## Chapter 2

## Short Answer Problems

1. The LMTD factor is required for heat exchangers when the two streams do not exhibit pure counter-current flow. Most exchangers such as S\&T and other arrangements fall into this category.
2. This statement is not generally true. In fact, condensers often have high heat transfer coefficients on the condensing side. One exception is for partial condensers where the majority of the condenser-side of the exchanger comprises of non-condensing gas - in this case the heat transfer coefficient on the condensing side may be limiting.
3. Turbulent flow on both sides of the heat exchanger - see Table 2.9
a. Shell-side $\Rightarrow h \propto \operatorname{Re}^{0.6} \therefore \frac{h_{2}}{h_{\text {basecase }}}=\left(\frac{1.5}{1}\right)^{0.6}=1.275$ or and increase of $27.5 \%$
b. Tube-side $\Rightarrow h \propto \operatorname{Re}^{0.8} \therefore \frac{h_{2}}{h_{\text {bassecase }}}=\left(\frac{1.5}{1}\right)^{0.8}=1.383$ or and increase of $38.3 \%$
4. Laminar flow on both sides of the heat exchanger --see Table 2.9
a. Shell-side $\Rightarrow h \propto \operatorname{Re}^{0.45} \therefore \frac{h_{2}}{h_{\text {basecase }}}=\left(\frac{1.5}{1}\right)^{0.45}=1.20$ or and increase of $20 \%$
b. Tube-side $\Rightarrow h \propto \operatorname{Re}^{1 / 3} \therefore \frac{h_{2}}{h_{\text {basecase }}}=\left(\frac{1.5}{1}\right)^{1 / 3}=1.145$ or and increase of $14.5 \%$
5. All else considered, the heat transfer coefficient for turbulent flow will be (much) higher than for laminar flow. For this reason when possible flow should be in the turbulent regime.
6. Refer to Section 2.2.1.7 - three reasons for placing a fluid on the tube side of a S\&T exchanger
2- If the fluid is corrosive
2- If the fluid causes severe fouling or scaling
2- If the fluid is at a much higher pressure than the other fluid
2- If the fluid has a much higher film coefficient
2- If the material in contact with the fluid must be an expensive alloy
7. For placing the fluid on the shell side - the opposite reasons given in Problem 6 apply. In addition, if the fluid requires a very low pressure drop through the exchanger.
8. The reason that fins are used in some heat exchangers is to increase the surface area in contact with a fluid that has a low heat transfer coefficient. Most often, fins are used in contact with gas flows or high viscosity liquids.
9. Definitions for
a. Baffle cut - this is the maximum distance between the edge of the baffle and the shell wall - usually expressed as a fraction or \% of the shell diameter.
b. Number of tube passes - this refers to the number of times the tube side fluid flows through the shell
c. Tube pitch - this is the distance between the centers of adjacent tubes in the tube bundle
d. Tube arrangement - this refers to the arrangement of the tubes in the tube bundle, such as square pitch, triangular pitch, and rotated square or triangular pitch.
10. Basic design equation for a S\&T exchanger with no phase change

$$
Q=U A \Delta T_{l n} F
$$



## Q

11. Draw $\mathrm{T}-\mathrm{Q}$ diagrams for
a. Condensing (pure) vapor using cooling water


Q
b. Distillation reboiler using condensing steam as the heating media

c. Process liquid stream cooled by a another process stream


## Problems to Solve

12. A process fluid (Stream 1) $\left(C_{p}=2100 \mathrm{~J} / \mathrm{kgK}\right)$ enters a heat exchanger at a rate of $3.4 \mathrm{~kg} / \mathrm{s}$ and at a temperature of $135^{\circ} \mathrm{C}$. This stream is to be cooled with another process stream (Stream 2) $\left(c_{p}=2450 \mathrm{~J} / \mathrm{kgK}\right)$ flowing at a rate of $2.65 \mathrm{~kg} / \mathrm{s}$ and entering the heat exchanger at a temperature of $55^{\circ} \mathrm{C}$. Determine the following (you may assume that the heat capacities of both streams are constant):
a. The exit temperature of Stream 1 if pure countercurrent flow occurs in the exchanger and the minimum approach temperature between the streams anywhere in the heat exchanger is $10^{\circ} \mathrm{C}$.
$\dot{m}_{1} c_{p, 1}=(3.4)(2100) /(1000)=7.14 \mathrm{~kW} / \mathrm{K}$ and $T_{1, i n}=135^{\circ} \mathrm{C}$
$\dot{m}_{2} c_{p, 2}=(2.65)(2450) /(1000)=6.4925 \mathrm{~kW} / \mathrm{K}$ and $T_{2, i n}=55^{\circ} \mathrm{C}$
Because $\dot{m}_{1} c_{p, 1}>\dot{m}_{2} c_{p, 2} \Delta T_{1}<\Delta T_{2}$ and the $10^{\circ} \mathrm{C}$ approach will occur at the left hand side of the diagram


Energy balance gives $\left(135-T_{1, \text { out }}\right)=\frac{(6.4925)}{(7.1400)}(125-55) \Rightarrow T_{1, \text { out }}=71.35^{\circ} \mathrm{C}$
b. The exit temperature of Stream 1 if pure co-current flow occurs in the exchanger and the minimum approach temperature between the streams anywhere in the heat exchanger is $10^{\circ} \mathrm{C}$. For this case the outlet temperatures for each stream are unknown but they differ by $10^{\circ} \mathrm{C}$.


Let the exit temperature for Stream $1=x$ And energy balance gives
$(135-x)=\frac{(6.4925)}{(7.1400)}((x-10)-55) \Rightarrow x=101.66^{\circ} \mathrm{C}$
and the corresponding exit temperature for Stream $2=x-10=91.66{ }^{\circ} \mathrm{C}$.
13. Repeat problem 12 except that

Stream 1: $C_{p}=2000+3(T-100) \mathrm{J} / \mathrm{kgK}$
Stream 2: $c_{p}=2425+5(T-50) \mathrm{J} / \mathrm{kgK}$
a. Based on the Cp values, it is not clear if the $10^{\circ} \mathrm{C}$ approach will be at the right hand or left hand side of the T-Q diagram. However, assume $10^{\circ} \mathrm{C}$ approach at the inlet of stream 1 and check the assumption.
The energy balance should be written in integral form as follows:

$$
\begin{aligned}
& \dot{m}_{1} \int_{T_{1, \text { out }}}^{135} c_{p, 1} d t=\dot{m}_{2} \int_{55}^{125} c_{p, 2} d t \\
& 3.4 \int_{T_{1, \text { out }}}^{135}\left(2000+3(T-100) d t=2.65 \int_{55}^{125}(2425+5(T-50) d t\right.
\end{aligned}
$$

$$
3.4\left[1700\left(135-T_{1, \text { out }}\right)+3\left(135^{2}-T_{1, \text { out }}^{2}\right) / 2\right]=2.65\left[2175(125-55)+5\left(125^{2}-55^{2}\right) / 2\right]
$$

$$
\Rightarrow T_{1, \text { out }}=63.30
$$

So our initial assumption was incorrect and the limiting end of the exchanger is now the right hand side - so the exit temperature for stream 1 is $65^{\circ} \mathrm{C}$ - reformulating the energy balance we can find the exit temperature for Stream 2 by

$$
\begin{aligned}
& \dot{m}_{1} \int_{65}^{135} c_{p, 1} d t=\dot{m}_{2} \int_{55}^{T_{2, \text { out }}} c_{p, 2} d t \\
& 3.4 \int_{65}^{135}\left(2000+3(T-100) d t=2.65 \int_{55}^{T_{2, \text { out }}}(2425+5(T-50) d t\right.
\end{aligned}
$$

$$
3.4\left[1700(135-65)+3\left(135^{2}-65^{2}\right) / 2\right]=2.65\left[2175\left(T_{2, \text { out }}-55\right)+5\left(T_{2, \text { out }}^{2}-55^{2}\right) / 2\right]
$$

$$
\Rightarrow T_{2, \text { out }}=123.5^{\circ} \mathrm{C}
$$


b. For co-current flow, the approach of $10^{\circ} \mathrm{C}$ must be at the exit. Let $x=$ $T_{1, \text { out, }}$, therefore

$$
\begin{aligned}
& \dot{m}_{1} \int_{x}^{135} c_{p, 1} d t=\dot{m}_{2} \int_{55}^{x-10} c_{p, 2} d t \\
& 3.4 \int_{x}^{135}\left(2000+3(T-100) d t=2.65 \int_{55}^{x-10}(2425+5(T-50) d t\right. \\
& 3.4\left[1700(135-x)+3\left(135^{2}-x^{2}\right) / 2\right]=2.65\left[2175(x-10-55)+5\left((x-10)^{2}-55^{2}\right) / 2\right] \\
& \Rightarrow T_{1, \text { out }}=100.65^{\circ} \mathrm{C}
\end{aligned}
$$

14. Find the number of shells for each case
a. Calculate $R$ and $P$

$$
P=\frac{\left(t_{2}-t_{1}\right)}{\left(T_{1}-t_{1}\right)}=\frac{(75-40)}{(130-40)}=\frac{35}{90}=0.3889 \quad \text { and } R=\frac{\left(T_{1}-T_{2}\right)}{\left(t_{2}-t_{1}\right)}=\frac{\dot{m} c_{p}}{\dot{M} C_{P}}=\frac{(130-90)}{(75-40)}=1.1429
$$

Now use Equation 2.16 to determine the number of shell passes

$$
N_{\text {shells }}=\frac{\ln \left[\frac{1-P R}{1-P}\right]}{\ln \left[\frac{1}{R}\right]}=\frac{\ln \left[\frac{1-(0.3889)(1.1429)}{1-0.3889}\right]}{\ln \left[\frac{1}{1.1429}\right]}=0.7138
$$

Therefore a 1-2 S\&T exchanger is needed
b. Calculate $R$ and $P$

$$
P=\frac{\left(t_{2}-t_{1}\right)}{\left(T_{1}-t_{1}\right)}=\frac{(73-40)}{(130-40)}=\frac{33}{90}=0.3667 \quad \text { and } R=\frac{\left(T_{1}-T_{2}\right)}{\left(t_{2}-t_{1}\right)}=\frac{\dot{m} c_{p}}{\dot{M} C_{P}}=\frac{(130-50)}{(73-40)}=2.4242
$$

Now use Equation 2.16 to determine the number of shell passes

$$
N_{\text {shells }}=\frac{\ln \left[\frac{1-P R}{1-P}\right]}{\ln \left[\frac{1}{R}\right]}=\frac{\ln \left[\frac{1-(0.3667)(2.4242)}{1-0.3667}\right]}{\ln \left[\frac{1}{2.4242}\right]}=1.9655
$$

Rounding up to give $N_{\text {shells }}=2$, therefore a 2-4 S\&T exchanger is needed
c. Calculate $R$ and $P$

$$
P=\frac{\left(t_{2}-t_{1}\right)}{\left(T_{1}-t_{1}\right)}=\frac{(114-80)}{(130-80)}=\frac{34}{50}=0.68 \quad \text { and } R=\frac{\left(T_{1}-T_{2}\right)}{\left(t_{2}-t_{1}\right)}=\frac{\dot{m} c_{p}}{\dot{M} C_{P}}=\frac{(130-90)}{(114-80)}=1.1765
$$

Now use Equation 2.16 to determine the number of shell passes

$$
N_{\text {shells }}=\frac{\ln \left[\frac{1-P R}{1-P}\right]}{\ln \left[\frac{1}{R}\right]}=\frac{\ln \left[\frac{1-(0 . .68)(1.1765)}{1-0.68}\right]}{\ln \left[\frac{1}{1.1765}\right]}=2.8920
$$

Rounding up to give $N_{\text {shells }}=3$, therefore a 3-6 S\&T exchanger is needed
15. At a given point in a thin walled heat exchanger, the shell-side fluid is at $100^{\circ} \mathrm{C}$ and the tube side fluid is at $20^{\circ} \mathrm{C}$, ignoring any fouling resistances and the resistance of the wall, determine the wall temperature for the following cases:
a. The inside and outside heat transfer coefficients are equal With the thin wall assumption we are justified in ignoring the radius effect when combining resistances and thus Equation 2.23 reduces to $U=\left[\frac{1}{h_{o}}+\frac{1}{h_{i}}\right]^{-1}$ and for this case $h_{o}=h_{i}=h \therefore U=\left[\frac{1}{h}+\frac{1}{h}\right]^{-1}=\frac{h}{2}$
The flux of energy through the inner and out films are related by $\frac{Q}{A}=h\left(100-T_{w}\right)=h\left(T_{w}-20\right) \Rightarrow T_{W}=120 / 2=60^{\circ} \mathrm{C}$
b. The inside coefficient has a value 3 times that of the outside coefficient

$$
\begin{aligned}
& h_{i}=3 h_{o} \text { and } \frac{Q}{A}=h_{o}\left(100-T_{w}\right)=h_{i}\left(T_{w}-20\right) \Rightarrow h_{o}\left(100-T_{w}\right)=3 h_{o}\left(T_{w}-20\right) \\
& T_{W}=\frac{100+(3)(20)}{4}=40^{\circ} \mathrm{C}
\end{aligned}
$$

c. The inside coefficient has a value $1 / 3$ that of the outside coefficient

$$
\begin{aligned}
& h_{i}=\frac{h_{o}}{3} \text { and } \frac{Q}{A}=h_{o}\left(100-T_{w}\right)=h_{i}\left(T_{w}-20\right) \Rightarrow h_{o}\left(100-T_{w}\right)=\frac{h_{o}}{3}\left(T_{w}-20\right) \\
& T_{W}=\frac{100+(1 / 3)(20)}{4 / 3}=80^{\circ} \mathrm{C}
\end{aligned}
$$

d. The inside coefficient is limiting $\Rightarrow T_{W}=100^{\circ} \mathrm{C}$ or $h_{i}=\frac{h_{o}}{3}$ and

$$
\begin{aligned}
& \frac{Q}{A}=h_{o}\left(100-T_{w}\right)=h_{i}\left(T_{w}-20\right) \Rightarrow h_{o}\left(100-T_{w}\right)=\frac{h_{o}}{n}\left(T_{w}-20\right) \text { where } n \gg 1 \\
& n h_{o}\left(100-T_{w}\right)=h_{o}\left(T_{w}-20\right) \Rightarrow T_{w}=\frac{n 100+20}{(n+1)} \underset{n \rightarrow \infty}{\rightarrow} 100^{\circ} \mathrm{C}
\end{aligned}
$$

16. In a heat exchanger, water ( $c_{p}=4200 \mathrm{~J} / \mathrm{kgK}$ ) flows at a rate of $1.5 \mathrm{~kg} / \mathrm{s}$ and toluene ( $c_{p}=1953 \mathrm{~J} / \mathrm{kgK}$ ) flows at a rate of $2.2 \mathrm{~kg} / \mathrm{s}$. The water enters at $50^{\circ} \mathrm{C}$ and leaves at $90^{\circ} \mathrm{C}$ and the toluene enters at $190^{\circ} \mathrm{C}$. For this situation, do the following:

$$
\begin{aligned}
& Q=\dot{m} c_{p} \Delta T \Rightarrow(1.5)(4200)(90-50)=252 \times 10^{3}=(2.2)(1953)\left(190-T_{\text {tol }, \text { out }}\right) \\
& T_{\text {tol }, \text { out }}=190-\frac{252,000}{(2.2)(1953)}=131.3^{\circ} \mathrm{C}
\end{aligned}
$$

a. Sketch the $T$ vs. $Q$ diagram if the flows are countercurrent

b. Sketch the $T$ vs. $Q$ diagram if the flows are concurrent

17. At a given location in a double pipe heat exchanger, the bulk temperature of the fluid in the annulus is $100^{\circ} \mathrm{C}$ and the bulk temperature of the fluid in the inner pipe is $20^{\circ} \mathrm{C}$. The tube wall is very thin and the resistance due to the metal wall may be ignored. Likewise fluid fouling resistances may also be ignored. If the individual heat transfer coefficients at this point in the heat exchanger are:

$$
h_{i}=100 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K} \text { and } h_{o}=500 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

a. What is the temperature of the wall at this location?

$$
\begin{aligned}
& \frac{Q}{A}=h_{o}\left(100-T_{w}\right)=h_{i}\left(T_{w}-20\right) \Rightarrow 500\left(100-T_{w}\right)=100\left(T_{w}-20\right) \\
& \therefore \mathrm{T}_{w}=\frac{(500)(100)+(20)(100)}{500+100}=86.67^{\circ} \mathrm{C}
\end{aligned}
$$

b. What is the heat flux across the wall at this location?

$$
\frac{Q}{A}=h_{o}\left(100-T_{w}\right)=500(100-86.67)=6.67 \mathrm{~kW} / \mathrm{m}^{2}
$$

c. If there was a fouling resistance of $0.001 \mathrm{~m}^{2} \mathrm{~K} / \mathrm{W}$ on the inside surface of the inner pipe, what would the temperature of the wall be at this location? From Equation 2.23 (or 2.24) and assuming $D_{0} \cong D_{i}$

$$
\begin{aligned}
& U_{o}=\left[\frac{1}{h_{o}}+R_{f o}+\frac{D_{o} \ln \left(D_{o} / D_{i}\right)}{2 k_{w}}+\frac{D_{o}}{D_{i}} R_{f i}+\frac{D_{o}}{D_{i}} \frac{1}{h_{i}}\right]^{-1}=\left[\frac{1}{h_{o}}+R_{f i}+\frac{1}{h_{i}}\right]^{-1} \\
& U_{o}=\left[\frac{1}{500}+0.001+\frac{1}{100}\right]^{-1}=76.92 \\
& \frac{Q}{A}=U \Delta T=(76.92)(100-20)=6.153 \mathrm{~kW} / \mathrm{m}^{2} \\
& \frac{Q}{A}=6153=h_{o}\left(100-T_{w}\right) \Rightarrow T_{W}=100-\frac{6153}{500}=87.69^{\circ} \mathrm{C}
\end{aligned}
$$

18. A single phase fluid ( $\dot{m}=2.5 \mathrm{~kg} / \mathrm{s}$ and $c_{p}=1800 \mathrm{~J} / \mathrm{kgK}$ ) is to be cooled from $175^{\circ} \mathrm{C}$ to $75^{\circ} \mathrm{C}$ by exchanging heat with another single phase fluid ( $\dot{m}=2.0 \mathrm{~kg} / \mathrm{s}$ and $c_{p}=2250 \mathrm{~J} / \mathrm{kgK}$ ) which is to be heated from $50^{\circ} \mathrm{C}$ to $150^{\circ} \mathrm{C}$. You have the choice of one of the following five heat exchangers:

|  | Shell Passes | Tube Passes |
| :--- | :---: | :---: |
| Exchanger 1 | 1 | 2 |
| Exchanger 2 | 2 | 4 |
| Exchanger 3 | 3 | 6 |
| Exchanger 4 | 4 | 8 |
| Exchanger 5 | 5 | 10 |

Which exchanger do you recommend using for this service - carefully explain your answer.

$$
P=\frac{\left(t_{2}-t_{1}\right)}{\left(T_{1}-t_{1}\right)}=\frac{(150-50)}{(175-50)}=\frac{100}{125}=0.80 \text { and } R=\frac{\left(T_{1}-T_{2}\right)}{\left(t_{2}-t_{1}\right)}=\frac{\dot{m} c_{p}}{\dot{M} C_{P}}=\frac{(175-75)}{(150-50)}=1.0
$$

Now use Equation 2.16 for $R=1$ to determine the number of shell passes

$$
N_{\text {shells }}=\frac{P}{1-P}=\frac{0.8}{1-0.8}=4
$$

So both Exchangers 4 and 5 are acceptable. This jibes with the results from Figures B.1.4 and 1.5 that give the F factors of 0.8 and 0.89 for Exchangers 4 and 5, respectively. Probably want to go with Exchanger 4 as this will be a little cheaper to construct.
19. What is the critical nucleate boiling flux for water at 20 atm?

