

**Chapter 2 Differentiation**

## 2.1 Tangent Lines and Their Slopes

1) Find the slope of the tangent line to the curve  $y = 4x - x^2$  at the point  $(-1, 0)$ .

- A) -1
- B) 2
- C) 6
- D)  $\frac{1}{2}$
- E) -2

Answer: C

Diff: 1

2) Find the equation of the tangent line to the curve  $y = 2x - x^2$  at the point  $(2, 0)$ .

- A)  $2x + y - 4 = 0$
- B)  $2x + y + 4 = 0$
- C)  $2x - y - 4 = 0$
- D)  $2x - y + 4 = 0$
- E)  $2x + y = 0$

Answer: A

Diff: 1

3) Find an equation of the line tangent to the curve  $y = 2x - \left(\frac{x}{10}\right)^2$  at the point where  $x = 2$ .

- A)  $25y = 49x - 1$
- B)  $5y = 49x + 1$
- C)  $25y = 49x + 1$
- D)  $25y = 41x + 1$
- E)  $25x = 49y + 1$

Answer: C

Diff: 2

4) Find an equation of the line tangent to the curve  $y = x^3 + 1$  at the point where  $x = 2$ .

- A)  $y = 12x + 15$
- B)  $y = 12x - 15$
- C)  $y = -12x - 15$
- D)  $y = -12x + 15$
- E)  $y = 15x + 12$

Answer: B

Diff: 2

5) Find an equation of the line tangent to the curve  $y = |x^2 - 8|$  at the point where  $x = 2$ .

A)  $y = -4x + 12$

B)  $y = 4x - 4$

C)  $y = -4x + 4$

D)  $y = 4x + 4$

E)  $y = 4x - 12$

Answer: C

Diff: 2

6) Find an equation of the line tangent to the curve  $y = \sqrt{x - 7}$  at the point where  $x = 11$ .

A)  $y = \frac{1}{4}x + \frac{3}{4}$

B)  $y = \frac{1}{4}x - \frac{3}{4}$

C)  $y = 4x - \frac{3}{4}$

D)  $y = 4x + \frac{3}{4}$

E)  $y = -\frac{1}{4}x - \frac{3}{4}$

Answer: B

Diff: 2

7) Find an equation of the line tangent to the curve  $y = \sqrt{10 - x^2}$  at the point  $(1, 3)$ .

A)  $y = -\frac{1}{3}x + \frac{10}{3}$

B)  $y = -\frac{1}{3}x - \frac{10}{3}$

C)  $y = \frac{1}{3}x + \frac{10}{3}$

D)  $y = \frac{1}{3}x - \frac{10}{3}$

E)  $y = 3x - 10$

Answer: A

Diff: 2

8) Let  $f(x)$  be a function such that  $\lim_{h \rightarrow 0} \frac{f(x) - f(x+h)}{h} = x^3 - 1$ . Find the slope of the line tangent to

the graph of  $f$  at the point  $(a, f(a))$ .

- A)  $3a^2$
- B)  $1 - a^3$
- C)  $-3a^2$
- D)  $a^3 - 1$
- E)  $\frac{1}{4}a^4 - a + C$

Answer: B

Diff: 1

9) Find the point(s) on the curve  $y = x^2$  such that the tangent lines to the curve at those points pass through  $(2, -12)$ .

- A)  $(6, 36)$  and  $(-2, 4)$
- B)  $(6, 36)$  and  $(2, 4)$
- C)  $(-6, 36)$  and  $(-2, 4)$
- D)  $(-6, 6)$  and  $(-2, 4)$
- E)  $(6, -36)$  and  $(-2, 4)$

Answer: A

Diff: 2

10) Find the standard equation of the circle with centre at  $(1, 3)$  which is tangent to the line  $5x - 12y = 8$ .

- A)  $(x - 1)^2 - (y - 3)^2 = 1$
- B)  $(x - 1)^2 + (y + 3)^2 = 9$
- C)  $(x - 1)^2 + (y - 3)^2 = 9$
- D)  $(x + 1)^2 + (y - 3)^2 = 8$
- E)  $(x - 1)^2 + (y - 3)^2 = 8$

Answer: C

Diff: 2

11) If the line  $4x - 9y = 0$  is tangent in the first quadrant to the graph of  $y = \frac{1}{3}x^3 + c$ , what is the

value of  $c$ ?

A)  $-\frac{16}{81}$

B)  $\frac{16}{81}$

C)  $\frac{18}{81}$

D)  $\frac{1}{81}$

E)  $\frac{81}{16}$

Answer: B

Diff: 3

## 2.2 The Derivative

1) Using the definition of the derivative, find the derivative of  $f(x) = \sqrt{x+2}$ .

A)  $\frac{1}{2\sqrt{x-2}}$

B)  $\frac{1}{\sqrt{x+2}}$

C)  $\frac{3}{2\sqrt{x+2}}$

D)  $\frac{1}{2\sqrt{x+2}}$

E)  $\frac{2}{2\sqrt{x+2}}$

Answer: D

Diff: 1

2) Find the derivative  $f'(x)$  of the function  $f(x) = \frac{1}{\sqrt{x}}$ .

A)  $-\frac{1}{2\sqrt{x^3}}$

B)  $\frac{1}{2\sqrt{x^3}}$

C)  $-\frac{1}{\sqrt{x^3}}$

D)  $-\frac{1}{3\sqrt{x^3}}$

E)  $\frac{1}{2\sqrt{x}}$

Answer: A

Diff: 2

3) Find the tangent line to the curve  $y = \frac{x}{4-x}$  at the origin.

A)  $y = -\frac{1}{4}x$

B)  $y = x$

C)  $y = \frac{1}{4}x$

D)  $y = -\frac{1}{2}x$

E)  $y = \frac{1}{2}x$

Answer: C

Diff: 2

4) Where is the function  $f(x) = |x - 3|$  differentiable?

A) at every  $x \in (-\infty, \infty)$

B) at every  $x \in (-\infty, 0) \cup (0, \infty)$

C) at every  $x \in (-\infty, 3) \cup (3, \infty)$

D) at every  $x \in (-\infty, 0) \cup (0, 3) \cup (3, \infty)$

E) none of the above

Answer: C

Diff: 2

5) Find the equation of the straight line that passes through the point P(0, -3) and is tangent to the curve  $y = x^3 - x - 1$ .

- A)  $y = -3$
- B)  $y = 2x - 3$
- C)  $y = -3x$
- D)  $y = -x - 3$
- E)  $y = x - 3$

Answer: B

Diff: 3

6) If  $f(x) = \frac{4}{5}(\sqrt{9-x})$ , calculate  $f'(5)$  by using the definition of the derivative.

- A)  $\frac{2}{5}$
- B)  $-\frac{1}{5}$
- C)  $-\frac{4}{5}$
- D)  $-\frac{1}{10}\sqrt{5}$
- E)  $\frac{1}{5}$

Answer: B

Diff: 2

7) Find the slope of the line tangent to the curve  $x^3 y = 1$  at the point  $\left(3, \frac{1}{27}\right)$ .

