MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the average velocity of the function over the given interval.

1) $y = x^2 + 3x$, $[1, 8]$

    A) $\dfrac{88}{7}$ B) $\dfrac{21}{2}$ C) 11   D) 12

1) _____

2) $y = 9x^3 + 5x^2 - 8$, $[-2, 8]$

    A) 492   B) $\dfrac{1245}{2}$ C) 615   D) 498

2) _____

3) $y = \sqrt{2x}$, $[2, 8]$

    A) 2   B) $\dfrac{1}{3}$ C) 7   D) $-\dfrac{3}{10}$

3) _____

4) $y = \dfrac{3}{x - 2}$, $[4, 7]$

    A) $-\dfrac{3}{10}$ B) 7   C) $\dfrac{1}{3}$ D) 2

4) _____

5) $y = 4x^2$, $\left[0, \dfrac{7}{4}\right]$

    A) $\dfrac{1}{3}$ B) 2   C) $-\dfrac{3}{10}$ D) 7

5) _____

6) $y = -3x^2 - x$, $[5, 6]$

    A) $\dfrac{1}{2}$ B) -2   C) $-\dfrac{1}{6}$ D) -34

6) _____

7) $h(t) = \sin(3t)$, $\left[0, \dfrac{\pi}{6}\right]$

    A) $-\dfrac{6}{\pi}$ B) $\dfrac{\pi}{6}$ C) $\dfrac{3}{\pi}$ D) $\dfrac{6}{\pi}$

7) _____

8) $g(t) = 4 + \tan t$, $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

    A) $-\dfrac{3}{2}$ B) $-\dfrac{4}{\pi}$ C) $\dfrac{4}{\pi}$ D) 0

8) _____

Use the table to find the instantaneous velocity of y at the specified value of x.

9) x = 1.

9) _____

x	y
0	0
0.2	0.02
0.4	0.08
0.6	0.18
0.8	0.32
1.0	0.5
1.2	0.72
1.4	0.98

A) 0.5   B) 2   C) 1   D) 1.5

10) x = 1.

10) _____

x	y
0	0
0.2	0.01
0.4	0.04
0.6	0.09
0.8	0.16
1.0	0.25
1.2	0.36
1.4	0.49

A) 1.5   B) 2   C) 1   D) 0.5

11) x = 1.

11) _____

x	y
0	0
0.2	0.12
0.4	0.48
0.6	1.08
0.8	1.92
1.0	3
1.2	4.32
1.4	5.88

A) 8   B) 6   C) 2   D) 4

12) x = 2.

x	y
0	10
0.5	38
1.0	58
1.5	70
2.0	74
2.5	70
3.0	58
3.5	38
4.0	10

A) 0   B) 4   C) 8   D) -8

12) _____

13) x = 1.

x	y
0.900	-0.05263
0.990	-0.00503
0.999	-0.0005
1.000	0.0000
1.001	0.0005
1.010	0.00498
1.100	0.04762

A) 0.5   B) -0.5   C) 0   D) 1

13) _____

Find the slope of the curve for the given value of x.

14) $y = x^2 + 5x$, x = 4

A) slope is $\dfrac{1}{20}$ B) slope is -39   C) slope is $-\dfrac{4}{25}$ D) slope is 13

14) _____

15) $y = x^2 + 11x - 15$, x = 1

A) slope is $\dfrac{1}{20}$ B) slope is -39   C) slope is $-\dfrac{4}{25}$ D) slope is 13

15) _____

16) $y = x^3 - 5x$, x = 1

A) slope is -3   B) slope is 1   C) slope is -2   D) slope is 3

16) _____

17) $y = x^3 - 2x^2 + 4$, x = 3

A) slope is 1   B) slope is -15   C) slope is 15   D) slope is 0

17) _____

18) $y = 2 - x^3$, x = -1

A) slope is 0   B) slope is 3   C) slope is -3   D) slope is -1

18) _____

Solve the problem.

19) Given $\lim\limits_{x \to 0^-} f(x) = L_l$, $\lim\limits_{x \to 0^+} f(x) = L_r$, and $L_l \neq L_r$, which of the following statements is true?

    I. $\lim\limits_{x \to 0} f(x) = L_l$

    II. $\lim\limits_{x \to 0} f(x) = L_r$

    III. $\lim\limits_{x \to 0} f(x)$ does not exist.

    A) none   B) I   C) III   D) II

19) _____

20) Given $\lim\limits_{x \to 0^-} f(x) = L_l$, $\lim\limits_{x \to 0^+} f(x) = L_r$, and $L_l = L_r$, which of the following statements is false?

    I. $\lim\limits_{x \to 0} f(x) = L_l$

    II. $\lim\limits_{x \to 0} f(x) = L_r$

    III. $\lim\limits_{x \to 0} f(x)$ does not exist.

    A) I   B) III   C) II   D) none

20) _____

21) If $\lim\limits_{x \to 0} f(x) = L$, which of the following expressions are true?

    I. $\lim\limits_{x \to 0^-} f(x)$ does not exist.

    II. $\lim\limits_{x \to 0^+} f(x)$ does not exist.

    III. $\lim\limits_{x \to 0^-} f(x) = L$

    IV. $\lim\limits_{x \to 0^+} f(x) = L$

   A) I and II only   B) I and IV only   C) III and IV only   D) II and III only

21) _____

22) What conditions, when present, are sufficient to conclude that a function $f(x)$ has a limit as x approaches some value of a?

    A) Either the limit of $f(x)$ as $x \to a$ from the left exists or the limit of $f(x)$ as $x \to a$ from the right exists

    B) $f(a)$ exists, the limit of $f(x)$ as $x \to a$ from the left exists, and the limit of $f(x)$ as $x \to a$ from the right exists.

    C) The limit of $f(x)$ as $x \to a$ from the left exists, the limit of $f(x)$ as $x \to a$ from the right exists, and at least one of these limits is the same as $f(a)$.

    D) The limit of $f(x)$ as $x \to a$ from the left exists, the limit of $f(x)$ as $x \to a$ from the right exists, and these two limits are the same.

22) _____

4

Use the graph to evaluate the limit.

23) $\lim\limits_{x \to -1} f(x)$

23) _____
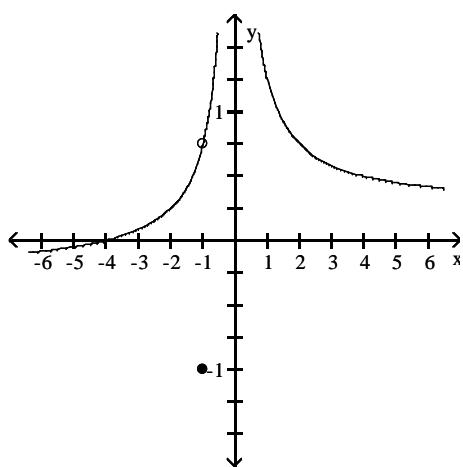
A) $\dfrac{3}{4}$ B) ∞ C) -1   D) $-\dfrac{3}{4}$

24) $\lim\limits_{x \to 0} f(x)$

24) _____
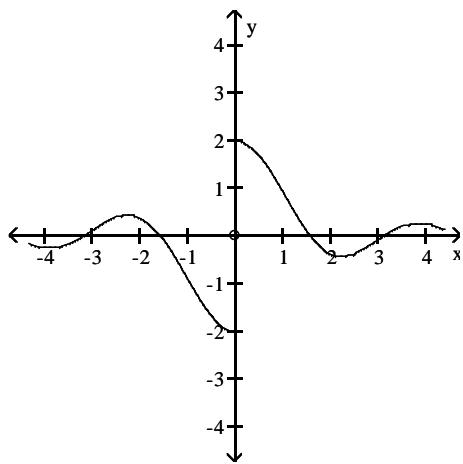
A) -2   B) 2   C) 0   D) does not exist

25) $\lim\limits_{x \to 0} f(x)$
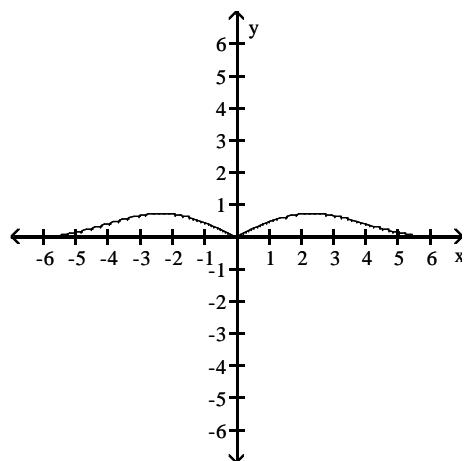
A) 0   B) does not exist   C) -1   D) 1

26) $\lim\limits_{x \to 0} f(x)$
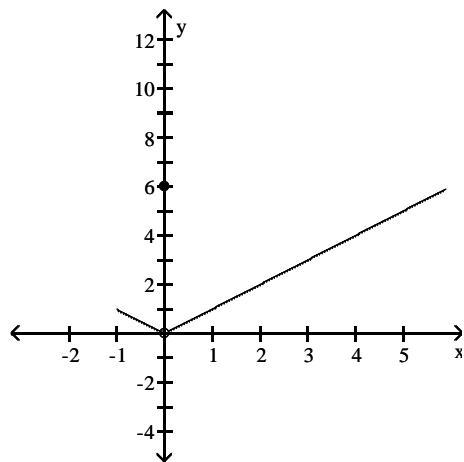
A) -1   B) 6   C) 0   D) does not exist

25) _____

26) _____

6

27) lim f(x)
     x→0
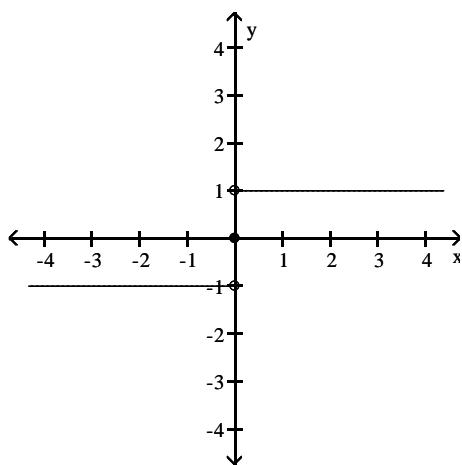
A) -1   B) does not exist   C) 1   D) ∞

28) lim f(x)
     x→0
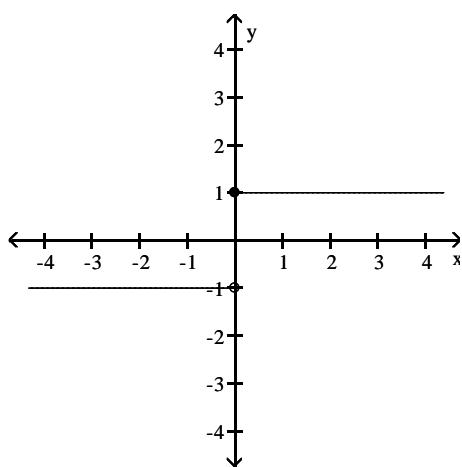
A) ∞   B) does not exist   C) -1   D) 1

7

29) $\lim\limits_{x \to 0} f(x)$
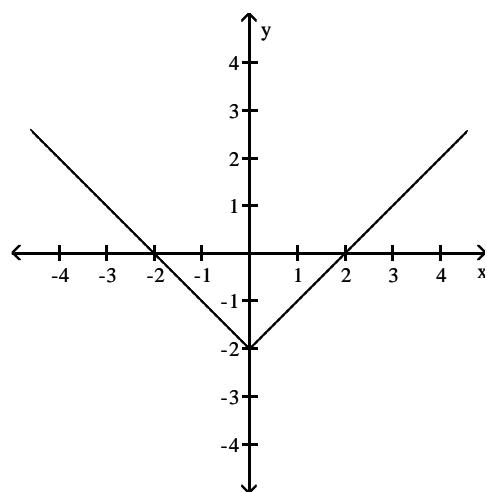
A) 2   B) does not exist   C) 0   D) -2

29) _____

30) $\lim\limits_{x \to 0} f(x)$
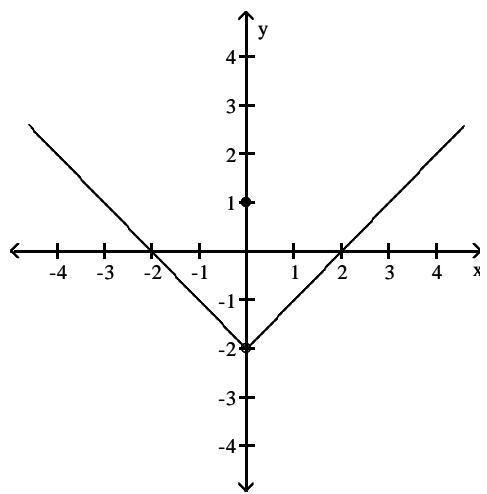
A) does not exist   B) 0   C) -2   D) 1

30) _____

8

31) $\lim\limits_{x \to 0} f(x)$

31) _____
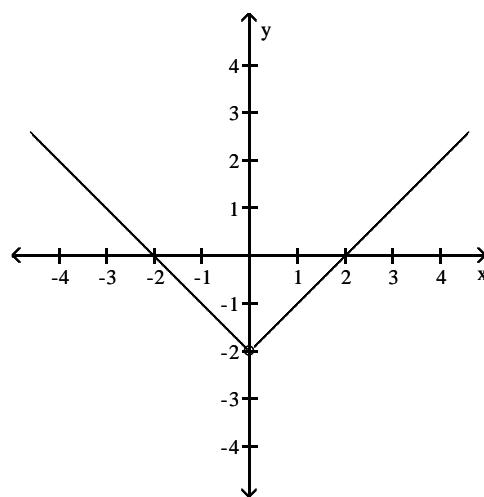
A) 2   B) -1   C) does not exist   D) -2

32) Find $\lim\limits_{x \to (-1)^-} f(x)$ and $\lim\limits_{x \to (-1)^+} f(x)$

32) _____
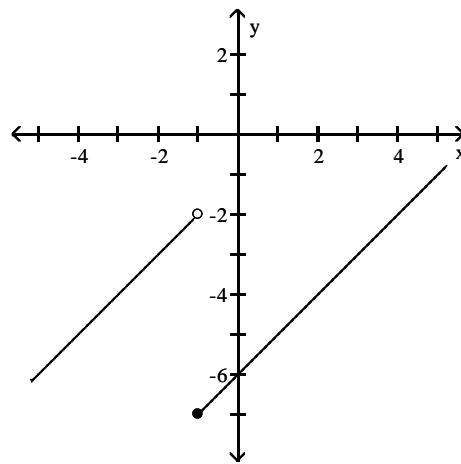
A) -7; -5   B) -7; -2   C) -5; -2   D) -2; -7

Use the table of values of f to estimate the limit.