The critical pressure for water, $\mathrm{Pc}=22.06 \times 10^{6} \mathrm{~Pa}$, using the correlation of Chichelli-Bonilla, Equation 2.46 gives

$$
\begin{aligned}
& {\left[\frac{Q}{A}\right]_{\max }=0.3673 P_{c}\left(\frac{P}{P_{c}}\right)^{0.35}\left(1-\frac{P}{P_{c}}\right)^{0.9}} \\
& {\left[\frac{Q}{A}\right]_{\max }=(0.3673)\left(22.06 \times 10^{6}\right)\left(\frac{(20)\left(1.013 \times 10^{5}\right)}{\left(22.06 \times 10^{6}\right)}\right)^{0.35}\left(1-\frac{(20)\left(1.013 \times 10^{5}\right)}{\left(22.06 \times 10^{6}\right)}\right)^{0.9}=3.22 \mathrm{MW} / \mathrm{m}^{2}}
\end{aligned}
$$

20. Water flows inside a long $\frac{3}{4}$-in 16 BWG tube at an average temperature of $110^{\circ} \mathrm{F}$. Determine the inside heat transfer coefficient for the following cases:
Properties of water at $110^{\circ} \mathrm{F}$ are
$\rho=61.9 \mathrm{lb} / \mathrm{ft}^{3}, c_{p}=1.00 \mathrm{Btu} / \mathrm{lb}_{\mathrm{m}}{ }^{\circ} \mathrm{F}, \mu=0.424 \times 10^{-3} \mathrm{l} \mathrm{b}_{\mathrm{m}} / \mathrm{ft} \mathrm{sec}, \mathrm{k}=0.366$
$\mathrm{Btu} / \mathrm{hr} \mathrm{ft}{ }^{\circ} \mathrm{F}, \mathrm{Pr}=\mathrm{c}_{\mathrm{p}} \mu / \mathrm{k}=(1.00)\left(0.424 \times 10^{-3}\right)(3600) /(0.366)=4.17$
$D_{i}$ for $\frac{3 " 1}{4} 16 B W G$ tube (Table 2.4) $=0.620$ inch
a. Velocity $=0.1 \mathrm{ft} / \mathrm{s}$
$\operatorname{Re}=\rho D u / \mu=(61.9)(0.620 / 12)(0.1) /\left(0.424 \times 10^{-3}\right)=754.3$ - laminar flow
Without any further information we will assume that the limiting condition $\mathrm{Nu}_{\infty}=3.66$ exists.
$H=(3.66) \mathrm{k} / \mathrm{D}=(3.66)(0.366) /(0.62 / 12)=25.9 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{2}{ }^{\circ} \mathrm{F}$
b. Velocity $=3 \mathrm{ft} / \mathrm{s}$
$\operatorname{Re}=\rho D u / \mu=(61.9)(0.620 / 12)(3) /\left(0.424 \times 10^{-3}\right)=22,630-$ turbulent flow Use Equation 2.26 and do not account for the viscosity correction as this will be small for water

$$
\begin{aligned}
& \mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023\left[1+\left(\frac{D_{i}}{L}\right)^{0.7}\right] \operatorname{Re}^{0.8} \operatorname{Pr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14}=(0.023)(22630)^{0.8}(4.17)^{1 / 3}=112.76 \\
& h_{i}=\mathrm{Nu} \frac{k}{D_{i}}=\frac{(112.76)(0.366)}{(0.62 / 12)}=798.8 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{2}{ }^{\circ} \mathrm{F}
\end{aligned}
$$

21. Use Equations 2.47 and 2.48 to estimate the heat flux and heat transfer coefficient for boiling acetone at 1 atm pressure for a temperature driving force of $10^{\circ} \mathrm{C}$.
For boiling acetone at 1 atm
$\rho_{l}=748.6 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{v}=4.3592 \mathrm{~kg} / \mathrm{m}^{3}, c_{p l}=2282 \mathrm{~J} / \mathrm{kgK}, \lambda=538.4 \mathrm{~kJ} / \mathrm{kg}, \mu_{l}=$
$2.36 \times 10^{-4} \mathrm{Ns} / \mathrm{m}^{2}, \mathrm{k}_{1}=0.1522 \mathrm{~W} / \mathrm{mK}, ~ \sigma=0.0193 \mathrm{~N} / \mathrm{m}$, and
$\operatorname{Pr} l_{1}=c_{p} \mid \mu_{\lambda} / k_{1}=(2282)\left(2.36 \times 10^{-4}\right) /(0.1522)=3.54$
Use a value of $C_{f}=0.01$ and $s=1.7$ (for substances other than water)

$$
\begin{aligned}
& \frac{Q}{A}=\mu_{l} \lambda\left(\frac{g\left(\rho_{l}-\rho_{v}\right)}{\sigma}\right)^{1 / 2}\left(\frac{c_{p l} \Delta T_{s}}{C_{f} \lambda \mathrm{Pr}_{l}^{s}}\right)^{3} \\
& =\left(2.36 \times 10^{-4}\right)\left(538.4 \times 10^{3}\right)\left[\frac{(9.81)(748.6-4.3592)}{(0.0193)}\right]^{1 / 2}\left(\frac{(2282)(10)}{(0.01)\left(538.4 \times 10^{3}\right)(3.54)^{1.7}}\right)^{3} \\
& \frac{Q}{A}=9432.9 \mathrm{~W} / \mathrm{m}^{2}
\end{aligned}
$$

and
$h=\mu_{l} \lambda\left(\frac{g\left(\rho_{l}-\rho_{v}\right)}{\sigma}\right)^{1 / 2}\left(\frac{c_{p l}}{C_{f} \lambda \mathrm{Pr}_{l}^{s}}\right)^{3} \Delta T_{s}^{2}=943.3 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
22. Use the Sieder-Tate equation to determine the inside heat transfer coefficient for a fluid flowing inside a 16 ft long, 1 -in tube (14 BWG) at a velocity of $2.5 \mathrm{ft} / \mathrm{s}$ that is being cooled and has an average temperature of $176^{\circ} \mathrm{F}$ and an average wall temperature of $104^{\circ} \mathrm{F}$.
$D i=0.834$ inch
Consider the following fluids:
a. Acetone (liquid)
$\rho=44.80 \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}, \mathrm{c}_{\mathrm{p}}=0.5706 \mathrm{Btu} / \mathrm{lb}_{\mathrm{m}}{ }^{\circ} \mathrm{F}, \mu_{176}=1.339 \times 10^{-4} \mathrm{lb} \mathrm{m} / \mathrm{ft} \mathrm{s}, \mu_{104}=$ $1.806 \times 10^{-4} \mathrm{lb} / \mathrm{ft} \mathrm{s}, \mathrm{k}=0.0838 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{\circ} \mathrm{F}$

$$
\begin{aligned}
& \operatorname{Pr}=\frac{c_{p} \mu}{k}=\frac{(0.5706)\left(1.339 \times 10^{-4}\right)}{(0.0838 / 3600)}=3.28 \\
& \operatorname{Re}=\frac{\rho D u}{\mu}=\frac{(44.80)(0.834 / 12)(2.5)}{\left(1.339 \times 10^{-4}\right)}=58,130 \\
& \mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023\left[1+\left(\frac{D_{i}}{L}\right)^{0.7}\right] \operatorname{Re}^{0.8} \operatorname{Pr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14} \\
& \mathrm{Nu}=0.023\left[1+\left(\frac{(0.834)}{(12)(16)}\right)^{0.7}\right](58,130)^{0.8}(3.28)^{1 / 3}\left(\frac{1.339}{1.806}\right)^{0.14}=217.1 \\
& h_{i}=\mathrm{Nu} \frac{k}{D_{i}}=\frac{(217.1)(0.0838)}{(0.834 / 12)}=261.8 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{20} \mathrm{~F}
\end{aligned}
$$

b. Iso-propanol (liquid)
$\rho=45.27 \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}, \mathrm{c}_{\mathrm{p}}=0.8037 \mathrm{Btu} / \mathrm{lb}_{\mathrm{m}}{ }^{\circ} \mathrm{F}, \mu_{176}=3.550 \times 10^{-4} \mathrm{lb} \mathrm{b} / \mathrm{ft} \mathrm{s}, \mu_{104}$
$=9.123 \times 10^{-4} \mathrm{lb} / \mathrm{ft} \mathrm{s}, \mathrm{k}=0.0708 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}{ }^{\circ} \mathrm{F}$
$\operatorname{Pr}=\frac{c_{p} \mu}{k}=\frac{(0.8037)\left(3.550 \times 10^{-4}\right)}{(0.0708 / 3600)}=14.51$
$\operatorname{Re}=\frac{\rho D u}{\mu}=\frac{(45.27)(0.834 / 12)(2.5)}{\left(3.55 \times 10^{-4}\right)}=22,160$
$\mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023\left[1+\left(\frac{D_{i}}{L}\right)^{0.7}\right] \operatorname{Re}^{0.8} \operatorname{Pr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14}$
$\mathrm{Nu}=0.023\left[1+\left(\frac{(0.834)}{(12)(16)}\right)^{0.7}\right](22,160)^{0.8}(14.51)^{1 / 3}\left(\frac{3.550}{9.123}\right)^{0.14}=150.5$
$h_{i}=\mathrm{Nu} \frac{k}{D_{i}}=\frac{(150.5)(0.0708)}{(0.834 / 12)}=153.3 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{2{ }^{2}} \mathrm{~F}$

## c. Methane at 1 atm pressure - use a velocity of $50 \mathrm{ft} / \mathrm{s}$

$\rho=0.0346 \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}, \mathrm{c}_{\mathrm{p}}=0.5676 \mathrm{Btu} / \mathrm{lb}_{\mathrm{m}}{ }^{\circ} \mathrm{F}, \mu_{176}=8.619 \times 10^{-6} \mathrm{lb} / \mathrm{ft} \mathrm{s}, \mu_{104}=$ $7.802 \times 10^{-6} \mathrm{lb} / \mathrm{ft} \mathrm{s}, \mathrm{k}=0.0245 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}{ }^{\circ} \mathrm{F}$
$\operatorname{Pr}=\frac{c_{p} \mu}{k}=\frac{(0.5676)\left(8.619 \times 10^{-6}\right)}{(0.0245 / 3600)}=0.719$
$\operatorname{Re}=\frac{\rho D u}{\mu}=\frac{(0.0356)(0.834 / 12)(50)}{\left(8.619 \times 10^{-6}\right)}=13,950$
$\mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023\left[1+\left(\frac{D_{i}}{L}\right)^{0.7}\right] \operatorname{Re}^{0.8} \operatorname{Pr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14}$
$\mathrm{Nu}=0.023\left[1+\left(\frac{(0.834)}{(12)(16)}\right)^{0.7}\right](13,950)^{0.8}(0.719)^{1 / 3}\left(\frac{8.619}{7.802}\right)^{0.14}=44.18$
$h_{i}=\mathrm{Nu} \frac{k}{D_{i}}=\frac{(44.18)(0.0245)}{(0.834 / 12)}=15.57 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{20} \mathrm{~F}$
d. Compare the results using the Dittus-Boelter equation

For a.

$$
\begin{aligned}
& \mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{0.3}=0.023(58,130)^{0.8}(3.28)^{0.3}=212.9 \\
& h_{i}=\mathrm{Nu} \frac{k}{D_{i}}=(212.9) \frac{(0.0838)}{(0.834 / 12)}=256.7 \mathrm{Btu} / \mathrm{hrft}^{20} \mathrm{~F} \sim 2 \% \text { difference }
\end{aligned}
$$

For b.

$$
\begin{aligned}
& \mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{0.3}=0.023(22,157)^{0.8}(14.51)^{0.3}=153.7 \\
& h_{i}=\mathrm{Nu} \frac{k}{D_{i}}=(153.7) \frac{(0.0708)}{(0.834 / 12)}=156.6 \mathrm{Btu} / \mathrm{hrft}^{20} \mathrm{~F} \sim-2 \% \text { difference }
\end{aligned}
$$

For $c$.

$$
\begin{aligned}
& \mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023 \operatorname{Re}^{0.8} \mathrm{Pr}^{0.3}=0.023(13,950)^{0.8}(0.72)^{0.3}=43.09 \\
& h_{i}=\mathrm{Nu} \frac{k}{D_{i}}=(43.09) \frac{(0.0245)}{(0.834 / 12)}=15.19 \mathrm{Btu} / \mathrm{hrft}^{20} \mathrm{~F} \sim 2 \% \text { difference }
\end{aligned}
$$

Therefore the corrections for tube entrance and wall viscosity are v. small for these cases.
23. Water at 5 atm pressure and $30^{\circ} \mathrm{C}$ flows at an inlet velocity of $1 \mathrm{~m} / \mathrm{s}$ into a square duct 2.5 m in length that has a cross section of 25 mm by 25 mm . The wall of the duct is maintained by condensing steam at a temperature of $120^{\circ} \mathrm{C}$. What will the exit temperature of the water be when it exits the duct?

$$
\begin{aligned}
& \rho=996 \mathrm{~kg} / \mathrm{m}^{3}, \mu_{30}=8.291 \times 10^{-4} \text { Pa.s, } \mu_{120}=2.19 \times 10^{-4} \text { Pa.s, } k=0.6130 \\
& \mathrm{~W} / \mathrm{m}^{\circ} \mathrm{C}, \mathrm{c}_{p}=4187 \mathrm{~J} / \mathrm{kg}^{\circ} \mathrm{C}
\end{aligned}
$$



$$
D_{H}=4 \frac{\text { flow area }}{\text { wetted perimeter }}=4 \frac{(25)(25)}{4(25)}=25 \mathrm{~mm}
$$

Assume flow is turbulent - check this when Re is found $\operatorname{Re}_{H}=\frac{\rho u D_{H}}{\mu}=\frac{(996)(1)(0.025)}{\left(8.291 \times 10^{-4}\right)}=30,030-$ turbulent use Seider-Tate Eqn with $\mathrm{ReH}_{\mathrm{H}}$
$\operatorname{Pr}=\frac{c_{p} \mu}{k}=\frac{(4187)\left(8.291 \times 10^{-4}\right)}{(0.6130)}=5.66$
$\mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023\left[1+\left(\frac{D_{H}}{L}\right)^{0.7}\right] \operatorname{Re}_{H}^{0.8} \operatorname{Pr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14}$
$\mathrm{Nu}=0.023\left[1+\left(\frac{0.025}{2.5}\right)^{0.7}\right](30,030)^{0.8}(5.66)^{1 / 3}\left(\frac{8.291 \times 10^{-4}}{2.19 \times 10^{-4}}\right)^{0.14}=196.2$
$h_{i}=\mathrm{Nu} \frac{k}{D_{i}}=195.2 \frac{(0.613)}{(0.025)}=4811 \mathrm{~W} / \mathrm{m}^{20} \mathrm{C}$
Since the wall temperature is constant, the outside heat transfer coefficient can be considered to be $\infty$ and $U=h_{i}$. The following analysis assumes that the physical properties of water are constant over the length of the channel.

$$
\begin{aligned}
& Q=\dot{m} c_{p}\left(T_{\text {out }}-30\right)=U A \frac{\left(T_{\text {out }}-30\right)}{\ln \frac{(120-30)}{\left(120-T_{\text {out }}\right)}} \\
& Q=(0.025)^{2}(1)(996)(4187)\left(T_{\text {out }}-30\right)=(4811)(4)(0.025)(2.5) \frac{\left(T_{\text {out }}-30\right)}{\ln \frac{(90)}{\left(120-T_{\text {out }}\right)}} \\
& \Rightarrow \ln \frac{(90)}{\left(120-T_{\text {out }}\right)}=\frac{(4811)(4)(0.025)(2.5)}{(0.025)^{2}(1)(996)(4187)}=0.4615 \Rightarrow T_{\text {out }}=63.3^{\circ} \mathrm{C}
\end{aligned}
$$

24. A shell-and-tube condenser contains six rows of five copper tubes per row on a square pitch. The tubes are $\frac{3}{4}$-in, 14 BWG and 3 m long. Cooling water flows through the tubes such that $h_{i}=2000 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$. The water flow is high so that the temperature on the tube side may be assumed to be constant at $35^{\circ} \mathrm{C}$. Pure saturated steam at 2 bar is condensing on the shell side. Determine the capacity of the condenser ( $Q$ in kW ) if the condenser tubes are oriented (a) vertically, and (b) horizontally.
a. Vertical Arrangement - assume $\operatorname{Re}_{\delta}<1800$ and use Equation 2.64 $D_{i}=0.584$ inch $=0.01483 \mathrm{~m}, D_{0}=3 / 4$ inch $=0.01905 \mathrm{~m}, h_{i}=2000$ $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}, k_{c u}=377 \mathrm{~W} / \mathrm{mK}$ (assume $T_{w}=100^{\circ} \mathrm{C}$ ). For saturated steam at 2 bar, $T_{\text {sat }}=120.2^{\circ} \mathrm{C}$
We need $T_{w}$ to be able to apply Eqn 264 or 2.66 - so follow the algorithm shown in Example 2.14:
i) guess $h_{0}$
ii) calculate film temperature and film properties
iii) apply equation for $h_{0}$ and iterate.

Assume $h_{0}=2000 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

$$
\begin{aligned}
& U_{o}=\left[\frac{1}{h_{o}}+\not_{f_{f o}}+\frac{D_{o} \ln \left(D_{o} / D_{i}\right)}{2 k_{w}}+\frac{D_{o}}{D_{i}} \not X_{K_{i}}+\frac{D_{o}}{D_{i}} \frac{1}{h_{i}}\right]^{-1} \text { and } \\
& \frac{Q}{A_{o}}=h_{o}\left(T_{\text {sat }}-T_{\text {wall }}\right)=U_{o}\left(T_{\text {sat }}-T_{\text {water }}\right) \text { rearranging gives } \\
& \frac{\left(120.2-T_{\text {wall }}\right)}{\frac{1}{2000}}=\frac{(120.2-35)}{\frac{1}{2000}+\frac{(0.01905) \ln (0.01905 / 0.01483)}{(2)(377)}+\frac{(0.01905)}{(0.01483)} \frac{1}{2000}} \\
& \left(120.2-T_{\text {wall }}\right)=\frac{(870.6)(120.2-35)}{(2000)} \Rightarrow T_{\text {wall }}=83.1^{\circ} \mathrm{C}
\end{aligned}
$$

$$
T_{\text {film }}=T_{\text {sat }}-0.75\left(T_{\text {sat }}-T_{\text {wall }}\right)=120.2-0.75(120.2-83.1)=92.4^{\circ} \mathrm{C}
$$

For water at $92.4^{\circ} \mathrm{C}, k_{l}=0.6721 \mathrm{~W} / \mathrm{mK}, \rho_{l}=963.2 \mathrm{~kg} / \mathrm{m}^{3}, \lambda=$ $2201.6 \mathrm{~kJ} / \mathrm{kg}$ ( $\mathrm{a}+2 \mathrm{bar}$ ), $\mu=3.062 \times 10^{-4} \mathrm{kgm} / \mathrm{s}, c_{p, I}=4215.8 \mathrm{~J} / \mathrm{kgK}$, $\rho_{v}=1.129 \mathrm{~kg} / \mathrm{m}^{3}$

$$
\begin{aligned}
& \lambda^{\prime}=\lambda+0.68 c_{p, l}\left(T_{\text {sat }}-T_{\text {wall }}\right)=2201.6+\frac{0.68(4215.8)(120.2-83.1)}{1000}=2307.9 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{Nu}=\frac{\bar{h}_{c} L}{k_{l}}=1.13\left[\frac{\rho_{l} g\left(\rho_{l}-\rho_{v}\right) \lambda^{\prime} L^{3}}{\mu_{l} k_{l}\left(T_{\text {sat }}-T_{w}\right)}\right]^{1 / 4} \\
& \mathrm{Nu}=1.13\left[\frac{(963.2)(9.81)(963.2-1.129)(2307.9)(3)^{3}}{\left(3.062 \times 10^{-4}\right)(0.6721)(120.3-83.1)}\right]^{1 / 4}=3317 \\
& \bar{h}_{c}=\mathrm{Nu} \frac{k_{l}}{L}=(3317) \frac{(0.6721)}{(3)}=743.0 \mathrm{Wm}^{2} \mathrm{~K}
\end{aligned}
$$

