

Because $\frac{1}{12} \approx 0.083$, the slope of the ramp is not steeper than recommended.

Example 8 – *Predicting Sales*

The sales for Best Buy were approximately $49.7 billion in 2009 and $50.3 billion in 2010. Using only this information, write a linear equation that gives the sales in terms of the year. Then predict the sales in 2013.

Example 8 – *Solution*

Let t = 9 represent 2009. Then the two given values are represented by the data points (9, 49.7) and (10, 50.3).

The slope of the line through these points is

$$m = \frac{50.3 - 49.7}{10 - 9}$$

$$= 0.6$$

You can find the equation that relates the sales y and the year t to be

y – 49.7 = 0.6(t – 9) Write in point-slope form.

Example 8 – *Solution*

cont'd

$y = 0.6t + 44.3.$ Write in slope-intercept form.

According to this equation, the sales for 2013 will be

$y = 0.6(13) + 44.3$

$= 7.8 + 44.3$

$= \$52.1$ billion. (See Figure 2.10.)

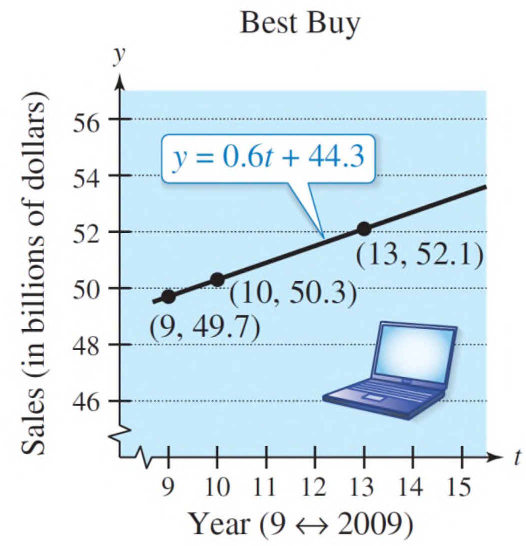
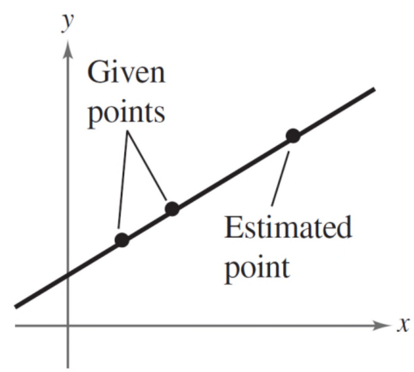


Figure 2.10

Applications

The prediction method illustrated in Example 8 is called **linear extrapolation.**

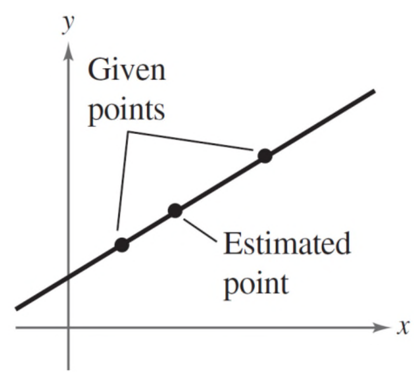
Note in Figure 2.11 that an extrapolated point does not lie between the given points.



Linear extrapolation

Figure 2.11

When the estimated point lies between two given points, as shown in Figure 2.12, the procedure is called **linear interpolation.**



Linear interpolation

Figure 2.12

Applications

Because the slope of a vertical line is not defined, its equation cannot be written in slope-intercept form.

However, every line has an equation that can be written in the **general form**

$$Ax + By + C = 0$$

where A and B are not both zero.

Applications

Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$
6. Two-point form: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

2 Functions and Their Graphs

2.2 Functions

Objectives

- Determine whether relations between two variables are functions, and use function notation.
- Find the domains of functions.
- Use functions to model and solve real-life problems.
- Evaluate difference quotients.

Introduction to Functions and Function Notation

Introduction to Functions and Function Notation

Many everyday phenomena involve two quantities that are related to each other by some rule of correspondence. The mathematical term for such a rule of correspondence is a **relation.**

In mathematics, equations and formulas often represent relations.

For instance, the simple interest I earned on \$1000 for 1 year is related to the annual interest rate r by the formula $I = 1000r$.

Introduction to Functions and Function Notation

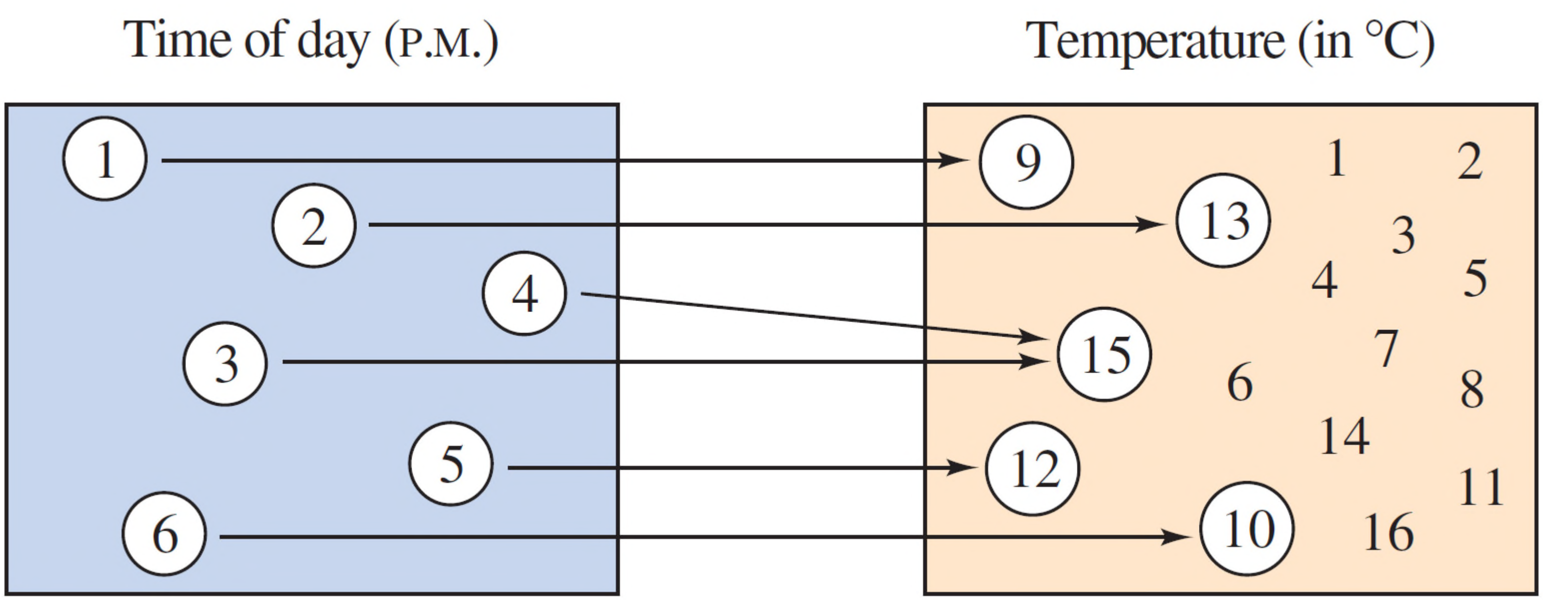
The formula $I = 1000r$ represents a special kind of relation that matches each item from one set with *exactly one* item from a different set. Such a relation is called a **function.**

Definition of Function

A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

Introduction to Functions and Function Notation

To help understand this definition, look at the function below, which relates the time of day to the temperature.



Set *A* is the domain.
Inputs: 1, 2, 3, 4, 5, 6

Set *B* contains the range.
Outputs: 9, 10, 12, 13, 15

Introduction to Functions and Function Notation

The following ordered pairs can represent this function. The first coordinate (*x*-value) is the input and the second coordinate (*y*-value) is the output.

$$\{(1, 9), (2, 13), (3, 15), (4, 15), (5, 12), (6, 10)\}$$

Characteristics of a Function from Set *A* to Set *B*

1. Each element in *A* must be matched with an element in *B*.
2. Some elements in *B* may not be matched with any element in *A*.
3. Two or more elements in *A* may be matched with the same element in *B*.
4. An element in *A* (the domain) cannot be matched with two different elements in *B*.

Introduction to Functions and Function Notation

Four common ways to represent functions are as follows.

Four Ways to Represent a Function

1. *Verbally* by a sentence that describes how the input variable is related to the output variable
2. *Numerically* by a table or a list of ordered pairs that matches input values with output values
3. *Graphically* by points on a graph in a coordinate plane in which the horizontal axis represents the input values and the vertical axis represents the output values
4. *Algebraically* by an equation in two variables

Introduction to Functions and Function Notation

To determine whether a relation is a function, you must decide whether each input value is matched with exactly one output value.

When any input value is matched with two or more output values, the relation is not a function.

Example 1 – *Testing for Functions*

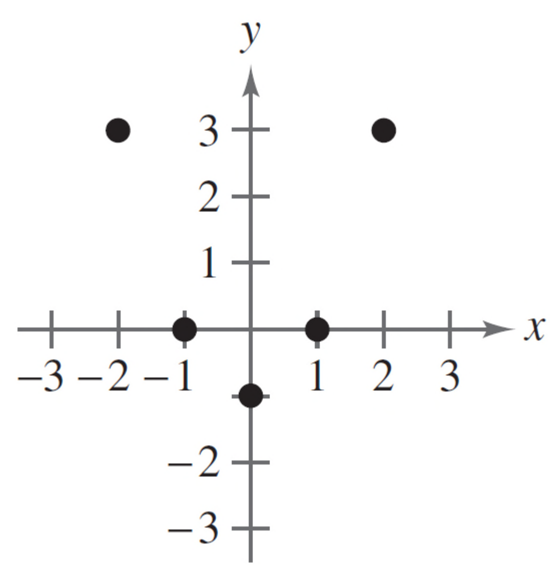
Determine whether the relation represents y as a function of x.

a. The input value x is the number of representatives from a state, and the output value y is the number of senators.

b.

Input, x	Output, y
2	11
2	10
3	8
4	5
5	1

c.



Example 1 – *Solution*

a. This verbal description *does* describe *y* as a function of *x*. Regardless of the value of *x*, the value of *y* is always 2. Such functions are called *constant functions*.

b. This table *does not* describe *y* as a function of *x*. The input value 2 is matched with two different *y*-values.

c. The graph *does* describe *y* as a function of *x*. Each input value is matched with exactly one output value.

Introduction to Functions and Function Notation

Representing functions by sets of ordered pairs is common in *discrete mathematics*.

In algebra, however, it is more common to represent functions by equations or formulas involving two variables. For instance, the equation

$$y = x^2$$

y is a function of x.

represents the variable y as a function of the variable x. In this equation, x is the **independent variable** and y is the **dependent variable.**

Introduction to Functions and Function Notation

The domain of the function is the set of all values taken on by the independent variable *x*, and the range of the function is the set of all values taken on by the dependent variable *y*.

Introduction to Functions and Function Notation

When using an equation to represent a function, it is convenient to name the function for easy reference.

For example, you know that the equation $y = 1 - x^2$ describes y as a function of x.

Suppose you give this function the name "f." Then you can use the following **function notation.**

Input	**Output**	**Equation**
x	$f(x)$	$f(x) = 1 - x^2$

Introduction to Functions and Function Notation

The symbol $f(x)$ is read as *the value of f at x* or simply *f of x*. The symbol $f(x)$ corresponds to the y-value for a given x. So, you can write $y = f(x)$.

Keep in mind that f is the *name* of the function, whereas $f(x)$ is the *value* of the function at x.

For instance, the function

$$f(x) = 3 - 2x$$

has *function values* denoted by $f(-1)$, $f(0)$, $f(2)$, and so on. To find these values, substitute the specified input values into the given equation.

Introduction to Functions and Function Notation

For $x = -1$, $f(-1) = 3 - 2(-1) = 3 + 2 = 5$.

For $x = 0$, $f(0) = 3 - 2(0) = 3 - 0 = 3$.

For $x = 2$, $f(2) = 3 - 2(2) = 3 - 4 = -1$.

Although f is often used as a convenient function name and x is often used as the independent variable, you can use other letters. For instance,

$f(x) = x^2 - 4x + 7$, $f(t) = t^2 - 4t + 7$, and $g(s) = s^2 - 4s + 7$

all define the same function. In fact, the role of the independent variable is that of a "placeholder."

Introduction to Functions and Function Notation

Consequently, the function could be described by

$$f(\square) = (\square)^2 - 4(\square) + 7.$$

A function defined by two or more equations over a specified domain is called a **piecewise-defined function.**

Example 4 – *A Piecewise-Defined Function*

Evaluate the function when $x = -1, 0$, and 1.

$$f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

Solution:

Because $x = -1$ is less than 0, use $f(x) = x^2 + 1$ to obtain

$$f(-1) = (-1)^2 + 1$$

$$= 2.$$

For $x = 0$, use $f(x) = x - 1$ to obtain

$$f(0) = (0) - 1$$

$$= -1.$$

Example 4 – *Solution*

cont'd

For $x = 1$, use $f(x) = x - 1$ to obtain

$$f(1) = (1) - 1$$

$$= 0.$$

The Domain of a Function

The Domain of a Function

The domain of a function can be described explicitly or it can be *implied* by the expression used to define the function.

