

NOT FOR SALE

Complete Solutions Manual  
for  
**SINGLE VARIABLE CALCULUS  
EARLY TRANSCENDENTALS**  
SEVENTH EDITION

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ISBN-13: 978-0-8400-4936-0

ISBN-10: 0-8400-4936-6

**Brooks/Cole**

20 Davis Drive  
 Belmont, CA 94002-3098  
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1 2 3 4 5 6 7 14 13 12 11 10

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## PREFACE

This *Complete Solutions Manual* contains solutions to all exercises in the text *Single Variable Calculus, Early Transcendentals*, Seventh Edition, by James Stewart. A student version of this manual is also available; it contains solutions to the odd-numbered exercises in each section, the review sections, the True-False Quizzes, and the Problem Solving sections, as well as solutions to all the exercises in the Concept Checks. No solutions to the projects appear in the student version. It is our hope that by browsing through the solutions, professors will save time in determining appropriate assignments for their particular class.

We use some nonstandard notation in order to save space. If you see a symbol that you don't recognize, refer to the Table of Abbreviations and Symbols on page iv.

We appreciate feedback concerning errors, solution correctness or style, and manual style. Any comments may be sent directly to [jeff.cole@anokaramsey.edu](mailto:jeff.cole@anokaramsey.edu), or in care of the publisher: Brooks/Cole, Cengage Learning, 20 Davis Drive, Belmont CA 94002-3098.

We would like to thank Stephanie Kuhns and Kathi Townes, of TECHarts, for their production services; and Elizabeth Neustaetter, of Brooks/Cole, Cengage Learning, for her patience and support. All of these people have provided invaluable help in creating this manual.

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## □ ABBREVIATIONS AND SYMBOLS

CD	concave downward
CU	concave upward
D	the domain of $f$
FDT	First Derivative Test
HA	horizontal asymptote(s)
I	interval of convergence
IP	inflection point(s)
R	radius of convergence
VA	vertical asymptote(s)
$\overset{\text{CAS}}{=}$	indicates the use of a computer algebra system.
$\overset{\text{H}}{=}$	indicates the use of l'Hospital's Rule.
$\overset{j}{=}$	indicates the use of Formula $j$ in the Table of Integrals in the back endpapers.
$\overset{s}{=}$	indicates the use of the substitution $\{u = \sin x, du = \cos x dx\}$ .
$\overset{c}{=}$	indicates the use of the substitution $\{u = \cos x, du = -\sin x dx\}$ .

# CONTENTS

## DIAGNOSTIC TESTS 1

---

## 1 FUNCTIONS AND MODELS 9

---

- 1.1 Four Ways to Represent a Function 9
- 1.2 Mathematical Models: A Catalog of Essential Functions 20
- 1.3 New Functions from Old Functions 27
- 1.4 Graphing Calculators and Computers 38
- 1.5 Exponential Functions 46
- 1.6 Inverse Functions and Logarithms 51
- Review 61

### Principles of Problem Solving 69

## 2 LIMITS AND DERIVATIVES 75

---

- 2.1 The Tangent and Velocity Problems 75
- 2.2 The Limit of a Function 78
- 2.3 Calculating Limits Using the Limit Laws 87
- 2.4 The Precise Definition of a Limit 97
- 2.5 Continuity 104
- 2.6 Limits at Infinity; Horizontal Asymptotes 115
- 2.7 Derivatives and Rates of Change 129
- 2.8 The Derivative as a Function 140
- Review 153

### Problems Plus 165

## 3 DIFFERENTIATION RULES 169

---

- 3.1 Derivatives of Polynomials and Exponential Functions 169
  - Applied Project* • *Building a Better Roller Coaster* 179
- 3.2 The Product and Quotient Rules 182

NOT FOR SALE

## vi □ CONTENTS

3.3	Derivatives of Trigonometric Functions	191
3.4	The Chain Rule	198
	<i>Applied Project</i> ◦ <i>Where Should a Pilot Start Descent?</i>	210
3.5	Implicit Differentiation	210
	<i>Laboratory Project</i> ◦ <i>Families of Implicit Curves</i>	224
3.6	Derivatives of Logarithmic Functions	225
3.7	Rates of Change in the Natural and Social Sciences	231
3.8	Exponential Growth and Decay	240
3.9	Related Rates	245
3.10	Linear Approximations and Differentials	253
	<i>Laboratory Project</i> ◦ <i>Taylor Polynomials</i>	260
3.11	Hyperbolic Functions	262
	Review	270

**Problems Plus 285****4 □ APPLICATIONS OF DIFFERENTIATION 297**

4.1	Maximum and Minimum Values	297
	<i>Applied Project</i> ◦ <i>The Calculus of Rainbows</i>	307
4.2	The Mean Value Theorem	309
4.3	How Derivatives Affect the Shape of a Graph	314
4.4	Indeterminate Forms and l'Hospital's Rule	338
4.5	Summary of Curve Sketching	350
4.6	Graphing with Calculus and Calculators	374
4.7	Optimization Problems	394
	<i>Applied Project</i> ◦ <i>The Shape of a Can</i>	415
4.8	Newton's Method	416
4.9	Antiderivatives	427
	Review	437

**Problems Plus 461****5 □ INTEGRALS 471**

5.1	Areas and Distances	471
5.2	The Definite Integral	481
	<i>Discovery Project</i> ◦ <i>Area Functions</i>	492
5.3	The Fundamental Theorem of Calculus	494

INSTRUCTOR USE ONLY

NOT FOR SALE

- 5.4 Indefinite Integrals and the Net Change Theorem 505  
 5.5 The Substitution Rule 511  
 Review 521