Use this new value of $h_{0}=\bar{h}_{c}$ and iterate - results shown in Table below.
Converged Solution gives:
$h_{0}=620.2 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$ and $U_{0}=442.3 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
$Q=U_{o} A_{o} \Delta T=(442.3)(30)(\pi)(0.01905)(3)(120.2-35)=203.0 \mathrm{~kW}$
Check for $\mathrm{Re}_{\delta}$
$\operatorname{Re}_{\delta}=\frac{4 \dot{m}}{\mu_{l} W}=\frac{4\left(Q / \lambda^{\prime}\right)}{\mu_{l}\left(\pi D_{o}\right) n_{\text {tubes }}}=\frac{4(203.0 / 2375.13)}{\left(3.85 \times 10^{-4}\right)(\pi)(0.01905)(30)}=495<1800 \mathrm{so}$
correct equation used - if this were not the case, you would not need to iterate using Eqn 2.64.
$\underline{Q}=203 \mathrm{~kW}$

## Results for Iterative Solution to Problem 24 - Vertical Condenser Tubes

| $\theta$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Iteration 1 |  | Iteration 2 |  | Iteration 3 |  | Iteration 4 |  | Iteration 5 |  |
| Guess $h_{\text {。 }}$ | 2000 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ | 743.0 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ | 634.342 W | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ | 621.848 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ | 620.416 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ |
| $h_{i}$ | 2000 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ | 2000 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ | 2000 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ | 2000 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ | 2000 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ |
| $T_{\text {sat }}$ | 120.2 | C | 120.2 | C | 120.2 C | C | 120.2 | C | 120.2 | C |
| $T_{\text {water }}$ | 35 | C | 35 | C | 35 C | C | 35 | C | 35 | C |
| $k_{c u}$ | 377 | $\mathrm{W} / \mathrm{mK}$ | 377 | $\mathrm{W} / \mathrm{mK}$ | 377 | $\mathrm{W} / \mathrm{mK}$ | 377 | $\mathrm{W} / \mathrm{mK}$ | 377 | $\mathrm{W} / \mathrm{mK}$ |
| D | 0.01905 | m | 0.01905 m | m | 0.01905 m | m | 0.01905 | m | 0.01905 | m |
| $D_{i}$ | 0.01483 | m | 0.01483 m | m | 0.01483 m | m | 0.01483 | m | 0.01483 | m |
| $U_{0}$ | 870.621 |  | 501.4 |  | 449.429 |  | 443.122 |  | 442.394 |  |
| $T_{\text {wall }}$ | 83.1116 |  | 62.708 |  | 59.836 |  | 59.4875 |  | 59.4472 |  |
| $T_{\text {film }}$ | 92.3837 |  | 77.081 |  | 74.927 |  | 74.6656 |  | 74.6354 |  |
| $\lambda$ | 2201.6 | $\mathrm{kJ} / \mathrm{kg}$ | 2201.6 | $\mathrm{kJ} / \mathrm{kg}$ | 2201.6 k | $\mathrm{kJ} / \mathrm{kg}$ | 2201.6 | $\mathrm{kJ} / \mathrm{kg}$ | 2201.6 | $\mathrm{kJ} / \mathrm{kg}$ |
| $c_{p l}$ | 4215.8 | $\mathrm{J} / \mathrm{kgK}$ | 4201.9 J | J/kgK | 4200.5 J | $\mathrm{J} / \mathrm{kgK}$ | 4200.4 | J/kgK | 4200.4 | $\mathrm{J} / \mathrm{kgK}$ |
| $\mu_{1}$ | $3.06 \mathrm{E}-04$ | $\mathrm{kgm} / \mathrm{s}$ | 3.72E-04 | $\mathrm{kgm} / \mathrm{s}$ | 3.83E-04 k | $\mathrm{kgm} / \mathrm{s}$ | $3.84 \mathrm{E}-04$ | $\mathrm{kgm} / \mathrm{s}$ | $3.85 \mathrm{E}-04$ | kgm/s |
| $\rho_{1}$ | 963.2 | $\mathrm{kg} / \mathrm{m}^{3}$ | 973.2 | $\mathrm{kg} / \mathrm{m}^{3}$ | 974.5 k | $\mathrm{kg} / \mathrm{m}^{3}$ | 974.7 | $\mathrm{kg} / \mathrm{m}^{3}$ | 974.7 | $\mathrm{kg} / \mathrm{m}^{3}$ |
| $\rho_{v}$ | 1.129 | $\mathrm{kg} / \mathrm{m}^{3}$ | 1.129 | $\mathrm{kg} / \mathrm{m}^{3}$ | 1.129 k | $\mathrm{kg} / \mathrm{m}^{3}$ | 1.129 | $\mathrm{kg} / \mathrm{m}^{3}$ | 1.129 | $\mathrm{kg} / \mathrm{m}^{3}$ |
| $k_{1}$ | 0.6721 | $\mathrm{W} / \mathrm{mK}$ | 0.662 | $\mathrm{W} / \mathrm{mK}$ | 0.6604 | $\mathrm{W} / \mathrm{mK}$ | 0.6602 | $\mathrm{W} / \mathrm{mK}$ | 0.6601 | $\mathrm{W} / \mathrm{mK}$ |
| $\lambda^{\prime}$ | 2307.92 | $\mathrm{kJ} / \mathrm{kg}$ | 2365.87 | $\mathrm{kJ} / \mathrm{kg}$ | 2374.02 k | kJ/kg | 2375.01 | $\mathrm{kJ} / \mathrm{kg}$ | 2375.13 | $\mathrm{kJ} / \mathrm{kg}$ |
|  |  |  |  |  |  |  |  |  |  |  |
| Nu | 3316.7 |  | 2874.7 |  | 2824.9 |  | 2819.2 |  | 2818.5 |  |
| $h_{c}$ | 743.0 |  | 634.3 |  | 621.8 |  | 620.4 |  | 620.2 |  |

## b. Horizontal Arrangement - use Equation 2.67

$\mathrm{Nu}=\frac{\bar{h}_{c} D_{o}}{k_{l}}=0.728\left[\frac{\rho_{l} g\left(\rho_{l}-\rho_{v}\right) \lambda^{\prime} D_{o}^{3}}{\mu_{l} k_{l}\left(T_{\text {sat }}-T_{w}\right)}\right]^{1 / 4}$ Following the same algorithm from part a and Example 2.14 - assume a value of $h_{0}=2000 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

$$
\frac{Q}{A_{o}}=h_{o}\left(T_{\text {sat }}-T_{\text {wall }}\right)=U_{o}\left(T_{\text {sat }}-T_{\text {water }}\right) \text { rearranging gives }
$$

$$
\frac{\left(120.2-T_{\text {wall }}\right)}{\frac{1}{2000}}=\frac{(120.2-35)}{\frac{1}{2000}+\frac{(0.01905) \ln (0.01905 / 0.01483)}{(2)(377)}+\frac{(0.01905)}{(0.01483)} \frac{1}{2000}}
$$

$$
\left(120.2-T_{\text {wall }}\right)=\frac{(870.6)(120.2-35)}{(2000)} \Rightarrow T_{\text {wall }}=83.1^{\circ} \mathrm{C}
$$

$T_{\text {film }}=T_{\text {sat }}-0.75\left(T_{\text {sat }}-T_{\text {wall }}\right)=120.2-0.75(120.2-83.1)=92.4^{\circ} \mathrm{C}$
For water at $92.4^{\circ} \mathrm{C}, \mathrm{k}_{l}=0.6721 \mathrm{~W} / \mathrm{mK}, \rho_{l}=963.2 \mathrm{~kg} / \mathrm{m}^{3}, \lambda=2201.6$ $\mathrm{kJ} / \mathrm{kg}$ (at 2 bar ), $\mu_{\mathrm{l}}=3.062 \times 10^{-4} \mathrm{kgm} / \mathrm{s}, c_{p, 1}=4215.8 \mathrm{~J} / \mathrm{kgK}, \rho_{v}=1.129$ $\mathrm{kg} / \mathrm{m}^{3}$

$$
\begin{aligned}
& \lambda^{\prime}=\lambda+0.68 c_{p, l}\left(T_{\text {sat }}-T_{\text {wall }}\right)=2201.6+\frac{0.68(4215.8)(120.2-83.1)}{1000}=2307.9 \mathrm{~kJ} / \mathrm{kg} \\
& \left.\mathrm{Nu}=\frac{\bar{h}_{c} D_{o}}{k_{l}}=0.728\left[\frac{\rho_{l} g\left(\rho_{l}-\rho_{v}\right) \lambda^{\prime} D_{o}^{3}}{\mu_{l} k_{l}\left(T_{\text {sat }}-T_{w}\right)}\right]^{1 / 4}\right]^{1 / 4}=48.1 \\
& \mathrm{Nu}=0.728\left[\frac{(963.2)(9.81)(963.2-1.129)(2307.9)(0.01905)^{3}}{\left(3.062 \times 10^{-4}\right)(0.6721)(120.3-83.1)}\right]^{1 / 4} \\
& \bar{h}_{c}=\mathrm{Nu} \frac{k_{l}}{D_{o}}=48.1 \frac{(0.6721)}{(0.01905)}=1695.8 \mathrm{Wm}^{2} \mathrm{~K}
\end{aligned}
$$

Starting the second iteration
$\frac{\left(120.2-T_{\text {wall }}\right)}{\frac{1}{(1695.8)}}=\frac{(120.2-35)}{\frac{1}{(1695.8)}+\frac{(0.01905) \ln (0.01905 / 0.01483)}{(2)(377)}+\frac{(0.01905)}{(0.01483)} \frac{1}{2000}}$
$\left(120.2-T_{\text {wall }}\right)=\frac{(807.6)(120.2-35)}{(1695.8)} \Rightarrow T_{\text {wall }}=79.6^{\circ} \mathrm{C}$
$T_{\text {film }}=T_{\text {sat }}-0.75\left(T_{\text {sat }}-T_{\text {wall }}\right)=120.2-0.75(120.2-79.6)=89.8^{\circ} \mathrm{C}$

For water at $89.8^{\circ} \mathrm{C}, k_{l}=0.6706 \mathrm{~W} / \mathrm{mK}, \rho_{l}=965.0 \mathrm{~kg} / \mathrm{m}^{3}, \lambda=2201.6$ $\mathrm{kJ} / \mathrm{kg}$ (at 2 bar ), $\mu \mathrm{m}=3.158 \times 10^{-4} \mathrm{kgm} / \mathrm{s}, c_{p, I}=4213.3 \mathrm{~J} / \mathrm{kgK}, \rho_{v}=1.129$ $\mathrm{kg} / \mathrm{m}^{3}$

$$
\begin{aligned}
& \lambda^{\prime}=\lambda+0.68 c_{p, l}\left(T_{\text {sat }}-T_{\text {wall }}\right)=2201.6+\frac{0.68(4213.3)(120.2-79.6)}{1000}=2317.8 \mathrm{~kJ} / \mathrm{kg} \\
& \mathrm{Nu}=\frac{\bar{h}_{c} D_{o}}{k_{l}}=0.728\left[\frac{\rho_{l} g\left(\rho_{l}-\rho_{v}\right) \lambda^{\prime} D_{o}^{3}}{\mu_{l} k_{l}\left(T_{\text {sat }}-T_{w}\right)}\right]^{1 / 4} \\
& \mathrm{Nu}=0.728\left[\frac{(965.0)(9.81)(965.0-1.129)(2317.8)(0.01905)^{3}}{\left(3.158 \times 10^{-4}\right)(0.6706)(120.3-79.6)}\right]^{1 / 4}=46.8 \\
& \bar{h}_{c}=\mathrm{Nu} \frac{k_{l}}{D_{o}}=46.8 \frac{(0.6706)}{(0.01905)}=1646.0 \mathrm{Wm}^{2} \mathrm{~K}
\end{aligned}
$$

Additional iterations are shown in the following table

## Converged Solution gives:

$$
\begin{aligned}
& h_{o}=1635.3 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K} \text { and } U_{o}=793.6 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K} \\
& Q=U_{o} A_{o} \Delta T=(793.6)(30)(\pi)(0.01905)(3)(120.2-35)=364.18 \mathrm{~kW}
\end{aligned}
$$

## Check for $\mathrm{Re}_{\delta}$

$$
\operatorname{Re}_{\delta}=\frac{4 \dot{m}}{\mu_{l} W}=\frac{4\left(Q / \lambda^{\prime}\right)}{\mu_{l}(2) L_{\text {tube }} n_{\text {tubes }}}=\frac{4(364.18 / 2320.03)}{\left(3.182 \times 10^{-4}\right)(2)(3)(30)}=439<1800 \text { so correct }
$$

## equation used

## $Q=364.18 \mathrm{~kW}$

## Results for Iterative Solution to Problem 24 - Horizontal Condenser Tubes


25. Liquid dimethyl ether (DME) flows across the outside of a bank of tubes. It enters at $100^{\circ} \mathrm{C}$ and leaves at $50^{\circ} \mathrm{C}$. The DME enters at a flowrate of $20 \mathrm{~kg} / \mathrm{s}$, the shell diameter is $15-\mathrm{in}$, the baffle spacing is 6in, the baffle cut $(B C)$ is $15 \%$, and 1 -in OD tubes on a 1.25 -in pitch are used. The fluid inside the tubes may be assumed to be at a constant temperature of $35^{\circ} \mathrm{C}$ (cooling water) and the inside coefficient is expected to be much higher than the shell-side coefficient and thus the wall temperature may be taken as $35^{\circ} \mathrm{C}$.
Use Kern's method to determine the average heat transfer coefficient for the shell side for the following arrangements:
$B C=15 \%, D_{0}=0.0254 \mathrm{~m}, \mathrm{p}=1.25$ inch $=0.03175 \mathrm{~m}, D_{\text {shell }}=15$ inch $=$ $0.381 \mathrm{~m}, L_{b}=6$ inch $=0.1524 \mathrm{~m}$,

## Properties of DME

| DME @ $50^{\circ} \mathrm{C}$ | $\underline{\text { DME @ } 100^{\circ} \mathrm{C}}$ | DME - average property |
| :--- | :--- | :--- |
| $\rho=612.1 \mathrm{~kg} / \mathrm{m}^{3}$ | $\rho=494.3 \mathrm{~kg} / \mathrm{m}^{3}$ | $\rho=553.2 \mathrm{~kg} / \mathrm{m}^{3}$ |
| $c_{\rho}=2460 \mathrm{~J} / \mathrm{kgK}$ | $c_{p}=2803 \mathrm{~J} / \mathrm{kgK}$ | $c_{\rho}=2631.5 \mathrm{~J} / \mathrm{kgK}$ |
| $k=0.1296 \mathrm{~W} / \mathrm{mK}$ | $k=0.1014 \mathrm{~W} / \mathrm{mK}$ | $k=0.1155 \mathrm{~W} / \mathrm{mK}$ |
| $\mu=99.13 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ | $\mu=84.02 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ | $\mu=91.58 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ |
| $\mu 35^{\circ} \mathrm{C}=108.8 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ |  | $\operatorname{Pr}=\frac{c_{p} \mu}{k}=2.087$ |

a. square pitch

From Equations 2.37-2.43
$D_{H, s}=\frac{1.273 p^{2}-D_{o}^{2}}{D_{o}}=\frac{(1.273)(0.03175)^{2}-(0.0254)^{2}}{(0.0254)}=0.02512 \mathrm{~m}$
$A_{s}=\frac{D_{s} L_{b}\left(p-D_{o}\right)}{p}=\frac{(0.381)(0.1524)(0.03175-0.0254)}{(0.03175)}=0.01161 \mathrm{~m}^{2}$
Shell-side superficial mass velocity, $G_{s}=\frac{\dot{m}_{s}}{A_{s}}=\frac{20}{0.01161}=1722.2 \mathrm{~kg} / \mathrm{m}^{2} \mathrm{~s}$
Shell-side velocity, $u_{s}=\frac{\dot{m}_{s}}{\rho A_{s}}=\frac{G_{s}}{\rho}=\frac{(1722.2)}{(553.2)}=3.1132 \mathrm{~m} / \mathrm{s}$

Shell-side Reynolds number,

$$
\operatorname{Re}_{s}=\frac{G_{s} D_{H, s}}{\mu}=\frac{u_{s} \rho D_{H, s}}{\mu}=\frac{(3.1132)(553.2)(0.02512)}{\left(91.58 \times 10^{-6}\right)}=472,440
$$

The average heat transfer coefficient for the shell side of the exchanger is given by:
$\mathrm{Nu}=\frac{h_{s} D_{H, s}}{k_{f}}=j_{h} \operatorname{RePr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14}$
where for $100<\operatorname{Re}<1 \times 10^{6}$

$$
\begin{aligned}
& j_{h}=1.2492(B C)^{-0.329} \operatorname{Re}^{-0.4696}=(1.2492)(15)^{-0.329}(472,530)^{-0.4696}=0.001109 \\
& \therefore \mathrm{Nu}=\frac{h_{s} D_{H, s}}{k_{f}}=j_{h} \operatorname{Re}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14}=(0.001109)(472,530)(2.087)^{1 / 3}\left[\frac{\left(91.58 \times 10^{-6}\right)}{108.8 \times 10^{-6}}\right]^{0.14} \\
& \mathrm{Nu}=653.7 \Rightarrow h_{s}=\mathrm{Nu} \frac{k_{f}}{D_{H, s}}=(653.7) \frac{(0.1155)}{(0.02512)}=3005 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
\end{aligned}
$$

## $h_{s}=3005 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

b. equilateral triangular pitch

Following same equations as in part a. except we use Equation 2.38

$$
\begin{aligned}
& D_{H, s}=\frac{1.103 p^{2}-D_{o}^{2}}{D_{o}}=\frac{(1.103)(0.03175)^{2}-(0.0254)^{2}}{(0.0254)}=0.01838 \mathrm{~m} \\
& A_{s}=\frac{D_{s} L_{b}\left(p-D_{o}\right)}{p}=\frac{(0.381)(0.1524)(0.03175-0.0254)}{(0.03175)}=0.01161 \mathrm{~m}^{2}
\end{aligned}
$$

Shell-side superficial mass velocity, $G_{s}=\frac{\dot{m}_{s}}{A_{s}}=\frac{20}{0.01161}=1722.2 \mathrm{~kg} / \mathrm{m}^{2} \mathrm{~s}$
Shell-side velocity, $u_{s}=\frac{\dot{m}_{s}}{\rho A_{s}}=\frac{G_{s}}{\rho}=\frac{(1722.2)}{(553.2)}=3.1132 \mathrm{~m} / \mathrm{s}$
Shell-side Reynolds number,
$\operatorname{Re}_{s}=\frac{G_{s} D_{H, s}}{\mu}=\frac{u_{s} \rho D_{H, s}}{\mu}=\frac{(3.1132)(553.2)(0.01838)}{\left(91.58 \times 10^{-6}\right)}=345,560$
The average heat transfer coefficient for the shell side of the exchanger is given by:

$$
\mathrm{Nu}=\frac{h_{s} D_{H, s}}{k_{f}}=j_{h} \operatorname{RePr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14}
$$

where for $100<\operatorname{Re}<1 \times 10^{6}$
$j_{h}=1.2492(B C)^{-0.329} \mathrm{Re}^{-0.4696}=(1.2492)(15)^{-0.329}(345,750)^{-0.4696}=0.001285$

$\mathrm{Nu}=553.8 \Rightarrow h_{s}=\mathrm{Nu} \frac{k_{f}}{D_{H, s}}=(553.8) \frac{(0.1155)}{(0.01838)}=3481 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
$\underline{h}_{s}=3482 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
Triangular pitch gives $\sim 16 \%$ increase in $h_{s}$
26. A double pipe heat exchanger is comprised of a length of 1 in sch 40 pipe inside an equal length of 3 in sch 40 pipe. Water flows at a velocity of 1.393 $\mathrm{m} / \mathrm{s}$ in the annular region between the pipes and enters the heat exchanger at $30^{\circ} \mathrm{C}$. Oil flows in the inner pipe at an average velocity of $1 \mathrm{~m} / \mathrm{s}$ and enters at $100^{\circ} \mathrm{C}$. The water and oil flow counter currently.