The **implied domain** is the set of all real numbers for which the expression is defined. For instance, the function

$$f(x) = \frac{1}{x^2 - 4}$$

Domain excludes *x*-values that result in division by zero.

has an implied domain consisting of all real x other than $x = \pm 2$.

These two values are excluded from the domain because division by zero is undefined.

The Domain of a Function

Another common type of implied domain is that used to avoid even roots of negative numbers. For example, the function

$$f(x) = \sqrt{x}$$

Domain excludes *x*-values that result in even roots of negative numbers.

is defined only for $x \geq 0$. So, its implied domain is the interval $[0, \infty)$.

In general, the domain of a function *excludes* values that would cause division by zero *or* that would result in the even root of a negative number.

Example 7 – *Finding the Domain of a Function*

Find the domain of each function.

a. *f* : {(–3, 0), (–1, 4), (0, 2), (2, 2), (4, –1)}

b. $g(x) = \dfrac{1}{x + 5}$

c. Volume of a sphere: $V = \frac{4}{3}\pi r^3$

d. $h(x) = \sqrt{4 - 3x}$

Example 7 – *Solution*

a. The domain of f consists of all first coordinates in the set of ordered pairs.

Domain = {–3, –1, 0, 2, 4}

b. Excluding x-values that yield zero in the denominator, the domain of g is the set of all real numbers x except $x = –5$.

c. Because this function represents the volume of a sphere, the values of the radius r must be positive.

So, the domain is the set of all real numbers r such that $r > 0$.

Example 7 – *Solution*

cont'd

d. This function is defined only for *x*-values for which

$$4 - 3x \geq 0.$$

By solving this inequality, you can conclude that $x \leq \frac{4}{3}$.

So, the domain is the interval $\left(-\infty, \frac{4}{3}\right]$.

The Domain of a Function

In Example 7(c), note that the domain of a function may be implied by the physical context.

For instance, from the equation

$$V = \frac{4}{3}\pi r^3$$

you would have no reason to restrict *r* to positive values, but the physical context implies that a sphere cannot have a negative or zero radius.

Applications

Example 8 – *The Dimensions of a Container*

You work in the marketing department of a soft-drink company and are experimenting with a new can for iced tea that is slightly narrower and taller than a standard can. For your experimental can, the ratio of the height to the radius is 4.

a. Write the volume of the can as a function of the radius *r*.

b. Write the volume of the can as a function of the height *h*.

Example 8 – *Solution*

a. $V(r) = \pi r^2 h$ Write V as a function of r.

$= \pi r^2(4r)$

$= 4\pi r^3$

b. $V(h) = \pi r^2 h$ Write V as a function of h.

$$= \pi \left(\frac{h}{4}\right)^2 h$$

$$= \frac{\pi h^3}{16}$$

Difference Quotients

Difference Quotients

One of the basic definitions in calculus employs the ratio

$$\frac{f(x+h)-f(x)}{h}, \quad h \neq 0.$$

This ratio is called a **difference quotient,** as illustrated in Example 11.

Example 11 – *Evaluating a Difference Quotient*

For $f(x) = x^2 - 4x + 7$, find $\dfrac{f(x+h) - f(x)}{h}$.

Solution:

$$\frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^2 - 4(x+h) + 7] - (x^2 - 4x + 7)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 4x - 4h + 7 - x^2 + 4x - 7}{h}$$

$$= \frac{2xh + h^2 - 4h}{h}$$

$$= \frac{h(2x + h - 4)}{h}$$

$$= 2x + h - 4, \quad h \neq 0$$

Difference Quotients

Summary of Function Terminology

Function: A **function** is a relationship between two variables such that to each value of the independent variable there corresponds exactly one value of the dependent variable.

Function Notation: $y = f(x)$

f is the *name* of the function.

y is the **dependent variable.**

x is the **independent variable.**

$f(x)$ is the *value of the function at x*.

Domain: The **domain** of a function is the set of all values (inputs) of the independent variable for which the function is defined. If x is in the domain of f, then f is said to be *defined* at x. If x is not in the domain of f, then f is said to be *undefined* at x.

Range: The **range** of a function is the set of all values (outputs) assumed by the dependent variable (that is, the set of all function values).

Implied Domain: If f is defined by an algebraic expression and the domain is not specified, then the **implied domain** consists of all real numbers for which the expression is defined.

2 Functions and Their Graphs

2.3

Analyzing Graphs of Functions

Objectives

- Use the Vertical Line Test for functions.
- Find the zeros of functions.
- Determine intervals on which functions are increasing or decreasing and determine relative maximum and relative minimum values of functions.
- Determine the average rate of change of a function.
- Identify even and odd functions.

The Graph of a Function

The Graph of a Function

We have studied functions from an algebraic point of view. In this section, you will study functions from a graphical perspective.

The **graph of a function** f is the collection of ordered pairs $(x, f(x))$ such that x is in the domain of f.

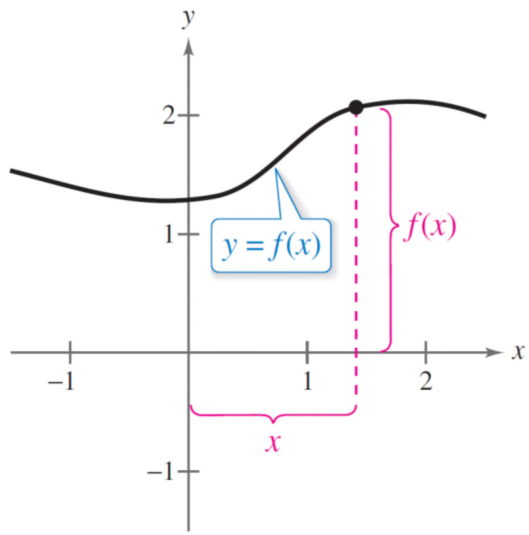
The Graph of a Function

As you study this section, remember that

x = the directed distance from the y-axis

$y = f(x)$ = the directed distance from the x-axis

as shown below.



Example 1 – *Finding the Domain and Range of a Function*

Use the graph of the function f, shown in Figure 2.13, to find

(a) the domain of f,

(b) the function values $f(-1)$ and $f(2)$, and

(c) the range of f.

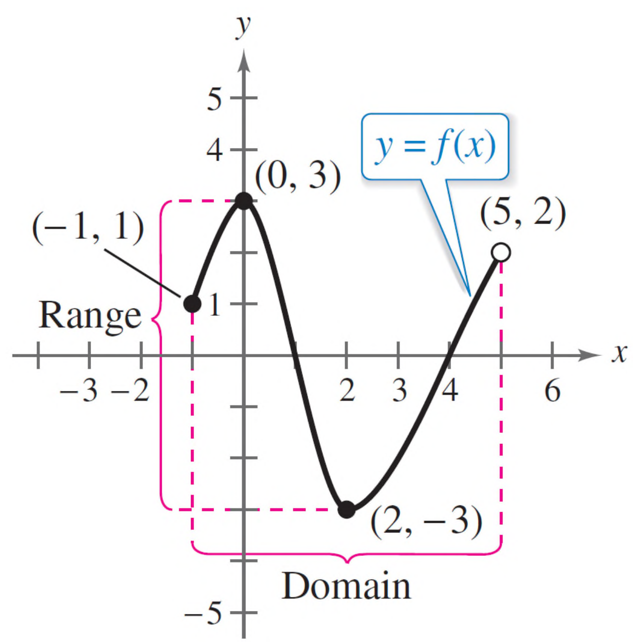


Figure 2.13

Example 1 – *Solution*

a. The closed dot at (–1, 1) indicates that $x = -1$ is in the domain of f, whereas the open dot at (5, 2) indicates that $x = 5$ is not in the domain.

So, the domain of f is all x in the interval $[-1, 5)$.

b. Because (–1, 1) is a point on the graph of f, it follows that $f(-1) = 1$. Similarly, because (2, –3) is a point on the graph of f, it follows that $f(2) = -3$.

c. Because the graph does not extend below $f(2) = -3$ or above $f(0) = 3$, the range of f is the interval $[-3, 3]$.

The Graph of a Function

The use of dots (open or closed) at the extreme left and right points of a graph indicates that the graph does not extend beyond these points.

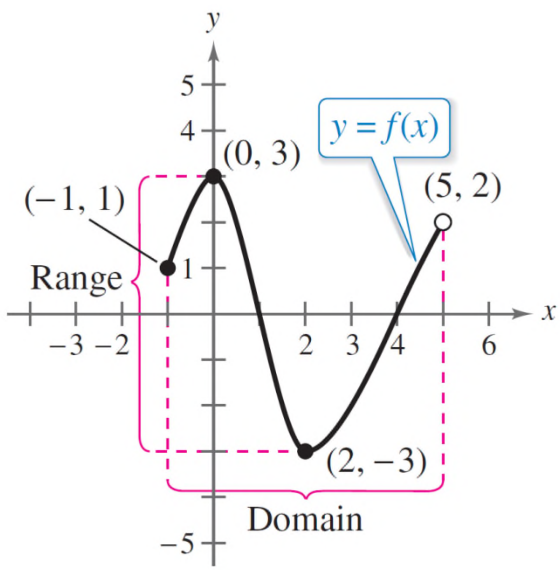


Figure 2.13

If such dots are not on the graph, then assume that the graph extends beyond these points.

The Graph of a Function

By the definition of a function, at most one *y*-value corresponds to a given *x*-value.

This means that the graph of a function cannot have two or more different points with the same *x*-coordinate, and no two points on the graph of a function can be vertically above or below each other.

It follows, then, that a vertical line can intersect the graph of a function at most once.

The Graph of a Function

This observation provides a convenient visual test called the **Vertical Line Test** for functions.

Vertical Line Test for Functions

A set of points in a coordinate plane is the graph of y as a function of x if and only if no *vertical* line intersects the graph at more than one point.

Zeros of a Function

Zeros of a Function

If the graph of a function of x has an x-intercept at $(a, 0)$, then a is a **zero** of the function.

Zeros of a Function

The **zeros of a function** f of x are the x-values for which $f(x) = 0$.

Example 3 – *Finding the Zeros of a Function*

Find the zeros of each function.

a. $f(x) = 3x^2 + x - 10$ **b.** $g(x) = \sqrt{10 - x^2}$ **c.** $h(t) = \dfrac{2t - 3}{t + 5}$

Solution:

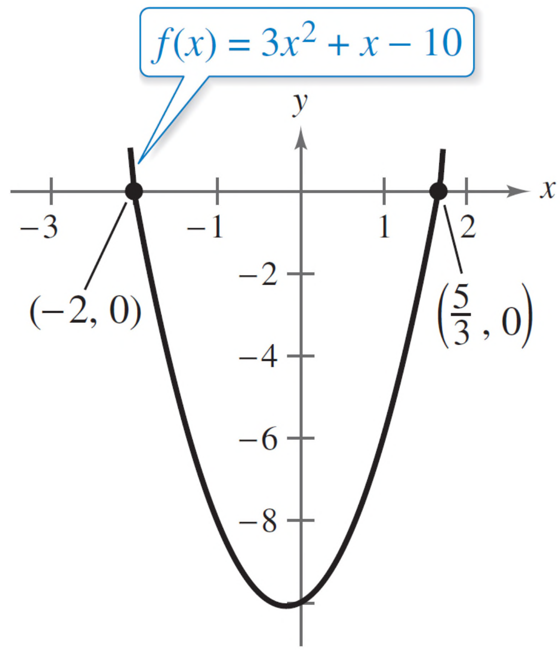
To find the zeros of a function, set the function equal to zero and solve for the independent variable.

a. $3x^2 + x - 10 = 0$		Set $f(x)$ equal to 0.
$(3x - 5)(x + 2) = 0$		Factor.
$3x - 5 = 0$	⇨ $x = \frac{5}{3}$	Set 1st factor equal to 0.
$x + 2 = 0$	⇨ $x = -2$	Set 2nd factor equal to 0.

Example 3 – *Solution*

cont'd

The zeros of *f* are $x = \frac{5}{3}$ and $x = –2$. In Figure 2.14, note that the graph of *f* has $(\frac{5}{3}, 0)$ and $(–2, 0)$ as its *x*-intercepts.



Zeros of *f*: $x = –2$, $x = \frac{5}{3}$

Figure 2.14

Example 3 – *Solution*

cont'd

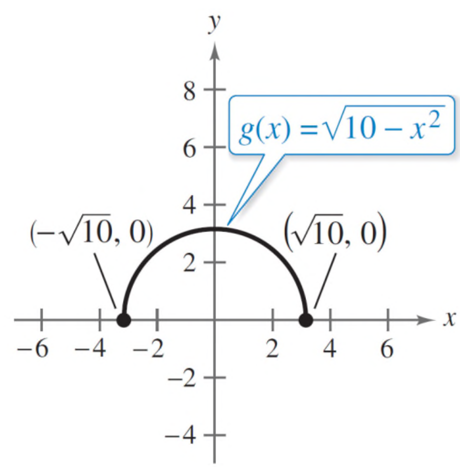
b. $\sqrt{10 - x^2} = 0$ Set $g(x)$ equal to 0.