- A)  $\frac{1}{27}$
- B)  $-\frac{2}{27}$
- C)  $-\frac{1}{27}$
- D)  $\frac{2}{27}$
- E)  $\frac{1}{9}$

Answer: C

Diff: 2

8) If  $f(x) = \frac{32}{\sqrt{8-x^3}}$ , calculate  $f'(-2)$  directly from the definition of the derivative.

- A) 3
- B)  $3\sqrt{2}$
- C) -3
- D) 4
- E) 2

Answer: D

Diff: 2

9) Let  $g(x)$  be a function such that  $\frac{g(x+h) - g(x)}{h} = -\frac{1}{x(x+h)}$ . Find  $g'(x)$ .

- A)  $\frac{2x+h}{x^2(x+h)^2}$
- B)  $-\frac{1}{x^2}$
- C)  $-\frac{1}{x(x+h)}$
- D)  $\frac{2}{x^3}$
- E)  $\lim_{h \rightarrow 0} \frac{2x+h}{x^2(x+h)^2}$

Answer: B

Diff: 1

10) Calculate the derivative of  $g(t) = t^{101} + t^{-99}$  using the general power rule.

- A)  $101 t^{100} - 99 t^{-100}$
- B)  $101 t^{101} - 99 t^{-99}$
- C)  $-101 t^{100} - 99 t^{-98}$
- D)  $100 t^{100} - 98 t^{-98}$
- E)  $101 t^{100} + 99 t^{-100}$

Answer: A

Diff: 2

11) If  $f(x)$  is an even, differentiable function, then  $f'(x)$

- A) is an odd function.
- B) is an even function.
- C) is neither odd nor even.
- D) may be either even or odd or neither.

Answer: A

Diff: 3

12) True or False: If the curve  $y = f(x)$  has a tangent line at  $(a, f(a))$ , then  $f$  is differentiable at  $x = a$ .

Answer: FALSE

Diff: 3

13) True or False: If  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = -\infty$ , then the graph of  $f$  has a tangent line at  $x = a$ .

Answer: TRUE

Diff: 3

14) True or False: If  $f$  is continuous at  $x = a$ , then  $f$  is differentiable at  $x = a$ .

Answer: FALSE

Diff: 3

15) True or False: If  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  exists, then  $f$  is continuous at  $x = a$ .

Answer: TRUE

Diff: 3

16) True or False: The domain of the derivative of a function is the same as the domain of the function.

Answer: FALSE

Diff: 3

### 2.3 Differentiation Rules

1) Differentiate  $f(x) = 10x^5$ .

A)  $10x^4$

B)  $50x^4$

C)  $55x^4$

D)  $50x^3$

E)  $50x$

Answer: B

Diff: 1

2) Find  $\frac{dy}{dx}$  if  $y = 4x^4 + 3x^3 + x - 6$ .

A)  $16x^3 - 9x^2 + 1$

B)  $16x^4 + 9x^3 + 1$

C)  $16x^3 + 9x^2 + 1$

D)  $16x^3 + 9x^2 - 6$

E)  $16x^3 + 9x^2 - 5$

Answer: C

Diff: 1



3) Differentiate the function  $f(x) = (2x^3 + 5)(3x^2 - x)$ .

- A)  $30x^4 - 8x^3 + 30x - 5$
- B)  $30x^4 - 8x^3 + 30x + 5$
- C)  $30x^4 + 8x^3 + 30x - 5$
- D)  $30x^4 + 8x^3 - 30x - 5$
- E)  $36x^3 - 6x^2$

Answer: A

Diff: 2

4) Find the equation of the tangent line to the curve  $y = (2 - \sqrt{x})(1 + \sqrt{x} + 3x)$  at the point  $(1, 5)$ .

- A)  $x - y + 4 = 0$
- B)  $x + y - 6 = 0$
- C)  $x - y - 4 = 0$
- D)  $6x - y - 1 = 0$
- E)  $x + y + 4 = 0$

Answer: A

Diff: 2

5) Find the points on the curve  $y = x^4 - 6x^2 + 4$  where the tangent line is horizontal.

- A)  $(\sqrt{3}, -5)$  and  $(-\sqrt{3}, -5)$
- B)  $(0, 4)$ ,  $(\sqrt{2}, -5)$ , and  $(-\sqrt{3}, -5)$
- C)  $(0, 4)$ ,  $(-\sqrt{3}, 5)$ , and  $(\sqrt{3}, -5)$
- D)  $(0, 4)$ ,  $(\sqrt{3}, -5)$ , and  $(-\sqrt{3}, -5)$
- E)  $(\sqrt{3}, 5)$  and  $(-\sqrt{3}, 5)$

Answer: D

Diff: 2

6) Given  $g(x) = \begin{cases} x^4 + 2 & \text{if } x < -1 \\ 10 & \text{if } x = -1 \\ -1 - 4x & \text{if } x > -1 \end{cases}$

which of the following statements is true?

- A)  $g$  is differentiable at  $x = -1$
- B)  $g$  is not differentiable at  $x = -1$
- C)  $g'(-1) = -4$
- D)  $g$  is continuous at  $x = -1$
- E)  $g$  is continuous from the left at  $x = -1$

Answer: B

Diff: 3

7) Lines passing through the point (0, 2) are tangent to the graph of  $y = -|x^3|$ . Find the points of tangency.

- A) (1, -1) and (-1, 1)
- B) (2, -8) and (-2, -8)
- C) (1, -1) and (-2, -8)
- D) (2, -8) and (-1, 1)
- E) (1, 1) and (-1, -1)

Answer: A

Diff: 3

8) Where does the normal line to the curve  $y = x - x^2$  at the point (1, 0) intersect the curve a second time?

- A) (-2, -6)
- B)  $(-\frac{1}{2}, -\frac{3}{4})$
- C) (-1, -2)
- D) (0, 0)
- E) It does not intersect the curve a second time.

Answer: C

Diff: 3

9) Which of the following statements is always true?

- A) If  $f$  is continuous at  $c$ , then it must be differentiable at  $c$ .
- B) If  $f$  is differentiable at  $c$ , then it must be continuous at  $c$ .
- C) If  $f$  is not differentiable at  $c$ , then it must be discontinuous at  $c$ .
- D) If  $\lim_{h \rightarrow 0} f(c + h) = f(c)$ , then  $f$  must be differentiable at  $c$ .
- E) All of the above

Answer: B

Diff: 2

10) How many tangent lines to the graph of  $y = x^4 - 15x^2 - 10$  pass through the point (0, 2)?

- A) 0
- B) 1
- C) 2
- D) 3
- E) 4

Answer: E

Diff: 3

11) Let  $f(x) = \begin{cases} k^2x^2 - 1 & \text{if } -\infty < x < 1 \\ 3x^4 - k^2x - 2k & \text{if } 1 \leq x < \infty \end{cases}$ .