## Properties of Water and Oil

| Water @ $30^{\circ} \mathrm{C}$ | $\underline{\text { Oil @ } 100^{\circ} \mathrm{C}}$ | $\underline{\text { Oil @ } 50^{\circ} \mathrm{C}}$ |
| :--- | :--- | :--- |
| $\rho=998 \mathrm{~kg} / \mathrm{m}^{3}$ | $\rho=690 \mathrm{~kg} / \mathrm{m}^{3}$ | $\rho=727 \mathrm{~kg} / \mathrm{m}^{3}$ |
| $c_{p}=4216 \mathrm{~J} / \mathrm{kgK}$ | $c_{p}=2421 \mathrm{~J} / \mathrm{kgK}$ | $c_{p}=2201 \mathrm{~J} / \mathrm{kgK}$ |
| $k=0.60 \mathrm{~W} / \mathrm{mK}$ | $k=0.1179 \mathrm{~W} / \mathrm{mK}$ | $k=0.1295 \mathrm{~W} / \mathrm{mK}$ |
| $\mu=700 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ | $\mu=511 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ | $\mu=945 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ |

Determine how long the pipe lengths must be in order for the oil to leave the exchanger at $50^{\circ} \mathrm{C}$ ? You may assume that both fluids are clean and that there is no fouling resistance.

Approach - using Equations 2.27 and 2.28 - this makes the problem noniterative

## Inside HT Coefficient - oil

Average properties are $\rho=708.5 \mathrm{~kg} / \mathrm{m}^{3}, c_{p}=2311 \mathrm{~J} / \mathrm{kgK}, k=0.1237 \mathrm{~W} / \mathrm{mK}$, $\mu=72.8 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}, u=1 \mathrm{~m} / \mathrm{s}$

For a $1^{\prime \prime}$ sch 40 pipe, $D_{i}=1.049$ inch $=0.02665 \mathrm{~m}, D_{0}=1.315$ inch $=0.03340 \mathrm{~m}$
For a $3^{\prime \prime}$ sch 40 pipe $D_{i}=3.068$ inch $=0.07793 \mathrm{~m}$

$$
\begin{aligned}
& A_{i}=\frac{\pi D_{i}^{2}}{4}=\frac{\pi}{4}(0.02665)^{2}=5.576 \times 10^{-4} \mathrm{~m}^{2} \\
& \dot{m}_{\text {oil }}=A_{i} \rho_{\text {oil }} u_{i}=\left(5.576 \times 10^{-4}\right)(708.5)(1)=0.3950 \mathrm{~kg} / \mathrm{s}
\end{aligned}
$$

$\operatorname{Re}_{i}=\frac{D_{i} u_{i} \rho_{\text {oil }}}{\mu_{\text {oil }}}=\frac{(0.02665)(1)(708.5)}{\left(728 \times 10^{-6}\right)}=25,930$
$\operatorname{Pr}_{\text {oil }}=\frac{c_{p, o i l} \mu_{\text {oil }}}{k_{\text {oil }}}=\frac{(2311)\left(728 \times 10^{-6}\right)}{(0.1237)}=13.60$
$\mathrm{Nu}_{i}=\frac{h_{i} D_{i}}{k_{\text {oil }}}=0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{0.3}=170.9$
$h_{i}=\mathrm{Nu}_{i} \frac{k_{\text {oil }}}{D_{i}}=170.9 \frac{(0.1237)}{(0.02665)}=794 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## Outside HT Coefficient - water

$A_{o}=\frac{\pi}{4}\left(D_{i-o}^{2}-D_{o-i}^{2}\right)=\frac{\pi}{4}\left(0.07793^{2}-0.03340\right)^{2}=3.8932 \times 10^{-3} \mathrm{~m}^{2}$
$\dot{m}_{\text {water }}=A_{o} \rho_{\text {water }} u_{o}=\left(3.8932 \times 10^{-3}\right)(998)(1.393)=5.412 \mathrm{~kg} / \mathrm{s}$

## Energy Balance on System

$$
\begin{aligned}
& \dot{m}_{\text {water }} c_{p, \text { water }}\left(T_{w, \text { out }}-30\right)=\dot{m}_{\text {oil }} c_{p, \text { oil }}(100-50) \\
& \Rightarrow T_{w, \text { out }}=30+\frac{\dot{m}_{\text {oil }} c_{p, \text { oil }}}{\dot{m}_{\text {water }} c_{p, \text { water }}}(50)=30+\frac{(0.3950)(2311)}{(5.412)(4216)}(50)=32.0^{\circ} \mathrm{C}
\end{aligned}
$$

So the water properties given at $30^{\circ} \mathrm{C}$ will be close enough For an annulus - Equation 2.28
$D_{H}=4 \frac{\text { flow area }}{\text { wetted perimeter }}=4 \frac{\pi\left(D_{i-o}^{2}-D_{o-i}^{2}\right)}{4} \frac{1}{\pi\left(D_{i-o}+D_{o-i}\right)}$
$D_{H}=D_{i-o}-D_{o-i}=(0.07793)-(0.03340)=0.04453 \mathrm{~m}$
$\operatorname{Re}_{o}=\frac{D_{H} u_{o} \rho_{\text {water }}}{\mu_{\text {water }}}=\frac{(0.04453)(1.393)(998)}{\left(700 \times 10^{-6}\right)}=88,430$
$\operatorname{Pr}_{\text {water }}=\frac{c_{p, \text { water }} \mu_{\text {water }}}{k_{\text {water }}}=\frac{(4216)\left(700 \times 10^{-6}\right)}{(0.60)}=4.92$
$\mathrm{Nu}_{o}=\frac{h_{o} D_{H}}{k_{\text {water }}}=0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{0.4}\left(\frac{D_{i-o}}{D_{o-i}}\right)^{0.45}=(0.023)(88,430)^{0.8}(4.92)^{0.4}\left(\frac{0.07793}{0.03340}\right)^{0.45}=577.2$
$h_{o}=\mathrm{Nu}_{o} \frac{k_{o i l}}{D_{H}}=577.2 \frac{(0.60)}{(0.04453)}=7778 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## From Equation 2.23 eliminating terms that are zero gives

$$
\begin{aligned}
& U_{o}=\left[\frac{1}{h_{o}}+\frac{D_{o}}{D_{i}} \frac{1}{h_{i}}\right]^{-1}=\left[\frac{1}{7778}+\frac{(0.07793)}{(0.03340)} \frac{1}{(794)}\right]^{-1}=325.9 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K} \\
& 100^{\circ} \mathrm{C} \overbrace{}^{50^{\circ} \mathrm{C}} 30^{\circ} \mathrm{C} \\
& 32^{\circ} \mathrm{C} T_{l m}=\frac{(100-32)-(50-30)}{\ln \frac{(100-32)}{(50-30)}}=39.22^{\circ} \mathrm{C} \\
& Q=\dot{m} c_{p} \Delta T=(0.3950)(2311)(100-50)=45.64 \times 10^{3} \mathrm{~W} \\
& Q=U_{o} A_{o} \Delta T_{l m} \Rightarrow A_{o}=\frac{Q}{U_{o} \Delta T_{l m}}=\frac{45.64 \times 10^{3}}{(325.9)(39.22)}=3.57 \mathrm{~m}^{2} \\
& A_{o}=\pi L D_{o-i} \Rightarrow L=\frac{(3.57)}{\pi(0.03340)}=34.0 \mathrm{~m} \\
& \underline{L}=34 \mathrm{~m}
\end{aligned}
$$

27. A 3 m long 1.25-in BWG 14 copper tube is used to condense ethanol at 3 bar pressure. Cooling water at $30^{\circ} \mathrm{C}$ flows through the inside of the tube at a high rate such that the wall temperature may be assumed to be at $30^{\circ} \mathrm{C}$ and the inside coefficient $h_{\gg} h_{0}$.
$P_{\text {sat, ethanol }}=3$ bar, $T_{\text {sat }}=108.7^{\circ} \mathrm{C}$
Determine how much vapor will condense ( $\mathrm{kg} / \mathrm{h}$ ) for clean (no fouling resistance) if,
a. The tube is oriented vertically assume $\operatorname{Re}_{8}<1800$ and use Equation 2.64
$D_{i}=1.084$ inch $=0.02753 \mathrm{~m}, D_{0}=1.25$ inch $=0.03175 \mathrm{~m}, h_{i}=2000$ $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$. We need $T_{w}$ to be able to apply Eqn 264 or 2.66 - so follow the algorithm shown in Example 2.14:
i) guess $h_{0}=$ skip this step since from the problem statement, $T_{\text {wall }}$ $=30^{\circ} \mathrm{C}$
ii) calculate film temperature and film properties

$$
T_{f}=T_{\text {sat }}-0.75\left(T_{\text {sat }}-T_{\text {wall }}\right)=108.7-0.75(108.7-30)=49.7^{\circ} \mathrm{C}
$$

Properties of ethanol at $49.7^{\circ} \mathrm{C}: k_{l}=0.1616 \mathrm{~W} / \mathrm{mK}, c_{p, l}=2667 \mathrm{~J} / \mathrm{kgK}, \mu_{l}=$ $6.928 \times 10^{-4}, \rho_{\rho}=763.2 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{\mathrm{v}}=4.585 \mathrm{~kg} / \mathrm{m}^{3}, \lambda=782.5 \mathrm{~kJ} / \mathrm{kg}$
$\lambda^{\prime}=\lambda+0.68 c_{p, l}\left(T_{\text {sat }}-T_{w}\right)=782.5+(0.68)(2.667)(108.7-30)=925.2 \mathrm{~kJ} / \mathrm{kg}$
$\mathrm{Nu}=\frac{\overline{\bar{h}}_{c} L}{k_{l}}=1.13\left[\frac{\rho_{l} g\left(\rho_{l}-\rho_{v}\right) \lambda^{\prime} L^{3}}{\mu_{l} k_{l}\left(T_{\text {sat }}-T_{w}\right)}\right]^{1 / 4}$
$\mathrm{Nu}=1.13\left[\frac{(763.2)(9.81)(763.2-4.585)\left(925.2 \times 10^{3}\right)(3)^{3}}{\left(6.928 \times 10^{-4}\right)(0.1616)(108.7-30)}\right]^{1 / 4}=12,729$
$\bar{h}_{c}=\operatorname{Nu} \frac{k_{l}}{L}=(12,729) \frac{(0.1616)}{(3)}=685.7 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
$A_{0}=\pi D_{0} L=\pi(0.03175)(3)=0.2992 \mathrm{~m}^{2}$
$Q=U_{o} A_{o}\left(T_{\text {sat }}-T_{\text {wall }}\right)=(685.7)(0.2992)(108.7-30)=16.15 \mathrm{~kW}$
$\dot{m}=\frac{Q}{\lambda^{\prime}}=\frac{16.15}{925.2}(3600)=62.83 \mathrm{~kg} / \mathrm{h}$
Check for Res
$\operatorname{Re}_{\delta}=\frac{4 \dot{m}}{\mu_{l} W}=\frac{4 \dot{m}}{\mu_{l}\left(\pi D_{o}\right)}=\frac{4(62.83 / 3600)}{\left(6.928 \times 10^{-4}\right)(\pi)(0.03175)}=1010<1800$
$\underline{m}=62.83 \mathrm{~kg} / \mathrm{h}$
b. The tube is oriented horizontally
$N u=\frac{\bar{h}_{c} D_{o}}{k_{l}}=0.728\left[\frac{\rho_{l} g\left(\rho_{l}-\rho_{v}\right) \lambda^{\prime} D_{o}^{3}}{\mu_{l} k_{l}\left(T_{\text {sat }}-T_{w}\right)}\right]^{1 / 4}$
$N u=0.728\left[\frac{(763.2)(9.81)(763.2-4.585)\left(925.2 \times 10^{3}\right)(0.03175)^{3}}{\left(6.928 \times 10^{-4}\right)(0.1616)(108.7-30)}\right]^{1 / 4}=289.9$
$\bar{h}_{c}=(289.9)(0.1616) /(0.03175)=1475.7 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
$A_{0}=\pi D_{0} L=\pi(0.03175)(3)=0.2992 \mathrm{~m}^{2}$
$Q=U_{o} A_{o}\left(T_{\text {sat }}-T_{\text {wall }}\right)=(1475.7)(0.2992)(108.7-30)=34.75 \mathrm{~kW}$
$\dot{m}=\frac{Q}{\lambda^{\prime}}=\frac{34.75}{925.2}(3600)=135.2 \mathrm{~kg} / \mathrm{h}$
Check for $\mathrm{Re}_{\delta}$
$\operatorname{Re}_{\delta}=\frac{4 \dot{m}}{\mu_{l} W}=\frac{4 \dot{m}}{\mu_{l}(2 L)}=\frac{4(135.2 / 3600)}{\left(6.928 \times 10^{-4}\right)(2)(0.03175)}=36.1<1800$
$\underline{m}=135.2 \mathrm{~kg} / \mathrm{h}$
28. Air ( 1 atm and $30^{\circ} \mathrm{C}$ ) flows crosswise over a bare copper tube (1-in BWG 16). The approach velocity of the air is $20 \mathrm{~m} / \mathrm{s}$. Water enters the tube at $140^{\circ} \mathrm{C}$ and leaves the tube at an average temperature of $80^{\circ} \mathrm{C}$. The average velocity of the water in the tubes is $1 \mathrm{~m} / \mathrm{s}$.
Determine the length of tube required to cool the water to the desired $80^{\circ} \mathrm{C}$.

Properties of water and air

| Water @ $140^{\circ} \mathrm{C}$ | Water @ $80^{\circ} \mathrm{C}$ | Air @ $30^{\circ} \mathrm{C}$ | Air @ $70^{\circ} \mathrm{C}$ |
| :--- | :--- | :--- | :--- |
| $\rho=925.6 \mathrm{~kg} / \mathrm{m}^{3}$ | $\rho=971.4 \mathrm{~kg} / \mathrm{m}^{3}$ | $\rho=1.1491 \mathrm{~kg} / \mathrm{m}^{3}$ | $\rho=1.0148 \mathrm{~kg} / \mathrm{m}^{3}$ |
| $c_{\rho}=4305 \mathrm{~J} / \mathrm{kgK}$ | $c_{\rho}=4205 \mathrm{~J} / \mathrm{kgK}$ | $c_{\rho}=1003 \mathrm{~J} / \mathrm{kgK}$ | $c_{\rho}=1005 \mathrm{~J} / \mathrm{kgK}$ |
| $k=0.6855 \mathrm{~W} / \mathrm{mK}$ | $k=0.6641 \mathrm{~W} / \mathrm{mK}$ | $k=0.0263 \mathrm{~W} / \mathrm{mK}$ | $k=0.0291 \mathrm{~W} / \mathrm{mK}$ |
| $\mu=193.2 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ | $\mu=357.8 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ | $\mu=18.7 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ | $\mu=20.5 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$ |

Using average properties of water: $\rho=948.5 \mathrm{~kg} / \mathrm{m}^{3} c_{p}=4255 \mathrm{~J} / \mathrm{kgK} k=0.6748$ $\mathrm{W} / \mathrm{mK}, \mu=276 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}$

Assume that the heat transfer coefficient for the air is very low compared to the inside coefficient and hence the wall temperature of the tube will be the temperature of the water (this will be checked later). The average wall temperature is $(140+80) / 2=110^{\circ} \mathrm{C}$. The film temperature $=\left(T_{\text {air }}+T_{\text {wall }}\right) / 2=$ $70^{\circ} \mathrm{C}$ and properties are given above. For air at the average wall temperature $\left(110^{\circ} \mathrm{C}\right)$, we have, $\rho=0.9088 \mathrm{~kg} / \mathrm{m}^{3}, c_{\rho}=1008 \mathrm{~J} / \mathrm{kgK}, k=0.0319 \mathrm{~W} / \mathrm{mK}, \mu=22.3 \times 10^{-6}$ $\mathrm{kg} / \mathrm{ms}$

For a 1-in BWG 16 tube $D_{i}=0.870$ inch $=0.02210 \mathrm{~m}, D_{0}=0.0254 \mathrm{~m}, k_{c u}=380$ W/mK,

## Outside HT Coefficient

For flow across a single tube, we use the equation due to Zhukaukas, Equation 2.36

$$
\mathrm{Nu}_{f}=\frac{h D_{o}}{k_{f}}=C \operatorname{Re}_{f}^{m} \operatorname{Pr}_{f}^{0.365}\left(\frac{\mathrm{Pr}_{\text {Bulk }}}{\mathrm{Pr}_{w}}\right)^{1 / 4}
$$

Where
$\operatorname{Pr}_{\text {Bulk }}=\left(\frac{c_{p} \mu}{k}\right)_{\text {air,30 }}=\frac{(1003)\left(18.7 \times 10^{-6}\right)}{(0.0263)}=0.713$
$\operatorname{Pr}_{\text {fllm }}=\left(\frac{c_{p} \mu}{k}\right)_{\text {air }, 70^{\circ} \mathrm{C}}=\frac{(1005)\left(20.5 \times 10^{-6}\right)}{(0.0291)}=0.708$
$\operatorname{Pr}_{\text {Bulk }}=\left(\frac{c_{p} \mu}{k}\right)_{\text {air } 110^{\circ} \mathrm{C}}=\frac{(1008)\left(22.3 \times 10^{-6}\right)}{(0.0319)}=0.705$
Note that the PRs at all conditions are very similar so there should be little error in taking an average wall temperature
$\operatorname{Re}=\frac{D_{o} u_{\text {bulk }} \rho_{f}}{\mu_{f}}=\frac{(0.0254)(20)(1.0148)}{\left(20.5 \times 10^{-6}\right)}=25,150$
from Table $2.5-C=0.26$ and $m=0.6$
$\mathrm{Nu}_{f}=\frac{h D_{o}}{k_{f}}=C \operatorname{Re}_{f}^{m} \operatorname{Pr}_{f}^{0.365}\left(\frac{\operatorname{Pr}_{\text {Bulk }}}{\operatorname{Pr}_{w}}\right)^{1 / 4}=0.26(25150)^{0.6}(0.708)^{0.365}\left(\frac{0.713}{0.705}\right)^{1 / 4}=100.4$
$h_{o}=(100.4) \frac{(0.0291)}{(0.0254)}=115.1 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## Inside HT Coefficient

Use Dittus-Boelter, Equation 2.27, since L is unknown and wall temp is essentially the same as the bulk temp of the water.
$\mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{n} \quad(n=0.4$ for heating fluid $)$
$\operatorname{Re}=\frac{\rho_{\text {water }} u D_{i}}{\mu_{\text {water }}}=\frac{(948.5)(1)(0.0221)}{\left(276 \times 10^{-6}\right)}=75,950$
$\operatorname{Pr}=\left(\frac{c_{p} \mu}{k}\right)_{\text {water }, 10^{\circ} \mathrm{C}}=\frac{(4255)\left(276 \times 10^{-6}\right)}{(0.6748)}=1.74$
$\mathrm{Nu}=\frac{h_{i} D_{i}}{k}=(0.023)(75,950)^{0.8}(1.74)^{0.4}=230.5$
$h_{i}=(230.5) \frac{(0.6748)}{(0.0221)}=7038 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## Overall HT Coefficient

$$
\begin{aligned}
U_{o} & =\left[\frac{1}{h_{o}}+\not \mathscr{K}_{f o}+\frac{D_{o} \ln \left(D_{o} / D_{i}\right)}{2 k_{w}}+\frac{D_{o}}{D_{i}} \not \mathscr{K}_{f_{i}}+\frac{D_{o}}{D_{i}} \frac{1}{h_{i}}\right]^{-1} \\
U_{o} & =\left[\frac{1}{(115.1)}+\frac{(0.0254) \ln (0.0254 / 0.0221)}{2(380)}+\frac{(0.0254)}{(0.0221)} \frac{1}{(7038)}\right]^{-1}=112.9 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
\end{aligned}
$$

Assumption about wall temperature is correct all resistance is on the outside of the tube.