$10 - x^2 = 0$ Square each side.

$10 = x^2$ Add x^2 to each side.

$\pm\sqrt{10} = x$ Extract square roots.

The zeros of g are $x = -\sqrt{10}$ and $x = \sqrt{10}$. In Figure 2.15, note that the graph of g has $(-\sqrt{10}, 0)$ and $(\sqrt{10}, 0)$ as its x-intercepts.



Zeros of g: $x = \pm\sqrt{10}$

Figure 2.15

Example 3 – *Solution*

cont'd

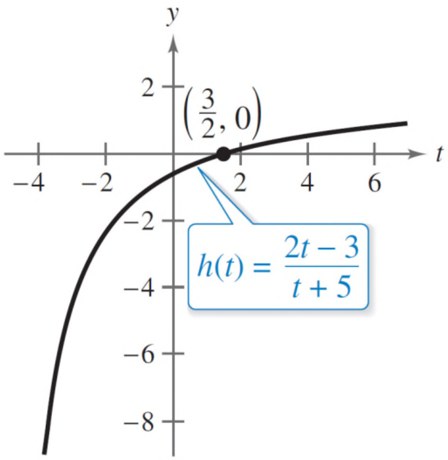
c. $\dfrac{2t-3}{t+5} = 0$ Set $h(t)$ equal to 0.

$2t - 3 = 0$ Multiply each side by $t + 5$

$2t = 3$ Add 3 to each side.

$t = \dfrac{3}{2}$ Divide each side by 2.

The zero of h is $t = \frac{3}{2}$. In Figure 2.16, note that the graph of h has $(\frac{3}{2}, 0)$ as its t-intercept.



Zero of h: $t = \frac{3}{2}$

Figure 2.16

Increasing and Decreasing Functions

Increasing and Decreasing Functions

The more you know about the graph of a function, the more you know about the function itself. Consider the graph shown in Figure 2.17.

As you move from *left to right*, this graph falls from $x = -2$ to $x = 0$, is constant from $x = 0$ to $x = 2$, and rises from $x = 2$ to $x = 4$.

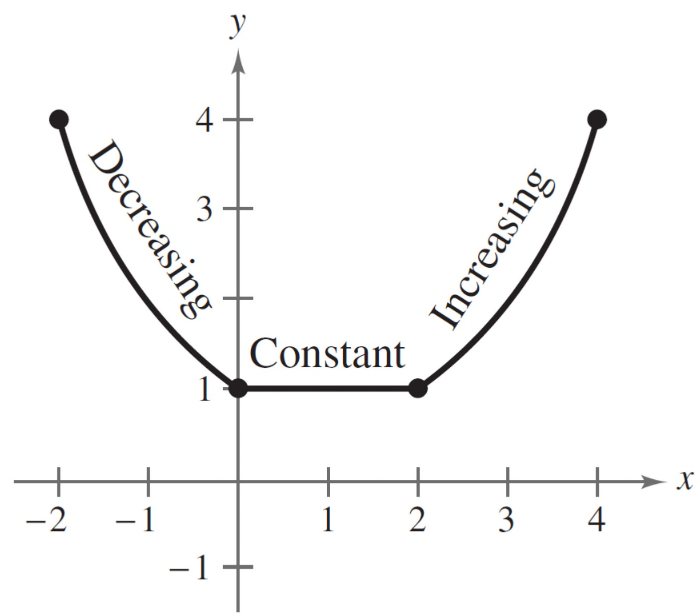


Figure 2.17

Increasing and Decreasing Functions

Increasing, Decreasing, and Constant Functions

A function f is **increasing** on an interval when, for any x_1 and x_2 in the interval,

$$x_1 < x_2 \quad \text{implies} \quad f(x_1) < f(x_2).$$

A function f is **decreasing** on an interval when, for any x_1 and x_2 in the interval,

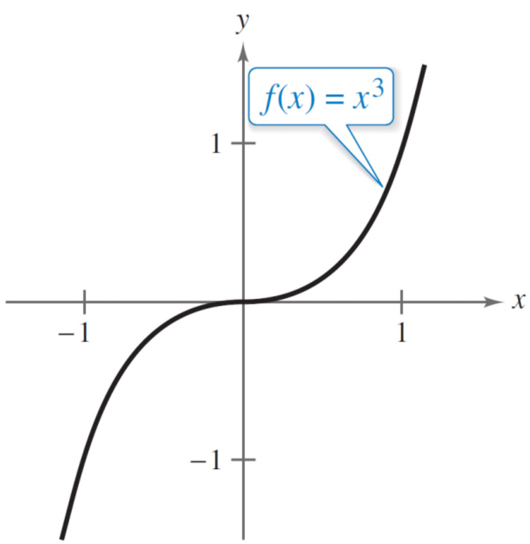
$$x_1 < x_2 \quad \text{implies} \quad f(x_1) > f(x_2).$$

A function f is **constant** on an interval when, for any x_1 and x_2 in the interval,

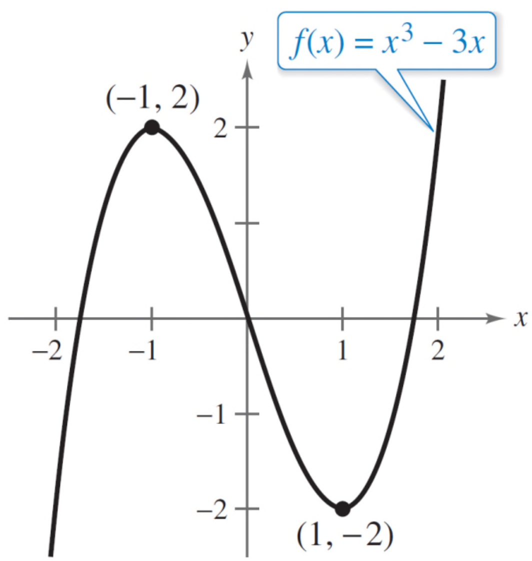
$$f(x_1) = f(x_2).$$

Example 4 – *Describing Function Behavior*

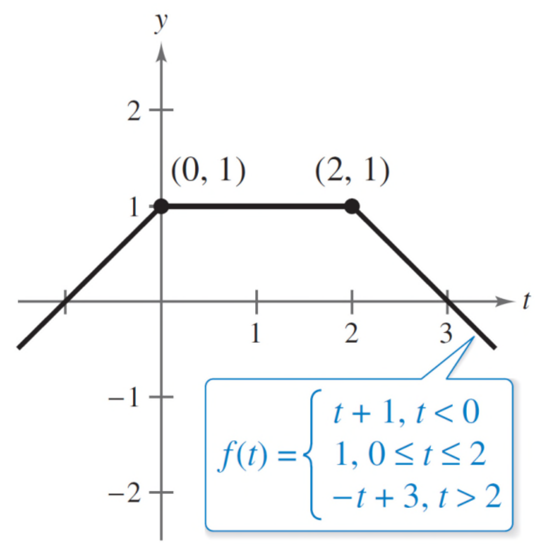
Use the graphs to describe the increasing, decreasing, or constant behavior of each function.



(a)



(b)



(c)

Example 4 – *Solution*

a. This function is increasing over the entire real line.

b. This function is increasing on the interval $(-\infty, -1)$, decreasing on the interval $(-1, 1)$ and increasing on the interval $(1, \infty)$.

c. This function is increasing on the interval $(-\infty, 0)$, constant on the interval $(0, 2)$, and decreasing on the interval $(2, \infty)$.

Increasing and Decreasing Functions

To help you decide whether a function is increasing, decreasing, or constant on an interval, you can evaluate the function for several values of *x*.

However, you need calculus to determine, for certain, all intervals on which a function is increasing, decreasing, or constant.

Increasing and Decreasing Functions

The points at which a function changes its increasing, decreasing, or constant behavior are helpful in determining the **relative minimum** or **relative maximum** values of the function.

Definitions of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f when there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \le f(x).$$

A function value $f(a)$ is called a **relative maximum** of f when there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \ge f(x).$$

Increasing and Decreasing Functions

Figure 2.18 shows several different examples of relative minima and relative maxima.

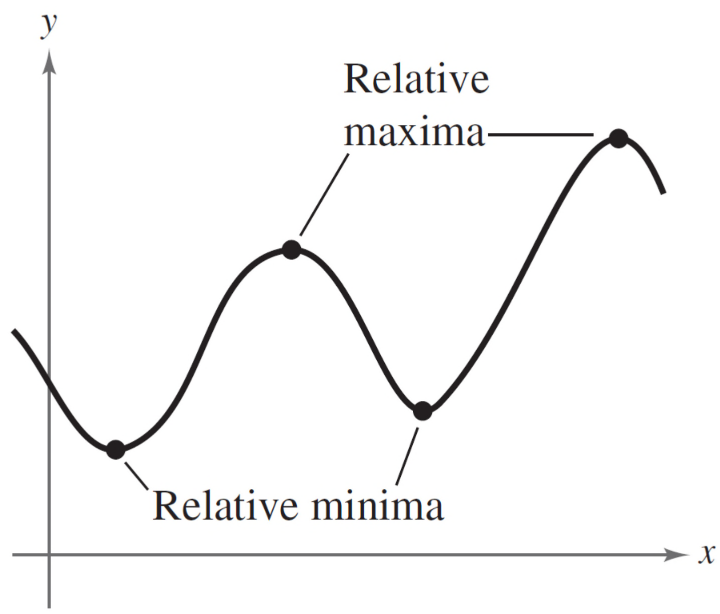


Figure 2.18

Increasing and Decreasing Functions

We will study a technique for finding the *exact point* at which a second-degree polynomial function has a relative minimum or relative maximum.

For the time being, however, you can use a graphing utility to find reasonable approximations of these points.

Average Rate of Change

Average Rate of Change

We have learned that the slope of a line can be interpreted as a *rate of change*.

For a nonlinear graph whose slope changes at each point, the **average rate of change** between any two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is the slope of the line through the two points (see Figure 2.20).

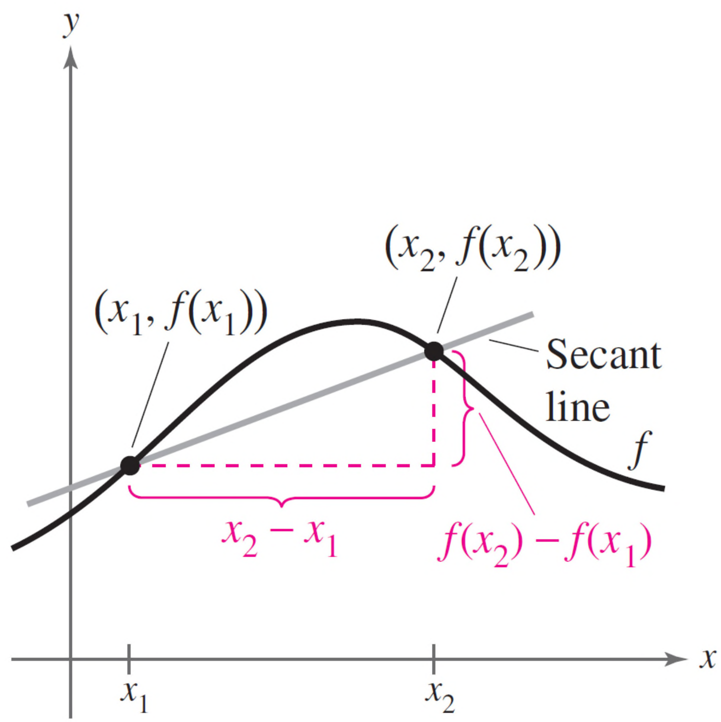


Figure 2.20

Average Rate of Change

The line through the two points is called the **secant line**, and the slope of this line is denoted as m_{sec}.

Average rate of change of f from x_1 to x_2 $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$= \frac{\text{change in } y}{\text{change in } x}$$

$$= m_{sec}$$

Example 6 – *Average Rate of Change of a Function*

Find the average rates of change of $f(x) = x^3 - 3x$

(a) from $x_1 = -2$ to $x_2 = -1$ and

(b) from $x_1 = 0$ to $x_2 = 1$ (see Figure 2.21).