Find all values of the real number  $k$  so that  $f$  is differentiable at  $x = 1$ .

- A) -2 and 1
- B) 2 and -1
- C) -2 and 2
- D) only -2
- E) only 2

Answer: D

Diff: 3

12) There are lines that pass through the point  $(-1, 3)$  and are tangent to the curve  $xy = 1$ . Find all their slopes.

- A) -1 and -9
- B) -1 and 9
- C) 1 and 9
- D) 1 and -9
- E) none of the above

Answer: A

Diff: 2

## 2.4 The Chain Rule

1) Find the derivative of  $\sqrt{4x - 6}$ .

A)  $\frac{1}{2\sqrt{4x-6}}$

B)  $\frac{-1}{2\sqrt{4x-6}}$

C)  $\frac{2}{\sqrt{4x-6}}$

D)  $\frac{-2}{\sqrt{4x-6}}$

E)  $\frac{1}{\sqrt{4x-6}}$

Answer: C

Diff: 1

2) Find the derivative of  $f(x) = \frac{1}{(3x^2 + 5)^4}$ .

A)  $-\frac{24x}{(3x^2 + 5)^5}$

B)  $\frac{24x}{(3x^2 + 5)^5}$

C)  $\frac{12x}{(3x^2 + 5)^3}$

D)  $-\frac{12x}{(3x^2 + 5)^3}$

E)  $-\frac{4}{(3x^2 + 5)^5}$

Answer: A

Diff: 1

3) Differentiate the following function:  $f(x) = \left(\frac{3x-1}{x^2+3}\right)^2$ .

A)  $\frac{2(3x-1)(3x^2+2x+9)}{(x^2+3)^3}$

B)  $\frac{2(3x-1)(-3x^2+2x-9)}{(x^2+3)^3}$

C)  $\frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}$

D)  $\frac{3(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}$

E) none of the above

Answer: C

Diff: 2

4) Differentiate the following function:  $f(x) = \left(\frac{x+2}{x-3}\right)^3$ .

A)  $14 \frac{(x+2)^2}{(x-3)^3}$

B)  $-15 \frac{(x+2)^2}{(x-3)^4}$

C)  $-16 \frac{(x-2)^2}{(x-3)^3}$

D)  $17 \frac{(x+2)^2}{(x+3)^4}$

E)  $3 \frac{(x+2)^2}{(x+3)^2}$

Answer: B

Diff: 2

5) Find an equation of the line tangent to the curve  $y = (x^3 + 2)^9$  at the point  $(-1, 1)$ .

A)  $27x - y + 28 = 0$

B)  $27x + y + 26 = 0$

C)  $27y - x - 28 = 0$

D)  $27y + x - 26 = 0$

E)  $9x - y + 10 = 0$

Answer: A

Diff: 2

6) Use the values in the table below to evaluate  $(f \circ g)'(-2)$

x	f(x)	f'(x)	g(x)	g'(x)
1	-2	6	3	0
-2	10	4	1	5
5	2	-8	0	8

Answer: 30

Diff: 2

7) Assuming all indicated derivatives exist,  $(f \circ g)'(c)$  is equal to

A)  $f'(g(c)) g'(c)$

B)  $f'(c) g(c) + f(c) g'(c)$

C)  $f'(c) g'(c)$

D)  $f'(c) \circ g'(c)$

E)  $f'(g'(c))$

Answer: A

Diff: 1

8) Let  $f(x) = (x - 2)(x^2 + 4x - 7)$ . Find all the points on this curve where the tangent line is horizontal.

A)  $\left(\frac{3}{5}, -\frac{238}{125}\right)$  and  $(-3, 50)$

B)  $\left(\frac{5}{3}, -\frac{238}{125}\right)$  and  $(3, 14)$

C)  $\left(\frac{5}{3}, -\frac{22}{27}\right)$  and  $(3, 14)$

D)  $\left(\frac{5}{3}, -\frac{22}{27}\right)$  and  $(-3, 50)$

E)  $(-2, 44)$

Answer: D

Diff: 2

9) Find  $\frac{d}{dx} \left\{ \frac{x^3 - 3x^2 + 2}{(x-1)^3} \right\}$ . Simplify your answer.

A)  $\frac{6 + 6x - 18x^2}{(x-1)^3}$

B)  $\frac{18x^2 - 6}{(x-1)^3}$

C)  $\frac{6}{(x-1)^3}$

D)  $-\frac{6}{(x-1)^3}$

E)  $\frac{x^2 - 1}{(x-1)^3}$

Answer: C

Diff: 2

10) Where does the function  $f(x) = |x^2 - x^3|$  fail to be differentiable?

A)  $f(x)$  is differentiable everywhere.

B) at  $x = 0$

C) at  $x = 1$

D) at  $x = 0$  and  $x = 1$

E) none of the above

Answer: C

Diff: 2

11) True or False: The function  $f(x) = \begin{cases} x^2 \operatorname{sgn} x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$  is differentiable at  $x = 0$ .

Answer: TRUE

Diff: 2

## 2.5 Derivatives of Trigonometric Functions

1) Differentiate  $y = \sin 4x$ .

- A)  $2\cos 4x$
- B)  $4\cos 2x$
- C)  $-4\cos 4x$
- D)  $4\cos 4x$
- E)  $\cos 4x$

Answer: D

Diff: 1

2) Find the derivative of  $y = \tan(\cos(x^2))$ .

- A)  $\sec^2(-\sin(2x))$
- B)  $2x \cos(x^2)$
- C)  $\sec^2(-2x \sin(x^2))$
- D)  $\sec^2(x^2) \cos(x^2) - \tan(x^2) \sin(x^2)$
- E)  $-2x \sec^2(\cos(x^2)) \sin(x^2)$

Answer: E

Diff: 2

3) Find the derivative of  $f(t) = \cos^3(5t)$ .

- A)  $15 \cos^2(5t) \sin(5t)$
- B)  $-3 \cos^2(5t) \sin(5t)$
- C)  $-15 \cos^2(5t) \sin(5t)$
- D)  $15 \cos^2(5t)$
- E)  $3\cos^2(5t)$

Answer: C

Diff: 1

4) Differentiate  $y = x^3 \sin 2x$ .

- A)  $x^2 \sin 2x + x^3 \cos 2x$
- B)  $3x^2 \sin 2x + x^3 \cos 2x$
- C)  $3x^2 \sin 2x - 2x^3 \cos 2x$
- D)  $3x^2 \sin 2x + 2x^3 \cos 2x$
- E)  $3x^2 \cos(2x)$

Answer: D

Diff: 1

5) If  $\frac{d}{dx}\{g(\sin(x))\} = \cot(x)$ , find  $g'(x)$ .