## Heat Balance

$$
\begin{aligned}
& Q=\dot{m} c_{p} \Delta T=\frac{\pi D_{i}^{2}}{4} u_{i} \rho_{\text {water }} c_{p, \text { water }}\left(T_{\text {in }}-T_{\text {out }}\right) \\
& Q=\frac{\pi(0.0221)^{2}}{4}(1.0)(948.5)(4255)(140-80)=92,890 \mathrm{~W} \\
& Q=U_{o} A_{o} \Delta T_{l m} \Rightarrow A_{o}=\frac{Q}{U_{o} \Delta T_{l m}}=\frac{(92,890)}{(112.9) \frac{[(140-30)-(80-30)]}{\ln \frac{(140-30)}{(80-30)}}=10.81 \mathrm{~m}^{2}} \\
& L=\frac{A_{o}}{\pi D_{o}}=\frac{(10.81)}{\pi(0.0254)}=135.5 \mathrm{~m}
\end{aligned}
$$

## Length of tube $=135.5 \mathrm{~m}$

29. Oil flows inside a thin walled copper tube of diameter, $D_{i}=30 \mathrm{~mm}$. Steam condenses on the outside of the tube and the tube wall temperature may be assumed to be constant at the temperature of the steam $\left(150^{\circ} \mathrm{C}\right)$. The oil enters the tube at $30^{\circ} \mathrm{C}$ and a flow rate of 1.6 $\mathrm{kg} / \mathrm{s}$. The properties of the oil are as follows:

|  | $\frac{30^{\circ} \mathrm{C}}{}$ | $\frac{50^{\circ} \mathrm{C}}{150^{\circ} \mathrm{C}}$ | $\underline{882}$ | 864 |
| :--- | :--- | :--- | :--- | :--- |
| Density, $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ | 886 | 0.2 | 0.2 | 0.18 |
| Thermal conductivity, $k(\mathrm{~W} / \mathrm{mK})$ | 0.2 | 2000 | 1950 |  |
| Specific heat capacity, $c_{p}(\mathrm{~J} / \mathrm{kgK})$ | 2000 | $5 \times 10^{-3}$ | $4 \times 10^{-3}$ | $4 \times 10^{-4}$ |
| Viscosity, $\mu(\mathrm{kg} / \mathrm{ms})$ |  |  |  |  |

a. Calculate the inside heat transfer coefficient $h_{i}$ using the appropriate correlation. You should assume that the bulk oil temperature changes from 30 to $50^{\circ} \mathrm{C}$ along the tube.
Use average bulk temperature of oil of $40^{\circ} \mathrm{C}$ with the following properties:
$\rho=884 \mathrm{~kg} / \mathrm{m}^{3}, \mathrm{k}=0.20 \mathrm{~W} / \mathrm{mK}, c_{p}=2000 \mathrm{~J} / \mathrm{kgK}, \mu=4.5 \times 10^{-3} \mathrm{~kg} / \mathrm{ms}$

Since there is a significant change in viscosity between the bulk and the wall, use the Sieder-Tate expression, Equation 2.26 with L>>Di (check this at end of calculation)

## Inside HT Coefficient

$$
\begin{aligned}
& \mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023\left[1+\left(\frac{D_{i}}{L}\right)^{0.7}\right] \operatorname{Re}^{0.8} \operatorname{Pr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14} \\
& u_{i n}=\frac{4 \dot{m}}{\pi D_{i}^{2} \rho}=\frac{(4)(1.6)}{\pi(0.030)^{2}(886)}=2.5548 \mathrm{~m} / \mathrm{s} \\
& \operatorname{Re}_{i n}=\frac{\rho u D_{i}}{\mu}=\frac{(884)(2.5548)(0.030)}{\left(5 \times 10^{-3}\right)}=13,580 \\
& \operatorname{Pr}_{\text {bulk }}=\frac{c_{p} \mu}{k}=\frac{(2000)\left(4.5 \times 10^{-3}\right)}{(0.20)}=45
\end{aligned}
$$

$\mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023\left[1+\left(\frac{D_{i}}{L}\right)^{0.7}\right] \operatorname{Re}^{0.8} \operatorname{Pr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14}$
$\mathrm{Nu}=0.023(1)(13580)^{0.8}(45)^{1 / 3}\left(\frac{4.5 \times 10^{-3}}{4.0 \times 10^{-4}}\right)^{0.14}=232.4$
$h_{i}=\mathrm{Nu} \frac{k}{D_{i}}=(232.4) \frac{(0.20)}{(0.030)}=1549.6 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## $\underline{h_{i}}=1550 \mathrm{~W} / \mathrm{m} 2 \mathrm{~K}$

b. Using the result from part a., calculate the length of tube required to heat the oil from 30 to $50^{\circ} \mathrm{C}$.

## Energy Balance

$$
\begin{aligned}
& Q=m_{\text {oil }} c_{p, \text { oil }} \Delta T_{\text {oil }}=(1.6)(2000)(50-30)=64,000 \mathrm{~W} \\
& \Delta \mathrm{~T}_{l m}=\frac{(150-30)-(150-50)}{\ln \frac{150-30}{150-50}}=109.7^{\circ} \mathrm{C} \\
& A_{i}=\frac{Q}{U_{i} \Delta \mathrm{~T}_{l m}}=\frac{(64,000)}{(1549.6)(109.7)}=0.3765 \mathrm{~m} \\
& A_{i}=\pi D_{i} L \Rightarrow L=\frac{A_{i}}{\pi D_{i}}=\frac{(0.3765)}{\pi(0.030)}=3.99 \mathrm{~m}
\end{aligned}
$$

Check for $D_{i} / L$ correction term in Equation 2.26

$$
\begin{aligned}
& {\left[1+\left(\frac{D_{i}}{L}\right)^{0.7}\right]=\left[1+\left(\frac{0.030}{3.99}\right)^{0.7}\right]=1.0326} \\
& \Rightarrow h_{i, \text { new }}=(1.0326)(1549.6)=1600.1
\end{aligned}
$$

## Iterating gives,

$$
\begin{aligned}
& A_{i}=\frac{Q}{U_{i} \Delta \mathrm{~T}_{l m}}=\frac{(64,000)}{(1600.1)(109.7)}=0.3646 \mathrm{~m} \\
& A_{i}=\pi D_{i} L \Rightarrow L=\frac{A_{i}}{\pi D_{i}}=\frac{(0.3646)}{\pi(0.030)}=3.87 \mathrm{~m}
\end{aligned}
$$

Further iteration does not change the answer
$L=3.87 \mathrm{~m}$
30. Follow the approach used in Example 2.12, solve the following problem. An organic liquid (acetic acid at 1 bar) is to be vaporized inside a set of vertical 3/4-in BWG 16 tubes using condensing steam on the outside of the tubes to provide the energy for vaporization. The major resistance to heat transfer is expected to be on the inside of the tubes and the wall temperature, as a first approximation, may be assumed to be at the temperature of the condensing steam, which for this case is $125^{\circ} \mathrm{C}$. It may be assumed that the value of the vapor quality, $x$, varies from 0.05 to 0.95 in the tube. Determine the length of the tubes required to vaporize the acetic acid.

The physical parameters for acetic acid are:
$D_{i}=0.62$ inch $=0.015748 \mathrm{~m}$
$\rho_{v}=1.893 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{l}=939.7 \mathrm{~kg} / \mathrm{m}^{3}, \mu_{v}=11.32 \times 10^{-6} \mathrm{~kg} / \mathrm{ms}, \mu_{l}=390.1 \times 10^{-6}$
$\mathrm{kg} / \mathrm{ms}, T_{\text {sat }}=117.6^{\circ} \mathrm{C}, P_{c}=57.9 \mathrm{bar}, M=60, \mathrm{k}_{\mathrm{l}}=0.1423 \mathrm{~W} / \mathrm{mK}, \lambda=405.0$
$\mathrm{kJ} / \mathrm{kg}, c_{p, I}=2.434 \mathrm{~kJ} / \mathrm{kgK}, \quad c_{p, v}=1.319 \mathrm{~kJ} / \mathrm{kgK} \dot{m}=0.04 \mathrm{~kg} / \mathrm{s} / \mathrm{tube}, \mathrm{T}_{\mathrm{w}}=125^{\circ} \mathrm{C}$
$\operatorname{Re}_{l}=\frac{\rho_{l} v D_{i}}{\mu_{l}}=\frac{4 \dot{m}}{\pi \mu_{l} D_{i}}=\frac{(4)(0.04)}{\pi\left(390.1 \times 10^{-6}\right)(0.015748)}=8290$
$\operatorname{Pr}=\frac{c_{p l \mu_{l}}}{k_{l}}=\frac{(2434)\left(390.1 \times 10^{-6}\right)}{(0.1423)}=6.673$

From Equation 2.49, the pool boiling coefficient is given by
$h_{c b}=f h_{l}+s h_{p b}$
Where from Equation 2.50 and 2.51 we have

$$
\begin{aligned}
& f=1+24,000 B_{o}^{1.16}+\frac{1.37}{X_{t p}^{0.86}} \\
& X_{t p}=\left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\rho_{v}}{\rho_{l}}\right)^{0.5}\left(\frac{\mu_{l}}{\mu_{v}}\right)^{0.1}
\end{aligned}
$$

From Equation 2.53,

$$
s=\frac{1}{1+1.15 \times 10^{-6} f^{2} \mathrm{Re}_{l}^{1.17}}
$$

and from Equation 2.55,
$h_{p b}^{0.33}=55 P_{r}^{0.12}\left(-\log _{10} P_{r}\right)^{-0.55}(M)^{-0.5}\left(T_{w}-T_{s a t}\right)^{0.67}$
$h_{p b}^{0.33}=(55)\left[\frac{1}{57.9}\right]^{0.12}\left(-\log _{10}\left[\frac{1}{57.9}\right]\right)^{-0.55}(60)^{-0.5}(125-117.6)^{0.67}=12.21$
$h_{p b}=(12.21)^{1 / 0.33}=1964 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
The convective heat transfer coefficient, $h_{l}$, is given by the SiederTate expression, Equation 2.26, assuming that $L \gg D$ and $\mu_{w} \cong \mu_{1}$ then,
$\mathrm{Nu}=\frac{h_{i} D_{i}}{k}=0.023\left[1+\left(\frac{D_{i}}{L}\right)^{0.7}\right] \operatorname{Re}^{0.8} \operatorname{Pr}^{1 / 3}=(0.023)(8290)^{0.8}(6.673)^{1 / 3}=59.06$ $h_{i}=(59.06)(0.1423) /(0.015748)=533.7 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## From Equation 2.45,

$$
X_{t p}=\left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\rho_{v}}{\rho_{l}}\right)^{0.5}\left(\frac{\mu_{l}}{\mu_{v}}\right)^{0.1}=\left(\frac{1-x}{x}\right)^{0.9}\left(\frac{1.893}{939.7}\right)^{0.5}\left(\frac{390.1 \times 10^{-6}}{11.32 \times 10^{-6}}\right)^{0.1}=0.06395\left(\frac{1-x}{x}\right)^{0.9}
$$

The value of $x$ in the above equation varies from 0.01 to 0.99 and hence $X_{t p}$ will also vary over a wide range. In order to take account of the change in $x$, the problem will be discretized with respect to $x$ and solved for each increment of $\Delta x=0.1$. The calculations for $x=0.01$ to 0.1 will be covered and then the results for the other increments will be summarized. In order to calculate $f$ in Equation 2.44, the value of Bo must be found from Equation 2.46; however, the heat flux $(q / A)$ is unknown. Using the discretization scheme shown in the Figure below the value of $B_{0}$ may be found in terms of $\Delta z_{i}$


Therefore, for the first increment
$B_{o}=\frac{Q / A}{\lambda G}=\frac{\dot{m}\left(x_{1}-x_{0}\right) \lambda}{\pi D_{i} \Delta z_{1}} \frac{1}{\lambda} \frac{\pi D_{i}^{2}}{4 \dot{m}}=\frac{\left(x_{1}-x_{0}\right) D_{i}}{4 \Delta z_{1}}=\frac{(0.1-0.01)(0.015748)}{4 \Delta z_{1}}=\frac{3.5433 \times 10^{-4}}{\Delta z_{1}}$
$\bar{x}_{1}=\frac{(0.1+0.01)}{2}=0.055$, and $X_{t p}=0.06395\left(\frac{1-0.055}{0.055}\right)^{0.9}=0.8268$
By guessing a value for $\Delta z_{1}$, the values of $f$ and $s$ from Equations 2.44 and 2.47 can be calculated and used in Equation 2.43 to calculate $h_{c b}$. Now an energy balance on the first increment of tube gives,

$$
\begin{equation*}
h_{c c} \pi D_{i} \Delta z_{1}\left(T_{w}-T_{s a t}\right)=\dot{m} \lambda\left(x_{1}-x_{0}\right) \tag{a}
\end{equation*}
$$

and rearranging

$$
\begin{equation*}
\Delta z_{1}=\frac{\dot{m} \lambda\left(x_{1}-x_{0}\right)}{h_{c b} \pi D_{i}\left(T_{w}-T_{s a t}\right)} \tag{b}
\end{equation*}
$$

The solution for the first increment is found by iterating between Equations 2.43 and (b) until a constant value of $\Delta z_{1}$ is obtained.

For the first iteration, choose a value of $\Delta z_{1}=0.5 \mathrm{~m}$
$B_{o}=\frac{\left(3.5433 \times 10^{-4}\right)}{(0.5)}=7.0866 \times 10^{-4}$
$f=1+24,000\left(B_{0}\right)^{1.16}+\frac{1.37}{\left(X_{t p}\right)^{0.86}}=1+24,000\left(7.0866 \times 10^{-4}\right)^{1.16}+\frac{1.37}{(0.8268)^{0.86}}=7.9434$ and

$$
s=\frac{1}{1+1.15 \times 10^{-6} f^{2} \mathrm{Re}_{l}^{1.17}}=\frac{1}{1+1.15 \times 10^{-6}(7.9434)^{2}(8290)^{1.17}}=0.2639
$$

$h_{c b}=f h_{l}+s h_{p b}=(7.9434)(533.7)+(0.2639)(1964)=4758 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
$h_{c b}=f h_{l}+s h_{p b}$
Substitute into Equation (b) to get

$$
\Delta z_{1}=\frac{\dot{m} \lambda\left(x_{1}-x_{0}\right)}{h_{c b} \pi D_{i}\left(T_{w}-T_{\text {sat }}\right)}=\frac{(0.04)\left(405.0 \times 10^{3}\right)(0.1-0.01)}{(4758) \pi(0.015748)(125-117.6)}=0.837 \mathrm{~m}
$$

Use this value of $\Delta z_{1}$ in Bo and continue to iterate until converged, this gives $\Delta z_{1}=1.141 \mathrm{~m}$. The results for the remaining increments are as follows,

| $x_{i}-x_{i-1}$ | $B_{0, i}$ | $\bar{x}_{i}$ | $x_{t, i}$ | $f_{i}$ | $s_{i}$ | $h_{c b, i}$ <br> $\left(\mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}\right)$ | $q / A$ <br> $\left(\mathrm{~W} / \mathrm{m}^{2}\right)$ | $\Delta z_{i}$ <br> $(\mathrm{~m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0.01-0.1$ | 0.000310 | 0.055 | 0.8268 | 4.660 | 0.5103 | 3489 | 25,820 | 1.141 |
| $0.1-0.2$ | 0.000412 | 0.150 | 0.3047 | 7.648 | 0.2789 | 4630 | 34,259 | 0.956 |
| $0.2-0.3$ | 0.000564 | 0.250 | 0.1719 | 11.315 | 0.1502 | 6334 | 46,870 | 0.699 |
| $0.3-0.4$ | 0.000769 | 0.350 | 0.1116 | 15.887 | 0.0823 | 8640 | 63,938 | 0.512 |
| $0.4-0.5$ | 0.001045 | 0.450 | 0.0766 | 21.849 | 0.0453 | 11,749 | 86,946 | 0.377 |
| $0.5-0.6$ | 0.001433 | 0.550 | 0.0534 | 30.094 | 0.0244 | 16,109 | 119,208 | 0.275 |
| $0.6-0.7$ | 0.002021 | 0.650 | 0.0366 | 42.515 | 0.0124 | 22,714 | 168,087 | 0.195 |
| $0.7-0.8$ | 0.003038 | 0.750 | 0.0238 | 63.965 | 0.0055 | 34,149 | 252,704 | 0.130 |
| $0.8-0.9$ | 0.005324 | 0.850 | 0.0134 | 112.108 | 0.0018 | 59,836 | 442,785 | 0.074 |
| $0.9-0.99$ | 0.015101 | 0.945 | 0.0049 | 318.018 | 0.0002 | 169,727 | $1,395,534$ | 0.023 |
|  |  |  |  |  |  |  |  |  |

The flux in the last cell is very high compared with the maximum heat flux for pool boiling but this will not alter the length of the tube very much, i.e., even if the flux is calculated using the relationship below.
$\left[\frac{Q}{A}\right]_{\max }=0.3673 P_{c}\left(\frac{P}{P_{c}}\right)^{0.35}\left(1-\frac{P}{P_{c}}\right)^{0.9}=0.3673\left(57.9 \times 10^{5}\right)\left(\frac{1}{57.9}\right)^{0.35}\left(1-\frac{1}{57.9}\right)^{0.9}$
$\left[\frac{Q}{A}\right]_{\max }=505,800 \mathrm{~W} / \mathrm{m}^{2}$
$L_{\text {tube }}=4.38 \mathrm{~m}$ or 14.4 ft
31. Repeat Problem 28 to find the length of a tube with 2-in diameter, $1 / 16$-in thick, annular fins spaced a distance of $3 / 16$-in apart ().
$r_{\text {fin }}=1$ inch $=0.0254 \mathrm{~m}, h=h_{0}=115.1 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}, \delta=1 / 16$ inch $=$ $0.0015875 \mathrm{~m}, \mathrm{kcu}=380 \mathrm{~W} / \mathrm{mK}$
From Figure 2.33:

$$
\begin{aligned}
& b=r_{\text {fn }} \sqrt{\frac{2 h}{k \delta}}=(0.0254) \sqrt{\frac{2(115.1)}{(380)(0.0015875)}}=0.496 \\
& a=\frac{r_{\text {ube }}}{r_{\text {fin }}}=0.5
\end{aligned}
$$

From Figure 2.33, $\varepsilon_{\text {fin }} \sim 0.97$
The enhancement in heat transfer for the tube with fins is given by Equation 2.76:
Enhancment in heat transfer $=\frac{A_{\text {base }}+\varepsilon_{\text {fin }} A_{\text {fin }}}{A_{\text {bare }}}=\frac{\pi D_{o}\left(L_{\text {bare }}\right)+\varepsilon_{\text {fin }}\left(2 \pi\left(r_{\text {fin }}^{2}-r_{\text {tube }}^{2}\right)+\pi r_{\text {fin }} L_{\text {fin }}\right)}{\pi D_{o}\left(L_{\text {bare }}+L_{\text {fin }}\right)}$ where
$L_{\text {bare }}=3 / 16$ inch, $L_{\text {fin }}=1 / 16$ inch , $D_{o}=1$ inch, $r_{\text {fin }}=1$ inch, $r_{\text {ube }}=1 / 2$ inch, $\varepsilon_{\text {fin }}=0.97$
Enhancment in heat transfer $=6.8125$
Therefore, the new tube length, $L=135.5 / 6.8125=19.89 \mathrm{~m}$
32. An air heater consists of a shell and tube heat exchanger with 24 longitudinal fins on the outside of the tubes. The tubes are 1.5 in 14 BWG, and the fins are 0.75 mm thick and are 15 mm "long." There are a total of 8 fins spaced uniformly around the circumference of each tube. The tubes are made from carbon steel and their length is 3 m . Air at $0.8 \mathrm{~kg} / \mathrm{s}$ is being heated from $30^{\circ} \mathrm{C}$ to $200^{\circ} \mathrm{C}$ in the shell, and the heat transfer coefficient may be taken as $h_{0}=25 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$ (without fins). Steam is condensing at $254^{\circ} \mathrm{C}$ in the tubes, and at that temperature $\lambda=1700 \mathrm{~kJ} / \mathrm{kg}$. The tube side heat transfer coefficient may be taken as $h_{i}=6000 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$ and no fouling occurs for this service. Because the outside heat transfer coefficient is limiting, the wall temperature will be close to $254^{\circ} \mathrm{C}$ and carbon steel, $k s s=42.3 \mathrm{~W} / \mathrm{mK}$
For 1.5 inch 14 BWG, Do $=1.5$ inch $=0.0381 \mathrm{~m}$ and $\mathrm{Di}_{\mathrm{I}}=1.334$ inch $=0.033884$ m
a. Calculate the overall heat transfer coefficient with and without fins.