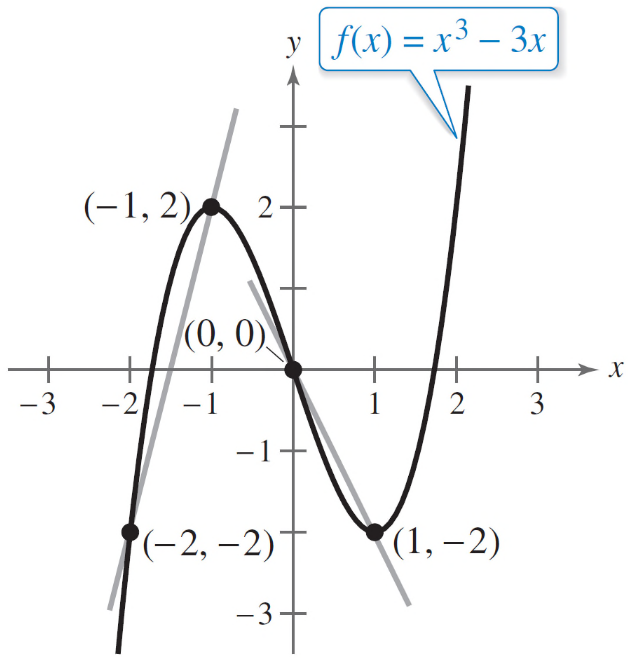


Figure 2.21

Example 6(a) – *Solution*

The average rate of change of f from $x_1 = –2$ to $x_2 = –1$ is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(-1) - f(-2)}{-1 - (-2)}$$

$$= \frac{2 - (-2)}{1}$$

$$= 4.$$

Secant line has positive slope.

Example 6(b) – *Solution*

cont'd

The average rate of change of *f* from $x_1 = 0$ to $x_2 = 1$ is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(0)}{1 - 0}$$

$$= \frac{-2 - 0}{1}$$

$$= -2.$$

Secant line has negative slope.

Even and Odd Functions

Even and Odd Functions

We have studied different types of symmetry of a graph. In the terminology of functions, a function is said to be **even** when its graph is symmetric with respect to the *y*-axis and **odd** when its graph is symmetric with respect to the origin.

The symmetry tests yield the following tests for even and odd functions.

Tests for Even and Odd Functions

A function $y = f(x)$ is **even** when, for each x in the domain of f, $f(-x) = f(x)$.

A function $y = f(x)$ is **odd** when, for each x in the domain of f, $f(-x) = -f(x)$.

Example 8 – *Even and Odd Functions*

(a) The function $g(x) = x^3 - x$ is odd because $g(-x) = -g(x)$, as follows.

$$g(-x) = (-x)^3 - (-x)$$ Substitute $-x$ for x.

$$= -x^3 + x$$ Simplify.

$$= -(x^3 - x)$$ Distributive Property

$$= -g(x)$$ Test for odd function

Example 8 – *Even and Odd Functions* cont'd

(b) The function $h(x) = x^2 + 1$ is even because $h(-x) = h(x)$, as follows.

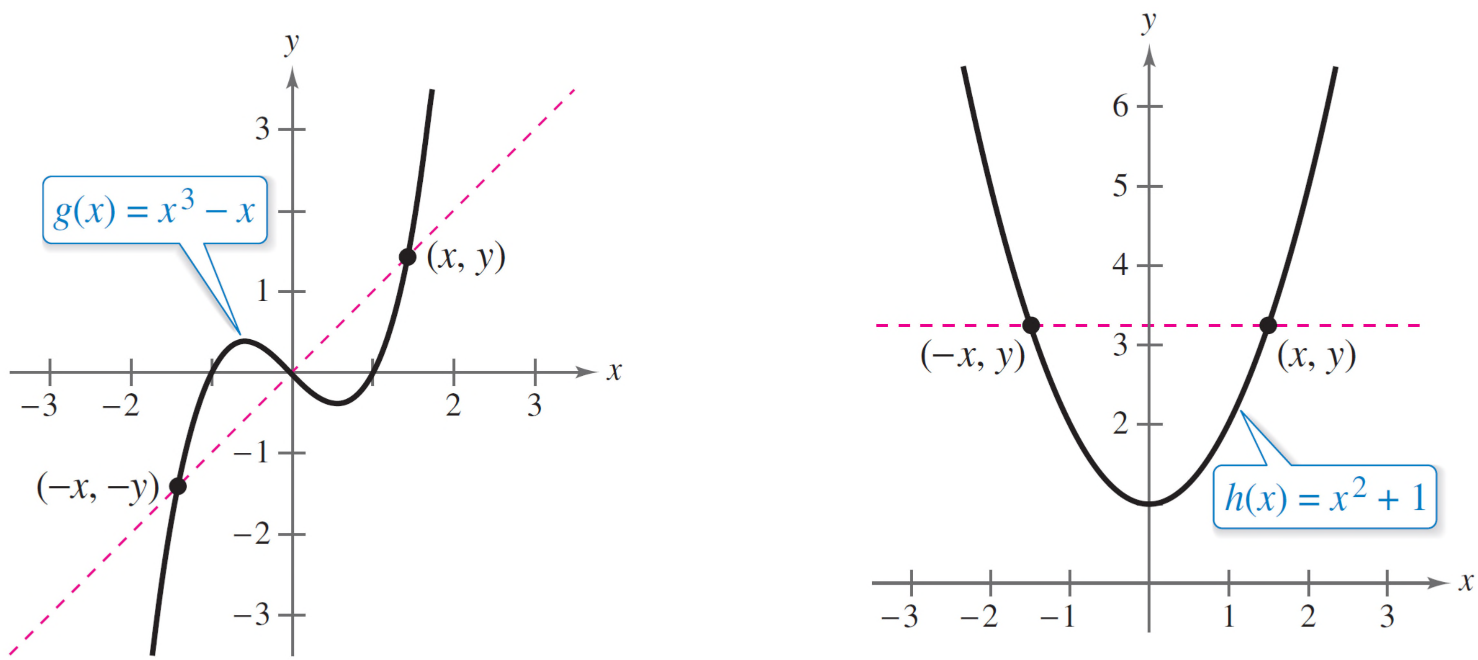
$$h(-x) = (-x)^2 + 1 \qquad \text{Substitute } -x \text{ for } x.$$

$$= x^2 + 1 \qquad \text{Simplify.}$$

$$= h(x) \qquad \text{Test for even function}$$

Example 8 – *Even and Odd Functions* cont'd

Figure 2.22 shows the graphs and symmetry of these two functions.



(a) Symmetric to origin: Odd Function

(b) Symmetric to *y*-axis: Even Function

Figure 2.22

2 Functions and Their Graphs

2.4 A Library of Parent Functions

Objectives

- Identify and graph linear and squaring functions.
- Identify and graph cubic, square root, and reciprocal function.
- Identify and graph step and other piecewise-defined functions.
- Recognize graphs of parent functions.

Linear and Squaring Functions

Linear and Squaring Functions

One of the goals of this text is to enable you to recognize the basic shapes of the graphs of different types of functions.

For instance, you know that the graph of the **linear function** $f(x) = ax + b$ is a line with slope $m = a$ and y-intercept at $(0, b)$.

Linear and Squaring Functions

The graph of the linear function has the following characteristics.

- The domain of the function is the set of all real numbers.
- When $m \neq 0$, the range of the function is the set of all real numbers.
- The graph has an x-intercept of $(-b/m, 0)$ and a y-intercept of $(0, b)$.
- The graph is increasing when $m > 0$, decreasing when $m < 0$, and constant when $m = 0$.

Example 1 – *Writing a Linear Function*

Write the linear function f for which $f(1) = 3$ and $f(4) = 0$.

Solution:

To find the equation of the line that passes through $(x_1, y_1) = (1, 3)$ and $(x_2, y_2) = (4, 0)$ first find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 1}$$

$$= \frac{-3}{3}$$

$$= -1$$

Example 1 – *Solution* cont'd

Next, use the point-slope form of the equation of a line.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 3 = -1(x - 1)$ Substitute for x_1, y_1 and m

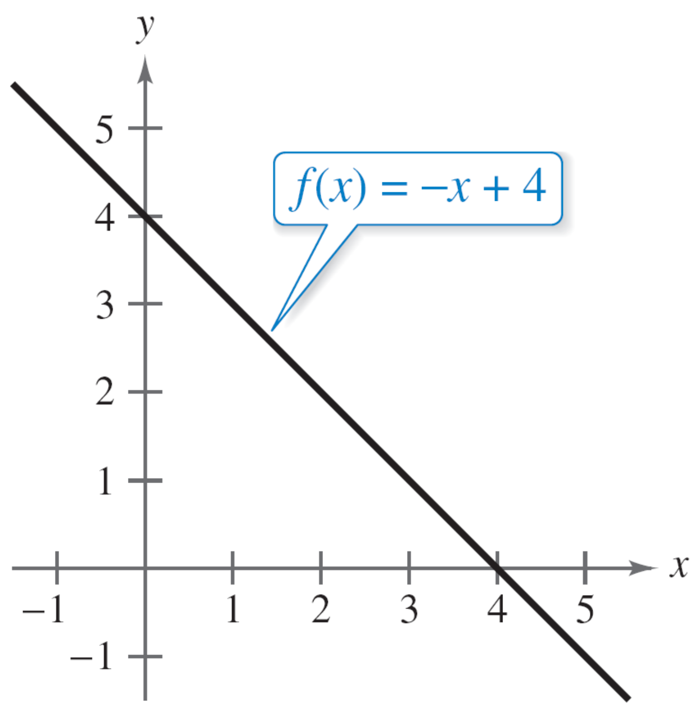
$y = -x + 4$ Simplify

$f(x) = -x + 4$ Function notation

Example 1 – *Solution*

cont'd

The figure below shows the graph of this function.



Linear and Squaring Functions

There are two special types of linear functions, the **constant function** and the **identity function.**

A constant function has the form

$$f(x) = c$$

and has the domain of all real numbers with a range consisting of a single real number c.

The graph of a constant function is a horizontal line, as shown in Figure 2.23. The identity function has the form $f(x) = x$.

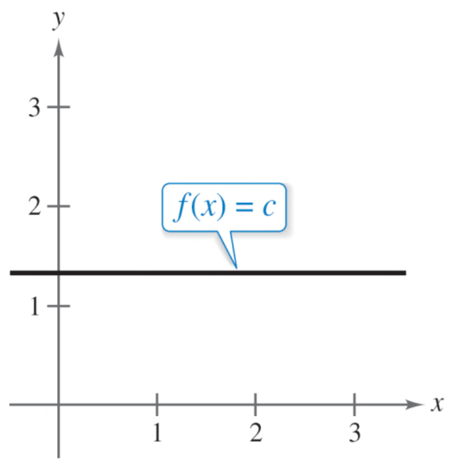


Figure 2.23

Linear and Squaring Functions

Its domain and range are the set of all real numbers. The identity function has a slope of $m = 1$ and a y-intercept at (0, 0).

The graph of the identity function is a line for which each x-coordinate equals the corresponding y-coordinate. The graph is always increasing, as shown in Figure 2.24.

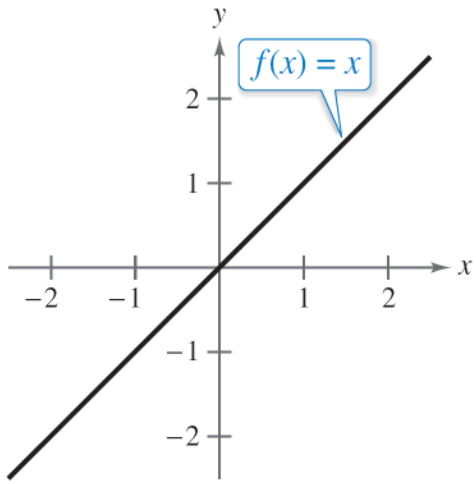


Figure 2.24

Linear and Squaring Functions

The graph of the **squaring function**

$$f(x) = x^2$$

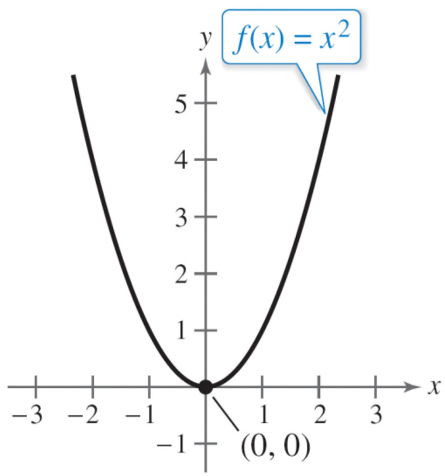
is a U-shaped curve with the following characteristics.

- The domain of the function is the set of all real numbers.
- The range of the function is the set of all nonnegative real numbers.
- The function is even.
- The graph has an intercept at (0, 0).

Linear and Squaring Functions

- The graph is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$
- The graph is symmetric with respect to the *y*-axis.
- The graph has a relative minimum at (0, 0).

The figure below shows the graph of the squaring function.



Cubic, Square Root, and Reciprocal Functions

Cubic, Square Root, and Reciprocal Functions

The following summarizes the basic characteristics of the graphs of the **cubic, square root,** and **reciprocal functions.**

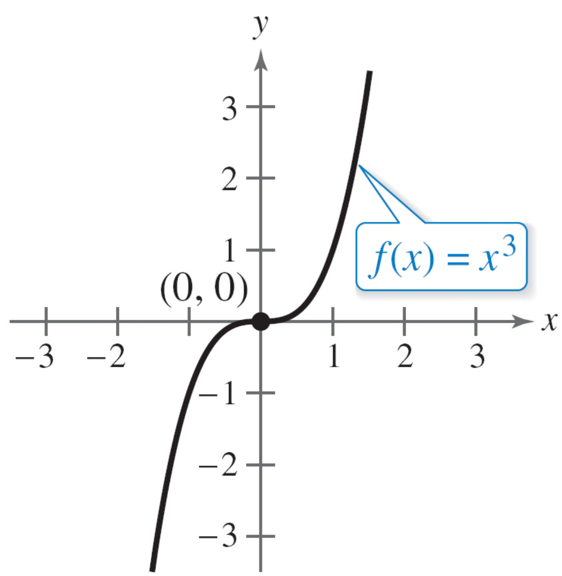
1. The graph of the *cubic* function $f(x) = x^3$ has the following characteristics.