Answer: By chain rule  $\frac{d}{dx}\{g(\sin(x))\} = g'(\sin(x)) \cos(x)$ .

Therefore we obtain:

$g'(\sin(x)) \cos(x) = \cot(x)$ . It follows that  $g'(\sin(x)) = \frac{\cot(x)}{\cos(x)}$ . But  $\frac{\cot(x)}{\cos(x)} = \frac{\frac{\cos(x)}{\sin(x)}}{\cos(x)} = \frac{1}{\sin(x)}$

and hence  $g'(\sin(x)) = \frac{1}{\sin(x)}$ . Now replacing  $\sin(x)$  by  $x$ , we obtain  $g'(x) = \frac{1}{x}$ .

Diff: 2

6) Find the derivative of  $y = \tan(x^2)$ .

- A)  $x \sec^2(x^2)$
- B)  $4x \sec^2(x^2)$
- C)  $2x \sec^2(x^2)$
- D)  $2x \sec(x^2) \tan(x^2)$
- E)  $\sec^2(x^2)$

Answer: C

Diff: 2

7) Find the derivative of the following function:  $y = \tan^2(\cos x)$ .

- A)  $-2\sin x[\tan(\cos x)][\sec^2(\cos x)]$
- B)  $2\sin x[\tan(\cos x)][\sec^2(\cos x)]$
- C)  $-2\sin x[\sec(\cos x)][\tan^2(\cos x)]$
- D)  $-2\sin x[\tan(\sin x)][\sec^2(\cos x)]$
- E)  $-2\tan(x)\cos(x)\sin(x)$

Answer: A

Diff: 2

8) Let  $y = \frac{\cos(x)}{1+\sin(x)}$ . A simplified expression for  $\frac{dy}{dx}$  is given by

- A)  $\frac{1}{1+\sin(x)}$
- B)  $\frac{-\sin(x)}{\cos(x)}$
- C)  $-\frac{1}{1+\sin(x)}$
- D)  $-\sin(x) - \csc^2(x)$
- E)  $\frac{-\sin(x) - \cos^2(x)}{1+\sin(x)}$

Answer: C

Diff: 2



9) Find the slope of the curve  $y = \cos\left(\frac{\pi \tan x}{6}\right)$  at the point where  $x = \frac{\pi}{2}$ .

A)  $-\frac{\pi}{6}$

B)  $-\frac{\pi}{4}$

C)  $-\frac{\pi}{3}$

D)  $-\frac{\pi}{2}$

E) The slope is not defined at  $x = \frac{\pi}{2}$ .

Answer: A

Diff: 2

10) Find all points in the interval  $[0, \pi]$  where the curve  $y = 2 \sin^2 x - \sin(2x)$  has a horizontal tangent line.

A)  $\left(\frac{\pi}{8}, 1 - \frac{1}{\sqrt{2}}\right)$  and  $\left(\frac{5\pi}{8}, 1 - \frac{1}{\sqrt{2}}\right)$

B)  $\left(\frac{\pi}{8}, 1 - \frac{1}{\sqrt{2}}\right)$  and  $\left(\frac{7\pi}{8}, 1 - \frac{1}{\sqrt{2}}\right)$

C)  $\left(\frac{\pi}{8}, 1 - \frac{1}{\sqrt{2}}\right)$  and  $\left(\frac{3\pi}{8}, 1 - \frac{1}{\sqrt{2}}\right)$

D)  $\left(\frac{3\pi}{8}, 1 - \frac{1}{\sqrt{2}}\right)$  and  $\left(\frac{5\pi}{8}, 1 - \frac{1}{\sqrt{2}}\right)$

E) The tangent line is never horizontal.

Answer: A

Diff: 3

## 2.6 Higher-Order Derivatives

1) Find  $y''$  if  $y = x^5$ .

A)  $12x^3$

B)  $5x^3$

C)  $15x^3$

D)  $20x^3$

E)  $10x^3$

Answer: D

Diff: 1

2) Find the second derivative of  $g(x) = \frac{4}{\sqrt{t}}$ .

- A)  $g''(t) = 2t^{-5/2}$
- B)  $g''(t) = -3t^{-5/2}$
- C)  $g''(t) = 3t^{-5/2}$
- D)  $g''(t) = -2t^{-5/2}$
- E)  $g''(t) = 4t^{-5/2}$

Answer: C  
Diff: 1

3) True or False: Assuming all indicated derivatives exist,  $(F \circ G(x))'' = F''(G(x))(G'(x))^2 + F'(G(x))G''(x)$ .

Answer: TRUE  
Diff: 2

4) Find  $f'''(2)$  given that  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = x^3 - 7$ .

- A) 12
- B) 0
- C) 6
- D) 5
- E) 10

Answer: A  
Diff: 1

5) let  $y = \frac{8}{x} \sqrt{324x^2 + 216 + \frac{36}{x^2}}$ ,  $x > 0$ . Show that  $\frac{d^2y}{dx^2} = \frac{288}{x^4}$ .

Answer: First observe that the expression  $324x^2 + 126 + \frac{36}{x^2}$  is a perfect square.

Indeed  $324x^2 + 216 + \frac{36}{x^2} = \left(18x + \frac{6}{x}\right)^2$ , hence we have

$$y = \frac{8}{x} \sqrt{\left(18x + \frac{6}{x}\right)^2} = \frac{8}{x} \left|18x + \frac{6}{x}\right| = \frac{8}{x} \left(18x + \frac{6}{x}\right), \text{ since } x > 0.$$

It follows that  $y = 144 + \frac{48}{x^2}$  or  $y = 144 + 48x^{-2}$ .

Therefore  $\frac{dy}{dx} = 0 - 96x^{-3}$  and  $\frac{d^2y}{dx^2} = 288x^{-4} = \frac{288}{x^4}$ .

Diff: 3

6) Calculate the third derivative of  $f(x) = \sin^2 x$ .

- A)  $-2\sin(2x)$
- B)  $-4\sin(2x)$
- C)  $-2\cos(2x)$
- D)  $-4\sin x$
- E)  $-2\sin^2(x)$

Answer: B

Diff: 2

7) Find a formula for the  $n$ th derivative  $y^{(n)}$  of the function  $y = \sqrt{x+5}$ .

A)  $y^{(n)} = (-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n} (x+5)^{-(2n-1)/2}$

B)  $y^{(n)} = -\frac{1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n} (x+5)^{-(2n-1)/2}$

C)  $y^{(n)} = (-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n} (x+5)^{-(2n-1)/2}$

D)  $y^{(n)} = (-1)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n} (x+5)^{-(2n-1)/2}$

E)  $y^{(n)} = (-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{2^n} (x+5)^{-(2n-1)/2}$

Answer: A

Diff: 3

8) Find the second derivative of the function  $f(x) = \frac{\cos x}{x}$ .