33) Let $f(x) = x^2 + 8x - 2$, find $\lim\limits_{x \to 2} f(x)$.

33) _____

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)						

A)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	5.043	5.364	5.396	5.404	5.436	5.763

; limit = ∞

B)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	5.043	5.364	5.396	5.404	5.436	5.763

; limit = 5.40

C)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	16.810	17.880	17.988	18.012	18.120	19.210

; limit = 18.0

D)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	16.692	17.592	17.689	17.710	17.808	18.789

; limit = 17.70

34) Let $f(x) = \dfrac{x - 4}{\sqrt{x} - 2}$, find $\lim\limits_{x \to 4} f(x)$.

34) _____

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

A)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	5.07736	5.09775	5.09978	5.10022	5.10225	5.12236

; limit = 5.10

B)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	3.97484	3.99750	3.99975	4.00025	4.00250	4.02485

; limit = 4.0

C)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	1.19245	1.19925	1.19993	1.20007	1.20075	1.20745

; limit = 1.20

D)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	1.19245	1.19925	1.19993	1.20007	1.20075	1.20745

; limit = ∞

35) Let f(x) = x² - 5, find lim f(x).
                                 x→0

35) _____

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

A)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-4.9900	-4.9999	-5.0000	-5.0000	-4.9999	-4.9900

; limit = -5.0

B)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-2.9910	-2.9999	-3.0000	-3.0000	-2.9999	-2.9910

; limit = -3.0

C)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit = -15.0

D)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit = ∞

36) Let f(x) = $\dfrac{x + 2}{x^2 + 5x + 6}$, find lim f(x).
                                              x→-2

36) _____

x	-2.1	-2.01	-2.001	-1.999	-1.99	-1.9
f(x)						

A)

x	-2.1	-2.01	-2.001	-1.999	-1.99	-1.9
f(x)	1.0111	0.9101	0.9010	0.8990	0.8901	0.8091

; limit = 0.9

B)

x	-2.1	-2.01	-2.001	-1.999	-1.99	-1.9
f(x)	-1.1111	-1.0101	-1.0010	-0.9990	-0.9901	-0.9091

; limit = -1

C)

x	-2.1	-2.01	-2.001	-1.999	-1.99	-1.9
f(x)	1.1111	1.0101	1.0010	0.9990	0.9901	0.9091

; limit = 1

D)

x	-2.1	-2.01	-2.001	-1.999	-1.99	-1.9
f(x)	1.2111	1.1101	1.1010	1.0990	1.0901	1.0091

; limit = 1.1

37) Let $f(x) = \dfrac{x^2 + 3x + 2}{x^2 - 2x - 3}$, find $\lim\limits_{x \to -1} f(x)$.

x	-1.1	-1.01	-1.001	-0.999	-0.99	-0.9
f(x)						

A)

x	-1.1	-1.01	-1.001	-0.999	-0.99	-0.9	; limit = -1.5
f(x)	-1.3810	-1.4876	-1.4988	-1.5013	-1.5126	-1.6316	

B)

x	-1.1	-1.01	-1.001	-0.999	-0.99	-0.9	; limit = -0.15
f(x)	-0.1195	-0.1469	-0.1497	-0.1503	-0.1531	-0.1821	

C)

x	-1.1	-1.01	-1.001	-0.999	-0.99	-0.9	; limit = -0.25
f(x)	-0.2195	-0.2469	-0.2497	-0.2503	-0.2531	-0.2821	

D)

x	-1.1	-1.01	-1.001	-0.999	-0.99	-0.9	; limit = -0.35
f(x)	-0.3195	-0.3469	-0.3497	-0.3503	-0.3531	-0.3821	

38) Let $f(x) = \dfrac{\sin(8x)}{x}$, find $\lim\limits_{x \to 0} f(x)$.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)		7.9914694			7.9914694	

A) limit = 7.5   B) limit does not exist
C) limit = 0   D) limit = 8

39) Let $f(\theta) = \dfrac{\cos(6\theta)}{\theta}$, find $\lim\limits_{\theta \to 0} f(\theta)$.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(θ)	-8.2533561					8.2533561

A) limit = 0   B) limit does not exist
C) limit = 8.2533561   D) limit = 6

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

40) It can be shown that the inequalities $1 - \dfrac{x^2}{6} < \dfrac{x \sin(x)}{2 - 2\cos(x)} < 1$ hold for all values of x close

to zero. What, if anything, does this tell you about $\dfrac{x \sin(x)}{2 - 2\cos(x)}$? Explain.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

41) Write the formal notation for the principle "the limit of a quotient is the quotient of the limits" and include a statement of any restrictions on the principle.   41) _____

   A) $\lim\limits_{x\to a} \dfrac{g(x)}{f(x)} = \dfrac{g(a)}{f(a)}$, provided that $f(a) \neq 0$.

   B) If $\lim\limits_{x\to a} g(x) = M$ and $\lim\limits_{x\to a} f(x) = L$, then $\lim\limits_{x\to a} \dfrac{g(x)}{f(x)} = \dfrac{\lim\limits_{x\to a} g(x)}{\lim\limits_{x\to a} f(x)} = \dfrac{M}{L}$, provided that

   $f(a) \neq 0$.

   C) If $\lim\limits_{x\to a} g(x) = M$ and $\lim\limits_{x\to a} f(x) = L$, then $\lim\limits_{x\to a} \dfrac{g(x)}{f(x)} = \dfrac{\lim\limits_{x\to a} g(x)}{\lim\limits_{x\to a} f(x)} = \dfrac{M}{L}$, provided that

   $L \neq 0$.

   D) $\lim\limits_{x\to a} \dfrac{g(x)}{f(x)} = \dfrac{g(a)}{f(a)}$.

42) Provide a short sentence that summarizes the general limit principle given by the formal notation   42) _____
   $\lim\limits_{x\to a} [f(x) \pm g(x)] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x) = L \pm M$, given that $\lim\limits_{x\to a} f(x) = L$ and $\lim\limits_{x\to a} g(x) = M$.

   A) The limit of a sum or a difference is the sum or the difference of the limits.
   B) The limit of a sum or a difference is the sum or the difference of the functions.
   C) The sum or the difference of two functions is the sum of two limits.
   D) The sum or the difference of two functions is continuous.

43) The statement "the limit of a constant times a function is the constant times the limit" follows from a combination of two fundamental limit principles. What are they?   43) _____
   A) The limit of a product is the product of the limits, and the limit of a quotient is the quotient of the limits.
   B) The limit of a function is a constant times a limit, and the limit of a constant is the constant.
   C) The limit of a product is the product of the limits, and a constant is continuous.
   D) The limit of a constant is the constant, and the limit of a product is the product of the limits.

Find the limit.

44) $\lim\limits_{x\to 15} \sqrt{3}$ 44) _____

   A) $\sqrt{15}$ B) 15   C) $\sqrt{3}$ D) 3

45) $\lim\limits_{x\to -9} (5x - 4)$ 45) _____

   A) -41   B) -49   C) 49   D) 41

46) $\lim\limits_{x\to -15} (6 - 5x)$ 46) _____

   A) 81   B) -81   C) 69   D) -69

13

Give an appropriate answer.

47) Let $\lim\limits_{x \to -8} f(x) = -9$ and $\lim\limits_{x \to -8} g(x) = -3$. Find $\lim\limits_{x \to -8} [f(x) - g(x)]$.

    A) -8   B) -9   C) -12   D) -6

47) _____

48) Let $\lim\limits_{x \to -7} f(x) = -10$ and $\lim\limits_{x \to -7} g(x) = 6$. Find $\lim\limits_{x \to -7} [f(x) \cdot g(x)]$.

    A) -4   B) -7   C) 6   D) -60

48) _____

49) Let $\lim\limits_{x \to -9} f(x) = -4$ and $\lim\limits_{x \to -9} g(x) = -2$. Find $\lim\limits_{x \to -9} \dfrac{f(x)}{g(x)}$.

    A) $\dfrac{1}{2}$ B) -9   C) -2   D) 2

49) _____

50) Let $\lim\limits_{x \to 5} f(x) = 169$. Find $\lim\limits_{x \to 5} \sqrt{f(x)}$.

    A) 169   B) 3.6056   C) 13   D) 5

50) _____

51) Let $\lim\limits_{x \to 5} f(x) = -3$ and $\lim\limits_{x \to 5} g(x) = 1$. Find $\lim\limits_{x \to 5} [f(x) + g(x)]^2$.

    A) -4   B) -2   C) 4   D) 10

51) _____

52) Let $\lim\limits_{x \to 10} f(x) = 1024$. Find $\lim\limits_{x \to 10} \sqrt[5]{f(x)}$.

    A) 10   B) 4   C) 5   D) 1024

52) _____

53) Let $\lim\limits_{x \to -6} f(x) = -1$ and $\lim\limits_{x \to -6} g(x) = -1$. Find $\lim\limits_{x \to -6} \left[\dfrac{-5f(x) - 10g(x)}{7 + g(x)} \right]$.

    A) $-\dfrac{5}{6}$ B) $\dfrac{5}{2}$ C) -6   D) $-\dfrac{65}{7}$

53) _____

Find the limit.

54) $\lim\limits_{x \to 2} (x^3 + 5x^2 - 7x + 1)$

    A) 15   B) does not exist   C) 29   D) 0

54) _____

55) $\lim\limits_{x \to 2} (2x^5 - 3x^4 + 4x^3 + x^2 + 5)$

    A) 153   B) 25   C) -7   D) 57

55) _____

56) $\lim\limits_{x \to -1} \dfrac{x}{3x + 2}$

    A) 0   B) does not exist   C) $-\dfrac{1}{5}$ D) 1

56) _____

57) $\lim\limits_{x \to 0} \dfrac{x^3 - 6x + 8}{x - 2}$

    A) 4   B) Does not exist   C) 0   D) -4

57) _____

58) $\lim\limits_{x \to 1} \dfrac{3x^2 + 7x - 2}{3x^2 - 4x - 2}$

    A) $-\dfrac{7}{4}$ B) 0   C) $-\dfrac{8}{3}$ D) Does not exist

58) _____

59) $\lim\limits_{x \to 1} (x + 2)^2(x - 3)^3$

    A) -8   B) 576   C) -72   D) 64

59) _____

60) $\lim\limits_{x \to 7} \sqrt{x^2 + 12x + 36}$

    A) 13   B) does not exist   C) 169   D) ±13

60) _____

61) $\lim\limits_{x \to -6} \sqrt{4x + 45}$

    A) $\sqrt{21}$ B) -21   C) 21   D) $-\sqrt{21}$

61) _____

62) $\lim\limits_{h \to 0} \dfrac{2}{\sqrt{3h + 4} + 2}$

    A) Does not exist   B) 1/2   C) 2   D) 1

62) _____

63) $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x} - 1}{x}$

    A) 1/4   B) 1/2   C) Does not exist   D) 0

63) _____

Determine the limit by sketching an appropriate graph.

64) $\lim\limits_{x \to 7^-} f(x)$, where $f(x) = \begin{cases} -2x + 4 & \text{for } x < 7 \\ 4x + 5 & \text{for } x \geq 7 \end{cases}$

    A) 33   B) 6   C) 5   D) -10

64) _____

65) $\lim\limits_{x \to 3^+} f(x)$, where $f(x) = \begin{cases} -2x + 0 & \text{for } x < 3 \\ 3x + 1 & \text{for } x \geq 3 \end{cases}$

    A) 2   B) 10   C) -6   D) 1

65) _____

66) $\lim\limits_{x \to 3^+} f(x)$, where $f(x) = \begin{cases} x^2 + 6 & \text{for } x \neq 3 \\ 0 & \text{for } x = 3 \end{cases}$

    A) 3   B) 9   C) 15   D) 0

66) _____

67) $\lim\limits_{x \to 6^-} f(x)$, where $f(x) = \begin{cases} \sqrt{9 - x^2} & 0 \le x < 3 \\ 3 & 3 \le x < 6 \\ 6 & x = 6 \end{cases}$

67) _____

    A) 0   B) 3   C) 6   D) Does not exist

68) $\lim\limits_{x \to -4^+} f(x)$, where $f(x) = \begin{cases} 2x & -4 \le x < 0, \text{ or } 0 < x \le 1 \\ 2 & x = 0 \\ 0 & x < -4 \text{ or } x > 1 \end{cases}$

68) _____

    A) Does not exist   B) -0   C) 2   D) -8

Find the limit, if it exists.

69) $\lim\limits_{x \to 0} \dfrac{x^3 + 12x^2 - 5x}{5x}$

69) _____

    A) Does not exist   B) -1   C) 0   D) 5

70) $\lim\limits_{x \to 1} \dfrac{x^4 - 1}{x - 1}$

70) _____

    A) Does not exist   B) 0   C) 4   D) 2

71) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

71) _____

    A) 2   B) 4   C) Does not exist   D) 1

72) $\lim\limits_{x \to -8} \dfrac{x^2 + 17x + 72}{x + 8}$

72) _____

    A) Does not exist   B) 272   C) 1   D) 17

73) $\lim\limits_{x \to 6} \dfrac{x^2 + 3x - 54}{x - 6}$

73) _____

    A) 3   B) Does not exist   C) 0   D) 15

74) $\lim\limits_{x \to 1} \dfrac{x^2 + 8x - 9}{x^2 - 1}$

74) _____

    A) 0   B) - 4   C) Does not exist   D) 5

75) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^2 - 6x + 5}$

75) _____

    A) $-\dfrac{1}{2}$ B) 0   C) $-\dfrac{1}{4}$ D) Does not exist

76) $\lim\limits_{x \to 3} \dfrac{x^2 - 9x + 18}{x^2 - 8x + 15}$

76) _____

    A) Does not exist   B) $-\dfrac{3}{2}$ C) $\dfrac{3}{2}$ D) $-\dfrac{9}{2}$

77) $\lim\limits_{h \to 0} \dfrac{(x + h)^3 - x^3}{h}$

    77) _____

    A) $3x^2 + 3xh + h^2$ B) $3x^2$ C) 0   D) Does not exist

78) $\lim\limits_{x \to 10} \dfrac{|10 - x|}{10 - x}$

    78) _____

    A) 0   B) Does not exist   C) 1   D) -1

Provide an appropriate response.