- The domain of the function is the set of all real numbers.
- The range of the function is the set of all real numbers.
- The function is odd.
- The graph has an intercept at (0, 0).

Cubic, Square Root, and Reciprocal Functions

- The graph is increasing on the interval $(-\infty, \infty)$.
- The graph is symmetric with respect to the origin.

The figure shows the graph of the cubic function.



Cubic function

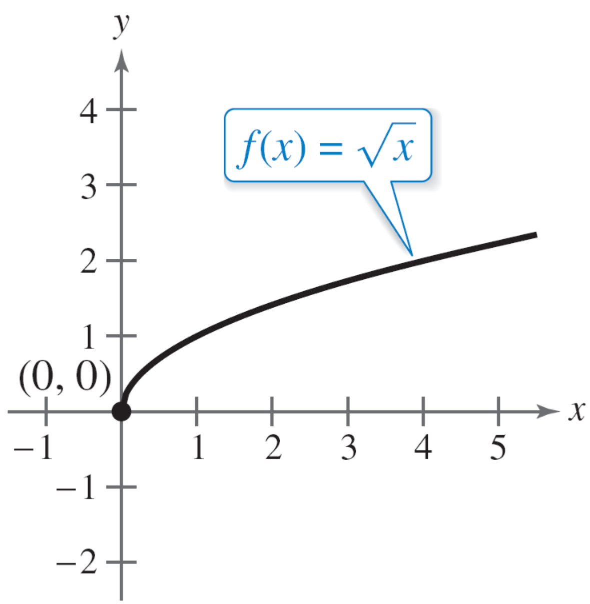
Cubic, Square Root, and Reciprocal Functions

2. The graph of the *square root* function $f(x) = \sqrt{x}$ has the following characteristics.

- The domain of the function is the set of all nonnegative real numbers.
- The range of the function is the set of all nonnegative real numbers.
- The graph has an intercept at (0, 0).
- The graph is increasing on the interval $(0, \infty)$

Cubic, Square Root, and Reciprocal Functions

The figure shows the graph of the square root function.



Square root function

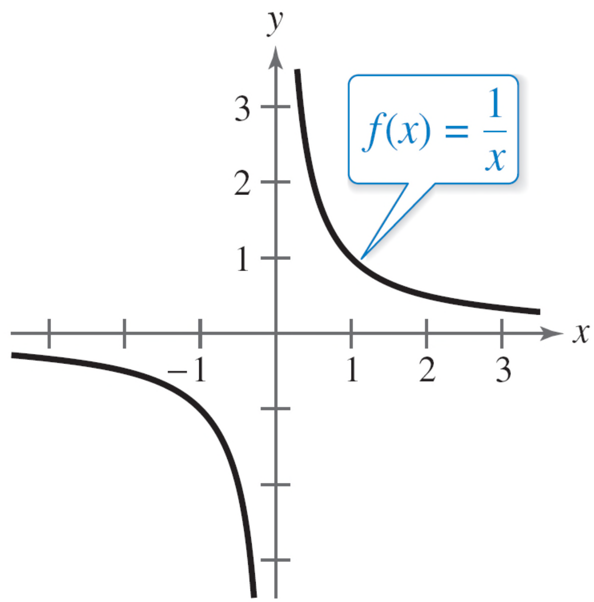
Cubic, Square Root, and Reciprocal Functions

3. The graph of the *reciprocal* function $f(x) = \frac{1}{x}$ has the following characteristics.

- The domain of the function is $(-\infty, 0) \cup (0, \infty)$
- The range of the function is $(-\infty, 0) \cup (0, \infty)$
- The function is odd.
- The graph does not have any intercepts.
- The graph is decreasing on the intervals $(-\infty, 0)$ and $(0, \infty)$.
- The graph is symmetric with respect to the origin.

Cubic, Square Root, and Reciprocal Functions

The figure shows the graph of the reciprocal function.



Reciprocal function

Step and Piecewise-Defined Functions

Step and Piecewise-Defined Functions

Functions whose graphs resemble sets of stairsteps are known as **step functions.**

The most famous of the step functions is the **greatest integer function,** which is denoted by $[\![x]\!]$ and defined as

$f(x) = [\![x]\!] =$ *the greatest integer less than or equal to x*.

Some values of the greatest integer function are as follows.

$[\![-1]\!]$ = (greatest integer ≤ –1) = –1

$[\![-\frac{1}{2}]\!]$ = (greatest integer ≤ $-\frac{1}{2}$) = –1

Step and Piecewise-Defined Functions

$[\![\frac{1}{10}]\!]$ = (greatest integer ≤ $\frac{1}{10}$) = 0

$[\![1.5]\!]$ = (greatest integer ≤ 1.5) = 1

$[\![1.9]\!]$ = (greatest integer ≤ 1.9) = 1

The graph of the greatest integer function

$f(x) = [\![x]\!]$

has the following characteristics, as shown in Figure 2.25.

- The domain of the function is the set of all real numbers.

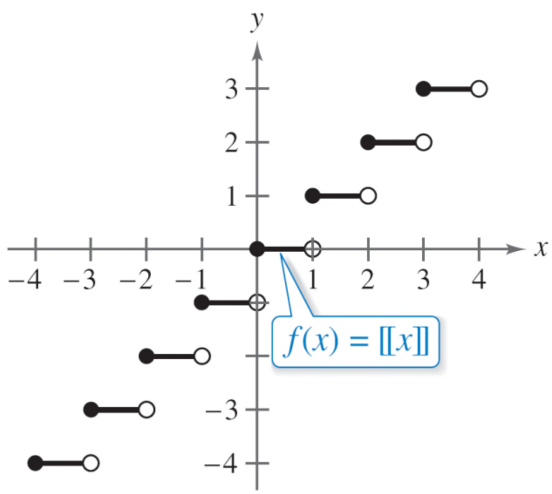


Figure 2.25

Step and Piecewise-Defined Functions

- The range of the function is the set of all integers.
- The graph has a *y*-intercept at (0, 0) and *x*-intercepts in the interval [0, 1).
- The graph is constant between each pair of consecutive integers values of *x*.
- The graph jumps vertically one unit at each integer value of *x*.

Example 2 – *Evaluating a Step Function*

Evaluate the function when $x = -1$, 2 and $\frac{3}{2}$.

$$f(x) = [\![x]\!] + 1$$

Solution:

For $x = -1$, the greatest integer ≤ -1 is -1, so

$$f(-1) = [\![-1]\!] + 1$$

$$= -1 + 1$$

$$= 0$$

Example 2 – *Solution*

cont'd

For $x = 2$, the greatest integer ≤ 2 is 2, so

$$f(2) = [\![2]\!] + 1$$

$$= 2 + 1$$

$$= 3.$$

For $x = \frac{3}{2}$, the greatest integer $\leq \frac{3}{2}$ is 1, so

$$f\left(\frac{3}{2}\right) = \left[\!\!\left[\frac{3}{2}\right]\!\!\right] + 1$$

$$= 1 + 1$$

$$= 2$$

Example 2 – *Solution*

cont'd

Verify your answers by examining the graph $f(x) = [\![x]\!] + 1$ shown in Figure 2.26.

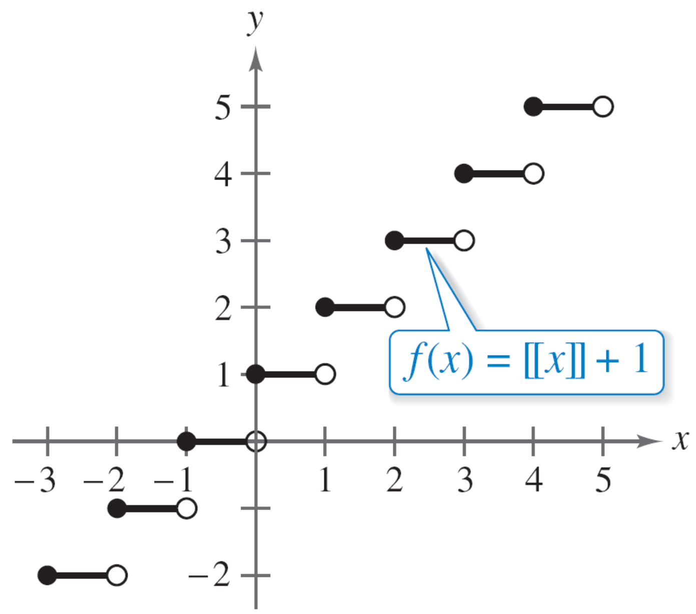
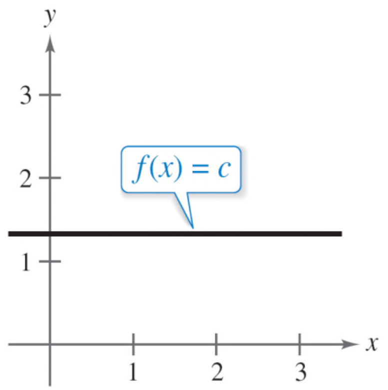


Figure 2.26

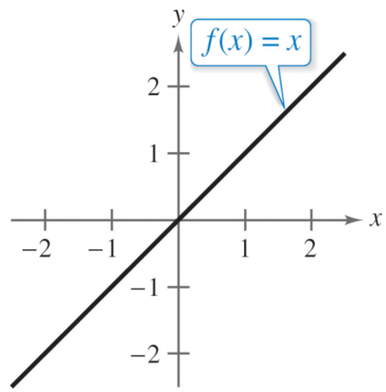
Parent Functions

Parent Functions

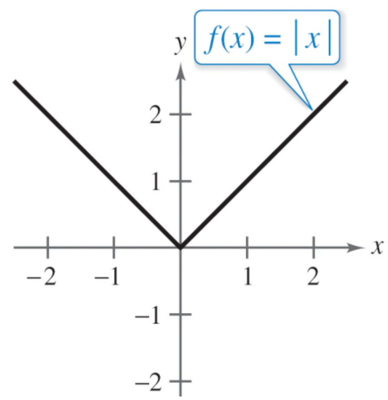
The eight graphs shown below represent the most commonly used functions in algebra.



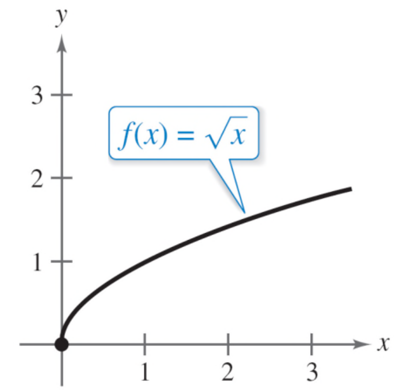
(a) Constant Function



(b) Identity Function

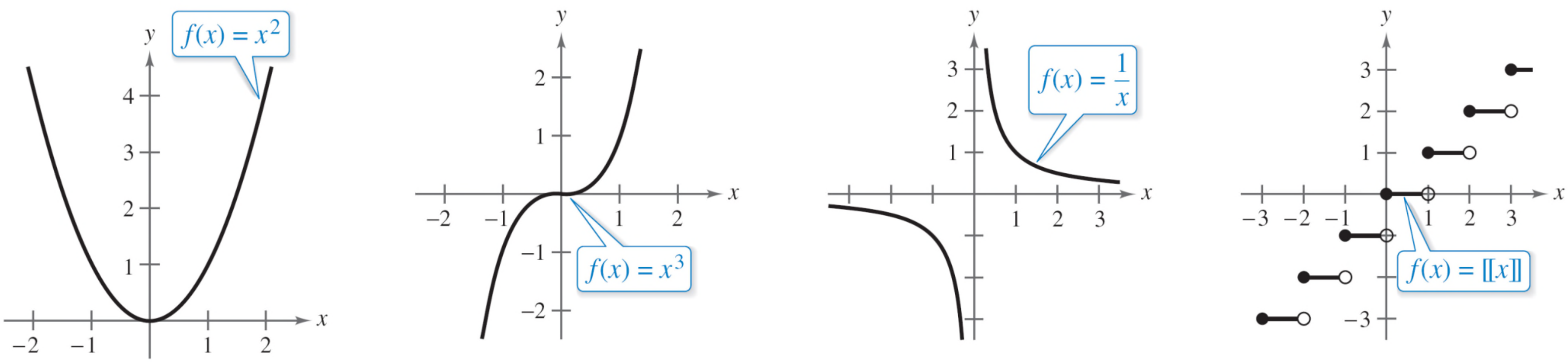


(c) Absolute Value Function



(d) Square Root Function

Parent Functions



(e) Quadratic Function

(f) Cubic Function

(g) Reciprocal Function

(h) Greatest Integer Function

Familiarity with the basic characteristics of these simple graphs will help you analyze the shapes of more complicated graphs—in particular, graphs obtained from these graphs by the rigid and non-rigid transformations.