For bare (finless) tubes
From Equation, 2.23

$$
\begin{aligned}
& U_{o}=\left[\frac{1}{h_{o}}+\not K_{f o}+\frac{D_{o} \ln \left(D_{o} / D_{i}\right)}{2 k_{w}}+\frac{D_{o}}{D_{i}} \not K_{f i}+\frac{D_{o}}{D_{i}} \frac{1}{h_{i}}\right]^{-1} \\
& U_{o}=\left[\frac{1}{25}+\frac{(0.0381) \ln \left(0.0381 / 0.033884_{i}\right)}{2(42.3)}++\frac{0.0381}{0.033884} \frac{1}{(6000)}\right]^{-1}=24.85 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
\end{aligned}
$$

$$
\underline{U}_{0}=24.85 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

## For finned tubes

Evaluate the fin effectiveness factor.


For each fin, $\delta=0.00075 \mathrm{~m}, L=0.015 \mathrm{~m}, L_{\text {tube }}=3 \mathrm{~m}$, From Equation 2.79 m $L_{c}=L+\delta / 2=0.015+0.00075 / 2=0.01538 \mathrm{~m}$
From Equation 2.69,

$$
\begin{aligned}
& m=\sqrt{\frac{2 h_{o}}{\delta k}}=\sqrt{\frac{2(25)}{(0.00075)(42.3)}}=39.70 \\
& m L_{c}=(39.70)(0.01538)=0.6104
\end{aligned}
$$

From Equation 2.68, we have
$\varepsilon_{\text {fin }}=\frac{\tanh \left(m L_{c}\right)}{m L_{c}}=\frac{\tanh (0.6104)}{(0.6104)}=0.8919$
The enhancement in heat transfer is given by Equation 2.76, so considering all 8 fins, we have

$$
\begin{aligned}
& A_{\text {fin }}=(8)(2) L_{C} L_{\text {tube }}=(16)(0.01538)(3)=0.738 \mathrm{~m}^{2} \\
& A_{\text {base }}=\pi D_{o} L_{\text {tube }}-8 \delta L_{\text {tube }}=\pi(0.0381)(3)-8(0.00075)(3)=0.34108 \mathrm{~m}^{2} \\
& A_{\text {bare }}=\pi D_{o} L_{\text {tube }}=\pi(0.0381)(3)=0.35908 \mathrm{~m}^{2}
\end{aligned}
$$

Enhancement in heat transfer $=\frac{A_{\text {base }}+\varepsilon_{\text {fin }} A_{\text {fin }}}{A_{\text {bare }}}=\frac{(0.34108)+(0.8919)(0.738)}{(0.35908)}=2.78$
$h_{o, \text { enhanced }}=(25)(2.78)=69.57 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
$U_{o}=\left[\frac{1}{h_{o}}+\not R_{f o}+\frac{D_{o} \ln \left(D_{o} / D_{i}\right)}{2 k_{w}}+\frac{D_{o}}{D_{i}} \not K_{f i}+\frac{D_{o}}{D_{i}} \frac{1}{h_{i}}\right]^{-1}$
$U_{o}=\left[\frac{1}{69.57}+\frac{(0.0381) \ln \left(0.0381 / 0.033884_{i}\right)}{2(42.3)}++\frac{0.0381}{0.033884} \frac{1}{(6000)}\right]^{-1}=68.43 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## $\underline{U}_{0} 68.43 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

b. Calculate heat transfer area and the number of tubes needed with and without fins.
For Air at $30^{\circ} \mathrm{C}-c_{p}=1003.4 \mathrm{~J} / \mathrm{kgK}$ and at $200^{\circ} \mathrm{C} c_{p}=1020.4 \mathrm{~J} / \mathrm{kgK}$.
Use an average $c_{p}$ value of $(1003.4+1020.4) / 2=1011.9 \mathrm{~J} / \mathrm{kg}$

$$
Q=\dot{m}_{a i r} \bar{c}_{p, a i r} \Delta T_{\text {air }}=(0.8)(1011.9)(200-30)=137.6 \mathrm{~kW}
$$

$Q=\dot{m}_{\text {steam }} \lambda \Rightarrow \dot{m}_{\text {steam }}=\frac{137.6}{1700}=0.08095 \mathrm{~kg} / \mathrm{s}$
$\Delta T_{l m}=\frac{(254-30)-(254-200)}{\ln \frac{(254-30)}{(254-200)}}=119.5^{\circ} \mathrm{C}$
$F=1$, since steam is condensing on one side

## For bare (finless) tubes

$$
\begin{aligned}
& A_{o}=\frac{Q}{U_{o} \Delta T_{1 m}}=\frac{(137,600)}{(24.85)(119.5)}=46.34 \mathrm{~m}^{2} \\
& A_{o}=\pi D_{o} L_{\text {tube }} n_{\text {tubes }} \Rightarrow n_{\text {tubes }}=\frac{A_{o}}{\pi D_{o} L_{\text {tube }}}=\frac{(46.34)}{\pi(0.0381)(3)}=129
\end{aligned}
$$

## For finned tubes

$A_{o}=\frac{Q}{U_{o} \Delta T_{1 m}}=\frac{(137,600)}{(68.43)(119.5)}=16.83 \mathrm{~m}^{2}$ (this is the bare surface area of tubes)
$A_{o}=\pi D_{o} L_{\text {tube }} n_{\text {tubes }} \Rightarrow n_{\text {tubes }}=\frac{A_{o}}{\pi D_{o} L_{\text {tube }}}=\frac{(46.34)}{\pi(0.0381)(3)}=47$

## Design Algorithm and Worked Examples for Rating of Heat Exchangers

33. Your assignment is to design a replacement condenser for an existing distillation column. Space constraints dictate a vertical-tube condenser with a maximum height (equals tube length) of 3 m . An organic is condensed at a rate of $12,000 \mathrm{~kg} / \mathrm{h}$ at a temperature of $75^{\circ} \mathrm{C}$, and cooling water is used, entering at $30^{\circ} \mathrm{C}$ and exiting at $40^{\circ} \mathrm{C}$. The person you are replacing has done some preliminary calculations suggesting that a 1-4 exchanger (water in tubes) using 1 inch 16 BWG copper tubes on 1.25 inch equilateral triangular pitch with a 37 inch shell diameter would be suitable. However, there are only partial calculations to support this claim, and the person who performed the original design is unavailable. Complete the detailed heat transfer calculations to evaluate the suitability of this heat exchanger design.

## Data for condensing organic:

$\rho_{f}=800 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{v}=5.3 \mathrm{~kg} / \mathrm{m}^{3}, \lambda=800 \mathrm{~kJ} / \mathrm{kg}, k_{f}=0.15 \mathrm{~W} / \mathrm{m} \mathrm{K}, \mu_{f}=400 \times 10^{-6}$ $\mathrm{kg} / \mathrm{ms}, c_{p l}=2600 \mathrm{~J} / \mathrm{kgK}$
heat transfer coefficient for tube side $\approx 6000 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
typical fouling coefficient for plant cooling water $=1200 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
assume no fouling on condensing side

## Properties for water:

$\rho_{f}=994 \mathrm{~kg} / \mathrm{m}^{3}, c_{p}=4187 \mathrm{~J} / \mathrm{kg}^{\circ}{ }^{\circ}, k_{f}=0.6195 \mathrm{~W} / \mathrm{m} \mathrm{K}, \mu_{f}=748 \times 10^{-6} \mathrm{~kg} / \mathrm{m} \mathrm{s}$

## Properties of tubes

$D_{o}=0.0254 \mathrm{~m}, \mathrm{D}_{i}=0.834$ inch $=0.02118 \mathrm{~m}, k_{c u}=382.9 \mathrm{~W} / \mathrm{mK}\left(a+35^{\circ} \mathrm{C}\right)$

## Energy Balance

Shell-side: $Q=\dot{m}_{\text {organic }} \lambda_{\text {organic }}=\left(\frac{12000}{3600}\right)\left(800 \times 10^{3}\right)=2,667 \mathrm{~kW}$
Tube-side:
$Q=2,667 \times 10^{3}=\dot{m}_{\text {water }} c_{p, \text { water }} \Delta T_{\text {water }} \Rightarrow \dot{m}_{\text {water }}=\frac{2,667 \times 10^{3}}{(4187)(40-30)}=63.69 \mathrm{~kg} / \mathrm{s}$

## Estimate the Shell Side Coefficient, $h_{o}$

Using Equation 2.64 - assume $T_{w}=(40+30) / 2=35^{\circ} \mathrm{C}$ All resistance on shell side.
$\lambda^{\prime}=\lambda+0.68 c_{p, l}\left(T_{\text {sat }}-T_{w}\right)=800 \times 10^{3}+0.68(2600)(75-35)=870.7 \times 10^{3} \mathrm{~J} / \mathrm{kg}$
$N u=\frac{\bar{h}_{c} L}{k_{l}}=1.13\left[\frac{\rho_{l} g\left(\rho_{l}-\rho_{v}\right) \lambda^{\prime} L^{3}}{\mu_{l} k_{l}\left(T_{\text {sat }}-T_{w}\right)}\right]^{1 / 4}=1.13\left[\frac{(800)(9.81)(800-5.3)\left(870.7 \times 10^{3}\right)(3)^{3}}{\left(400 \times 10^{-3}\right)(0.15)(75-35)}\right]^{1 / 4}$
$N u=17,765 \Rightarrow \bar{h}_{c}=(17,765) \frac{(0.15)}{(3)}=888.3 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## Estimate overall coefficient using given $h_{i}$

$U_{o}=\left[\frac{1}{h_{o}}+\not \mathscr{K}_{f_{o}}+\frac{D_{o} \ln \left(D_{o} / D_{i}\right)}{2 k_{w}}+\frac{D_{o}}{D_{i}} R_{f_{i}}+\frac{D_{o}}{D_{i}} \frac{1}{h_{i}}\right]^{-1}$
$U_{o}=\left[\frac{1}{888.3}+\frac{(0.0254) \ln (0.0254 / 0.02118)}{(2)(382.9)}+\frac{(0.0254)}{(0.02118)} \frac{1}{1200}+\frac{(0.0254)}{(0.02118)} \frac{1}{6096}\right]^{-1}=429.6 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## Estimate Number of tubes

$\Delta T_{l m}=\frac{(75-30)-(75-40)}{\ln \frac{(75-30)}{(75-40)}}=\frac{10}{\ln \frac{45}{35}}=39.79^{\circ} \mathrm{C}$
$A_{o}=\frac{Q}{U_{o} \Delta T_{l m}}=\frac{\left(2667 \times 10^{3}\right)}{(429.6)(39.79)}=156.0 \mathrm{~m}^{2}$ calculate the total number of tubes and the tubes per pass
$A_{o}=n_{\text {tube }} \pi D_{o} L_{\text {tube }} \Rightarrow n_{\text {tube }}=\frac{A_{o}}{\pi D_{o} L_{\text {tube }}}=\frac{(156)}{\pi(0.0254)(3)}=652$
$n_{\text {pass }}=\frac{n_{\text {tube }}}{4}=163$

## Reevaluate $h_{i}$

$$
u_{i}=\frac{4 \dot{m}_{\text {water }}}{n_{\text {pass }} \pi D_{i}^{2}}=\frac{(4)(63.69)}{(163)(994) \pi(0.02118)^{2}}=1.12 \mathrm{~m} / \mathrm{s}
$$

$$
\begin{aligned}
& \operatorname{Re}_{\text {tube }}=\frac{\rho_{\text {water }} D_{i} u_{i}}{\mu_{\text {water }}}=\frac{(994)(0.02118)(1.12)}{\left(748 \times 10^{-6}\right)}=31,400 \\
& \operatorname{Pr}=\frac{c_{p, \text { water }} \mu_{\text {water }}}{k_{\text {water }}}=\frac{(4187)\left(748 \times 10^{-6}\right)}{(0.6195)}=5.055 \\
& \mathrm{Nu}=\frac{h_{i} D_{i}}{k_{\text {water }}}=0.023 \operatorname{Re}^{0.8} \mathrm{Pr}^{1 / 3}=(0.023)(31,400)^{0.8}(5.055)^{1 / 3}=156.2 \\
& h_{i}=\mathrm{Nu} \frac{k_{\text {water }}}{D_{i}}=(156.2) \frac{(0.6195)}{(0.02118)}=4570
\end{aligned}
$$

## Estimate $U_{0}$ and $T_{w}$ and the recalculate $h_{0}$

$U_{o}=\left[\frac{1}{888.3}+\frac{(0 . .0254) \ln (0.0254 / 0.02118)}{(2)(382.9)}+\frac{(0.0254)}{(0.02118)} \frac{1}{1200}+\frac{(0.0254)}{(0.02118)} \frac{1}{4570}\right]^{-1}=418 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
$U_{o}\left(T_{\text {sat }}-T_{\text {water }}\right)=h_{o}\left(T_{\text {sat }}-T_{w}\right)$
$\therefore T_{W}=T_{\text {sat }}-\frac{U_{o}}{h_{o}}\left(T_{\text {sat }}-T_{\text {water }}\right)=75-\frac{418}{888.3}(75-35)=56.2$
$\lambda^{\prime}=\lambda+0.68 c_{p, l}\left(T_{\text {sat }}-T_{w}\right)=800 \times 10^{3}+0.68(2600)(75-56.2)=870.7 \times 10^{3} \mathrm{~J} / \mathrm{kg}$
The iteration process continues until the following solution is reached:
$h_{i}=4807 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}, h_{0}=1088.5 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}, U_{0}=460.1 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}, A_{0}=145.7 \mathrm{~m} 2$, $n_{\text {tubes }}$ $=612$ with 153 tubes per pass.

From Table 2.6, max number of tubes in a 37 inch diameter shell using 1 inch tubes on a triangular pitch of 1.25 inch is 638 . Therefore, the suggested arrangement should work for this system.
34. A 1-2 shell and tube heat exchanger has the following dimensions:

Tube length $=20 \mathrm{ft}$
Tube diameter = 1-in BWG 14 carbon steel ( $k_{c s}=45 \mathrm{~W} / \mathrm{mK}$ )
Number of tubes in shell $=608$
Shell diameter $=35$-in
Tube arrangement $=$ triangular pitch, center-to-center $=1.25$ - in
Number of baffles $=19$
Baffle spacing $=1 \mathrm{ft}$
Baffle $=$ horizontal baffle with baffle cut $=25 \%$

The fluids in the shell and tube sides of the exchanger have the following properties:

|  | Shell Side | Tube Side |
| :--- | :---: | :---: |
| Inlet temperature $\left({ }^{\circ} \mathrm{C}\right)$ | 120 | 30 |
| Mass flowrate, $\dot{m}(\mathrm{~kg} / \mathrm{s})$ | 120 | 180 |
| Specific heat capacity, $c_{p}(\mathrm{~kJ} / \mathrm{kgK})$ | 2.0 | 4.2 |
| Thermal conductivity, $k(\mathrm{~W} / \mathrm{mK})$ | 0.2 | 0.61 |
| Density, $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ | 850 | 1000 |
| Viscosity, $\mu(\mathrm{kg} / \mathrm{ms})$ | $5.0 \times 10^{-4}$ | $0.72 \times 10^{-3}$ |

Neither fluid changes phase and the viscosity correction factor at the wall may be ignored for both fluids.
$D_{0}=0.0254 \mathrm{~m}, D_{i}=0.834$ inch $=0.021184 \mathrm{~m}, \mathrm{p}=1.25 \mathrm{inch}=0.03175 \mathrm{~m}, D_{s}=$
35 inch $=0.889 \mathrm{~m}, L_{\text {tube }}=20 \mathrm{ft}=6.096 \mathrm{~m}, n_{\text {baffle }}=19, L_{b}=L_{\text {tube }} /\left(n_{\text {baffle }}+1\right)=$ $6.096 /(19+1)=0.3048 \mathrm{~m}, ~ B C=25 \%, n_{\text {tube }}=608, n_{\text {pass }}=608 / 2=304$
a. The inside heat transfer coefficient - using the Seider-Tate relationship, Equation, 2.26)

$$
\begin{aligned}
& u_{i}=\frac{4 \dot{m}_{i}}{n_{\text {pass }} \rho \pi D_{i}^{2}}=\frac{(4)(180)}{(608 / 2)(1000) \pi(0.021184)^{2}}=1.68 \mathrm{~m} / \mathrm{s} \\
& \operatorname{Re}_{i}=\frac{u_{i} D_{i} \rho_{i}}{\mu_{i}}=\frac{(1.68)(0.021184)(1000)}{\left(0.72 \times 10^{-3}\right)}=49,400 \\
& \operatorname{Pr}_{i}=\frac{c_{p, i} \mu_{i}}{k_{i}}=\frac{(4200)\left(0.72 \times 10^{-3}\right)}{(0.61)}=4.96 \\
& \mathrm{Nu}_{i}=\frac{h_{i} D_{i}}{k_{i}}=0.023\left[1+\left(\frac{D_{i}}{L_{\text {tube }}}\right)^{0.7}\right] \operatorname{Re}^{0.8} \operatorname{Pr}^{1 / 3}\left(\frac{\mu_{i}}{\mu_{w}}\right)^{0.14} \\
& \mathrm{Nu}_{i}=0.023\left[1+\left(\frac{(0.021184)}{6.096}\right)^{0.7}\right](49,400)^{0.8}(4.96)^{1 / 3}(1)=227.4 \\
& h_{i}=\mathrm{Nu}_{i} \frac{k_{i}}{D_{i}}=(227.4) \frac{(0.61)}{(0.021184)}=6550 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
\end{aligned}
$$

b. The shell side heat transfer coefficient - use Kern's method $D_{H, S}($ traingular pitch $)=\frac{1.103 p^{2}-D_{o}^{2}}{D_{o}}=\frac{1.103(0.03175)^{2}-(0.0254)^{2}}{(0.0254)}=0.018375 \mathrm{~m}$ $A_{s}=D_{s} L_{b} \frac{\left(p-D_{o}\right)}{p}=(0.889)(0.3048) \frac{(0.03175-0.0254)}{(0.03175)}=0.05419 \mathrm{~m}^{2}$
$G_{s}=\frac{\dot{m}_{s}}{A_{s}}=\frac{(120)}{(0.05419)}=2215 \mathrm{~kg} / \mathrm{m}^{2} \mathrm{~s}$ and $u_{s}=\frac{G_{s}}{\rho_{s}}=\frac{2215}{850}=2.61 \mathrm{~m} / \mathrm{s}$
$\operatorname{Re}_{s}=\frac{G_{s} D_{H, s}}{\mu}=\frac{(2215)(0.018375)}{\left(5 \times 10^{-4}\right)}=81,400$
$j_{h}=1.2492(B C)^{-0.329} \operatorname{Re}_{s}^{-0.4696}=1.2492(25)^{-0.329}(81,400)^{-0.4696}=0.002141$
$\operatorname{Pr}=\left[\frac{c_{p} \mu}{k}\right]_{\text {shell }}=\frac{(2000)\left(5 \times 10^{-4}\right)}{(0.2)}=5.0$
$\mathrm{Nu}=\frac{h_{s} D_{H, s}}{k_{f}}=j_{h} \operatorname{Re}_{s} \operatorname{Pr}^{1 / 3}\left(\frac{\mu}{\mu_{w}}\right)^{0.14}=(0.002141)(81400)(5.0)^{1 / 3}(1)^{0.14}=298$
$h_{s}=\mathrm{Nu} \frac{k_{f}}{D_{H, s}}=(298) \frac{(0.2)}{(0.018375)}=3244 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$

## c. The overall heat transfer coefficient (assuming that fouling may be

 ignored)$$
\begin{aligned}
U_{o} & =\left[\frac{1}{h_{o}}+\not R_{f o}+\frac{D_{o} \ln \left(D_{o} / D_{i}\right)}{2 k_{w}}+\frac{D_{o}}{D_{i}} \not K_{f i}+\frac{D_{o}}{D_{i}} \frac{1}{h_{i}}\right]^{-1} \\
U_{o} & =\left[\frac{1}{3244}+\frac{(0.0254) \ln (0.0254 / 0.021184)}{2(45)}+\frac{(0.0254)}{(0.021184)} \frac{1}{(6550)}\right]^{-1}=1843 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
\end{aligned}
$$

## d. The exit temperatures of both fluids

## Energy Balance gives

$$
\begin{equation*}
Q=m_{i} c_{p, i}\left(T_{i, \text { out }}-30\right)=m_{s} c_{p, s}\left(120-T_{s, \text { out }}\right) \tag{a}
\end{equation*}
$$

## Design Equation



$$
\begin{equation*}
\Delta T_{l m}=\frac{\left(120-T_{i, \text { out }}\right)-\left(T_{s, \text { out }}-30\right)}{\ln \frac{\left(120-T_{i, \text { out }}\right)}{\left(T_{s, \text { out }}-30\right)}} \tag{b}
\end{equation*}
$$

$$
\begin{equation*}
P=\frac{\left(T_{s, \text { out }}-30\right)}{(120-30)} \quad \text { and } R=\frac{\dot{m} c_{p}}{\dot{M} C_{p}}=\frac{(180)(4200)}{(120)(2000)}=3.15 \tag{c}
\end{equation*}
$$

$$
\begin{equation*}
F_{1-2}=\frac{\sqrt{\left(R^{2}+1\right)} \ln \left[\frac{1-P}{1-P R}\right]}{(R-1) \ln \left[\frac{2-P\left(R+1-\sqrt{\left(R^{2}+1\right)}\right)}{2-P\left(R+1+\sqrt{\left(R^{2}+1\right)}\right)}\right]} \tag{d}
\end{equation*}
$$

$A_{o}=\pi D_{o} L_{\text {tube }} n_{\text {tube }}=\pi(0.0254)(6.096)(608)=295.8 \mathrm{~m}^{2}$
$Q=U_{o} A_{o} \Delta T_{l m} F_{12}$
In Equations (a-f) there are three unknowns, $Q, T_{i, \text { out }}$, and $T_{s, ~ o u t . ~}^{\text {. }}$
Substituting relationships $(b-e)$ into $(f)$ gives one equation and there are
two equations given in relationship (a). These all must be solved simultaneously. The results are as follows:

$$
T_{i, \text { out }}=52.15^{\circ} \mathrm{C}, T_{\text {s,out }}=50.23^{\circ} \mathrm{C}, P=0.2461, F_{12}=0.781, Q=16.7 \mathrm{MW}
$$

35. A 1-2 heat exchanger is used to cool oil in the tubes from $91^{\circ} \mathrm{C}$ to $51^{\circ} \mathrm{C}$ using cooling water at $30^{\circ} \mathrm{C}$. In the design case the water exits at $40^{\circ} \mathrm{C}$. The resistances on the water and oil sides are equal. What are the new cooling water outlet temperature and the required cooling water flowrate if the oil throughput must be increased by $25 \%$ but the outlet temperature must be maintained at $51^{\circ} \mathrm{C}$ ?