79) It can be shown that the inequalities $-x \le x \cos\left(\dfrac{1}{x}\right) \le x$ hold for all values of $x \ge 0$.

    79) _____

    Find $\lim\limits_{x \to 0} x \cos\left(\dfrac{1}{x}\right)$ if it exists.

    A) 1   B) 0.0007   C) 0   D) does not exist

80) The inequality $1 - \dfrac{x^2}{2} < \dfrac{\sin x}{x} < 1$ holds when x is measured in radians and $|x| < 1$.

    80) _____

    Find $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$ if it exists.

    A) does not exist   B) 1   C) 0.0007   D) 0

81) If $x^3 \le f(x) \le x$ for x in $[-1, 1]$, find $\lim\limits_{x \to 0} f(x)$ if it exists.

    81) _____

    A) does not exist   B) -1   C) 1   D) 0

Compute the values of f(x) and use them to determine the indicated limit.

82) If $f(x) = x^2 + 8x - 2$, find $\lim\limits_{x \to 2} f(x)$.

    82) _____

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)						

A)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	5.043	5.364	5.396	5.404	5.436	5.763

; limit = ∞

B)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	16.692	17.592	17.689	17.710	17.808	18.789

; limit = 17.70

C)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	5.043	5.364	5.396	5.404	5.436	5.763

; limit = 5.40

D)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	16.810	17.880	17.988	18.012	18.120	19.210

; limit = 18.0

83) If $f(x) = \dfrac{x^4 - 1}{x - 1}$, find $\lim\limits_{x \to 1} f(x)$.

83) _____

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)						

A)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	1.032	1.182	1.198	1.201	1.218	1.392

; limit = ∞

B)
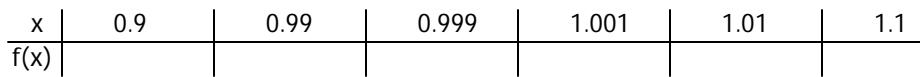
x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	3.439	3.940	3.994	4.006	4.060	4.641

; limit = 4.0

C)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	4.595	5.046	5.095	5.105	5.154	5.677

; limit = 5.10

D)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	1.032	1.182	1.198	1.201	1.218	1.392

; limit = 1.210

84) If $f(x) = \dfrac{x^3 - 6x + 8}{x - 2}$, find $\lim\limits_{x \to 0} f(x)$.

84) _____

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

A)
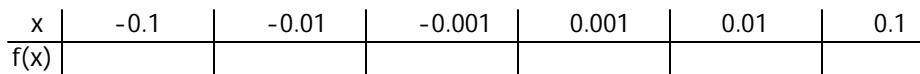
x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-2.18529	-2.10895	-2.10090	-2.99910	-2.09096	-2.00574

; limit = -2.10

B)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.22843	-1.20298	-1.20030	-1.19970	-1.19699	-1.16858

; limit = ∞

C)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-4.09476	-4.00995	-4.00100	-3.99900	-3.98995	-3.89526

; limit = -4.0

D)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.22843	-1.20298	-1.20030	-1.19970	-1.19699	-1.16858

; limit = -1.20

85) If $f(x) = \dfrac{x - 4}{\sqrt{x} - 2}$, find $\lim\limits_{x \to 4} f(x)$.

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

A)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	3.97484	3.99750	3.99975	4.00025	4.00250	4.02485

; limit = 4.0

B)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	1.19245	1.19925	1.19993	1.20007	1.20075	1.20745

; limit = 1.20

C)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	1.19245	1.19925	1.19993	1.20007	1.20075	1.20745

; limit = ∞

D)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	5.07736	5.09775	5.09978	5.10022	5.10225	5.12236

; limit = 5.10

86) If $f(x) = x^2 - 5$, find $\lim\limits_{x \to 0} f(x)$.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

A)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit = ∞

B)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-2.9910	-2.9999	-3.0000	-3.0000	-2.9999	-2.9910

; limit = -3.0

C)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit = -15.0

D)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-4.9900	-4.9999	-5.0000	-5.0000	-4.9999	-4.9900

; limit = -5.0

87) If $f(x) = \dfrac{\sqrt{x+1}}{x+1}$, find $\lim\limits_{x \to 1} f(x)$.

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)						

A)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.21764	0.21266	0.21219	0.21208	0.21160	0.20702

; limit = ∞

B)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.72548	0.70888	0.70728	0.70693	0.70535	0.69007

; limit = 0.7071

C)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	0.21764	0.21266	0.21219	0.21208	0.21160	0.20702

; limit = 0.21213

D)

x	0.9	0.99	0.999	1.001	1.01	1.1
f(x)	2.15293	2.13799	2.13656	2.13624	2.13481	2.12106

; limit = 2.13640

88) If $f(x) = \sqrt{x} - 2$, find $\lim\limits_{x \to 4} f(x)$.

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

A)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	1.47736	1.49775	1.49977	1.50022	1.50225	1.52236

; limit = 1.50

B)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	-0.02516	-0.00250	-0.00025	0.00025	0.00250	0.02485

; limit = 0

C)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	3.9000	2.9000	1.9000	2.0000	3.0000	4.0000

; limit = ∞

D)

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	3.9000	2.9000	1.9000	2.0000	3.0000	4.0000

; limit = 1.95

For the function f whose graph is given, determine the limit.

89) Find $\lim\limits_{x \to -1^-} f(x)$ and $\lim\limits_{x \to -1^+} f(x)$.

89) _____
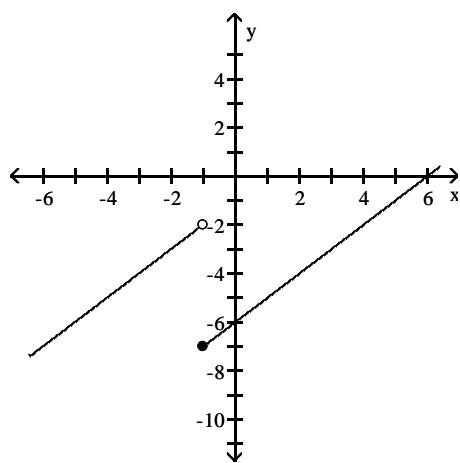
A) -7; -5   B) -5; -2   C) -7; -2   D) -2; -7

90) Find $\lim\limits_{x \to 2^-} f(x)$ and $\lim\limits_{x \to 2^+} f(x)$.

90) _____
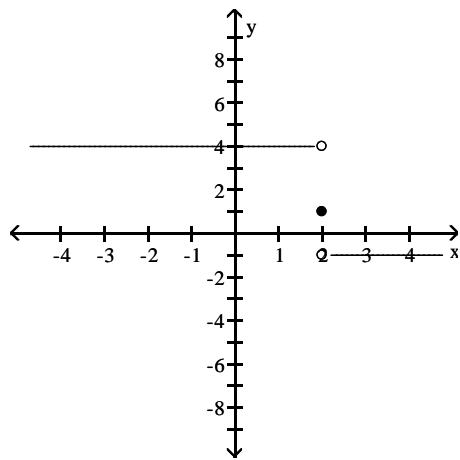
A) does not exist; does not exist   B) 4; -1
C) -1; 4   D) 1; 1

91) Find lim f(x).
  x→2⁻

A) -1   B) 4   C) 1.3   D) 2.3

92) Find lim f(x).
  x→1⁻
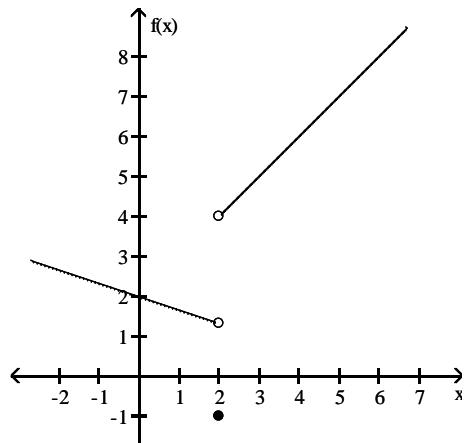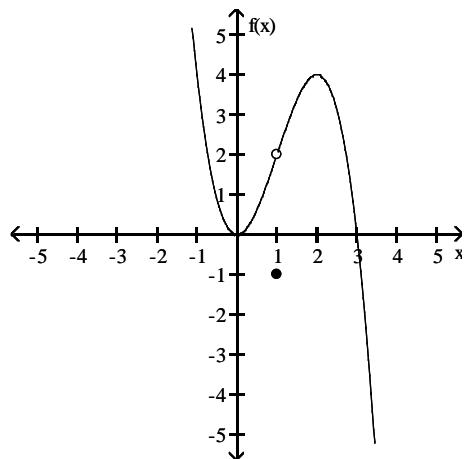
A) 2   B) $\frac{1}{2}$ C) -1   D) does not exist

93) Find $\lim\limits_{x \to 1^+} f(x)$.
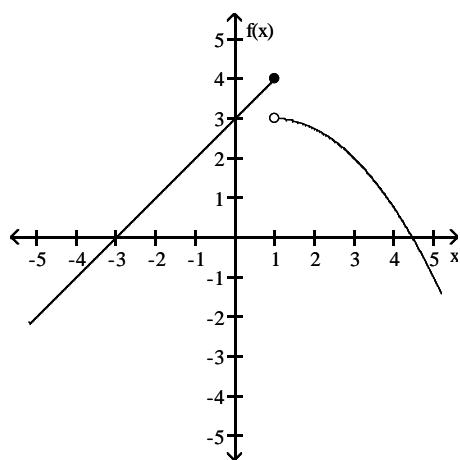
A) does not exist   B) 4   C) $3\frac{1}{2}$ D) 3

93) _____
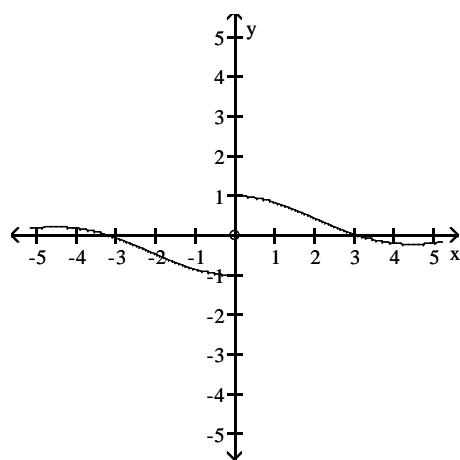
94) Find $\lim\limits_{x \to 0} f(x)$.

A) does not exist   B) 0   C) 1   D) -1

94) _____

23
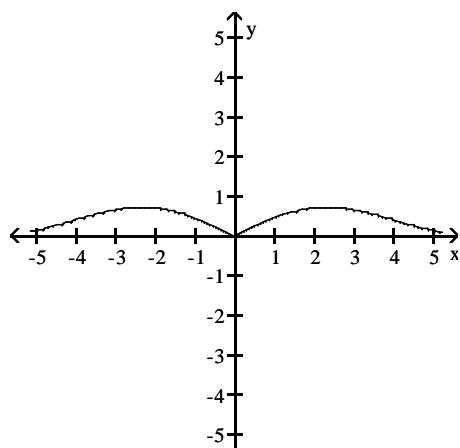
95) Find $\lim\limits_{x\to 0} f(x)$.

A) does not exist   B) 1   C) -1   D) 0

96) Find $\lim\limits_{x\to 0} f(x)$.
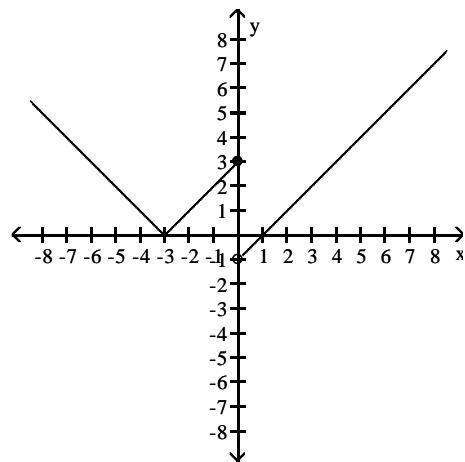
A) -3   B) does not exist   C) 0   D) 3

97) Find $\lim\limits_{x \to -1} f(x)$.
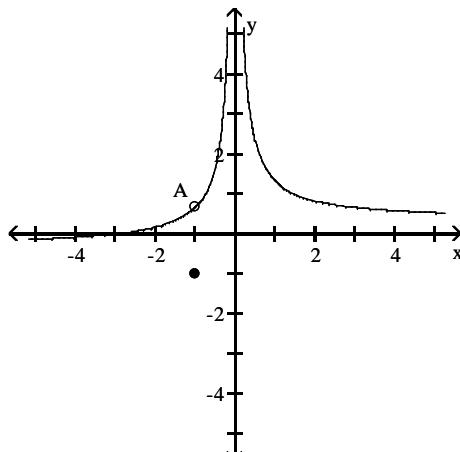
A) does not exist   B) $\dfrac{2}{3}$ C) $-\dfrac{2}{3}$ D) -1

97) _____

98) Find $\lim\limits_{x \to -\infty} f(x)$.
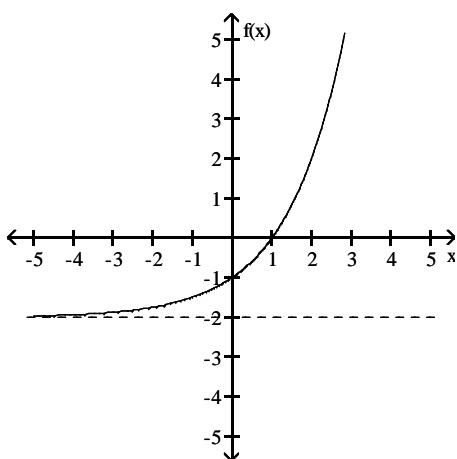
A) -2   B) 0   C) does not exist   D) $-\infty$

98) _____

Find the limit.