2 Functions and Their Graphs

2.5 Transformations of Functions

Objectives

- Use vertical and horizontal shifts to sketch graphs of functions.
- Use reflections to sketch graphs of functions.
- Use nonrigid transformations to sketch graphs of functions.

Shifting Graphs

Shifting Graphs

Many functions have graphs that are simple transformations of the parent graphs.

For example, you can obtain the graph of

$$h(x) = x^2 + 2$$

by shifting the graph of $f(x) = x^2$ *up* two units, as shown in Figure 2.28.

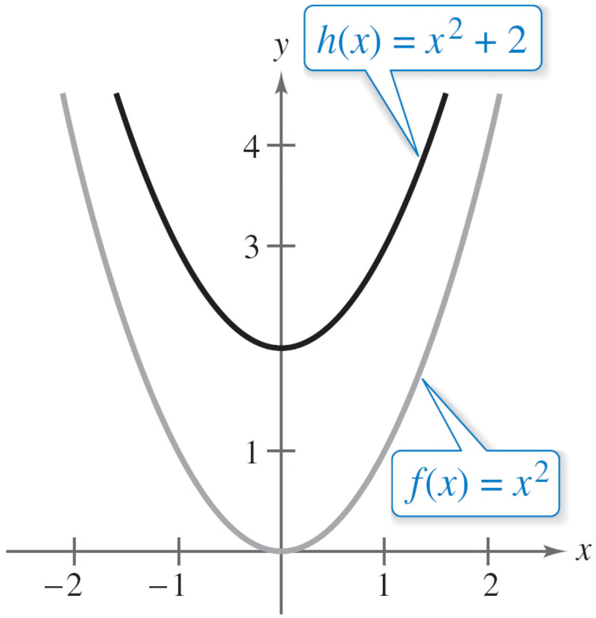


Figure 2.28

Shifting Graphs

In function notation, *h* and *f* are related as follows.

$h(x) = x^2 + 2 = f(x) + 2$ Upward shift of two units

Similarly, you can obtain the graph of

$g(x) = (x - 2)^2$

by shifting the graph of $f(x) = x^2$ to the *right* two units, as shown in Figure 2.29.

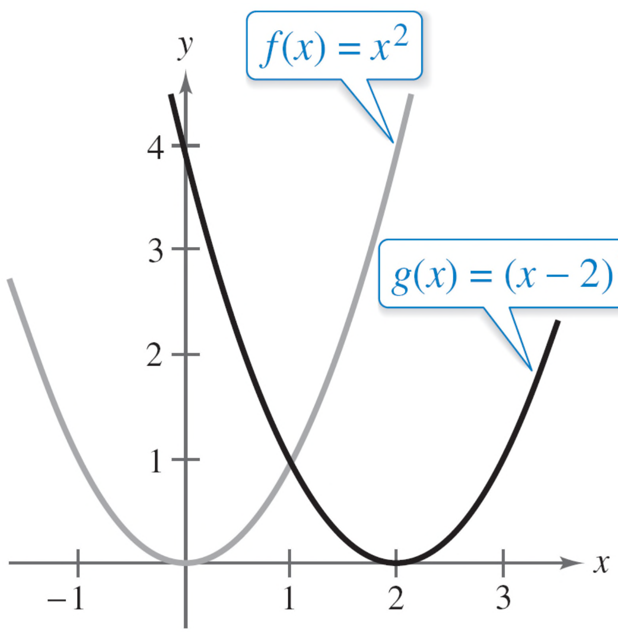


Figure 2.29

Shifting Graphs

In this case, the functions g and f have the following relationship.

$$g(x) = (x - 2)^2 = f(x - 2)$$

Right shift of two units

The following list summarizes this discussion about horizontal and vertical shifts.

Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift c units *up:* $h(x) = f(x) + c$
2. Vertical shift c units *down:* $h(x) = f(x) - c$
3. Horizontal shift c units to the *right:* $h(x) = f(x - c)$
4. Horizontal shift c units to the *left:* $h(x) = f(x + c)$

Shifting Graphs

Some graphs can be obtained from combinations of vertical and horizontal shifts, as demonstrated in Example 1(b).

Vertical and horizontal shifts generate a *family of functions*, each with the same shape but at a different location in the plane.

Example 1 – *Shifts in the Graphs of a Function*

Use the graph of $f(x) = x^3$ to sketch the graph of each function.

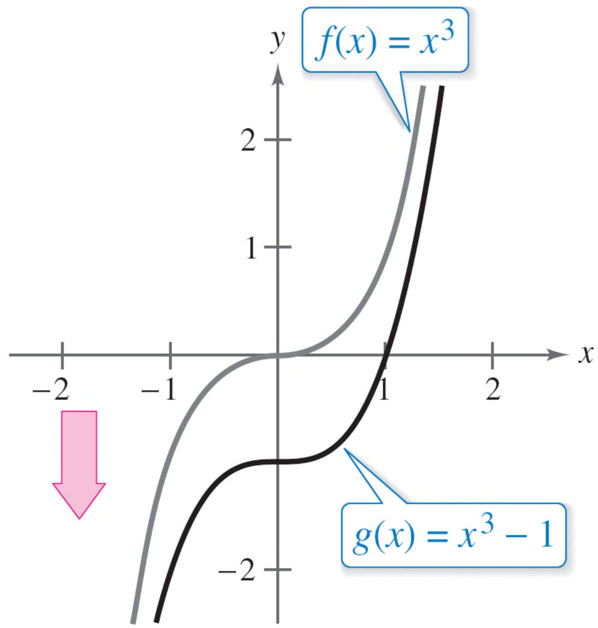
a. $g(x) = x^3 - 1$ **b.** $h(x) = (x + 2)^3 + 1$

Solution:

a. Relative to the graph of $f(x) = x^3$, the graph of

$$g(x) = x^3 - 1$$

is a downward shift of one unit, as shown in the figure at the right.



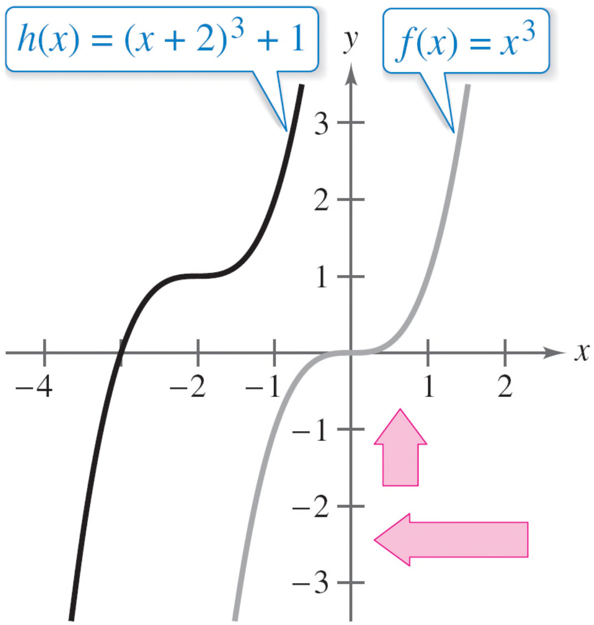
Example 1 – *Solution*

cont'd

b. Relative to the graph of $f(x) = x^3$, the graph of

$$h(x) = (x + 2)^3 + 1$$

involves a left shift of two units and an upward shift of one unit, as shown in figure below.



Shifting Graphs

In Example 1(b), you obtain the same result when the vertical shift precedes the horizontal shift *or* when the horizontal shift precedes the vertical shift.

Reflecting Graphs

Reflecting Graphs

Another common type of transformation is a **reflection.**

For instance, if you consider the *x*-axis to be a mirror, the graph of

$$h(x) = -x^2$$

is the mirror image (or reflection) of the graph of

$$f(x) = x^2,$$

as shown in Figure 2.30.

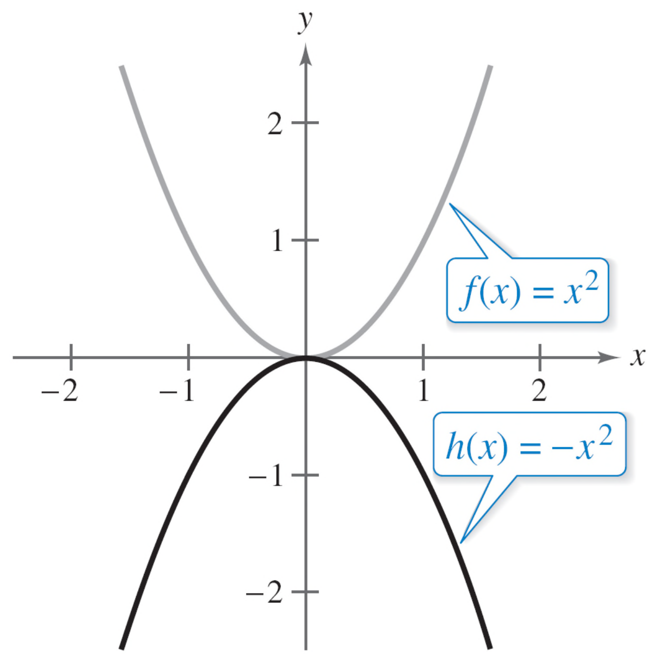


Figure 2.30

Reflecting Graphs

Reflections in the Coordinate Axes

Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

1. Reflection in the x-axis: $h(x) = -f(x)$

2. Reflection in the y-axis: $h(x) = f(-x)$

Example 2 – *Writing Equations from Graphs*

The graph of the function

$$f(x) = x^4$$

is shown in Figure 2.31.

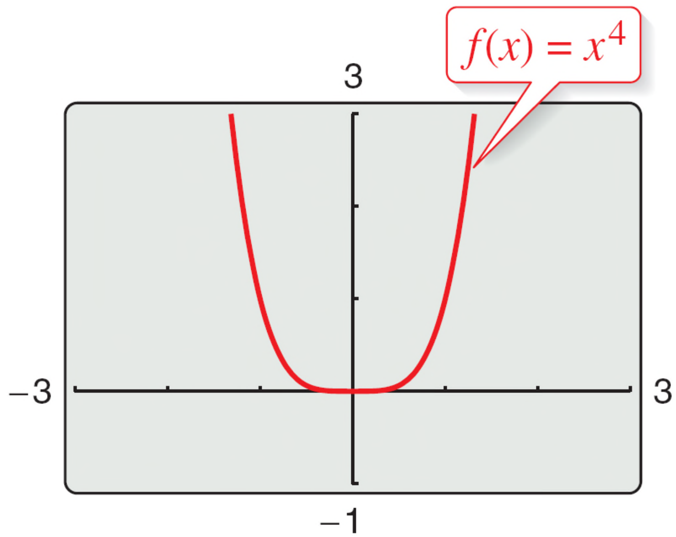
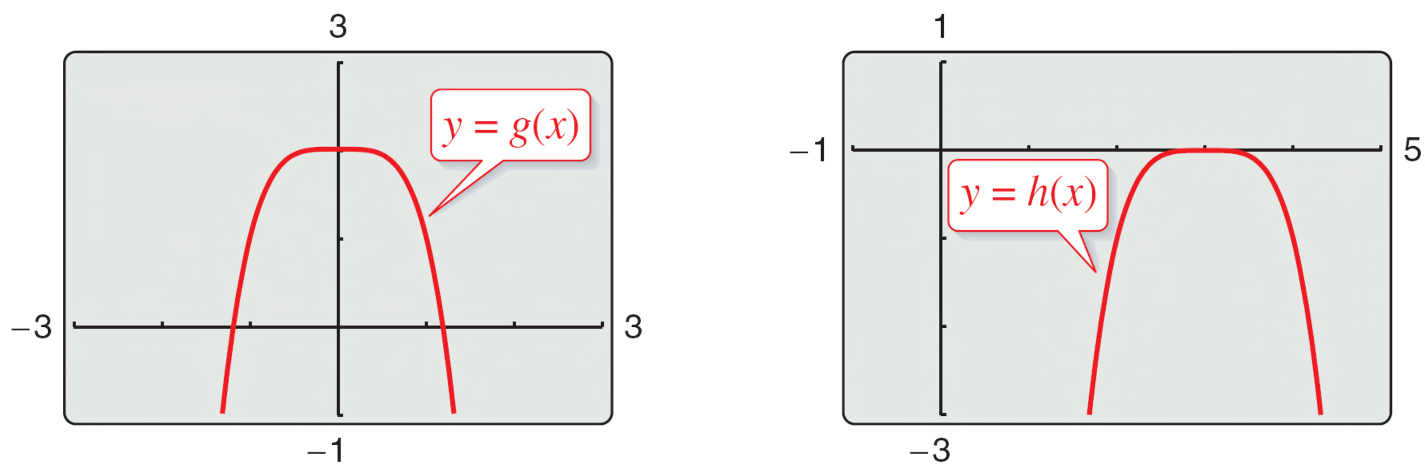


Figure 2.31

Example 2 – *Writing Equations from Graphs* cont'd

Each of the graphs below is a transformation of the graph of *f*. Write an equation for each of these functions.