A)  $-\frac{\cos x}{x} + \frac{2\sin x}{x^2} + \frac{2\cos x}{x^3}$

B)  $-\frac{\cos x}{x} - \frac{2\sin x}{x^2} + \frac{2\cos x}{x^3}$

C)  $-\frac{\cos x}{x} + \frac{2\sin x}{x^2} - \frac{2\cos x}{x^3}$

D)  $\frac{\cos x}{x} + \frac{2\sin x}{x^2} + \frac{2\cos x}{x^3}$

E)  $\frac{\cos x}{x} + \frac{2\sin x}{x^2} - \frac{2\cos x}{x^3}$

Answer: A

Diff: 2

9) Find the second derivative of the function  $f(x) = x^3 \sin(2x)$ .

- A)  $-6x \sin(2x) + 8x^2 \cos(2x) - 4x^3 \sin(2x)$
- B)  $6x \sin(2x) + 8x^2 \cos(2x) - 4x^3 \sin(2x)$
- C)  $6x \sin(2x) + 12x^2 \cos(2x) - 4x^3 \sin(2x)$
- D)  $-6x \sin(2x) + 12x^2 \cos(2x) + 4x^3 \sin(2x)$
- E)  $6x \sin(2x) + 12x^2 \cos(2x) + 4x^3 \sin(2x)$

Answer: C

Diff: 3

10) Find the second derivative of the function  $f(x) = \frac{x^2 \sin(x) \cos(x)}{\sin(2x)}$ .

- A) 1
- B)  $x$
- C)  $-\frac{x \cos(x) \sin(x)}{\cos(2x)}$
- D)  $-\frac{\sin(x) \cos(x)}{2 \sin(2x)}$
- E) 0

Answer: A

Diff: 2

## 2.7 Using Differentials and Derivatives

1) A spherical balloon is being inflated. Find the rate of change of volume with respect to the radius when the radius is 5 cm.

- A)  $200\pi \text{ cm}^3/\text{cm}$
- B)  $100\pi \text{ cm}^3/\text{cm}$
- C)  $300\pi \text{ cm}^3/\text{cm}$
- D)  $400\pi \text{ cm}^3/\text{cm}$
- E)  $500\pi \text{ cm}^3/\text{cm}$

Answer: B

Diff: 1

2) Find the rate of change of the volume of a cube with respect to its edge length  $x$  when  $x = 4$  m.

- A)  $40 \text{ m}^3/\text{m}$
- B)  $42 \text{ m}^3/\text{m}$
- C)  $48 \text{ m}^3/\text{m}$
- D)  $50 \text{ m}^3/\text{m}$
- E)  $8 \text{ m}^3/\text{m}$

Answer: C

Diff: 1

3) A spherical balloon is being inflated. Find the rate of increase of the surface area ( $S = 4\pi r^2$ ) with respect to the radius when  $r = 2$  m.

- A)  $16\pi \text{ m}^2/\text{m}$
- B)  $8\pi \text{ m}^2/\text{m}$
- C)  $12 \text{ m}^2/\text{m}$
- D)  $24 \text{ m}^2/\text{m}$
- E)  $4\pi \text{ m}^2/\text{m}$

Answer: A

Diff: 1

4) Find the rate of change of the area of a circle with respect to its circumference C.

- A)  $\frac{1}{\pi}C$
- B)  $\frac{1}{2\pi}C$
- C)  $\frac{3}{2\pi}C$
- D)  $\frac{3}{\pi}C$
- E)  $\frac{\pi}{2}C$

Answer: B

Diff: 1

5) The electrical resistance R of a wire of unit length and cross-sectional radius x is given by  $R = \frac{K}{x^2}$ , where K is a non-zero constant real number. By approximately what percentage does the

resistance R change if the diameter of the wire is decreased by 6%?

- A) -6%
- B) -9%
- C) 12%
- D) 6%
- E) -12%

Answer: C

Diff: 2

6) The cost in dollars for a company to produce x pairs of shoes is  $C(x) = 2000 + 3x + 0.01x^2 + 0.0002x^3$ . Find the marginal cost function.

- A)  $C'(x) = 1 + 0.02x + 0.0006x^2$
- B)  $C'(x) = 1 + 0.01x + 0.0002x^2$
- C)  $C'(x) = 3 + 0.02x + 0.0003x^2$
- D)  $C'(x) = 3 + 0.02x + 0.0006x^2$
- E)  $C'(x) = 3 + 0.01x + 0.0006x^2$

Answer: D

Diff: 2

7) The population (in thousands) of the city of Abbotsford is given by  $P(t) = 100[1 + (0.04)t + (0.003)t^2]$ , with  $t$  in years and with  $t = 0$  corresponding to 1980. What was the rate of change of  $P$  in 1986?

- A) 9.6 thousand per year
- B) 8.6 thousand per year
- C) 7.6 thousand per year
- D) 8.9 thousand per year
- E) 4.4 thousand per year

Answer: C

Diff: 2

8) The daily cost of production of  $x$  widgets in a widget factory is  $C$  dollars, where

$C = 40,000 + 2x + \frac{x^2}{100}$ . What is the cost per widget, and what value of  $x$  will make the cost per

widget as small as possible?

- A)  $\frac{C}{x}$  dollars,  $x = 2,000$
- B)  $\frac{C}{x^2}$  dollars,  $x = 4,000$
- C)  $\frac{C}{x}$  dollars,  $x = 8,000$
- D)  $\frac{C}{x^2}$  dollars,  $x = 400$
- E)  $\frac{C}{x}$  dollars,  $x = 4,000$

Answer: A

Diff: 2

9) If the cost of mining  $x$  kg of gold is  $C(x) = A + Bx + Cx^2$  dollars where  $A$ ,  $B$ , and  $C$  are positive constants, which of the following statements is true for a given positive value of  $x$ ?

- A) The marginal cost  $C'(x)$  is greater than the cost  $C(x + 1) - C(x)$  of mining 1 more kg.
- B) The marginal cost  $C'(x)$  is less than the cost  $C(x + 1) - C(x)$  of mining 1 more kg.
- C) The marginal cost  $C'(x)$  is equal to the cost  $C(x + 1) - C(x)$  of mining 1 more kg.
- D) There is not enough information to make any conclusion.
- E) None of the above

Answer: B

Diff: 2

10) By approximately what percentage does the volume of a cube change if the edge length changes by 1%?

- A) 3%
- B) 1%
- C) 2%
- D) 4%
- E)  $\frac{1}{3}$ %

Answer: A

Diff: 2

11) The pressure difference  $P$  between the ends of a small pipe of radius  $x$  is given by  $P = \frac{k}{x^4}$ ,

where  $k$  is a non-zero constant real number. Use differentials to determine by approximately what percentage the pressure changes if the radius of the pipe is decreased from 2 to 1.96 units.