99) $\lim\limits_{x \to -2} \dfrac{1}{x + 2}$

A) 1/2   B) Does not exist   C) $-\infty$ D) ∞

99) _____

100) $\lim\limits_{x \to -9^-} \dfrac{1}{x + 9}$

A) -1   B) 0   C) ∞ D) $-\infty$

100) _____

101) $\lim\limits_{x \to 10^+} \dfrac{1}{(x - 10)^2}$

A) -1   B) ∞ C) 0   D) $-\infty$

101) _____

102) $\lim\limits_{x \to -1^-} \dfrac{1}{x^2 - 1}$

102) _____

    A) -1   B) ∞   C) - ∞   D) 0

103) $\lim\limits_{x \to 3^+} \dfrac{1}{x^2 - 9}$

103) _____

    A) - ∞   B) 1   C) 0   D) ∞

104) $\lim\limits_{x \to (\pi/2)^+} \tan x$

104) _____

    A) 1   B) 0   C) ∞   D) - ∞

105) $\lim\limits_{x \to (-\pi/2)^-} \sec x$

105) _____

    A) 1   B) ∞   C) - ∞   D) 0

106) $\lim\limits_{x \to 0^+} (1 + \csc x)$

106) _____

    A) 0   B) 1   C) ∞   D) Does not exist

107) $\lim\limits_{x \to 0} (1 - \cot x)$

107) _____

    A) - ∞   B) ∞   C) 0   D) Does not exist

108) $\lim\limits_{x \to 1^-} \dfrac{x^2 - 6x + 5}{x^3 - x}$

108) _____

    A) – ∞   B) ∞   C) 0   D) - 2

109) $\lim\limits_{x \to 0^-} \dfrac{x^2 - 3x + 2}{x^3 - 4x}$

109) _____

    A) ∞   B) 0   C) Does not exist   D) – ∞

Find all vertical asymptotes of the given function.

110) $f(x) = \dfrac{2x}{x - 9}$

110) _____

    A) none   B) x = 9   C) x = 2   D) x = -9

111) $f(x) = \dfrac{x + 8}{x^2 - 36}$

111) _____

    A) x = -6, x = 6   B) x = 36, x = -8
    C) x = -6, x = 6, x = -8   D) x = 0, x = 36

112) $f(x) = \dfrac{x + 6}{x^2 + 9}$ 112) _____

   A) $x = -3, x = 3$ B) $x = -3, x = 3, x = -6$
   C) $x = -3, x = -6$ D) none

113) $g(x) = \dfrac{x + 11}{x^2 + 36x}$ 113) _____

   A) $x = 0, x = -6, x = 6$ B) $x = -36, x = -11$
   C) $x = 0, x = -36$ D) $x = -6, x = 6$

114) $f(x) = \dfrac{x(x - 1)}{x^3 + 49x}$ 114) _____

   A) $x = 0, x = -49$ B) $x = 0$
   C) $x = 0, x = -7, x = 7$ D) $x = -7, x = 7$

115) $R(x) = \dfrac{-3x^2}{x^2 + 5x - 24}$ 115) _____

   A) $x = -8, x = 3, x = -3$ B) $x = -8, x = 3$
   C) $x = -24$ D) $x = 8, x = -3$

116) $R(x) = \dfrac{x - 1}{x^3 + 2x^2 - 8x}$ 116) _____

   A) $x = -2, x = 0, x = 4$ B) $x = -4, x = 2$
   C) $x = -2, x = -30, x = 4$ D) $x = -4, x = 0, x = 2$

117) $f(x) = \dfrac{-2x(x + 2)}{3x^2 - 4x - 7}$ 117) _____

   A) $x = \dfrac{7}{3}, x = -1$ B) $x = \dfrac{3}{7}, x = -1$ C) $x = -\dfrac{3}{7}, x = 1$ D) $x = -\dfrac{7}{3}, x = 1$

118) $f(x) = \dfrac{x - 4}{16x - x^3}$ 118) _____

   A) $x = 0, x = 4$ B) $x = -4, x = 4$
   C) $x = 0, x = -4, x = 4$ D) $x = 0, x = -4$

119) $f(x) = \dfrac{-x^2 + 16}{x^2 + 5x + 4}$ 119) _____

   A) $x = 1, x = -4$ B) $x = -1, x = -4$ C) $x = -1, x = 4$ D) $x = -1$

Choose the graph that represents the given function without using a graphing utility.

120) $f(x) = \dfrac{x}{x-2}$

120) _____

A)

B)

C)

D)
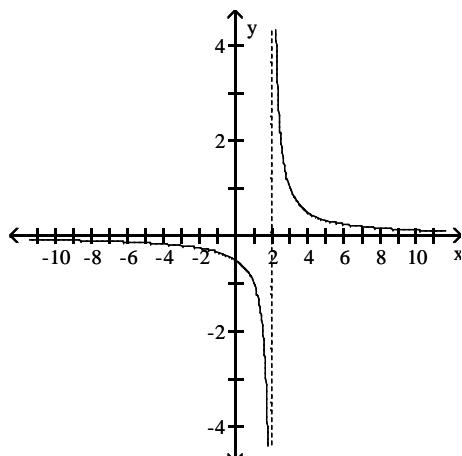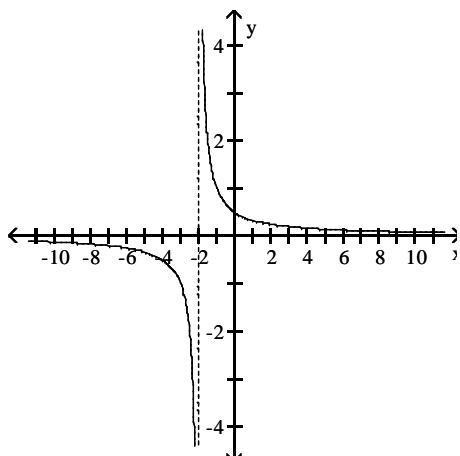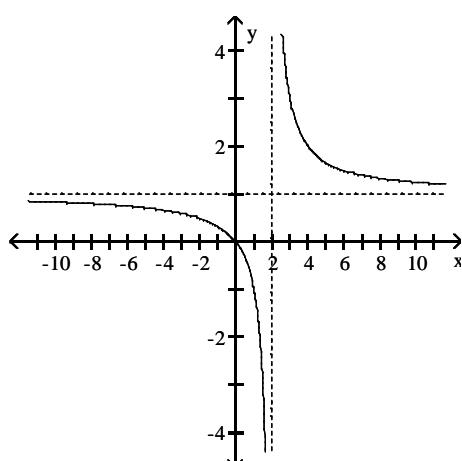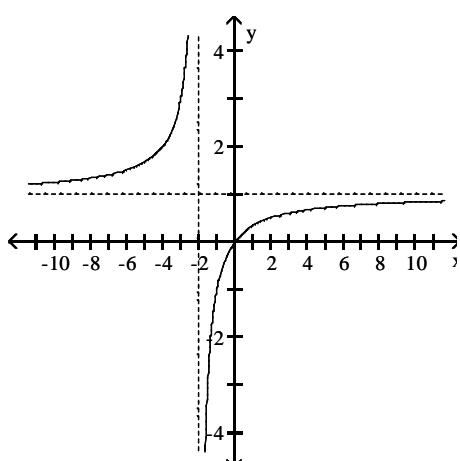
121) $f(x) = \dfrac{x}{x^2 + x + 1}$

A)
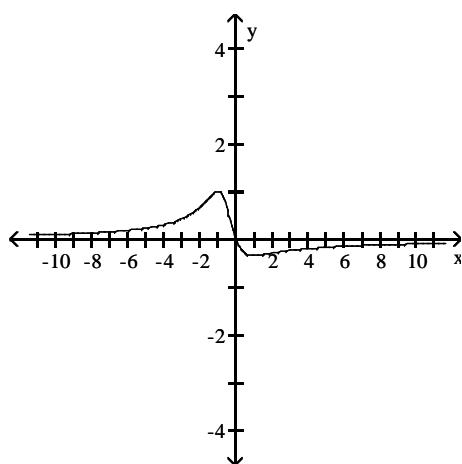
B)
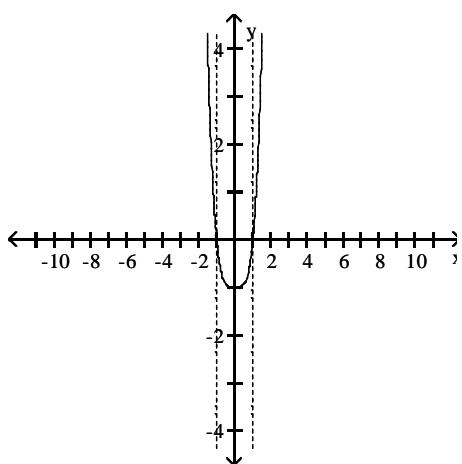
C)
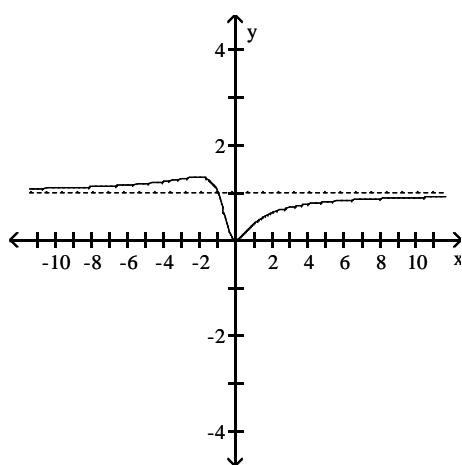
D)
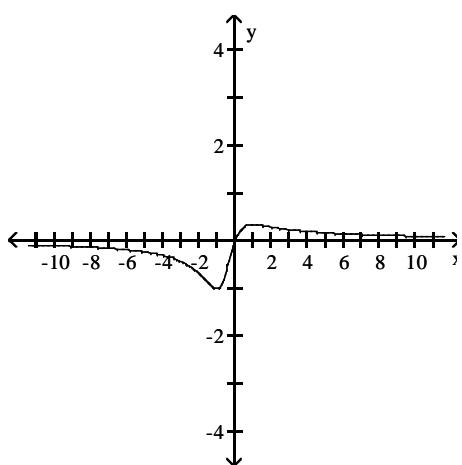
122) $f(x) = \dfrac{x^2 - 1}{x^3}$

122) _____

A)

B)

C)

D)
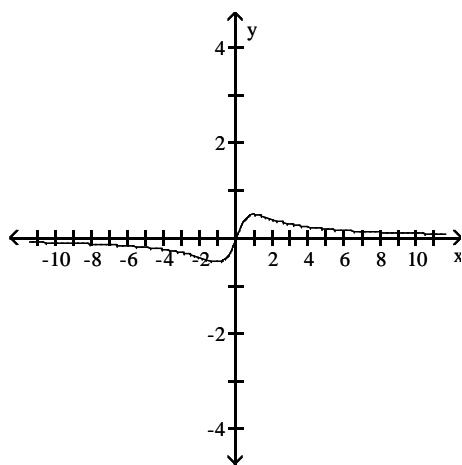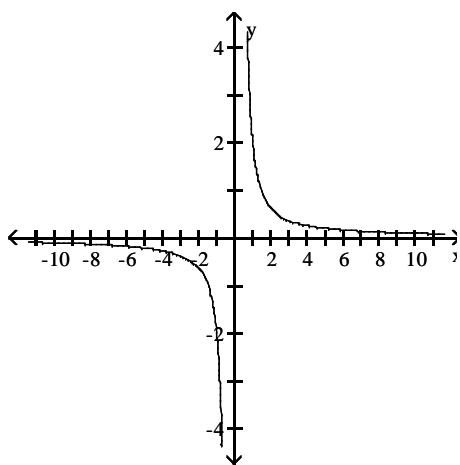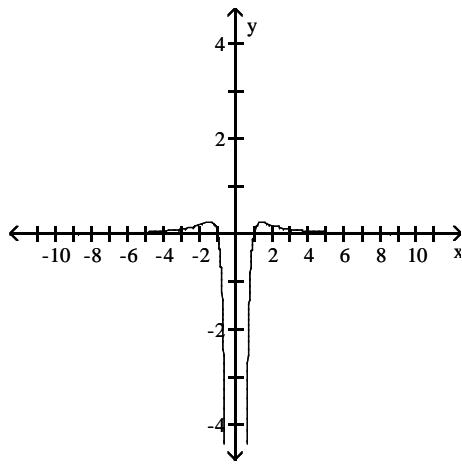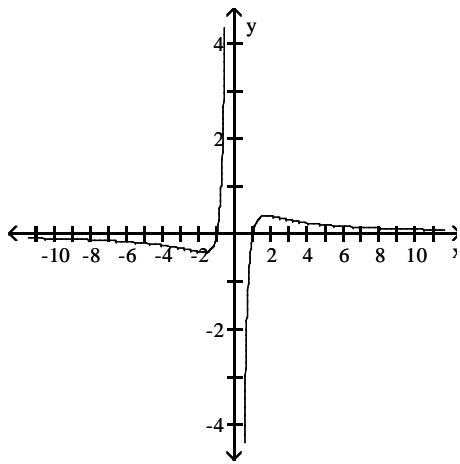
123) $f(x) = \dfrac{1}{x+1}$