Using ratios, new case $=2$, design case (base case) $=1$

$$
\begin{align*}
& \frac{Q_{2}}{Q_{1}}=\frac{m_{c w, 2}}{m_{c w, 1}} \frac{c_{p, \text {, }}, 2}{c} \frac{\left(T_{c w, o u t}-T_{c w, \text { in }}\right)_{2}}{\left(T_{c w, o u t}-T_{c w, \text { in }}\right)_{1}}=M_{c w} \frac{\left(T_{c w, o u t, 2}-30\right)}{10}  \tag{a}\\
& \frac{Q_{2}}{Q_{1}}=\frac{m_{\text {oil }, 2}}{m_{\text {oil, }, 2}} \frac{c_{p, o \rho} / l_{p}}{c_{p}} \frac{\left(T_{\text {oil }, 1}\right.}{} \frac{\left(T_{\text {oil, in }}-T T_{\text {out }}\right)_{2}}{\left(T_{\text {oil, in }}-T_{\text {oil,out }}\right)_{1}}=M_{\text {oil }}=1.25  \tag{b}\\
& \frac{Q_{2}}{Q_{1}}=\frac{U_{2} A\left(\frac{\Delta T_{l m, 2}}{U_{1}} \frac{\not A_{1}}{\Delta T_{l m}} \frac{U_{2}}{F_{1}}=\frac{\left(91-T_{c w, \text { out }, 2}\right)-(51-30)}{U_{1}} \frac{\left(91-T_{c w, \text { out }, 2}\right)}{(33.81) \ln \frac{(91-30)}{(51-30}}=U \frac{\left(70-T_{c w, \text { out }, 2}\right)}{(33.81) \ln \frac{\left(91-T_{c w, \text { out }, 2}\right)}{(21)}}\right.}{\text { (21) }} \tag{c}
\end{align*}
$$

Assume physical properties are constant, there are no change in the areas, and assume the LMTD correction factors, $F$ do not change.
Let base case inside and outside HT coefficients = $h$
Therefore, $U=\frac{\left[\frac{1}{h M_{c w}^{0.6}}+\frac{1}{h M_{o i l}^{0.8}}\right]^{-1}}{\left[\frac{1}{h}+\frac{1}{h}\right]^{-1}}=\frac{2}{\left[\frac{1}{M_{c w}^{0.6}}+\frac{1}{1.25^{0.8}}\right]}$ substitute in Eqn (c) to
give:

$$
\begin{equation*}
\frac{Q_{2}}{Q_{1}}=\frac{1}{(33.81)} \frac{\left(70-T_{c w, \text { out }, 2}\right)}{\ln \frac{\left(91-T_{c w, \text { out }, 2}\right)}{(21)}} \frac{2}{\left[\frac{1}{M_{c w}^{0.6}}+\frac{1}{1.25^{0.8}}\right]} \tag{d}
\end{equation*}
$$

Solving Eqns ( $a, b$, and d) we get:

## $M_{c w}=1.48, T_{c w, o u t, 2}=38.5^{\circ} \mathrm{C}$

Check assumption about $F$
Base case $P=(91-51) /(91-30)=0.656, R=(40-30) /(91-51)=0.25 \Rightarrow F_{12}=0.9360$
New case $P=0.656, R=(38.46-30) /(91-51)=0.2182 \Rightarrow F_{12}=0.9460$
Change in $F_{12}$ is only $1.1 \%$ so above answer is ok. If you do include the change in $F_{12}$, you should get the following answer
$M_{c w}=1.44, T_{c w, o u t, 2}=38.7^{\circ} \mathrm{C}$
36. Repeat Problem 35 if the oil side provides $80 \%$ of the total resistance to heat transfer. The approach is the same as in Problem 35, and Equations $a$ and $b$ remain the same. However, the estimation of the change in $U$ changes:

Let base case inside (oil-side) coefficients $=h$ therefore the water side coefficient $=4 h \Rightarrow U=(1 / h+1 / 4 h)^{-1}=5 h / 4$ and $(1 / h) /(1 / U)=4 / 5=80 \%$
Therefore, $U=\frac{\left[\frac{1}{4 h M_{c w}^{0.6}}+\frac{1}{h M_{o i l}^{0.8}}\right]^{-1}}{\left[\frac{1}{4 h}+\frac{1}{h}\right]^{-1}}=\frac{1.25}{\left[\frac{1}{4 M_{c w}^{0.6}}+\frac{1}{1.25^{0.8}}\right]}$ substitute in Eqn (c) to
give:

$$
\begin{equation*}
\frac{Q_{2}}{Q_{1}}=\frac{1}{(33.81)} \frac{\left(70-T_{c w, o u t, 2}\right)}{\ln \frac{\left(91-T_{c w, o u t, 2}\right)}{(21)}} \frac{1.25}{\left[\frac{1}{4 M_{c w}^{0.6}}+\frac{1}{1.25^{0.8}}\right]} \tag{d}
\end{equation*}
$$

Solving Eqns ( $a, b$, and d) we get:
$M_{c w}=1.61, T_{c w, o u t, 2}=37.8^{\circ} \mathrm{C}$
Taking into account the change in $F_{12}$ gives
$M_{c w}=1.49, T_{c w, \text { out }, 2}=38.4^{\circ} \mathrm{C}$
37. Repeat Problem 35 for the case when the oil throughput must be increased by $25 \%$ but the cooling water flow rate remains unchanged from the base case. Determine the new outlet temperatures for both the process and cooling water streams?


$$
\Delta T_{l m, 1}=\frac{(91-40)-(51-30)}{\ln \frac{(91-40)}{(51-30)}}=33.81^{\circ} \mathrm{C}
$$

Using ratios, new case $=2$, design case (base case) $=1$

$$
\begin{align*}
& \frac{Q_{2}}{Q_{1}}=\frac{m}{n \int_{c w, 2}} \frac{c_{p, 1}}{c} \int_{c w, 2} \frac{\left(T_{c w, \text { out }}-T_{c w, \text { in }}\right)_{2}}{\left(T_{c w, o u t}-T_{c w, \text { in }}\right)_{1}}=\frac{\left(T_{c w, o u t, 2}-30\right)}{10}  \tag{a}\\
& \frac{Q_{2}}{Q_{1}}=\frac{m_{\text {oil }, 2}}{m_{\text {oil }, 1}} \frac{c_{p, 0} / 2}{c_{p}} \frac{\left(T_{\text {oil }, \text { in }, 1}-T_{\text {oil,out }}\right)_{2}}{\left(T_{\text {oil }, \text { in }}-T_{\text {oil out }}\right)_{1}}=M_{\text {oil }} \frac{91-T_{\text {oil,out }, 2}}{91-51}=\left(\frac{1.25}{40}\right)\left(91-T_{\text {oil,out }, 2}\right) \tag{b}
\end{align*}
$$

Assume physical properties are constant, there are no change in the areas, and assume the LMTD correction factors, $F$ do not change.
Let base case inside and outside HT coefficients $=h$
Therefore, $U=\frac{\left[\frac{1}{h M_{c w}^{0.6}}+\frac{1}{h M_{o i l}^{0.8}}\right]^{-1}}{\left[\frac{1}{h}+\frac{1}{h}\right]^{-1}}=\frac{2}{\left[\frac{1}{1}+\frac{1}{1.25^{0.8}}\right]}=1.0890$ substitute in Eqn (c)
to give:

$$
\begin{equation*}
\frac{Q_{2}}{Q_{1}}=(1.0890) \frac{\left(121-T_{c w, \text { out }, 2}-T_{\text {oil.out }, 2}\right)}{(33.81) \ln \frac{\left(91-T_{c w, \text { out }, 2}\right)}{\left(T_{\text {oil.out }, 2}-30\right)}} \tag{d}
\end{equation*}
$$

Solving Eqns ( $a, b$, and d) we get:

## $T_{c w, o u t, 2}=41.4^{\circ} \mathrm{C}, T_{\text {oil,out }, 2}=54.4^{\circ} \mathrm{C}$

Check assumption about $F$
Base case $P=(91-51) /(91-30)=0.656, R=(40-30) /(91-51)=0.25 \Rightarrow F_{12}=0.9287$
New case $P=0.656, R=(38.46-30) /(91-51)=0.2115 \Rightarrow F_{12}=0.9423$
Change in $F_{12}$ is only $1.4 \%$ so above answer is ok. If you do include the change in $F_{12}$, you should get the following answer
$T_{c w, \text { out }, 2}=41.3^{\circ} \mathrm{C}, T_{\text {oil }, \text { out }, 2}=54.7^{\circ} \mathrm{C}$
38. For the situation in Problem 35, suppose that the process stream rate must be temporarily reduced while keeping the process exit temperature at $51^{\circ} \mathrm{C}$. Therefore, it will be necessary to reduce the flow of the cooling water stream. Determine the maximum scale-down of the process fluid that can occur without the exit cooling water temperature exceeding the limit of $45^{\circ} \mathrm{C}$ (when excessive fouling is known to occur)? Plot the results as the ratio of the process stream from the base case (x-axis) vs. the cooling water exit temperature.

$$
\begin{align*}
& \frac{Q_{2}}{Q_{1}}=\frac{m_{c w, 2}}{m_{c w, 1}} \frac{c_{p, /}, 2}{c / \frac{\left(T_{c w, o u t}-T_{c w, i n}\right)_{2}}{\left(T_{c w, o u t}-T_{c w, \text { in }}\right)_{1}}=M_{c w} \frac{\left(T_{c w, o u t, 2}-30\right)}{10}}  \tag{a}\\
& \frac{Q_{2}}{Q_{1}}=\frac{m_{o i l, 2}}{m_{o i l, 1}} \frac{c_{p, o f, 2}}{c_{p}} \frac{\left(T_{\text {oil, in }}-T_{\text {oil, out }}\right)_{2}}{\left(T_{o i l, \text { in }}-T_{o i l, o u t}\right)_{1}}=M_{\text {oil }}  \tag{b}\\
& \frac{Q_{2}}{Q_{1}}=\frac{U_{2} A}{U_{1}}=\frac{\Delta T_{l m, 2}}{\Delta T_{l m, 1}} \frac{F_{2}}{F_{1}}=\frac{U_{2}}{U_{1}} \frac{\left(91-T_{c w, o u t, 2}\right)-(51-30)}{(33.81) \ln \frac{\left(91-T_{c w, o u t, 2}\right)}{(21)}}=U \frac{\left(70-T_{c w, o u t, 2}\right)}{(33.81) \ln \frac{\left(91-T_{c w, o u t, 2}\right)}{(21)}} \tag{c}
\end{align*}
$$

Since both the flows of the cooling water and process streams change, we have:

$$
U=\frac{\left[\frac{1}{h M_{c w}^{0.6}}+\frac{1}{h M_{o i l}^{0.8}}\right]^{-1}}{\left[\frac{1}{h}+\frac{1}{h}\right]^{-1}}=\frac{2}{\left[\frac{1}{M_{c w}^{0.6}}+\frac{1}{0.5^{0.8}}\right]} \text { Substitute this into Equation (c) to }
$$

give:

$$
\begin{equation*}
\frac{Q_{2}}{Q_{1}}=\frac{2}{\left[\frac{1}{M_{c w}^{0.6}}+\frac{1}{0.5^{0.8}}\right]} \frac{\left(70-T_{c w, o u t, 2}\right)}{(33.81) \ln \frac{\left(91-T_{c w, o u t, 2}\right)}{(21)}} \tag{d}
\end{equation*}
$$

Set $M_{\text {oil }}=0.95,0.90$, etc. and solve for $M_{c w}$ and $T_{c w, o u t, 2}$ and plot the results.


Maximum scale-down $=0.54$ or $54 \%$
39. A reboiler is a heat exchanger used to add heat to a distillation column. In a typical reboiler, an almost pure material is vaporized at constant temperature, with the energy supplied by condensing steam at constant temperature. Suppose that steam is condensing at $254^{\circ} \mathrm{C}$ to vaporize an organic at $234^{\circ} \mathrm{C}$. It is desired to scale up the throughput of the distillation column by $25 \%$, meaning that $25 \%$ more organic (the process fluid) must be vaporized. What will be the new operating conditions in the reboiler (numerical value for temperature, qualitative answer for pressure)? Suggest at least two possible answers.

Consider the T-Q diagram for the reboiler


Without specifics about the heat transfer area, only approximate answers can be given. The performance equations are (1-base case, 2- new case, $M$ = process, $m=$ steam)

$$
\begin{aligned}
Q & =\frac{Q_{2}}{Q_{1}}=\frac{\dot{M}_{2} \chi_{s 2}}{\dot{M}_{1} \chi_{s 1}}=M=1.25 \\
Q & =\frac{Q_{2}}{Q_{1}}=\frac{\dot{m}_{2} \chi_{p 2}}{\dot{m}_{1} \chi_{p 1}}=m=1.25 \\
Q & =\frac{Q_{2}}{Q_{1}}=\frac{U_{2} A_{2} \Delta T_{l m, 2} Z_{2}}{U_{1} A_{1} \Delta T_{l m, 1} Z_{1}}=\frac{U_{2} \Delta T_{2}}{U_{1} \Delta T_{1}}=\frac{U_{2}}{U_{1}} \frac{\Delta T_{2}}{(20)}=1.25
\end{aligned}
$$

If we assume that neither heat transfer coefficient is a function of the temperature driving force or flowrate then $U_{2}=U_{1}$ and $\Delta T_{2}=25^{\circ} \mathrm{C}$.

Two simple solutions are possible:

1. Decrease process temperature by $5^{\circ} \mathrm{C}$ to $239^{\circ} \mathrm{C}$ by lowering the column pressure
2. Increase the steam temperature by $5^{\circ} \mathrm{C}$ to $259^{\circ} \mathrm{C}$ by desuperheating the steam less or by using higher pressure steam.
3. In a shell-and-tube heat exchanger, initially $h_{0}=500 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$ and $h_{i}=1500$ $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$. The fouling resistances and wall resistance may be assumed to be negligible. If the mass flowrate of the tube-side stream is increased by $30 \%$, what change in the mass flowrate of the shell-side stream is required to keep the overall heat transfer coefficient constant?

For shell-side $h_{o} \propto \operatorname{Re}_{o}^{0.6}$, for tube-side $h_{i} \propto \operatorname{Re}_{i}^{0.8}$ and $U \cong\left[\frac{1}{h_{o}}+\frac{1}{h_{i}}\right]^{-1}$
Using subscripts 1 and 2 to denote base case and new case, respectively, and $M$ to represent the ratios of mass flows, we have,

$$
\begin{aligned}
& \frac{h_{o, 2}}{h_{o, 1}} \propto\left(M_{\text {shell }}\right)^{0.6} \Rightarrow h_{o, 2}=500 M_{\text {shell }}^{0.6} \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K} \\
& \frac{h_{i, 2}}{h_{i, 1}} \propto\left(M_{\text {tube }}\right)^{0.8} \Rightarrow h_{o, 2}=1500(1.3)^{0.8}=1850.3 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K} \\
& \frac{U_{2}}{U_{1}} \cong \frac{\left[\frac{1}{h_{o, 2}}+\frac{1}{h_{i, 2}}\right]^{-1}}{\left[\frac{1}{h_{o, 1}}+\frac{1}{h_{i, 1}}\right]^{-1}}=\frac{\left[\frac{1}{500\left(M_{\text {shell }}\right)^{0.6}}+\frac{1}{1850.3}\right]^{-1}}{\left[\frac{1}{500}+\frac{1}{1500}\right]^{-1}}=1 \Rightarrow M_{\text {shell }}=0.9030
\end{aligned}
$$

Solving for Mshell gives that the shell side flow rate must be reduced to $90.3 \%$ of its original value.
41. A reaction occurs in a shell-and-tube reactor. One type of shell-and-tube reactor is essentially a heat exchanger with catalyst packed in the tubes. For an exothermic reaction, heat is removed by circulating a heat transfer fluid through the shell. In this situation, the reaction occurs isothermally at $510^{\circ} \mathrm{C}$. The Dowtherm always enters the shell coil at $350^{\circ} \mathrm{C}$. In the design case, it exits at $400^{\circ} \mathrm{C}$. In the base case, the heat transfer resistance on the reactor side is equal to that on the Dowtherm side. If it is required to increase throughput in the reactor by $25 \%$, what is the required Dowtherm flowrate and the new Dowtherm exit temperature. You should assume that the reaction temperature remains at $510^{\circ} \mathrm{C}$ ?