- A) 20 %
- B) -16 %
- C) 16 %
- D) -8 %
- E) 8 %

Answer: E

Diff: 2

## 2.8 The Mean-Value Theorem

1) Given  $f(x) = 5 - (4/x)$ , find all values of  $c$  in the open interval  $(1, 4)$  such that

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}.$$

- A) only  $\frac{5}{2}$
- B) only 2
- C) only 2 and 3
- D) only 3
- E) 2, 3, and  $\frac{5}{2}$

Answer: B

Diff: 1

2) Given  $f(x) = x^2 - 6x + 12$ , find the value of  $c$  in the open interval  $(4, 7)$  that satisfies

$$f'(c) = \frac{f(7) - f(4)}{7 - 4}.$$

- A)  $\frac{11}{2}$
- B)  $\frac{13}{2}$
- C) 5
- D) 6
- E)  $\frac{5}{2}$

Answer: A

Diff: 1

3) Where is the function  $f(x) = 3x^2 + 12x + 7$  increasing, and where is it decreasing?

- A) decreasing on  $(-\infty, -2)$  and increasing on  $(-2, \infty)$
- B) increasing on  $(-\infty, -2)$  and decreasing on  $(-2, \infty)$
- C) decreasing on  $(-\infty, -4)$  and increasing on  $(-4, \infty)$
- D) increasing on  $(-\infty, \infty)$
- E) increasing on  $(-\infty, -4)$  and decreasing on  $(-4, \infty)$

Answer: A

Diff: 1

4) Determine the open intervals on the  $x$ -axis on which the function  $f(x) = 4x^3 - x^4$  is increasing and those on which it is decreasing.

- A) increasing on  $(-\infty, 3)$  and decreasing on  $(3, \infty)$
- B) increasing on  $(-\infty, 0)$  and decreasing on  $(0, \infty)$
- C) increasing on  $(0, 3)$  and decreasing on  $(-\infty, 0)$  and  $(3, \infty)$
- D) decreasing on  $(0, 3)$  and increasing on  $(-\infty, 0)$  and  $(3, \infty)$
- E) decreasing on  $(-\infty, 3)$  and increasing on  $(3, \infty)$

Answer: A

Diff: 1

5) Determine the open intervals on the  $x$ -axis on which the function  $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$  is increasing and those on which it is decreasing.

- A) increasing on  $(-1, 0)$  and  $(2, \infty)$ , and decreasing on  $(-\infty, -1)$  and  $(0, 2)$
- B) decreasing on  $(-1, 0)$  and  $(2, \infty)$ , and increasing on  $(-\infty, -1)$  and  $(0, 2)$
- C) increasing on  $(-\infty, 2)$ , and decreasing on  $(2, \infty)$
- D) decreasing on  $(-\infty, -1)$ , and increasing on  $(-1, \infty)$
- E) decreasing on  $(-\infty, 2)$ , and increasing on  $(2, \infty)$

Answer: A

Diff: 2



6) If  $f'(x) = 0$  throughout an interval  $[a, b]$ , prove that  $f(x)$  is constant on the interval.

Answer: If  $a < x \leq b$ , the mean-value theorem assures us that  $f(x) - f(a) = f'(t)(x - a)$  for some  $t$  in  $(a, x)$ .

Since  $f'(t) = 0$ , we must have  $f(x) = f(a)$ . This is true for all  $x$  in  $[a, b]$ .

Diff: 3

7) True or False: The function  $f(x) = x^3$  is nondecreasing but is not increasing on its domain.

Answer: FALSE

Diff: 3

8) True or False: If  $f'(x) = 0$  at every point of the domain of  $f$ , then  $f$  is constant on its domain.

Answer: FALSE

Diff: 3

9) True or False: If function  $f$  is defined on  $[a, b]$  (where  $a < b$ ) and is differentiable on  $(a, b)$ , then there must exist a number  $c$  in  $(a, b)$  such that  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .

Answer: FALSE

Diff: 3

## 2.9 Implicit Differentiation

1) Use implicit differentiation to find  $\frac{dy}{dx}$  if  $x^3 + y^3 = 6xy$ .

A)  $\frac{2y + x^2}{y^2 - 2x}$

B)  $\frac{2y - x^2}{y^2 - 2x}$

C)  $\frac{2y - x^2}{y^2 + 2x}$

D)  $\frac{2y - 2x^2}{y^2 - 2x}$

E)  $\frac{2y - x^2}{y^2 - x}$

Answer: B

Diff: 1

2) Find  $\frac{dy}{dx}$  if  $y^3 + y^2 - 5y - x^2 = -4$ .

A)  $-\frac{4x}{3y^2 + 2y - 5}$

B)  $\frac{4x}{3y^2 + 2y - 5}$

C)  $\frac{2x}{3y^2 + 2y - 5}$

D)  $-\frac{2x}{3y^2 + 2y - 5}$

E) none of the above

Answer: C

Diff: 1

3) Find the slope of the curve  $x^{3/4} + y^{3/4} = 9$  at the point (1, 16).

A) 2

B) -2

C)  $\frac{1}{2}$

D)  $-\frac{1}{2}$

E) 1

Answer: B

Diff: 2

4) Find an equation of the line normal to the curve  $y^4 + 3x^3y^2 - 2x^5 = 2$  at the point (1, 1) on the curve.

A)  $10x - y - 11 = 0$

B)  $x - 2y + 1 = 0$

C)  $10x + y - 11 = 0$

D)  $x - 10y + 9 = 0$

E)  $8x + y - 9 = 0$

Answer: C

Diff: 2

5) Find an equation of the line tangent to the curve  $2xy^2 + x^2y = 6$  at the point (1, -2).

A)  $4x - 7y = 18$

B)  $4x + 7y = -10$

C)  $7x + 4y = -1$

D)  $7x - 4y = 15$

E) none of the above

Answer: A

Diff: 2

6) Find the slope of the curve  $\tan(2x + y) = x$  at  $(0, 0)$ .

- A) 1
- B) 2
- C) -2
- D) -1
- E) 0

Answer: D

Diff: 2

7) If  $x \sin y = \cos(x + y)$ , then calculate  $\frac{dy}{dx}$ .

- A)  $-\frac{\sin y - \sin(x + y)}{x \cos y + \sin(x + y)}$
- B)  $\frac{\sin y + \sin(x + y)}{x \cos y + \sin(x + y)}$
- C)  $-\frac{\sin y + \sin(x + y)}{x \cos y + \sin(x + y)}$
- D)  $\frac{\sin y - \sin(x + y)}{x \cos y + \sin(x + y)}$
- E)  $\frac{\sin y + \cos(x + y)}{x \cos y + \sin(x + y)}$

Answer: C

Diff: 2

8) Find all points on the graph  $x^2 + y^2 = 4x + 4y$  at which the tangent line is horizontal.

- A)  $(2, 2 - \sqrt{8})$  and  $(2, 2 + \sqrt{8})$
- B)  $(1, 2 - \sqrt{8})$  and  $(2, 2 + \sqrt{8})$
- C)  $(2, 2 - \sqrt{8})$  and  $(1, 2 + \sqrt{8})$
- D)  $(0, 0)$  and  $(0, 4)$
- E) The tangent line is never horizontal.