A)
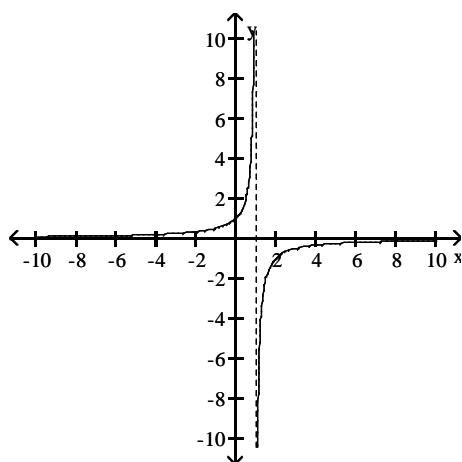
B)
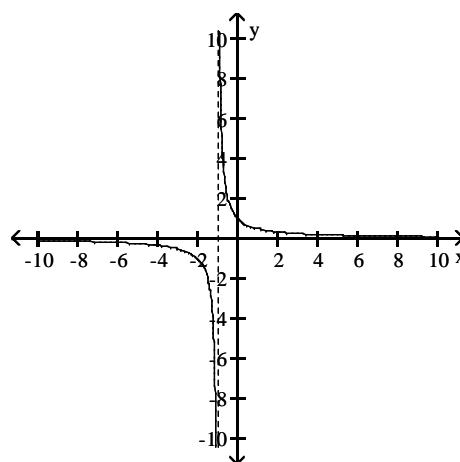
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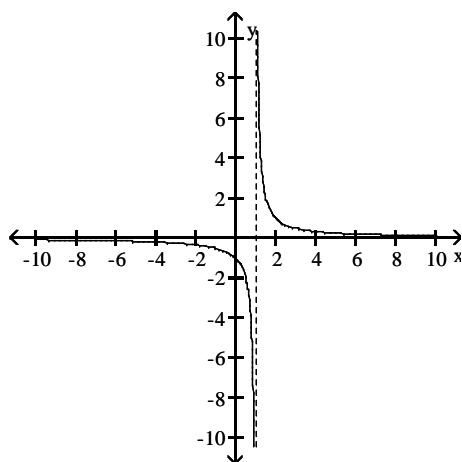
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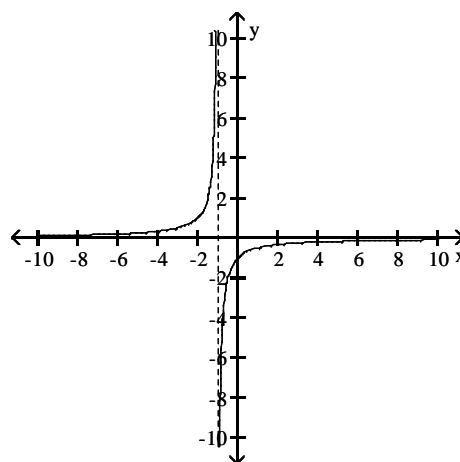
124) $f(x) = \dfrac{x - 1}{x + 1}$ 124) _____

A)
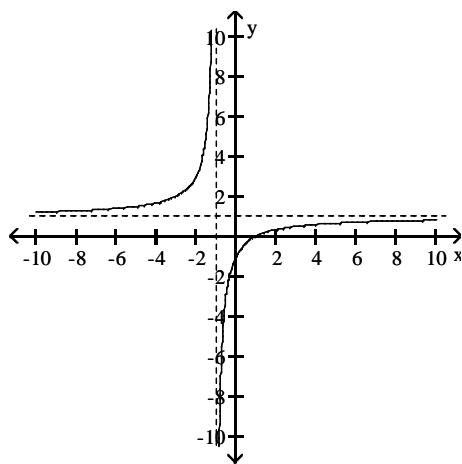
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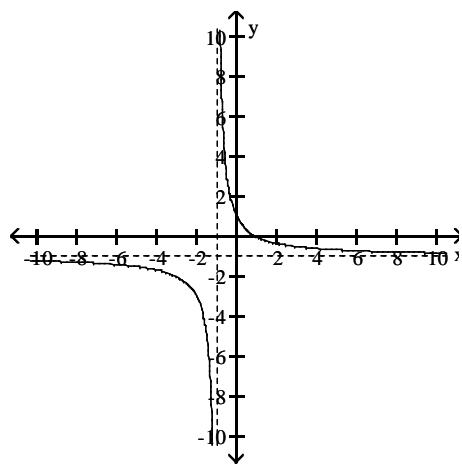
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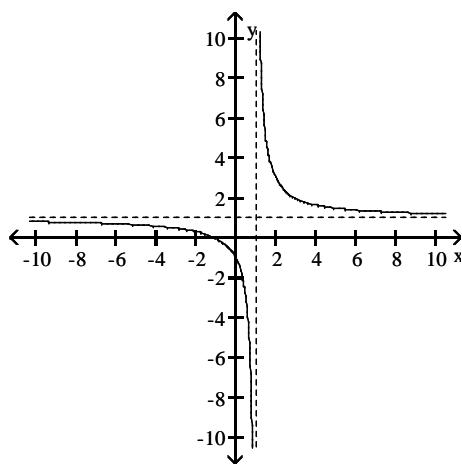
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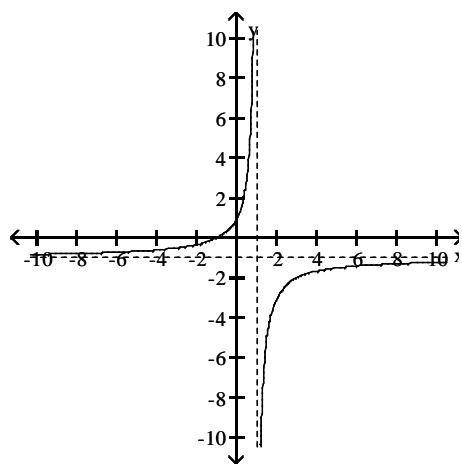
125) $f(x) = \dfrac{1}{(x+2)^2}$

125) _____

A)

B)

C)

D)

126) $f(x) = \dfrac{2x^2}{4 - x^2}$

126) _____
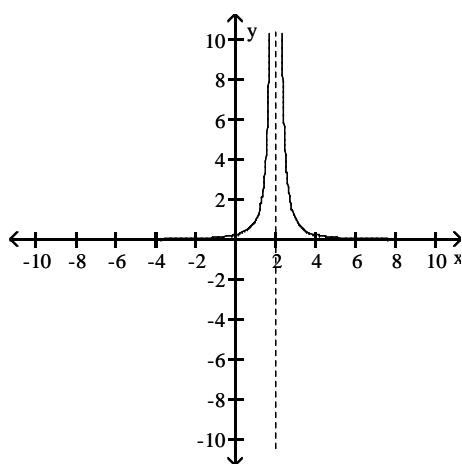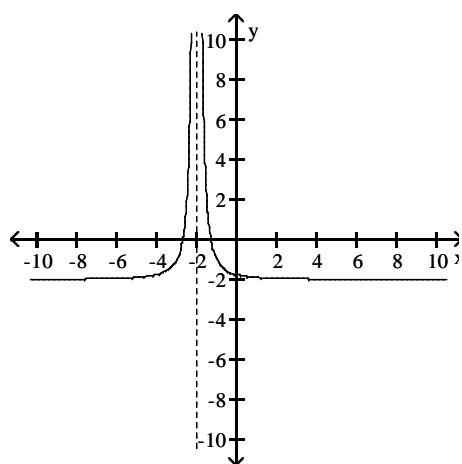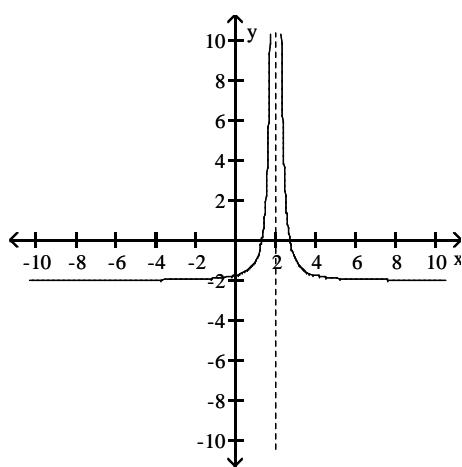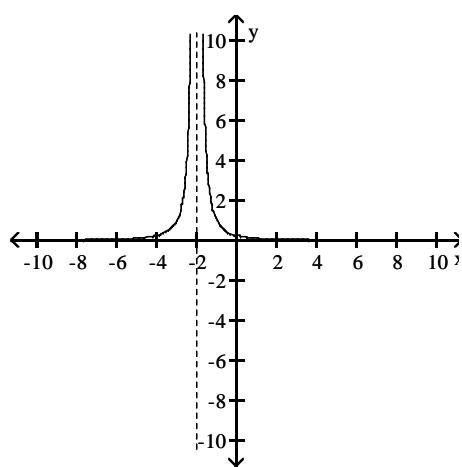
A)
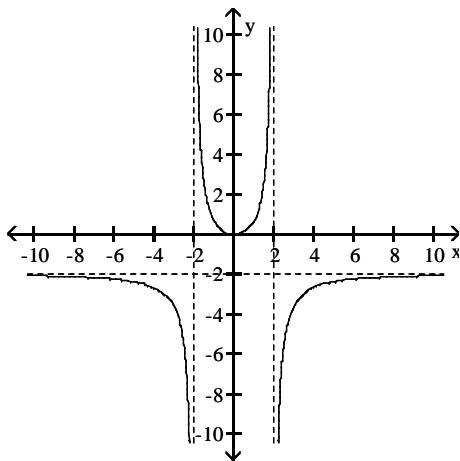
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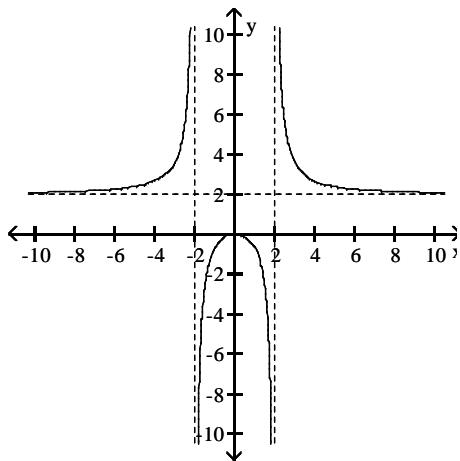
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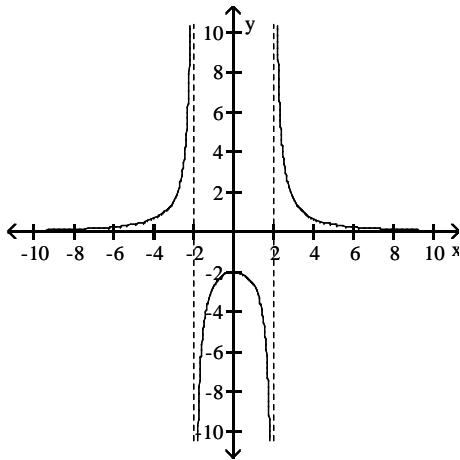
D)
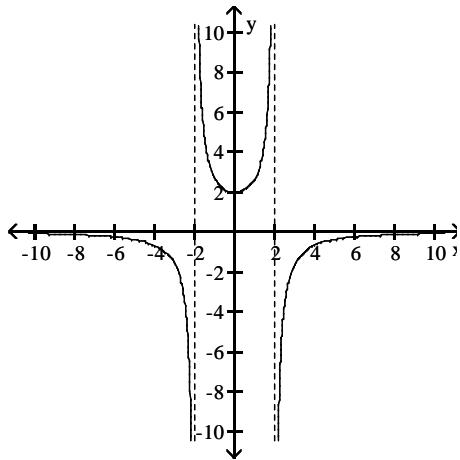
Find the limit.

127) $\displaystyle \lim_{x \to \infty} \dfrac{7}{x} - 5$

127) _____

A) -12   B) 2   C) 5   D) -5

128) $\displaystyle \lim_{x \to -\infty} \dfrac{3}{5 - (1/x^2)}$

128) _____

A) $\dfrac{3}{5}$ B) $-\infty$ C) 3   D) $\dfrac{3}{4}$

129) $\displaystyle \lim_{x \to -\infty} \dfrac{-4 + (3/x)}{7 - (1/x^2)}$

129) _____

A) $-\infty$ B) $-\dfrac{4}{7}$ C) ∞ D) $\dfrac{4}{7}$

34

130) $\lim\limits_{x \to \infty} \dfrac{x^2 - 4x + 19}{x^3 - 8x^2 + 13}$

130) _____

    A) $\dfrac{19}{13}$ B) 1   C) ∞ D) 0

131) $\lim\limits_{x \to -\infty} \dfrac{-9x^2 + 6x + 10}{-16x^2 - 8x + 19}$

131) _____

    A) $\dfrac{9}{16}$ B) ∞ C) $\dfrac{10}{19}$ D) 1

132) $\lim\limits_{x \to \infty} \dfrac{6x + 1}{15x - 7}$

132) _____

    A) $-\dfrac{1}{7}$ B) $\dfrac{2}{5}$ C) 0   D) ∞

133) $\lim\limits_{x \to \infty} \dfrac{2x^3 - 2x^2 + 3x}{-x^3 - 2x + 7}$

133) _____

    A) $\dfrac{3}{2}$ B) 2   C) -2   D) ∞

134) $\lim\limits_{x \to -\infty} \dfrac{5x^3 + 3x^2}{x - 7x^2}$

134) _____

    A) 5   B) $-\infty$ C) ∞ D) $-\dfrac{3}{7}$

135) $\lim\limits_{x \to -\infty} \dfrac{\cos 3x}{x}$

135) _____

    A) $-\infty$ B) 3   C) 1   D) 0

Divide numerator and denominator by the highest power of x in the denominator to find the limit.

136) $\lim\limits_{x \to \infty} \sqrt{\dfrac{9x^2}{4 + 49x^2}}$

136) _____

    A) $\dfrac{3}{7}$ B) does not exist   C) $\dfrac{9}{4}$ D) $\dfrac{9}{49}$

137) $\lim\limits_{x \to \infty} \sqrt{\dfrac{16x^2 + x - 3}{(x - 9)(x + 1)}}$

137) _____

    A) 16   B) 0   C) ∞ D) 4

138) $\lim\limits_{x \to \infty} \dfrac{-5\sqrt{x} + x^{-1}}{4x - 5}$

138) _____

    A) $-\dfrac{5}{4}$ B) $\dfrac{1}{4}$ C) ∞ D) 0

139) $\lim\limits_{x\to\infty} \dfrac{-4x^{-1} - 2x^{-3}}{-2x^{-2} + x^{-5}}$

A) 0   B) $-\infty$ C) ∞ D) 2

139) _____

140) $\lim\limits_{x\to-\infty} \dfrac{\sqrt[3]{x} - 3x + 6}{-4x + x^{2/3} - 6}$

A) $\dfrac{3}{4}$ B) $-\infty$ C) $\dfrac{4}{3}$ D) 0

140) _____

141) $\lim\limits_{t\to\infty} \dfrac{\sqrt{4t^2 - 8}}{t - 2}$

A) 8   B) 4   C) 2   D) does not exist

141) _____

142) $\lim\limits_{t\to\infty} \dfrac{\sqrt{64t^2 - 512}}{t - 8}$

A) does not exist   B) 512   C) 8   D) 64

142) _____

143) $\lim\limits_{x\to\infty} \dfrac{2x + 7}{\sqrt{7x^2 + 1}}$

A) ∞ B) $\dfrac{2}{7}$ C) 0   D) $\dfrac{2}{\sqrt{7}}$

143) _____

Find all horizontal asymptotes of the given function, if any.