(a) (b)

Example 2 – *Solution*

a. The graph of g is a reflection in the x-axis *followed by* an upward shift of two units of the graph of $f(x) = x^4$.

So, the equation for g is

$$g(x) = -x^4 + 2.$$

b. The graph of h is a horizontal shift of three units to the right *followed by* a reflection in the x-axis of the graph of $f(x) = x^4$.

So, the equation for h is

$$h(x) = -(x - 3)^4.$$

Reflecting Graphs

When sketching the graphs of functions involving square roots, remember that you must restrict the domain to exclude negative numbers inside the radical.

For instance, here are the domains of the functions:

$$\text{Domain of } g(x) = -\sqrt{x}: \quad x \geq 0$$

$$\text{Domain of } h(x) = \sqrt{-x}: \quad x \leq 0$$

$$\text{Domain of } k(x) = -\sqrt{x+2}: \quad x \geq -2$$

Nonrigid Transformations

Nonrigid Transformations

Horizontal shifts, vertical shifts, and reflections are **rigid transformations** because the basic shape of the graph is unchanged.

These transformations change only the *position* of the graph in the coordinate plane.

Nonrigid transformations are those that cause a *distortion*—a change in the shape of the original graph.

Nonrigid Transformations

For instance, a nonrigid transformation of the graph of $y = f(x)$ is represented by $g(x) = cf(x)$, where the transformation is a **vertical stretch** when $c > 1$ and a **vertical shrink** if $0 < c < 1$.

Another nonrigid transformation of the graph of $y = f(x)$ is represented by $h(x) = f(cx)$, where the transformation is a **horizontal shrink** when $c > 1$ and a **horizontal stretch** when $0 < c < 1$.

Example 4 – *Nonrigid Transformations*

Compare the graph of each function with the graph of $f(x) = |x|$.

a. $h(x) = 3|x|$ **b.** $g(x) = \frac{1}{3}|x|$

Solution:

a. Relative to the graph of $f(x) = |x|$, the graph of $h(x) = 3|x| = 3f(x)$ is a vertical stretch (each y-value is multiplied by 3) of the graph of f. (See Figure 2.32.)

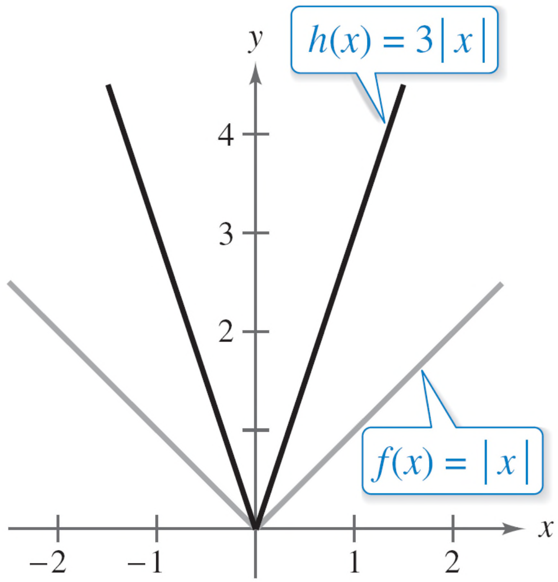


Figure 2.32

Example 4 – *Solution*

cont'd

b. Similarly, the graph of

$$g(x) = \tfrac{1}{3}|x| = \tfrac{1}{3}f(x)$$

is a vertical shrink (each *y*-value is multiplied by $\frac{1}{3}$) of the graph of *f*. (See Figure 2.33.)

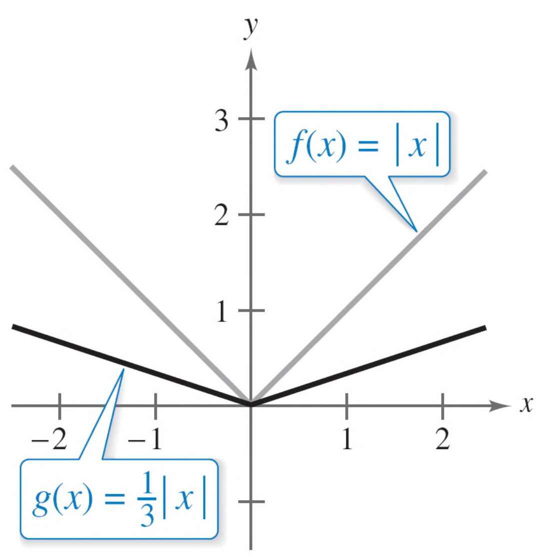


Figure 2.33

2 Functions and Their Graphs

2.6 Combinations of Functions: Composite Functions

Objectives

- Add, subtract, multiply, and divide functions.
- Find the composition of one function with another function.
- Use combinations and compositions of functions to model and solve real-life problems.

Arithmetic Combinations of Functions

Arithmetic Combinations of Functions

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two *functions* can be combined to create new functions.

For example, the functions given by $f(x) = 2x - 3$ and $g(x) = x^2 - 1$ can be combined to form the sum, difference, product, and quotient of f and g.

$$f(x) + g(x) = (2x - 3) + (x^2 - 1)$$

$$= x^2 + 2x - 4 \qquad \text{Sum}$$

Arithmetic Combinations of Functions

$f(x) - g(x) = (2x - 3) - (x^2 - 1)$

$= -x^2 + 2x - 2$ Difference

$f(x)g(x) = (2x - 3)(x^2 - 1)$

$= 2x^3 - 3x^2 - 2x + 3$ Product

$$\frac{f(x)}{g(x)} = \frac{2x - 3}{x^2 - 1}, \quad x \neq \pm 1$$ Quotient

Arithmetic Combinations of Functions

The domain of an **arithmetic combination** of functions *f* and *g* consists of all real numbers that are common to the domains of *f* and *g.*

In the case of the quotient $f(x)/g(x)$, there is the further restriction that $g(x) \neq 0$.

Arithmetic Combinations of Functions

Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the *sum, difference, product,* and *quotient* of f and g are defined as follows.

1. Sum: $(f + g)(x) = f(x) + g(x)$

2. Difference: $(f - g)(x) = f(x) - g(x)$

3. Product: $(fg)(x) = f(x) \cdot g(x)$

4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Example 1 – *Finding the Sum of Two Functions*

Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$, find $(f + g)(x)$. Then evaluate the sum when $x = 3$.

Solution:

The sum of f and g is

$$(f + g)(x) = f(x) + g(x) = (2x + 1) + (x^2 + 2x - 1)$$

$$= x^2 + 4x$$

When $x = 3$, the value of this sum is

$$(f + g)(3) = 3^2 + 4(3) = 21.$$

Composition of Functions

Composition of Functions

Another way of combining two functions is to form the **composition** of one with the other.

For instance, if $f(x) = x^2$ and $g(x) = x + 1$, the composition of f with g is

$$f(g(x)) = f(x + 1)$$

$$= (x + 1)^2.$$

This composition is denoted as $f \circ g$ and reads as "*f* composed with *g*."

Composition of Functions

Definition of Composition of Two Functions

The **composition** of the function f with the function g is

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. (See Figure 2.36.)

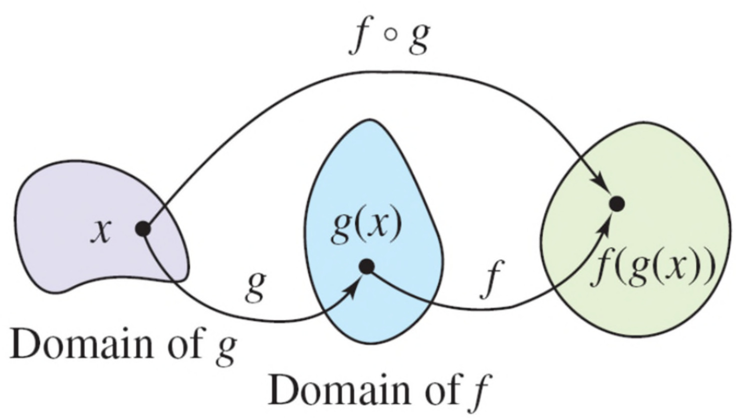


Figure 2.36

Example 5 – *Composition of Functions*

Given $f(x) = x + 2$ and $g(x) = 4 - x^2$, find the following.

a. $(f \circ g)(x)$ **b.** $(g \circ f)(x)$ **c.** $(g \circ f)(-2)$

Solution:

a. The composition of f with g is as follows.

$$(f \circ g)(x) = f(g(x))$$ Definition of $f \circ g$

$$= f(4 - x^2)$$ Definition of $g(x)$

$$= (4 - x^2) + 2$$ Definition of $f(x)$

$$= -x^2 + 6$$ Simplify.

Example 5 – *Solution*

cont'd

b. The composition of g with f is as follows.

$(g \circ f)(x) = g(f(x))$ — Definition of $g \circ f$

$= g(x + 2)$ — Definition of $f(x)$

$= 4 - (x + 2)^2$ — Definition of $g(x)$

$= 4 - (x^2 + 4x + 4)$ — Expand.

$= -x^2 - 4x$ — Simplify.

Note that, in this case, $(f \circ g)(x) \neq (g \circ f)(x)$.

Example 5 – *Solution*

cont'd

c. Using the result of part (b), write the following.

$(g \circ f)(-2) = -(-2)^2 - 4(-2)$ Substitute.

$= -4 + 8$ Simplify.

$= 4$ Simplify.

Composition of Functions

In Example 5, you formed the composition of two given functions.

In calculus, it is also important to be able to identify two functions that make up a given composite function.

For instance, the function $h(x) = (3x - 5)^3$ is the composition of $f(x) = x^3$ and $g(x) = 3x - 5$. That is,

$$h(x) = (3x - 5)^3 = [g(x)]^3 = f(g(x)).$$

Composition of Functions

Basically, to "decompose" a composite function, look for an "inner" function and an "outer" function. In the function *h*, $g(x) = 3x - 5$ is the inner function and $f(x) = x^3$ is the outer function.

Application

Example 8 – *Bacteria Count*

The number N of bacteria in a refrigerated food is given by

$$N(T) = 20T^2 - 80T + 500, \qquad 2 \le T \le 14$$

where T is the temperature of the food in degrees Celsius. When the food is removed from refrigeration, the temperature of the food is given by

$$T(t) = 4t + 2, \qquad 0 \le t \le 3$$

where t is the time in hours.

a. Find the composition $(N \circ T)(t)$ and interpret its meaning in context.

b. Find the time when the bacteria count reaches 2000.

Example 8(a) – *Solution*

$$(N \circ T)(t) = N(T(t)) = 20(4t + 2)^2 - 80(4t + 2) + 500$$

$$= 20(16t^2 + 16t + 4) - 320t - 160 + 500$$

$$= 320t^2 + 320t + 80 - 320t - 160 + 500$$

$$= 320t^2 + 420$$

The composite function $(N \circ T)(t)$ represents the number of bacteria in the food as a function of the amount of time the food has been out of refrigeration.

Example 8(b) – *Solution*

cont'd

The bacteria count will reach 2000 when $320t^2 + 420 = 2000$. Solve this equation to find that the count will reach 2000 when $t \approx 2.2$ hours.

Note that when you solve this equation, you reject the negative value because it is not in the domain of the composite function.

2 Functions and Their Graphs

2.7 Inverse Functions

Objectives

- Find inverse functions informally and verify that two functions are inverse functions of each other.
- Use graphs of functions to determine whether functions have inverse functions.
- Use the Horizontal Line Test to determine if functions are one-to-one.
- Find inverse functions algebraically.

Inverse Functions

Inverse Functions

We know that a set of ordered pairs can represent a function.

For instance, the function $f(x) = x + 4$ from the set $A = \{1, 2, 3, 4\}$ to the set $B = \{5, 6, 7, 8\}$ can be written as follows.

$$f(x) = x + 4: \{(1, 5), (2, 6), (3, 7), (4, 8)\}$$

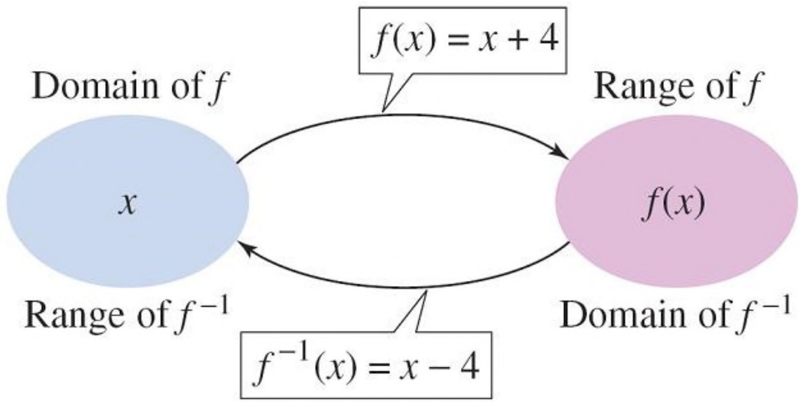
In this case, by interchanging the first and second coordinates of each of these ordered pairs, you can form the **inverse function** of f, which is denoted by f^{-1}.