Answer: A

Diff: 2

9) Given the curve  $x^3 + y^3 = 9xy$ , find (a) the equation of its tangent line at the point  $(2, 4)$  and (b) the equation of its tangent line with slope -1.

Answer: (a)  $5y = 4x + 12$  (b)  $y - \frac{9}{2} = -(x - \frac{9}{2})$

Diff: 3

10) Show that the curves given by  $y^2 + 4y + 4x + 3 = 0$  and  $y^2 = 4x + 1$  have a common tangent line and determine its equation.

Answer:  $y = -1 - 2x$

Diff: 3

11) Find the value of  $\frac{d^2y}{dx^2}$  at (2, 4) if  $x^3 + y^3 = 9xy$ .

A)  $-\frac{54}{125}$

B)  $\frac{108}{25}$

C)  $\frac{54}{25}$

D)  $\frac{78}{125}$

E)  $-\frac{54}{25}$

Answer: A

Diff: 3

12) If  $2x^2 + y^2 = 1$ , express  $\frac{d^2y}{dx^2}$  in terms of  $y$  alone.

A)  $-\frac{1}{y^2}$

B)  $-\frac{2}{y^3}$

C)  $-\frac{2}{y^2}$

D)  $\frac{1}{y^4}$

E)  $\frac{2}{y^2}$

Answer: B

Diff: 3

13) Find  $\frac{dy}{dx}$  if  $x^3y + \cos(y) = \sin(x)$ .

A)  $\frac{\cos(x) - 3x^2y}{x^3 - \sin(y)}$

B)  $\frac{\cos(x)}{3x^2y - \sin(x)}$

C)  $\frac{\cos(x) + 3x^2y}{x^3 - \sin(y)}$

D)  $\frac{-\cos(x) + \sin(y)}{3x^2y}$

E)  $\frac{\cos(x) - 3x^2y}{x^3 + \sin(y)}$

Answer: A

Diff: 2

14) Find the value of  $y''$  at  $\left(0, \frac{\pi}{3}\right)$  given that  $x + \sqrt{3} = 2\sin(2x + y)$ .

A)  $-\sqrt{3}$

B) -1

C)  $\sqrt{3}$

D) 1

E)  $y''$  is not defined at  $x = 0$

Answer: C

Diff: 2

15) Find the value of  $y''$  at  $(1, 2)$  given that  $x^3y + xy^2 = 6$ .

A)  $\frac{5}{8}$

B)  $\frac{8}{5}$

C) -1

D)  $\frac{1}{3}$

E) 3

Answer: B

Diff: 2

16) Find the points at which the curve  $5x^2 + 6xy + 5y^2 = 80$  has horizontal tangent lines.

A) (3, -5) and (-3, 5)

B) (3, 5), (5, -3), (-3, 5), and (-5, -3)

C) (5, -3) and (-5, 3)

D) (3, 5), (3, -5), (-3, 5), and (-3, -5)

E) The curve does not have horizontal tangent lines.

Answer: A

Diff: 2

17) At how many points does the curve  $x^3 y + xy^2 = 6$  have a horizontal tangent line?

- A) no points
- B) 1 point
- C) 2 points
- D) 3 points
- E) more than 3 points

Answer: A

Diff: 2

18) At how many points does the curve  $x^3 y + xy^2 = 6$  have a vertical tangent line?

- A) no points
- B) 1 point
- C) 2 points
- D) 3 points
- E) more than 3 points

Answer: B

Diff: 2

19) At how many points does the curve  $x^3 + y^3 = 1$  have a tangent line with a slope of -1?

- A) no points
- B) 1 point
- C) 2 points
- D) 3 points
- E) more than 3 points

Answer: B

Diff: 2

## 2.10 Antiderivatives and Initial-Value Problems

1) Evaluate  $\int (x + 2) dx$ .

A)  $\frac{x^2}{2} - 2x + C$

B)  $\frac{x^2}{2} + 2x + C$

C)  $x^2 + 3x + C$

D)  $\frac{x^2}{3} + 2x + C$

E)  $x^2 + 2x + C$

Answer: B

Diff: 1

2) Evaluate  $\int (2x^3 - 5x^2 + 3x + 1) dx$ .

A)  $\frac{1}{2}x^4 + \frac{5}{3}x^3 + \frac{3}{2}x^2 + x + C$

B)  $\frac{1}{3}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 - x + C$

C)  $\frac{1}{2}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + x + C$

D)  $\frac{1}{3}x^4 - \frac{5}{3}x^3 + \frac{3}{4}x^2 + x + C$

E)  $2x^4 - 5x^3 + 3x^2 + x + C$

Answer: C

Diff: 1

3) Evaluate  $\int 4\sqrt[3]{x^2} dx$ .

A)  $\frac{12}{5}x^{5/3} + C$

B)  $3x^{4/3} + C$

C)  $4x + C$

D)  $\frac{12}{9}x^{5/3} + C$

E)  $x + C$

Answer: A

Diff: 1

4) Calculate  $\int \left( \frac{1}{x^3} - \frac{1}{x^5} \right) dx$ .

A)  $\frac{1}{3x^4}(2x^2 - 1) + C$

B)  $\frac{1}{12x^6}(3x^2 - 2) + C$

C)  $\frac{1}{4x^4}(1 + 2x^2) + C$

D)  $\frac{1}{4x^4}(1 - 2x^2) + C$

E)  $\frac{1}{3x^4}(2x^2 + 1) + C$

Answer: D

Diff: 2

5) Evaluate  $\int \frac{3x^2 - 2x + 1}{\sqrt{x}} dx$ .

A)  $\frac{2}{15}\sqrt{x}(9x^2 - 10x + 15) + C$

B)  $\frac{1}{15}\sqrt{x}(9x^2 + 10x + 15) + C$

C)  $\frac{1}{15}\sqrt{x}(9x^2 - 10x + 15) + C$

D)  $\frac{3}{15}\sqrt{x}(9x^2 - 10x + 15) + C$

E)  $\frac{2}{15}\sqrt{x}(9x^2 + 10x + 15) + C$

Answer: A

Diff: 2

6) Evaluate  $\int \frac{\sin x}{\cos^2 x} dx$ .

A)  $\sec x + C$

B)  $\sin x + C$

C)  $\cos x + C$

D)  $\tan x + C$

E)  $\cot x + C$

Answer: A

Diff: 2

7) True or false: The function  $F(x) = \frac{1}{3}x^3 - (2 - x)^2 + 1$  is an antiderivative of the function  $f(x) = x^2$

$-2x + 4$ .

Answer: TRUE

Diff: 1

8) Solve the initial value problem  $\frac{dy}{dx} = \frac{1}{\sqrt{x-13}}$ ;  $y(17) = 2$ .