144) $h(x) = \dfrac{4x - 9}{x - 6}$

A) $y = 6$ B) $y = 0$

C) $y = 4$ D) no horizontal asymptotes

144) _____

145) $h(x) = 7 - \dfrac{5}{x}$

A) $y = 5$ B) $y = 7$

C) $x = 0$ D) no horizontal asymptotes

145) _____

146) $g(x) = \dfrac{x^2 + 7x - 2}{x - 2}$

A) $y = 0$ B) $y = 1$

C) $y = 2$ D) no horizontal asymptotes

146) _____

147) $h(x) = \dfrac{6x^2 - 5x - 6}{4x^2 - 3x + 3}$

A) $y = \dfrac{3}{2}$ B) $y = \dfrac{5}{3}$

C) $y = 0$ D) no horizontal asymptotes

147) _____

148) $h(x) = \dfrac{2x^4 - 4x^2 - 4}{3x^5 - 9x + 9}$

148) _____

    A) $y = \dfrac{4}{9}$ B) $y = 0$

    C) $y = \dfrac{2}{3}$ D) no horizontal asymptotes

149) $h(x) = \dfrac{8x^3 - 2x}{4x^3 - 6x + 3}$

149) _____

    A) $y = 2$ B) $y = \dfrac{1}{3}$

    C) $y = 0$ D) no horizontal asymptotes

150) $h(x) = \dfrac{6x^3 - 9x - 7}{4x^2 + 4}$

150) _____

    A) $y = 6$ B) $y = 0$

    C) $y = \dfrac{3}{2}$ D) no horizontal asymptotes

151) $g(x) = \dfrac{7x + 1}{x^2 - 36}$

151) _____

    A) no horizontal asymptotes   B) $y = 0$
    C) $y = 7$ D) $y = -6, y = 6$

152) $R(x) = \dfrac{-3x^2 + 1}{x^2 + 4x - 12}$

152) _____

    A) $y = 0$ B) $y = -6, y = 2$
    C) $y = -3$ D) no horizontal asymptotes

153) $f(x) = \dfrac{x^2 - 4}{16x - x^4}$

153) _____

    A) $y = 0$ B) no horizontal asymptotes
    C) $y = -1$ D) $y = -4, y = 4$

154) $f(x) = \dfrac{49x^4 + x^2 - 7}{x - x^3}$

154) _____

    A) $y = -49$ B) $y = -1, y = 1$
    C) no horizontal asymptotes   D) $y = 0$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Sketch the graph of a function y = f(x) that satisfies the given conditions.

155) $f(0) = 0$, $f(1) = 3$, $f(-1) = -3$, $\lim\limits_{x \to -\infty} f(x) = -2$, $\lim\limits_{x \to \infty} f(x) = 2$.

155) _____
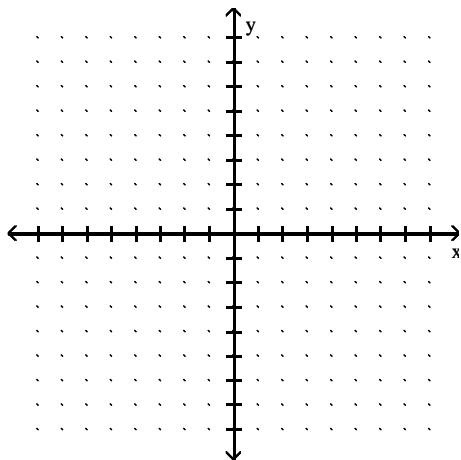
156) $f(0) = 0$, $f(1) = 2$, $f(-1) = 2$, $\lim\limits_{x \to \pm\infty} f(x) = -2$.

156) _____
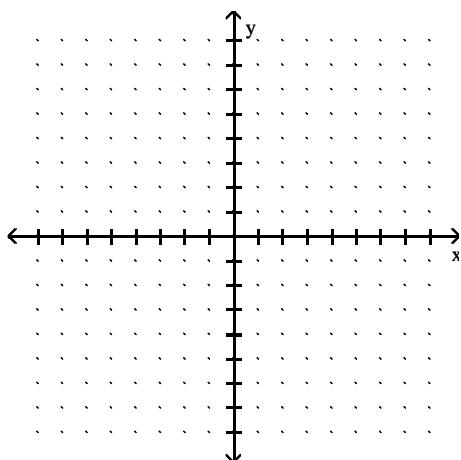
157) $f(0) = 4$, $f(1) = -4$, $f(-1) = -4$, $\lim\limits_{x \to \pm\infty} f(x) = 0$.

157) _____
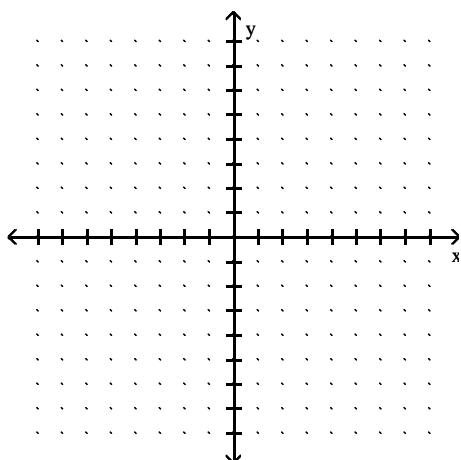
158) $f(0) = 0$, $\lim\limits_{x \to \pm\infty} f(x) = 0$, $\lim\limits_{x \to 6^-} f(x) = -\infty$, $\lim\limits_{x \to -6^+} f(x) = -\infty$, $\lim\limits_{x \to 6^+} f(x) = \infty$,   158) _____

$\lim\limits_{x \to -6^-} f(x) = \infty$.
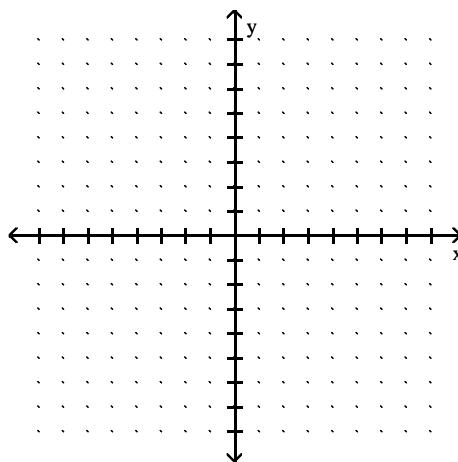
MULTIPLE CHOICE. **Choose the one alternative that best completes the statement or answers the question.**

Find all points where the function is discontinuous.

159)   159) _____

A) $x = 4$, $x = 2$ B) $x = 2$ C) None   D) $x = 4$

160)   160) _____

A) $x = -2$, $x = 1$ B) None   C) $x = 1$ D) $x = -2$

161)   161) _____

A) $x = -2$, $x = 0$ B) $x = 0$, $x = 2$
C) $x = 2$ D) $x = -2$, $x = 0$, $x = 2$

39

162)   162) _____

A) $x = -2, x = 6$ B) None   C) $x = -2$ D) $x = 6$

163)   163) _____

A) None   B) $x = 4$
C) $x = 1, x = 4, x = 5$ D) $x = 1, x = 5$

164)   164) _____

A) $x = 0, x = 1$ B) $x = 0$ C) $x = 1$ D) None

165)   165) _____

A) None   B) $x = 0, x = 3$ C) $x = 3$ D) $x = 0$

166)   166) _____

A) $x = 2$ B) $x = -2$ C) $x = -2, x = 2$ D) None

167) _____

A) x = 0   B) x = -2, x = 2
C) None   D) x = -2, x = 0, x = 2

Provide an appropriate response.

168) Is f continuous at f(1)?   168) _____

$$f(x) = \begin{cases} -x^2 + 1, & -1 \le x < 0 \\ 4x, & 0 < x < 1 \\ -2, & x = 1 \\ -4x + 8 & 1 < x < 3 \\ 1, & 3 < x < 5 \end{cases}$$


(1, -2)
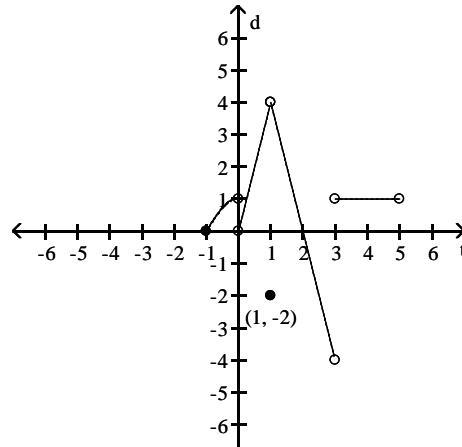
A) No   B) Yes

169) Is f continuous at f(0)?   169) _____

$$f(x) = \begin{cases} -x^2 + 1, & -1 \le x < 0 \\ 5x, & 0 < x < 1 \\ -5, & x = 1 \\ -5x + 10 & 1 < x < 3 \\ 1, & 3 < x < 5 \end{cases}$$

(1, -5)
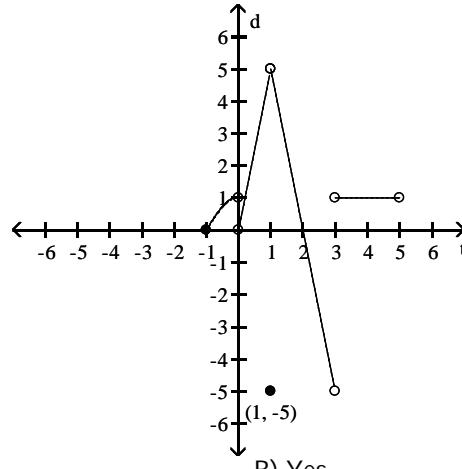
A) No   B) Yes

170) Is f continuous at x = 0?

$$f(x) = \begin{cases} x^3, & -2 < x \le 0 \\ -2x, & 0 \le x < 2 \\ 5, & 2 < x \le 4 \\ 0, & x = 2 \end{cases}$$

(2, 0)

A) No   B) Yes

170) _____
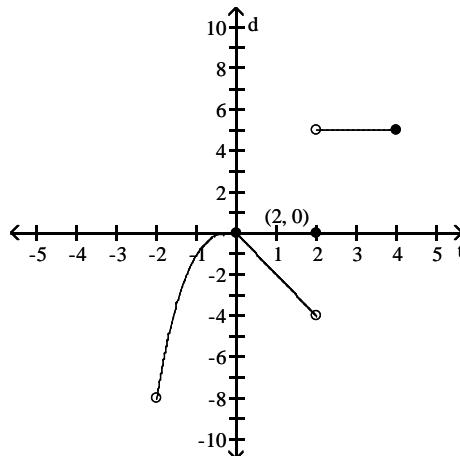
171) Is f continuous at x = 4?

$$f(x) = \begin{cases} x^3, & -2 < x \le 0 \\ -2x, & 0 \le x < 2 \\ 1, & 2 < x \le 4 \\ 0, & x = 2 \end{cases}$$

(2, 0)

A) Yes   B) No

171) _____
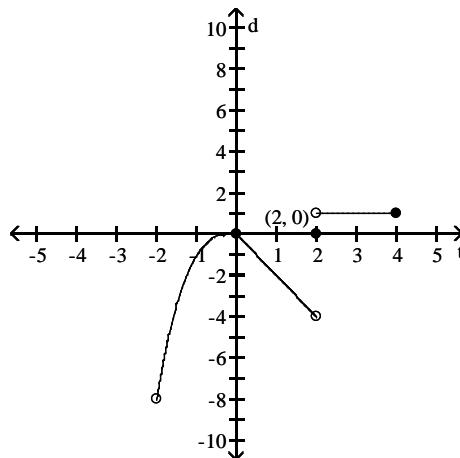
Find the intervals on which the function is continuous.

172) $y = \dfrac{2}{x + 5} - 2x$

A) discontinuous only when x = -5   B) continuous everywhere
C) discontinuous only when x = 5   D) discontinuous only when x = -7

172) _____

173) $y = \dfrac{1}{(x + 5)^2 + 10}$

A) discontinuous only when x = -5   B) discontinuous only when x = 35
C) continuous everywhere   D) discontinuous only when x = -40

173) _____

174) $y = \dfrac{x + 2}{x^2 - 6x + 8}$

A) discontinuous only when x = 2   B) discontinuous only when x = -4 or x = 2
C) discontinuous only when x = -2 or x = 4   D) discontinuous only when x = 2 or x = 4

174) _____

42

175) $y = \dfrac{3}{x^2 - 4}$

    A) discontinuous only when x = -4 or x = 4   B) discontinuous only when x = -2

    C) discontinuous only when x = 4   D) discontinuous only when x = -2 or x = 2

175) _____

176) $y = \dfrac{3}{|x| + 5} - \dfrac{x^2}{7}$

    A) discontinuous only when x = -7 or x = -5   B) continuous everywhere

    C) discontinuous only when x = -5   D) discontinuous only when x = -12

176) _____

177) $y = \dfrac{\sin (4\theta)}{2\theta}$

    A) discontinuous only when $\theta = 0$ B) discontinuous only when $\theta = \pi$

    C) continuous everywhere   D) discontinuous only when $\theta = \dfrac{\pi}{2}$

177) _____

178) $y = \dfrac{2 \cos \theta}{\theta + 4}$

    A) discontinuous only when $\theta = 4$ B) discontinuous only when $\theta = \dfrac{\pi}{2}$

    C) continuous everywhere   D) discontinuous only when $\theta = -4$

178) _____

179) $y = \sqrt{10x + 1}$

    A) continuous on the interval $\left(-\dfrac{1}{10}, \infty\right)$ B) continuous on the interval $\left[-\dfrac{1}{10}, \infty\right)$

    C) continuous on the interval $\left(-\infty, -\dfrac{1}{10}\right]$ D) continuous on the interval $\left[\dfrac{1}{10}, \infty\right)$

179) _____

180) $y = \sqrt[4]{2x - 4}$

    A) continuous on the interval $(2, \infty)$ B) continuous on the interval $(-\infty, 2]$

    C) continuous on the interval $[2, \infty)$ D) continuous on the interval $[-2, \infty)$

180) _____

181) $y = \sqrt{x^2 - 3}$

    A) continuous on the interval $[-\sqrt{3}, \sqrt{3}]$

    B) continuous everywhere

    C) continuous on the interval $[\sqrt{3}, \infty)$

    D) continuous on the intervals $(-\infty, -\sqrt{3}]$ and $[\sqrt{3}, \infty)$

181) _____

Provide an appropriate response.