## Design Case


$\Delta T_{l m, 1}=\frac{(510-350)-(510-400)}{\ln \frac{510-350}{510-400}}=133.4^{\circ} \mathrm{C}$
$U \cong h_{0}$

Taking ratios between the new case (2) and the design case (1) gives
$\frac{U_{2}}{U_{1}}=M_{D T}^{0.6}$
$\frac{Q_{2}}{Q_{1}}=1.25$ (reactor/tube side)
(a)
$\frac{Q_{2}}{Q_{1}}=\frac{m_{D T, 2} c_{p, D T, 2}\left(T_{D T, \text { out } 2}-T_{D T, \text { in, } 2}\right)}{m_{D T, 1} c_{p, D T, 1}\left(T_{D T, \text { out } 1}-T_{D T, i, 1,1}\right)}=M_{D T} \frac{\left(T_{D T, \text { out }, 2}-350\right)}{400-350}=M_{D T} \frac{\left(T_{D T, \text { out }, 2}-350\right)}{50}$
$\frac{Q_{2}}{Q_{1}}=\frac{U_{2} A_{2} \Delta T_{l m, 2}}{U_{1} A_{1} \Delta T_{l m, 1}}=\frac{U_{2}}{U_{1}} \frac{\left[(510-350)-\left(510-T_{D T, \text { out }, 2}\right)\right]}{(123.3) \ln \frac{(510-3500}{\left(510-T_{D T, \text { out }, 2}\right)}}=M_{D T}^{0.6} \frac{\left(T_{D T, \text { out }, 2}-350\right)}{(133.4) \ln \frac{(160)}{\left(510-T_{D T, \text { out }, 2}\right)}}$
Solving Eqns. a-c for MDT and $T_{D T, o u t, 2}$ gives,

## $M_{D T}=1.3974$ or $139.7 \%$ of the design flow of Dowtherm and

## $T_{\text {DT, out }, 2}=394.7^{\circ} \mathrm{C}$

Note: In solving this problem, we have only considered the heat transfer aspects of the solution, i.e., how can we remove $25 \%$ more heat from the reactor. How to increase the production by $25 \%$ is another matter. For a shell and tube type reactor this would need a combination of increased $P$ and/or $T$ and/or a more active catalyst. To solve this problem from both the process and cooling aspects requires a more involved and complicated approach that is avoided here.
42. In Problem 41, the reactor is now a fluidized bed where Dowtherm circulates through tubes in the reactor with the reaction in the shell. The Dowtherm then flows to a heat exchanger in which boiler feed water (bfw) is vaporized on the shell side to high-pressure steam at $254^{\circ} \mathrm{C}$. This removes the heat absorbed by the Dowtherm stream in the reactor so the Dowtherm can be recirculated to the reactor. So, the Dowtherm is in a closed loop. The resistance in the steam boiler is all on the Dowtherm side. In the base case of the reactor, the resistance on the reaction side is four times that on the Dowtherm side. The desired increase in production can be accomplished by adding $25 \%$ more catalyst to the bed and operating at the same temperature, that is what is assumed in this analysis.

## Design Conditions



FB Reactor - shell-side $\left(510^{\circ} \mathrm{C}\right)$
a. Write the six equations needed to model the performance of this system. Use subscripts 1 and 2 to represent design (base) case and the new case.

For Reactor (heat exchanger - Dowtherm in tubes)

$$
\begin{align*}
& \Delta T_{l m, 1}=\frac{(510-350)-(510-400)}{\ln \frac{(510-350)}{(510-400)}}=\frac{50}{\ln \frac{160}{110}}=133.4^{\circ} \mathrm{C} \\
& \frac{U_{2}}{U_{1}}=\frac{\left[\frac{1}{h}+\frac{1}{4 h M_{D T}^{0.8}}\right]^{-1}}{\left[\frac{1}{h}+\frac{1}{4 h}\right]^{-1}}=\frac{1.25}{\left[1+\frac{1}{4 M_{D T}^{0.8}}\right]} \\
& \frac{Q_{2}}{Q_{1}}=Q=\frac{m_{D T, 2} c_{p, 2} \Delta T_{D T, 2}}{m_{D T, 1} c_{p, 1} \Delta T_{D T, 1}}=M_{D T} \frac{\left(T_{D T, \text { out, },}-T_{D T, i n, 1}\right)}{(400-350)}=M_{D T} \frac{\left(T_{D T, o u t, 2}-T_{D T, i n, 1}\right)}{(50)} \tag{a}
\end{align*}
$$

$$
\begin{align*}
& Q=\frac{m_{p, 2} \xi_{2} \Delta H_{R, 2}}{m_{p, 1} \xi_{1} \Delta H_{R, 1}}=1.25  \tag{b}\\
& Q=\frac{U_{2} A_{2} \Delta T_{\operatorname{lm}, R, 2}}{U_{1} A_{1} \Delta T_{\operatorname{lm}, R, 1}}=\frac{U_{2}}{U_{1}} \frac{\left[\left(510-T_{D T, \text { in, } 2}\right)-\left(510-T_{D T, \text { out }, 2}\right)\right]}{(133.4) \ln \frac{\left(510-T_{D T, \text { in } 2}\right)}{\left(510-T_{D T, \text { out }, 2}\right)}} \\
& Q=\frac{1.25}{\left[1+\frac{1}{4 M_{D T}^{0.8}}\right]} \frac{\left[\left(T_{D T, \text { out }, 2}-T_{D T, \text { out }, 1}\right)\right]}{(133.4) \ln \frac{\left(510-T_{D T, \text { in }, 2}\right)}{\left(510-T_{D T, \text { out }, 2}\right)}} \tag{c}
\end{align*}
$$

For Heat Exchanger (heat exchanger - steam boiler)

$$
\begin{align*}
& \Delta T_{\text {lm, } 1}=\frac{(400-254)-(350-254)}{\ln \frac{(400-254)}{(350-254)}}=\frac{50}{\ln \frac{146}{96}}=119.3^{\circ} \mathrm{C} \\
& \frac{U_{2}}{U_{1}}=\frac{\left[\frac{1}{h M_{D T}^{0.8}}\right]^{-1}}{\left[\frac{1}{h}\right]^{-1}}=M_{D T}^{0.8} \\
& Q=\frac{m_{D T, 2} c_{p, 2} \Delta T_{D T, 2}}{m_{D T, 1} c_{p, 1} \Delta T_{D T, 1}}=M_{D T} \frac{\left(T_{D T, \text { out }, 2}-T_{D T, \text { in, } 1}\right)}{(400-350)}=M_{D T} \frac{\left(T_{D T, \text { out }, 2}-T_{D T, \text { in }, 1}\right)}{(50)}  \tag{d}\\
& Q=\frac{\dot{m}_{\text {water }, 2} \lambda_{\text {water }, 2}}{\dot{m}_{\text {water }, 1} \lambda_{\text {water }, 1}}=M_{\text {steam }}=1.25  \tag{e}\\
& Q=\frac{U_{2} A_{2} \Delta T_{\text {lm }, E x, 2}}{U_{1} A_{1} \Delta T_{\text {lm }, E x, 1}}=\frac{U_{2}}{U_{1}} \frac{\left[\left(T_{D T, \text { out }, 2}-254\right)-\left(T_{D T, \text { in }, 2}-254\right)\right]}{(119.3) \ln \frac{\left(T_{D T, \text { out }, 2}-254\right)}{\left(T_{D T, i n, 2}-254\right)}} \\
& Q=M_{D T}^{0.8} \frac{\left[\left(T_{D T, \text { out }, 2}-T_{D T, i n, 2}\right)\right]}{(119.3) \ln \frac{\left(T_{D T, \text { out }, 2}-254\right)}{\left(T_{D T, i n, 2}-254\right)}} \tag{f}
\end{align*}
$$

b. How many unknowns are there?

Q, $M_{D T}, T_{D T, o u t, 2,} T_{D T, i n, 2}, M_{\text {steam }}$.
Note that although there are 6 equations (a-f) only 5 of them are independent as Eqns, a and d are the same, i.e., the heat balance on the Dowtherm in the reactor and steam generator are the same for steady state operations.

Moreover, Eqns. b and e are explicit, i.e., $25 \%$ more heat must be removed from the reactor and this heat will produce $25 \%$ more steam in the exchanger.
Solving Equations $a, c$, and $f$ for $M_{D T}, T_{D T, o u t, 2, ~} T_{D T, i n, 2, ~ g i v e s, ~}^{\text {, }}$
$M_{D T}=1.677$
TDT, out, $2=372.4^{\circ} \mathrm{C}$
$T_{\text {DT, } \text { in }, 2}=335.1^{\circ} \mathrm{C}$
c. If the temperature of the reactor is to be maintained at $510^{\circ} \mathrm{C}$, determine the amount of process scale-up and all other unknowns for the following cases:
i) 10\% increase in Dowtherm flowrate

Set $M_{D T}=1.1$ and solve for remaining variables to give,
$Q=1.046$
$T_{D T, \text { out }, 2}=394.9^{\circ} \mathrm{C}$
$T_{\text {DT, in }, 2}=347.4^{\circ} \mathrm{C}$
ii) $25 \%$ increase in Dowtherm flowrate

Set $M_{D T}=1.25$ and solve for remaining variables to give,
$Q=1.107$
$T_{D T, \text { out }, 2}=388.1^{\circ} \mathrm{C}$
$T_{D T, \text { in }, 2}=343.8^{\circ} \mathrm{C}$
iii) 50\% increase in Dowtherm flowrate.

Set $M_{D T}=1.50$ and solve for remaining variables to give,
$Q=1.196$
$T_{D T, \text { out }, 2}=378.3^{\circ} \mathrm{C}$
$T_{D T, \text { in, } 2}=338.6^{\circ} \mathrm{C}$


Fractional Increase in Dowtherm Flow
43. It is necessary to decrease the capacity of an existing distillation column by $30 \%$. As a consequence, the amount of liquid condensed in the shell of the overhead condenser must decrease by the same amount (30\%). In this condenser, cooling water (in tubes) is available at $30^{\circ} \mathrm{C}$, and, under present operating conditions, exits the condenser at $40^{\circ} \mathrm{C}$. The maximum allowable return temperature without a financial penalty assessed to your process is $45^{\circ} \mathrm{C}$. Condensation takes place at $85^{\circ} \mathrm{C}$.

$$
1 \text { - design case, } 2 \text { - new (scaled-down)case }
$$


a. If the limiting resistance is on the cooling water side, what is the maximum scale-down possible based on the condenser conditions without incurring a financial penalty? What is the new outlet temperature of cooling water? By what factor must the cooling water flow change?
Design Case

$$
\Delta T_{l m, 1}=\frac{(85-30)-(85-40)}{\ln \frac{(85-30)}{(85-40)}}=\frac{10}{\ln \frac{55}{45}}=49.83^{\circ} \mathrm{C}
$$

## Energy Balances

$$
\begin{align*}
& \frac{Q_{2}}{Q_{1}}=Q=\frac{m_{c w, 2} c_{p, 2} \Delta T_{c w, 2}}{m_{c w, 1} c_{p, 1} \Delta T_{c w, 1}}=M_{c w} \frac{\left(T_{c w, o u t, 2}-30\right)}{(40-30)}=M_{c w} \frac{\left(T_{D T, \text { out }, 2}-30\right)}{(10)}  \tag{a}\\
& Q=\frac{m_{p, 2} \lambda_{2}}{m_{p, 1} \lambda_{1}}=M_{p}=0.70 \tag{b}
\end{align*}
$$

Design Equation
$\frac{U_{2}}{U_{1}}=\frac{\left[\frac{1}{h M_{c w}^{0.8}}\right]^{-1}}{\left[\frac{1}{h}\right]^{-1}}=M_{c w}^{0.8}$

$$
\begin{equation*}
Q=\frac{U_{2} A_{2} \Delta T_{\operatorname{lm}, R, 2}}{U_{1} A_{1} \Delta T_{\operatorname{lm}, R, 1}}=\frac{U_{2}}{U_{1}} \frac{\left[(85-30)-\left(85-T_{c w, \text { out }, 2}\right)\right]}{(49.83) \ln \frac{(85-30)}{\left(85-T_{c w, \text { out }, 2}\right)}}=M_{c w}^{0.8} \frac{\left(T_{c w, \text { out }, 2}-30\right)}{(49.83) \ln \frac{(55)}{\left(85-T_{c w, o u t, 2}\right)}} \tag{c}
\end{equation*}
$$

Solving Equations a-c for unknowns,
$M_{c w}=0.6474$ (a decrease of $\sim 36 \%$ from design case)

## $\underline{T}_{c w, \text { out }, 2}=40.81^{\circ} \mathrm{C}$

b. Repeat part a. if the resistances are such that the cooling water heat transfer coefficient is three times the condensing heat transfer coefficient. You may assume that the value of the condensing heat transfer coefficient does not change appreciably from the design case. Does your solution exceed the maximum cooling water return temperature of $45^{\circ} \mathrm{C}$ ? If so, can you suggest other options to decrease the condenser duty by $30 \%$ that would not violate the cooling water return temperature constraint?

Let $h_{0}=h$ then $h_{i}=3 h$. From Table 2.9, $h_{0}$ is a function of $\left(T_{\text {sat }}-T_{w}\right)^{-1 / 4}$ but problem says ignore this change.

## Energy Balances

$$
\begin{align*}
& \frac{Q_{2}}{Q_{1}}=Q=\frac{m_{c w, 2} c_{p, 2} \Delta T_{c w, 2}}{m_{c w, 1} c_{p, 1} \Delta T_{c w, 1}}=M_{c w} \frac{\left(T_{c w, o u t, 2}-30\right)}{(40-30)}=M_{c w} \frac{\left(T_{D T, o u t, 2}-30\right)}{(10)}  \tag{a}\\
& Q=\frac{m_{p, 2} \lambda_{2}}{m_{p, 1} \lambda_{1}}=M_{p}=0.70 \tag{b}
\end{align*}
$$

Design Equation - follow procedure included in Example 2.20

$$
\begin{align*}
& \frac{U_{2}}{U_{1}}=\frac{\left[\frac{1}{3 h M_{c w}^{0.8}}+\frac{1}{h}\right]^{-1}}{\left[\frac{1}{3 h}+\frac{1}{h}\right]^{-1}}=\frac{(1.3333)}{\left[1+\frac{1}{3 M_{c w}^{0.8}}\right]} \\
& Q=\frac{U_{2} A_{2} \Delta T_{\mathrm{lm}, R, 2}}{U_{1} A_{1} \Delta T_{\mathrm{lm}, R, 1}}=\frac{(1.3333)}{\left[1+\frac{1}{3 M_{c w}^{0.8}}\right]} \frac{\left(T_{c w, o u t, 2}-30\right)}{(49.83) \ln \frac{(55)}{\left(85-T_{c w, o u t, 2}\right)}} \tag{c}
\end{align*}
$$

Solving Equations a-c for unknowns,
$M_{c w}=0.3787$ (a decrease of $\sim 62 \%$ from design case)
$\underline{T}_{c w, o u t, 2}=48.49^{\circ} \mathrm{C}$ (this exceeds the maximum cooling water return temperature of $\sim 45^{\circ} \mathrm{C}$ )

A possible alternative to avoid $T_{c w, o u t, 2}>45^{\circ} \mathrm{C}$ is to decrease the pressure in the tower - this has the effect of reducing the condensation temperature.
44. A reaction occurs in a well-mixed fluidized bed reactor maintained at $450^{\circ} \mathrm{C}$. Heat is removed by Dowtherm $A^{\text {TM }}$ circulating through a coil in the fluidized bed. In the design or base case, The Dowtherm enters and exits the reactor at 320 and $390^{\circ} \mathrm{C}$, respectively. In the base case, the heat transfer resistance on the reactor side is two times that on the Dowtherm side. It may be assumed that for the fluidized bed, the heat transfer coefficient on the fluidized side is essentially constant and independent of the throughput.
The Dowtherm is cooled in an external exchanger that produces steam at $900 \mathrm{psig}\left(T_{\text {sat }}=279^{\circ} \mathrm{C}\right)$. The Dowtherm pump limits the maximum increase in Dowtherm flowrate, so the pump limits the heat removal rate based on its pump and system curves. For the current situation, the maximum increase in Dowtherm flowrate through the reactor and boiler is estimated to be $34 \%$, by how much can the process be scaled-up.
You may assume that the limiting heat transfer coefficient on the steam boiler is Dowtherm that flows through the tube-side of the exchanger.


## Reactor

## Energy Balances

$\frac{Q_{2}}{Q_{1}}=Q=\frac{m_{D T, 2} c_{p, 2} \Delta T_{D T, 2}}{m_{D T, 1} c_{p, 1} \Delta T_{D T, 1}}=M_{D T} \frac{\left(T_{D T, \text { out }, 2}-T_{D T, i n, 2}\right)}{(390-320)}=M_{D T} \frac{\left(T_{D T, \text { out }, 2}-T_{D T, i, 2,2}\right)}{(70)}$
$Q=\frac{m_{p, 2} \xi_{2} \Delta H_{R, 2}}{m_{p, 1} \xi_{1} \Delta H_{R, 1}}=$ reactor scale-up factor

## Design Equation

For the base case, $h_{0}=h_{i} / 2=h$, and

$$
\begin{align*}
& \Delta T_{l m, 1}=\frac{(450-320)-(450-390)}{\ln \frac{(450-320)}{(450-390)}}=\frac{70}{\ln \frac{130}{60}}=90.53^{\circ} \mathrm{C} \\
& \frac{U_{2}}{U_{1}}=\frac{\left[\frac{1}{h_{o, 2}}+\frac{1}{h_{i, 2}}\right]^{-1}}{\left[\frac{1}{h_{o, 1}}+\frac{1}{h_{i, 1}}\right]^{-1}}=\frac{\left[\frac{1}{h}+\frac{1}{2 h M_{D T}^{0.8}}\right]^{-1}}{\left[\frac{1}{h}+\frac{1}{2 h}\right]^{-1}}=\frac{1.5}{\left[1+\frac{1}{2 M_{D T}^{0.8}}\right]} \\
& Q=\frac{U_{2} A_{2} \Delta T_{\operatorname{lm}, R, 2}}{U_{1} A_{1} \Delta T_{\operatorname{lm}, R, 1}}=\frac{U_{2}}{U_{1}} \frac{\left[\left(450-T_{D T, i n, 2}\right)-\left(450-T_{D T, \text { out }, 2}\right)\right]}{(90.53) \ln \frac{\left(450-T_{D T, i n, 2}\right)}{\left(450-T_{D T, o u t, 2}\right)}} \\
& Q=\frac{1.5}{\left[1+\frac{1}{2 M_{c w}^{0.8}}\right]} \frac{\left(T_{D T, o u t, 2}-T_{D T, i n, 2}\right)}{(90.53) \ln \frac{\left(450-T_{D T, i n, 2}\right)}{\left(450-T_{D T, o u t, 2}\right)}} \tag{c}
\end{align*}
$$

## Boiler

## Energy Balances

$\frac{Q_{2}}{Q_{1}}=Q=\frac{m_{D T, 2} c_{p, 2} \Delta T_{D T, 2}}{m_{D T, 1} c_{p, 1} \Delta T_{D T, 1}}=M_{D T} \frac{\left(T_{D T, \text { out }, 2}-T_{D T, \text { in,2 }}\right)}{(390-320)}=M_{D T} \frac{\left(T_{D T, \text { out }, 2}-T_{D T, i n, 2}\right)}{(70)}$
$Q=\frac{m_{\text {steam }, 2} \lambda_{2}}{m_{\text {steam }, 1} \lambda_{1}}=M_{\text {steam }}$
(e)

## Design Equation

For the base case, $h_{i}=h, U=h$, and
$\Delta T_{l m, 1}=\frac{(390-279)-(320-279)}{\ln \frac{(390-279)}{(320-279)}}=\frac{70}{\ln \frac{111}{41}}=70.28^{\circ} \mathrm{C}$
$\frac{U_{2}}{U_{1}}=\frac{\left[\frac{1}{h_{o, 2}}+\frac{1}{h_{i, 2}}\right]^{-1}}{\left[\frac{1}{h_{o, 1}}+\frac{1}{h_{i, 1}}\right]^{-1}}=\frac{\left[\frac{1}{h M_{D T}^{0.8}}\right]^{-1}}{\left[\frac{1}{h}\right]^{-1}}=M_{D T}^{0.8}$
$Q=\frac{U_{2} A_{2} \Delta T_{\mathrm{lm}, R, 2}}{U_{1} A_{1} \Delta T_{\mathrm{lm}, R, 1}}=\frac{U_{2}}{U_{1}} \frac{\left[\left(T_{D T, \text { out }, 2}-279\right)-\left(T_{D T, i n, 2}-279\right)\right]}{(70.28) \ln \frac{\left(T_{D T, \text { out }, 2}-279\right)}{\left(T_{D T, i n, 2}-279\right)}}$
$Q=M_{c w}^{0.8} \frac{\left(T_{D T, \text { out }, 2}-T_{D T, \text { in }, 2}\right)}{(70.28) \ln \frac{\left(T_{D T, \text { out }, 2}-279\right)}{\left(T_{D T, \text { in }, 2}-279\right)}}$

Setting $M_{D T}=1.34$ (maximum increase in flow) and solving Equations ( $a, c$, and f) for 3 unknowns gives,
$Q=1.167$
$T_{D T, \text { out }, 2}=379.06^{\circ} \mathrm{C}$
$T_{\text {DT, }, \mathrm{in}, 2}=318.11^{\circ} \mathrm{C}$
Therefore the maximum scale-up of the reactor from the base case is 1.167 or $16.7 \%$.