A)  $y(x) = 2\sqrt{x-13} - 2$ , for  $x > 13$

B)  $y(x) = \sqrt{x-13} - 2$ , for  $x \neq 13$

C)  $y(x) = 4\sqrt{x-13} - 2$ , for  $x \neq 13$

D)  $y(x) = 13\sqrt{x-13} - 2$ , for  $x \geq 13$

E)  $y(x) = 2\sqrt{x-13} - 2$ , for  $x \geq 13$

Answer: A

Diff: 2



9) Find the curve  $y = F(x)$  that passes through  $(-1, 0)$  and satisfies  $\frac{dy}{dx} = 6x^2 + 6x$ .

- A)  $y = x^3 + 3x^2 - 2$
- B)  $y = 2x^3 + 3x^2 - 1$
- C)  $y = 3x^3 + 3x^2$
- D)  $y = 3x^3 + 2x^2 + 1$
- E)  $y = 2x^3 + 3x^2 + 1$

Answer: B

Diff: 3

10) The line  $x + y = \frac{4}{3}$  is tangent to the graph of  $y = F(x)$  where  $F$  is an antiderivative of  $-x^2$ .

Find  $F$ .

Answer:  $F(x) = \frac{1}{3}(2 - x^3)$  or  $F(x) = 2 - \frac{x^3}{3}$  line  $x + y = \frac{4}{3}$  is tangent to the graph of  $y = F(x)$

Diff: 3

11) Find all antiderivatives  $F(x)$  of  $-x^2$  such that the line  $x + y = \frac{4}{3}$  is tangent to the graph of  $y = F(x)$ .

A)  $F(x) = \frac{1}{3}(2 - x^3)$  and  $F(x) = 2 - \frac{x^3}{3}$

B)  $F(x) = \frac{1}{3}(2 - x^3)$

C)  $F(x) = 2 - \frac{x^3}{3}$

D)  $F(x) = A - \frac{x^3}{3}$  for any number  $A$

E) none of the above

Answer: A

Diff: 3

12) True or False: The functions  $F_1(x) = -\cos(4x) + 34$ ,  $F_2(x) = 2\sin^2(2x) - 15$ , and  $F_3(x) = 7 - 2\cos^2(2x)$  are all antiderivatives of  $f(x) = 8 \sin(2x) \cos(2x)$ .

Answer: TRUE

Diff: 2

13) Find the solution of the differential equation  $\frac{dy}{dx} = \frac{7}{3} \frac{(1-x)^2}{x^{2/3}}$  such that  $y = 85$  when

$x = 8$ .

A)  $y = 7x^{1/3} - \frac{7}{2}x^{4/3} + x^{7/3} - 1$

B)  $y = 7x^{1/3} - \frac{7}{2}x^{4/3} + x^{7/3} + 1$

C)  $y = 7x^{1/3} - \frac{7}{2}x^{4/3} + \frac{1}{2}x^{7/3} + 63$

D)  $y = -7x^{1/3} + \frac{7}{2}x^{4/3} - x^{7/3} + 171$

E) none of the above

Answer: A

Diff: 2

14) Evaluate  $\int x \left( 3\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx$

A)  $\frac{x^2}{2} (2x^{3/2} - 2\sqrt{x})^2 + C$

B)  $2 \left( 3\sqrt{x} - \frac{1}{\sqrt{x}} \right) + C$

C)  $3x^3 - 3x^2 + C$

D)  $3x^3 - 3x^2 + x + C$

E)  $3x^3 - x + C$

Answer: D

Diff: 1

### 2.11 Velocity and Acceleration

1) An object moves in a straight line with velocity  $v = 6t - 3t^2$  where  $v$  is measured in metres per second and  $t$  is in seconds.

(a) How far does the object move in the first second?

(b) The object is back where it started when  $t = 3$ . How far did it travel to get there?

A) (a) 2 m (b) 8 m

B) (a) 2 m (b) 0 m

C) (a) 2 m (b) 4 m

D) (a) 1 m (b) 2 m

E) (a) 2 m (b) 6 m

Answer: A

Diff: 1

2) A car is accelerating in a straight line so that its velocity in metres per second is equal to the square root of the time elapsed, measured in seconds. How far has the car travelled in the first hour?

- A) 144 km
- B) 120 km
- C) 244 km
- D) 344 km
- E) 169 km

Answer: A

Diff: 1

3) A rock is thrown upwards at 5 m/s from the top of a building 80 m high. When will it hit the ground (to the nearest tenth of a second)?

- A) 4.6 s
- B) 4.1 s
- C) 3.6 s
- D) 3.1 s
- E) 5.1 s

Answer: A

Diff: 2

4) A particle moves along the x-axis so that its velocity at time  $t$  is  $v(t) = t^2 - t - 6$  (in metres per second).

Find, to the nearest tenth of a metre,

- (a) the displacement of the particle during the time period  $1 \leq t \leq 4$ .
- (b) the distance travelled by the particle during this time period.

- A) (a) 4.5 m to the left, (b) 10.2 m
- B) (a) 4.5 m to the right, (b) 10.2 m
- C) (a) 12.0 m to the left, (b) 20.5 m
- D) (a) 12.0 m to the right, (b) 20.5 m
- E) none of the above

Answer: A

Diff: 2

5) The brakes of a car were fully applied for a distance of 125 m before the car came to a stop. Suppose that the car in question has a constant deceleration of  $10 \text{ m/s}^2$ . How fast was the car travelling when its brakes were first applied?

- A) 40 m/s
- B) 35 m/s
- C) 25 m/s
- D) 50 m/s
- E) 30 m/s

Answer: D

Diff: 3

6) A ball is thrown vertically upward from the top of a building that is 57 metres high. The height of the ball above ground at  $t$  seconds later is given by  $h = 57 + 12t - 9.8t^2$  m. Find the initial speed of the ball.

- A) 6 m/s
- B) 12 m/s
- C) 9.8 m/s
- D) 57 m/s
- E) 19.6 m/s

Answer: B

Diff: 1

7) At time  $t = 0$  seconds, a ball is thrown vertically upward from the top of a building that is 76 metres high. The height of the ball above ground  $t$  seconds later is given by  $h = 76 + 39.2t - 9.8t^2$  m (until the ball hits the ground). What is the maximum height reached by the ball?

- A) 120.6 m
- B) 115.2 m
- C) 112.3 m
- D) 98.7 m
- E) 138.3 m

Answer: B

Diff: 2

8) At time  $t = 0$  seconds, a ball is thrown vertically upward from the top of a building that is 76 metres high. The height of the ball above ground  $t$  seconds later is given by  $h = 76 + 39.2t - 9.8t^2$  m (until the ball hits the ground). At what speed does it hit the ground (to the nearest tenth of a metre per second)?

- A) 200.0 m/s
- B) 230.4 m/s
- C) 215.6 m/s
- D) 245.9 m/s
- E) 237.8 m/s

Answer: B

Diff: 2