182) Is f continuous on (-2, 4]?   182) _____

$$f(x) = \begin{cases} x^3, & -2 < x \le 0 \\ -4x, & 0 \le x < 2 \\ 6, & 2 < x \le 4 \\ 0, & x = 2 \end{cases}$$
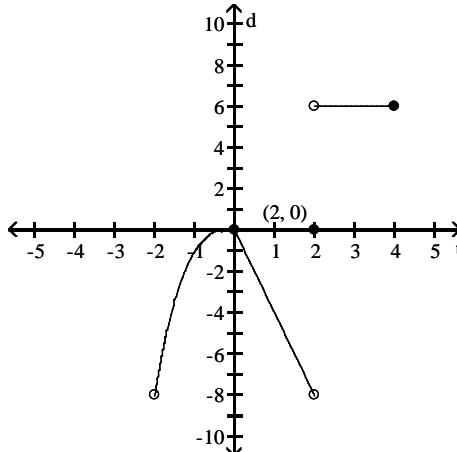
A) No   B) Yes

Find the limit, if it exists.

183) $\lim\limits_{x \to -3} (x^2 - 16 + \sqrt[3]{x^2 - 36})$ 183) _____

   A) -4   B) 4   C) Does not exist   D) -10

184) $\lim\limits_{x \to 7} \sqrt{x^2 + 8x + 16}$ 184) _____

   A) 121   B) ±11   C) Does not exist   D) 11

185) $\lim\limits_{x \to 3} \sqrt{x - 9}$ 185) _____

   A) Does not exist   B) 0   C) 2.44948974   D) -2.4494897

186) $\lim\limits_{x \to 8} \sqrt{x^2 - 9}$ 186) _____

   A) 27.5   B) ±$\sqrt{55}$ C) $\sqrt{55}$ D) Does not exist

187) $\lim\limits_{x \to -6^-} \sqrt{x^2 - 36}$ 187) _____

   A) 3   B) 0   C) $6\sqrt{3}$ D) Does not exist

188) $\lim\limits_{x \to 5^+} \dfrac{9\sqrt{(x - 5)^3}}{x - 5}$ 188) _____

   A) $9\sqrt{5}$ B) 0   C) 9   D) Does not exist

189) $\lim\limits_{t \to 1^+} \dfrac{\sqrt{(t + 25)(t - 1)^2}}{11t - 11}$ 189) _____

   A) $\dfrac{\sqrt{26}}{11}$ B) $\dfrac{1}{11}$ C) 0   D) Does not exist

44

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

190) Use the Intermediate Value Theorem to prove that $5x^3 + 4x^2 + 10x + 10 = 0$ has a solution between -1 and 0.

190) _____

191) Use the Intermediate Value Theorem to prove that $6x^4 + 9x^3 - 5x - 8 = 0$ has a solution between -2 and -1.

191) _____

192) Use the Intermediate Value Theorem to prove that $x(x - 9)^2 = 9$ has a solution between 8 and 10.

192) _____

193) Use the Intermediate Value Theorem to prove that $4 \sin x = x$ has a solution between $\dfrac{\pi}{2}$ and π.

193) _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find numbers a and b, or k, so that f is continuous at every point.

194)
$$f(x) = \begin{cases} -12, & x < -3 \\ ax + b, & -3 \le x \le 1 \\ 8, & x > 1 \end{cases}$$

194) _____

A) a = 5, b = 13   B) a = -12, b = 8   C) a = 5, b = 3   D) Impossible

195)
$$f(x) = \begin{cases} x^2, & x < -3 \\ ax + b, & -3 \le x \le 4 \\ x + 12, & x > 4 \end{cases}$$

195) _____

A) a = -1, b = 12   B) a = 1, b = 12   C) a = 1, b = -12   D) Impossible

196)
$$f(x) = \begin{cases} 3x + 8, & \text{if } x < -5 \\ kx + 7, & \text{if } x \ge -5 \end{cases}$$

196) _____

A) $k = \dfrac{7}{5}$ B) k = 4   C) $k = -\dfrac{7}{5}$ D) $k = \dfrac{14}{5}$

197)
$$f(x) = \begin{cases} x^2, & \text{if } x \le 7 \\ x + k, & \text{if } x > 7 \end{cases}$$

197) _____

A) k = 56   B) k = 42   C) k = -7   D) Impossible

198)

$$f(x) = \begin{cases} x^2, & \text{if } x \le 8 \\ kx, & \text{if } x > 8 \end{cases}$$

A) k = 64   B) $k = \dfrac{1}{8}$ C) k = 8   D) Impossible

Solve the problem.

199) Select the correct statement for the definition of the limit: $\lim\limits_{x \to x_0} f(x) = L$

means that _____
   A) if given any number $\varepsilon > 0$, there exists a number $\delta > 0$, such that for all x,
      $0 < |x - x_0| < \varepsilon$ implies $|f(x) - L| < \delta$.
   B) if given any number $\varepsilon > 0$, there exists a number $\delta > 0$, such that for all x,
      $0 < |x - x_0| < \delta$ implies $|f(x) - L| < \varepsilon$.
   C) if given any number $\varepsilon > 0$, there exists a number $\delta > 0$, such that for all x,
      $0 < |x - x_0| < \varepsilon$ implies $|f(x) - L| > \delta$.
   D) if given a number $\varepsilon > 0$, there exists a number $\delta > 0$, such that for all x,
      $0 < |x - x_0| < \delta$ implies $|f(x) - L| > \varepsilon$.

200) Identify the incorrect statements about limits.

I. The number L is the limit of f(x) as x approaches x_0 if f(x) gets closer to L as x approaches x_0.

II. The number L is the limit of f(x) as x approaches x_0 if, for any $\varepsilon > 0$, there corresponds a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - x_0| < \delta$.

III. The number L is the limit of f(x) as x approaches x_0 if, given any $\varepsilon > 0$, there exists a value of x for which $|f(x) - L| < \varepsilon$.

   A) II and III   B) I and III   C) I and II   D) I, II, and III

Use the graph to find a $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

201)

f(x) = x + 2
$x_0 = 2$
L = 4
$\varepsilon = 0.2$

NOT TO SCALE

   A) 0.1   B) 0.2   C) 2   D) 0.4

202)

$f(x) = 4x - 1$
$x_0 = 2$
$L = 7$
$\varepsilon = 0.2$

y = 4x - 1

7.2
7
6.8

1.95   2.05

0   2

x
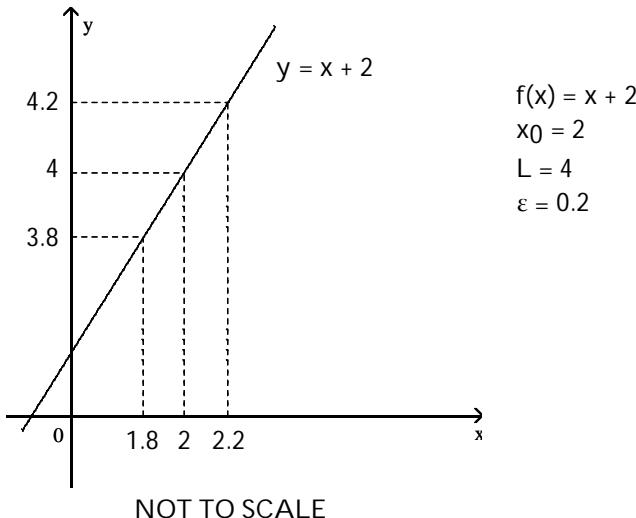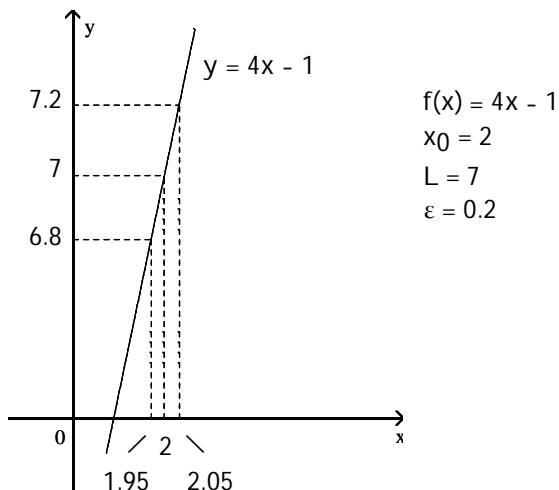
NOT TO SCALE

A) 0.1   B) 0.05   C) 5   D) 0.5

203)

$f(x) = -4x - 3$
$x_0 = -2$
$L = 5$
$\varepsilon = 0.2$

y = -4x - 3

5.2
5
4.8

-2.05   -1.95

-2   0

x
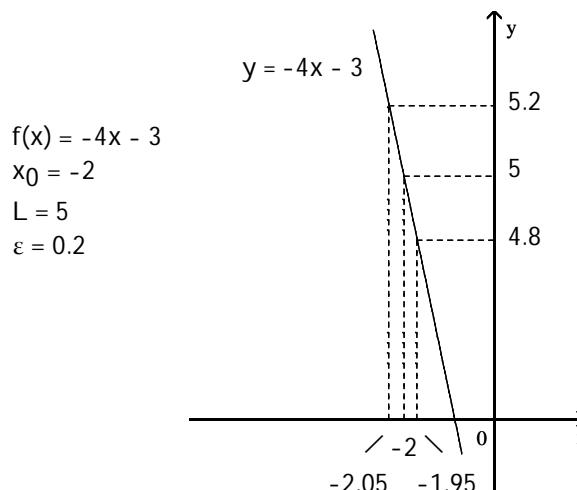
NOT TO SCALE

A) 0.5   B) 0.05   C) 13   D) -0.05

204)

$y = -x + 2$

$f(x) = -x + 2$
$x_0 = -2$
$L = 4$
$\varepsilon = 0.2$

4.2

4

3.8

-2.2 -2 -1.8   0   x

NOT TO SCALE

A) -0.2   B) 0.2   C) 0.4   D) 6

205)

$y = \frac{3}{2}x + 3$

4.7

4.5

4.3

$f(x) = \frac{3}{2}x + 3$

$x_0 = 1$
$L = 4.5$
$\varepsilon = 0.2$

0   0.9 1 1.1   x

NOT TO SCALE

A) -0.2   B) 0.2   C) 3.5   D) 0.1

206)

$$y = -\frac{3}{2}x + 2$$

$f(x) = -\frac{3}{2}x + 2$

$x_0 = -1$

$L = 3.5$

$\varepsilon = 0.2$

3.7

3.5

3.3

-1.1   -1   -0.9

0   x
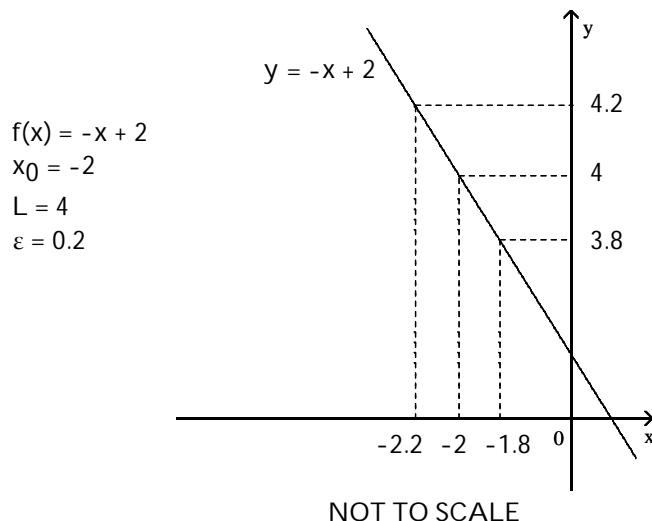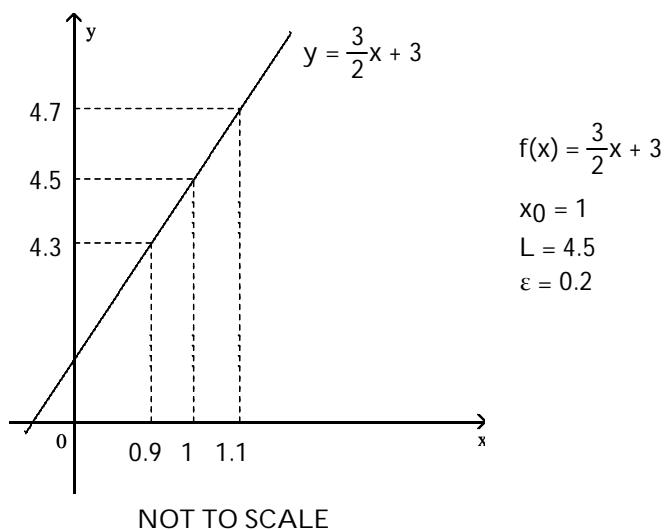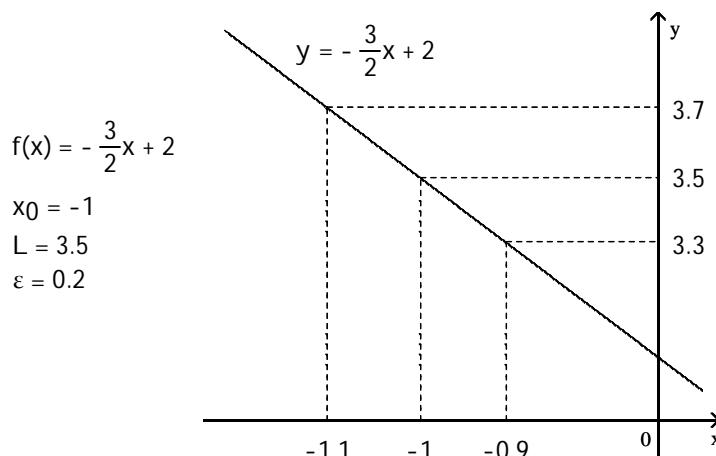
NOT TO SCALE

A) 4.5   B) 0.1   C) 0.2   D) -0.2

207)