Inverse Functions

It is a function from the set *B* to the set *A*, and can be written as follows.

$$f^{-1}(x) = x - 4: \{(5, 1), (6, 2), (7, 3), (8, 4)\}$$

Note that the domain of *f* is equal to the range of f^{-1}, and vice versa, as shown in Figure below. Also note that the functions *f* and f^{-1} have the effect of "undoing" each other.



Inverse Functions

In other words, when you form the composition of f with f^{-1} or the composition of f^{-1} with f, you obtain the identity function.

$$f(f^{-1}(x)) = f(x - 4) = (x - 4) + 4 = x$$

$$f^{-1}(f(x)) = f^{-1}(x + 4) = (x + 4) - 4 = x$$

Example 1 – *Finding Inverse Functions Informally*

Find the inverse of $f(x) = 4x$. Then verify that both $f(f^{-1}(x))$ and $f^{-1}(f(x))$ are equal to the identity function.

Solution:

The function f *multiplies* each input by 4. To “undo” this function, you need to *divide* each input by 4.

So, the inverse function of $f(x) = 4x$ is

$$f^{-1}(x) = \frac{x}{4}.$$

Example 1 – *Solution*

cont'd

Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ as follows.

$$f(f^{-1}(x)) = f\left(\frac{x}{4}\right)$$

$$= 4\left(\frac{x}{4}\right)$$

$$= x$$

$$f^{-1}(f(x)) = f^{-1}(4x)$$

$$= \frac{4x}{4}$$

$$= x$$

Inverse Functions

Definition of Inverse Function

Let f and g be two functions such that

$$f(g(x)) = x \qquad \text{for every } x \text{ in the domain of } g$$

and

$$g(f(x)) = x \qquad \text{for every } x \text{ in the domain of } f.$$

Under these conditions, the function g is the **inverse function** of the function f. The function g is denoted by f^{-1} (read "f-inverse"). So,

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x.$$

The domain of f must be equal to the range of f^{-1}, and the range of f must be equal to the domain of f^{-1}.

Inverse Functions

Do not be confused by the use of –1 to denote the inverse function f^{-1}.

In this section, whenever f^{-1} is written, it *always* refers to the inverse function of the function f and *not* to the reciprocal of $f(x)$.

If the function g is the inverse function of the function f, it must also be true that the function f is the inverse function of the function g.

For this reason, you can say that the functions f and g are *inverse functions of each other*.

The Graph of an Inverse Function

The Graph of an Inverse Function

The graphs of a function *f* and its inverse function f^{-1} are related to each other in the following way.

If the point (*a*, *b*) lies on the graph of *f*, then the point (*b*, *a*) must lie on the graph of f^{-1}, and vice versa.

This means that the graph of f^{-1} is a *reflection* of the graph of *f* in the line $y = x$, as shown in Figure 2.37.

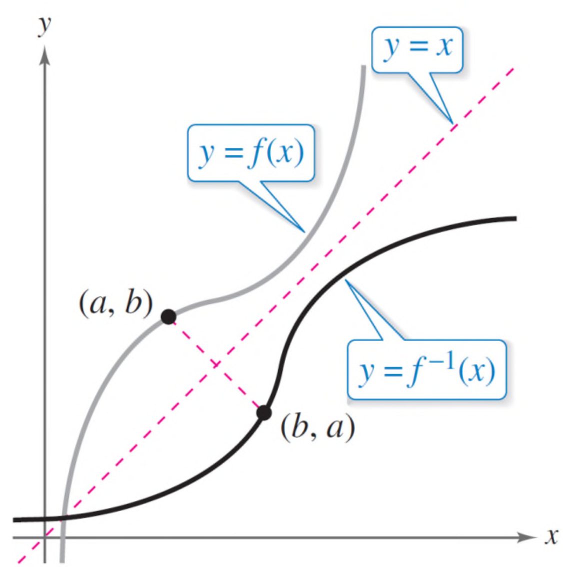


Figure 2.37

Example 3 – *Verifying Inverse Functions Graphically*

Sketch the graphs of the inverse functions $f(x) = 2x - 3$ and $f^{-1}(x) = \frac{1}{2}(x + 3)$ on the same rectangular coordinate system and show that the graphs are reflections of each other in the line $y = x$.

Example 3 – *Solution*

The graphs of *f* and f^{-1} are shown in Figure 2.38.

It appears that the graphs are reflections of each other in the line $y = x$.

You can further verify this reflective property by testing a few points on each graph.

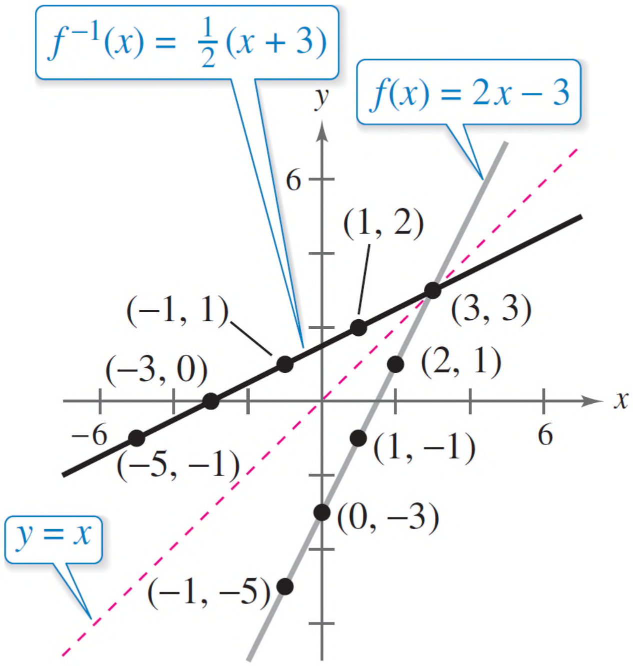


Figure 2.38

Example 3 – *Solution*

cont'd

Note in the following list that if the point (a, b) is on the graph of f, the point (b, a) is on the graph of f^{-1}.

Graph of $f(x) = 2x - 3$	*Graph of* $f^{-1}(x) = \frac{1}{2}(x + 3)$
(–1, –5)	(–5, –1)
(0, –3)	(–3, 0)
(1, –1)	(–1, 1)
(2, 1)	(1, 2)
(3, 3)	(3, 3)

One-to-One Functions

One-to-One Functions

The reflective property of the graphs of inverse functions gives you a nice *geometric* test for determining whether a function has an inverse function.

This test is called the **Horizontal Line Test** for inverse functions.

Horizontal Line Test for Inverse Functions

A function f has an inverse function if and only if no *horizontal* line intersects the graph of f at more than one point.

One-to-One Functions

If no horizontal line intersects the graph of *f* at more than one point, then no *y-value* is matched with more than one *x*-value.

This is the essential characteristic of what are called **one-to-one functions.**

One-to-One Functions

A function *f* is **one-to-one** when each value of the dependent variable corresponds to exactly one value of the independent variable. A function *f* has an inverse function if and only if *f* is one-to-one.

One-to-One Functions

Consider the function given by $f(x) = x^2$. The table on the left is a table of values for $f(x) = x^2$. The table on the right is the same as the table on the left but with the values in the columns interchanged.

x	$f(x) = x^2$
-2	4
-1	1
0	0
1	1
2	4
3	9

x	y
4	-2
1	-1
0	0
1	1
4	2
9	3

One-to-One Functions

The table on the right does not represent a function because the input $x = 4$ is matched with two different outputs: $y = –2$ and $y = 2$. So, $f(x) = x^2$ is not one-to-one and does not have an inverse function.

Example 5(a) – *Applying the Horizontal Line Test*

The graph of the function given by $f(x) = x^3 - 1$ is shown in Figure 2.40.

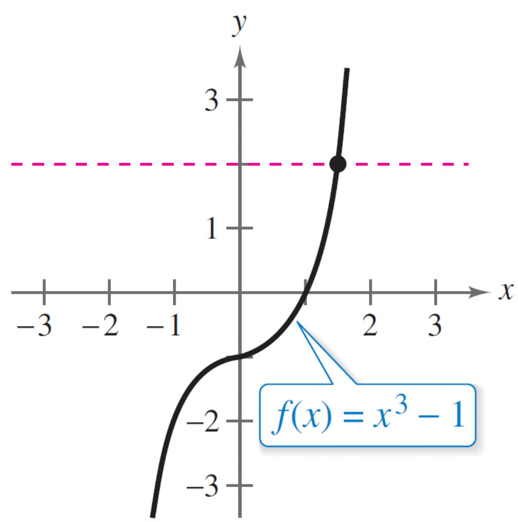


Figure 2.40

Because no horizontal line intersects the graph of *f* at more than one point, you can conclude that *f is* a one-to-one function and *does* have an inverse function.

Example 5(b) – *Applying the Horizontal Line Test* cont'd

The graph of the function given by $f(x) = x^2 - 1$ is shown in Figure 2.41.

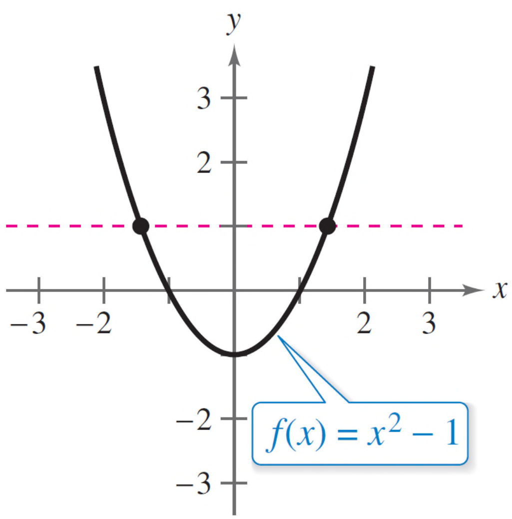


Figure 2.41

Because it is possible to find a horizontal line that intersects the graph of *f* at more than one point, you can conclude that *f is not* a one-to-one function and *does not* have an inverse function.

Finding Inverse Functions Algebraically

Finding Inverse Functions Algebraically

For relatively simple functions (such as the one in Example 1), you can find inverse functions by inspection. For more complicated functions, however, it is best to use the following guidelines.

The key step in these guidelines is Step 3—interchanging the roles of x and y. This step corresponds to the fact that inverse functions have ordered pairs with the coordinates reversed.

Finding Inverse Functions Algebraically

Finding an Inverse Function

1. Use the Horizontal Line Test to decide whether f has an inverse function.
2. In the equation for $f(x)$, replace $f(x)$ with y.
3. Interchange the roles of x and y, and solve for y.
4. Replace y with $f^{-1}(x)$ in the new equation.
5. Verify that f and f^{-1} are inverse functions of each other by showing that the domain of f is equal to the range of f^{-1}, the range of f is equal to the domain of f^{-1}, and $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Example 6 – *Finding an Inverse Function Algebraically*

Find the inverse function of

$$f(x) = \frac{5 - 3x}{2}.$$

Solution:

The graph of *f* is shown in Figure 2.42.

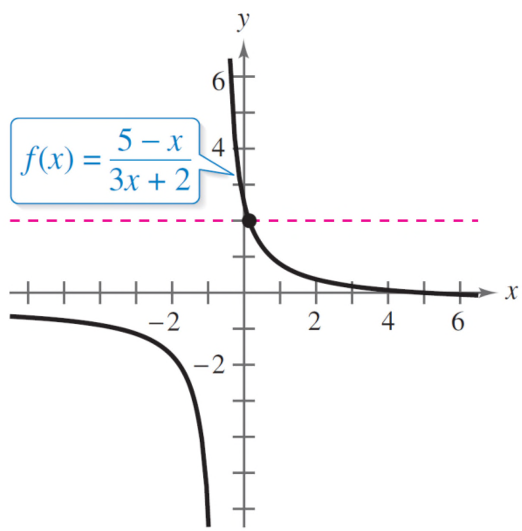


Figure 2.42

Example 6 – *Solution*

cont'd

This graph passes the Horizontal Line Test. So, you know that is one-to-one and has an inverse function.

$$f(x) = \frac{5 - 3x}{2}$$ Write original function.

$$y = \frac{5 - x}{3x + 2}$$ Replace $f(x)$ by y.

$$x = \frac{5 - 3y}{2}$$ Interchange x and y.

$$x(3y + 2) = 5 - y$$ Multiply each side by $3y + 2$.

Example 6 – *Solution*

cont'd

$$3xy + 2x = 5 - y$$ Distributive Property

$$3xy + y = 5 - 2x$$ Collect terms with *y*.

$$y(3x + 1) = 5 - 2x$$ Factor.

$$y = \frac{5 - 2x}{3x + 1}$$ Solve for *y*.

$$f^{-1}(x) = \frac{5 - 2x}{3x + 1}$$ Replace *y* by $f^{-1}(x)$.

Check that

$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.