$f(x) = \sqrt{x}$

$x_0 = 2$

$L = \sqrt{2}$

$\varepsilon = \frac{1}{4}$

$y = \sqrt{x}$

1.66

1.41

1.16

0   1.3575   2   2.7675   x
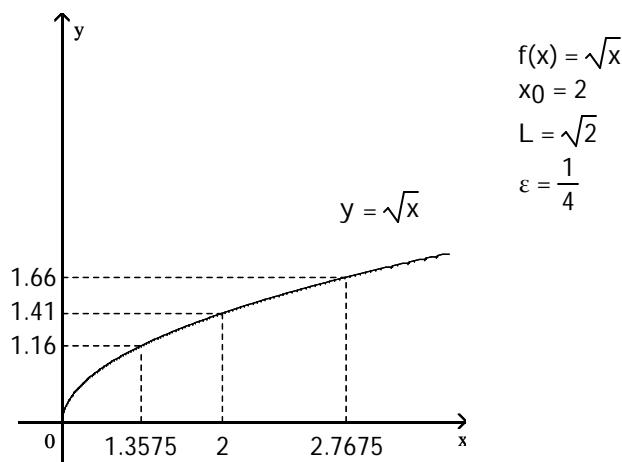
NOT TO SCALE

A) 0.7675   B) 0.6425   C) -0.59   D) 1.41

208)

$f(x) = \sqrt{x - 2}$
$x_0 = 3$
$L = 1$
$\varepsilon = \dfrac{1}{4}$

$y = \sqrt{x - 2}$

1.25
1
0.75

0   2.5625 3   3.5625   x

NOT TO SCALE

A) 1   B) 2   C) 0.4375   D) 0.5625

209)

$y = x^2$

$f(x) = x^2$
$x_0 = 2$
$L = 4$
$\varepsilon = 1$

5
4
3

0   2   x
  1.73   2.24

NOT TO SCALE

A) 0.51   B) 0.27   C) 2   D) 0.24

210)

$y = x^2 - 2$

8

7

6

0   3   x

2.83   3.16

NOT TO SCALE

$f(x) = x^2 - 2$
$x_0 = 3$
$L = 7$
$\varepsilon = 1$
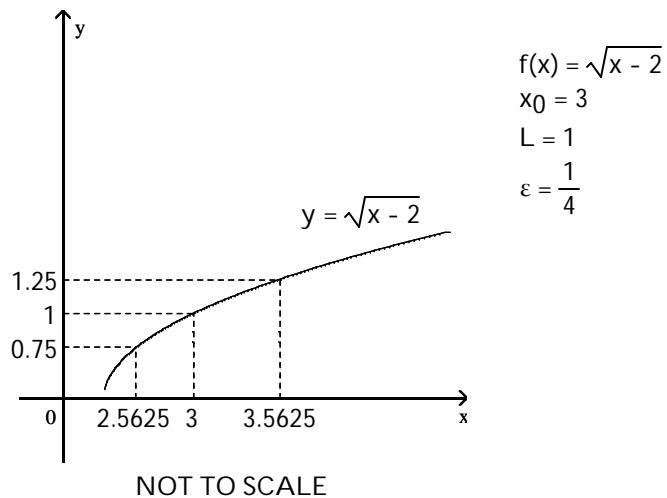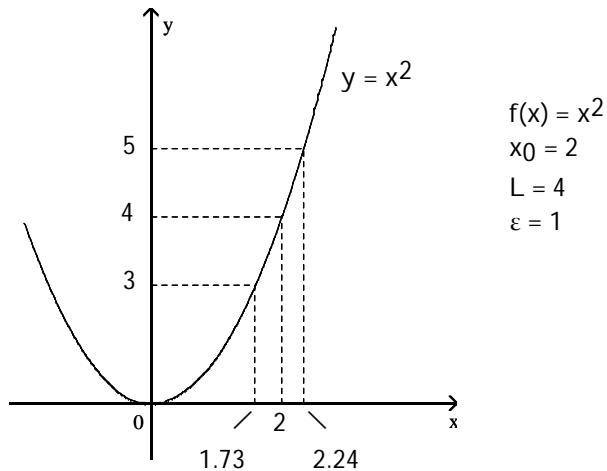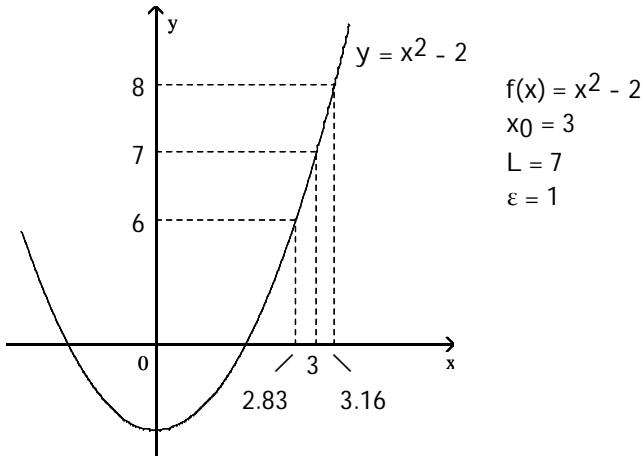
    A) 0.17   B) 4   C) 0.33   D) 0.16

A function f(x), a point x_0, the limit of f(x) as x approaches x_0, and a positive number ε is given. Find a number $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

    211) $f(x) = 3x + 10$, $L = 13$, $x_0 = 1$, and $\varepsilon = 0.01$ 211) _____

        A) 0.003333   B) 0.01   C) 0.016667   D) 0.006667

    212) $f(x) = 9x - 7$, $L = 2$, $x_0 = 1$, and $\varepsilon = 0.01$ 212) _____

        A) 0.001111   B) 0.002222   C) 0.000556   D) 0.01

    213) $f(x) = -3x + 9$, $L = 6$, $x_0 = 1$, and $\varepsilon = 0.01$ 213) _____

        A) 0.003333   B) -0.01   C) 0.013333   D) 0.006667

    214) $f(x) = -9x - 7$, $L = -34$, $x_0 = 3$, and $\varepsilon = 0.01$ 214) _____

        A) 0.000556   B) -0.003333   C) 0.002222   D) 0.001111

    215) $f(x) = 2x^2$, $L = 98$, $x_0 = 7$, and $\varepsilon = 0.4$ 215) _____

        A) 7.01427   B) 0.01427   C) 0.0143   D) 6.9857

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Prove the limit statement

    216) $\lim\limits_{x \to 5} (2x - 1) = 9$ 216) _____

    217) $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x - 5} = 10$ 217) _____

    218) $\lim\limits_{x \to 7} \dfrac{3x^2 - 19x - 14}{x - 7} = 23$ 218) _____

219) $\displaystyle\lim_{x \to 5} \frac{1}{x} = \frac{1}{5}$

219) _____

1) D
2) D
3) B
4) A
5) D
6) D
7) D
8) C
9) C
10) D
11) B
12) A
13) A
14) D
15) D
16) C
17) C
18) C
19) C
20) B
21) C
22) D
23) A
24) D
25) A
26) C
27) B
28) B
29) D
30) C
31) D
32) D
33) C
34) B
35) A
36) C
37) C
38) D
39) B

40) Answers may vary. One possibility: $\lim\limits_{x \to 0} 1 - \dfrac{x^2}{6} = \lim\limits_{x \to 0} 1 = 1$. According to the squeeze theorem, the function

$\dfrac{x \sin(x)}{2 - 2\cos(x)}$, which is squeezed between $1 - \dfrac{x^2}{6}$ and 1, must also approach 1 as x approaches 0. Thus,

$\lim\limits_{x \to 0} \dfrac{x \sin(x)}{2 - 2\cos(x)} = 1$.

41) C
42) A
43) D

44) C
45) B
46) A
47) D
48) D
49) D
50) C
51) C
52) B
53) B
54) A
55) D
56) D
57) D
58) C
59) C
60) A
61) A
62) B
63) B
64) D
65) B
66) C
67) B
68) D
69) B
70) C
71) B
72) C
73) D
74) D
75) A
76) C
77) B
78) B
79) C
80) B
81) D
82) D
83) B
84) C
85) A
86) D
87) B
88) B
89) D
90) B
91) C
92) A
93) D

94) A
95) D
96) B
97) B
98) A
99) B
100) D
101) B
102) B
103) D
104) D
105) B
106) C
107) D
108) D
109) A
110) B
111) A
112) D
113) C
114) B
115) B
116) D
117) A
118) D
119) D
120) C
121) D
122) D
123) B
124) A
125) D
126) A
127) D
128) A
129) B
130) D
131) A
132) B
133) C
134) C
135) D
136) A
137) D
138) D
139) C
140) A
141) C
142) C
143) D

144) C
145) B
146) D
147) A
148) B
149) A
150) D
151) B
152) C
153) A
154) C
155) Answers may vary. One possible answer:

156) Answers may vary. One possible answer:

157) Answers may vary. One possible answer:

158) Answers may vary. One possible answer:
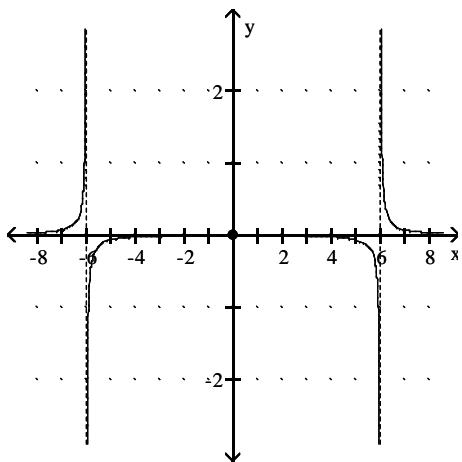
159) D
160) C
161) D
162) D
163) A
164) D
165) C
166) C
167) A
168) A
169) A
170) B
171) A
172) A
173) C
174) D
175) D
176) B
177) A
178) D
179) B

180) C
181) D
182) A
183) D
184) D
185) A
186) C
187) B
188) B
189) A
190) Let $f(x) = 5x^3 + 4x^2 + 10x + 10$ and let $y_0 = 0$. $f(-1) = -1$ and $f(0) = 10$. Since f is continuous on $[-1, 0]$ and since $y_0 = 0$ is between $f(-1)$ and $f(0)$, by the Intermediate Value Theorem, there exists a c in the interval $(-1, 0)$ with the property that $f(c) = 0$. Such a c is a solution to the equation $5x^3 + 4x^2 + 10x + 10 = 0$.
191) Let $f(x) = 6x^4 + 9x^3 - 5x - 8$ and let $y_0 = 0$. $f(-2) = 26$ and $f(-1) = -6$. Since f is continuous on $[-2, -1]$ and since $y_0 = 0$ is between $f(-2)$ and $f(-1)$, by the Intermediate Value Theorem, there exists a c in the interval $(-2, -1)$ with the property that $f(c) = 0$. Such a c is a solution to the equation $6x^4 + 9x^3 - 5x - 8 = 0$.
192) Let $f(x) = x(x - 9)^2$ and let $y_0 = 9$. $f(8) = 8$ and $f(10) = 10$. Since f is continuous on $[8, 10]$ and since $y_0 = 9$ is between $f(8)$ and $f(10)$, by the Intermediate Value Theorem, there exists a c in the interval $(8, 10)$ with the property that $f(c) = 9$. Such a c is a solution to the equation $x(x - 9)^2 = 9$.
193) Let $f(x) = \dfrac{\sin x}{x}$ and let $y_0 = \dfrac{1}{4}$. $f\left(\dfrac{\pi}{2}\right) \approx 0.6366$ and $f(\pi) = 0$. Since f is continuous on $\left[\dfrac{\pi}{2}, \pi\right]$ and since $y_0 = \dfrac{1}{4}$ is between $f\left(\dfrac{\pi}{2}\right)$ and $f(\pi)$, by the Intermediate Value Theorem, there exists a c in the interval $\left(\dfrac{\pi}{2}, \pi\right)$, with the property that $f(c) = \dfrac{1}{4}$. Such a c is a solution to the equation $4 \sin x = x$.

194) C
195) B
196) D
197) B
198) C
199) B
200) B
201) B
202) B
203) B
204) B
205) D
206) B
207) B
208) C
209) D
210) D
211) A
212) A
213) A
214) D
215) B

Answer Key
Testname: UNTITLED2

216)
    Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/2$. Then $0 < |x - 5| < \delta$ implies that

$$|(2x - 1) - 9| = |2x - 10|$$
$$= |2(x - 5)|$$
$$= 2|x - 5| < 2\delta = \varepsilon$$

    Thus, $0 < |x - 5| < \delta$ implies that $|(2x - 1) - 9| < \varepsilon$

217) Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$. Then $0 < |x - 5| < \delta$ implies that

$$\left|\frac{x^2 - 25}{x - 5} - 10\right| = \left|\frac{(x - 5)(x + 5)}{x - 5} - 10\right|$$
$$= |(x + 5) - 10| \quad \text{for } x \neq 5$$
$$= |x - 5| < \delta = \varepsilon$$

    Thus, $0 < |x - 5| < \delta$ implies that $\left|\dfrac{x^2 - 25}{x - 5} - 10\right| < \varepsilon$

218) Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/3$. Then $0 < |x - 7| < \delta$ implies that

$$\left|\frac{3x^2 - 19x - 14}{x - 7} - 23\right| = \left|\frac{(x - 7)(3x + 2)}{x - 7} - 23\right|$$
$$= |(3x + 2) - 23| \quad \text{for } x \neq 7$$
$$= |3x - 21|$$
$$= |3(x - 7)|$$
$$= 3|x - 7| < 3\delta = \varepsilon$$

    Thus, $0 < |x - 7| < \delta$ implies that $\left|\dfrac{3x^2 - 19x - 14}{x - 7} - 23\right| < \varepsilon$

219) Let $\varepsilon > 0$ be given. Choose $\delta = \min\{5/2, 25\varepsilon/2\}$. Then $0 < |x - 5| < \delta$ implies that

$$\left|\frac{1}{x} - \frac{1}{5}\right| = \left|\frac{5 - x}{5x}\right|$$
$$= \frac{1}{|x|} \cdot \frac{1}{5} \cdot |x - 5|$$
$$< \frac{1}{5/2} \cdot \frac{1}{5} \cdot \frac{25\varepsilon}{2} = \varepsilon$$

    Thus, $0 < |x - 5| < \delta$ implies that $\left|\dfrac{1}{x} - \dfrac{1}{5}\right| < \varepsilon$