**Problems Plus 531****6 □ APPLICATIONS OF INTEGRATION 537**

- 6.1 Areas Between Curves 537  
*Applied Project • The Gini Index 549*  
 6.2 Volumes 550  
 6.3 Volumes by Cylindrical Shells 566  
 6.4 Work 576  
 6.5 Average Value of a Function 582  
*Applied Project • Calculus and Baseball 585*  
*Applied Project • Where To Sit at the Movies 586*  
 Review 587

**Problems Plus 595****7 □ TECHNIQUES OF INTEGRATION 603**

- 7.1 Integration by Parts 603  
 7.2 Trigonometric Integrals 615  
 7.3 Trigonometric Substitution 624  
 7.4 Integration of Rational Functions by Partial Fractions 635  
 7.5 Strategy for Integration 652  
 7.6 Integration Using Tables and Computer Algebra Systems 664  
*Discovery Project • Patterns in Integrals 673*  
 7.7 Approximate Integration 675  
 7.8 Improper Integrals 689  
 Review 703

**Problems Plus 719****8 □ FURTHER APPLICATIONS OF INTEGRATION 727**

- 8.1 Arc Length 727  
*Discovery Project • Arc Length Contest 735*

INSTRUCTOR USE ONLY

NOT FOR SALE

## viii □ CONTENTS

- 8.2 Area of a Surface of Revolution 735  
     *Discovery Project* ◦ *Rotating on a Slant* 743
- 8.3 Applications to Physics and Engineering 744  
     *Discovery Project* ◦ *Complementary Coffee Cups* 757
- 8.4 Applications to Economics and Biology 758
- 8.5 Probability 761  
     Review 765

**Problems Plus 771****9 □ DIFFERENTIAL EQUATIONS 781**

---

- 9.1 Modeling with Differential Equations 781
- 9.2 Direction Fields and Euler's Method 784
- 9.3 Separable Equations 792  
     *Applied Project* ◦ *How Fast Does a Tank Drain?* 805  
     *Applied Project* ◦ *Which Is Faster, Going Up or Coming Down?* 806
- 9.4 Models for Population Growth 807
- 9.5 Linear Equations 817
- 9.6 Predator-Prey Systems 824  
     Review 829

**Problems Plus 837****10 □ PARAMETRIC EQUATIONS AND POLAR COORDINATES 843**

---

- 10.1 Curves Defined by Parametric Equations 843  
     *Laboratory Project* ◦ *Running Circles Around Circles* 857
- 10.2 Calculus with Parametric Curves 860  
     *Laboratory Project* ◦ *Bézier Curves* 874
- 10.3 Polar Coordinates 875  
     *Laboratory Project* ◦ *Families of Polar Curves* 890
- 10.4 Areas and Lengths in Polar Coordinates 893
- 10.5 Conic Sections 905
- 10.6 Conic Sections in Polar Coordinates 916  
     Review 922

**Problems Plus 935**

INSTRUCTOR USE ONLY



**11 □ INFINITE SEQUENCES AND SERIES 939**

- 11.1 Sequences 939
  - Laboratory Project • Logistic Sequences 952*
- 11.2 Series 956
- 11.3 The Integral Test and Estimates of Sums 971
- 11.4 The Comparison Tests 980
- 11.5 Alternating Series 985
- 11.6 Absolute Convergence and the Ratio and Root Tests 991
- 11.7 Strategy for Testing Series 998
- 11.8 Power Series 1002
- 11.9 Representations of Functions as Power Series 1011
- 11.10 Taylor and Maclaurin Series 1021
  - Laboratory Project • An Elusive Limit 1036*
- 11.11 Applications of Taylor Polynomials 1037
  - Applied Project • Radiation from the Stars 1051*
  - Review 1052

**Problems Plus 1065****□ APPENDIXES 1077**

- A Numbers, Inequalities, and Absolute Values 1077
- B Coordinate Geometry and Lines 1082
- C Graphs of Second-Degree Equations 1088
- D Trigonometry 1092
- E Sigma Notation 1100
- G The Logarithm Defined as an Integral 1104
- H Complex Numbers 1105

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## □ DIAGNOSTIC TESTS

### Test A Algebra

---

1. (a)  $(-3)^4 = (-3)(-3)(-3)(-3) = 81$  (b)  $-3^4 = -(3)(3)(3)(3) = -81$   
 (c)  $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$  (d)  $\frac{5^{23}}{5^{21}} = 5^{23-21} = 5^2 = 25$   
 (e)  $(\frac{2}{3})^{-2} = (\frac{3}{2})^2 = \frac{9}{4}$  (f)  $16^{-3/4} = \frac{1}{16^{3/4}} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{2^3} = \frac{1}{8}$
2. (a) Note that  $\sqrt{200} = \sqrt{100 \cdot 2} = 10\sqrt{2}$  and  $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$ . Thus  $\sqrt{200} - \sqrt{32} = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$ .  
 (b)  $(3a^3b^3)(4ab^2)^2 = 3a^3b^3 \cdot 16a^2b^4 = 48a^5b^7$   
 (c)  $\left(\frac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2} = \left(\frac{x^2y^{-1/2}}{3x^{3/2}y^3}\right)^2 = \frac{(x^2y^{-1/2})^2}{(3x^{3/2}y^3)^2} = \frac{x^4y^{-1}}{9x^3y^6} = \frac{x^4}{9x^3y^6y} = \frac{x}{9y^7}$
3. (a)  $3(x+6) + 4(2x-5) = 3x + 18 + 8x - 20 = 11x - 2$   
 (b)  $(x+3)(4x-5) = 4x^2 - 5x + 12x - 15 = 4x^2 + 7x - 15$   
 (c)  $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - \sqrt{a}\sqrt{b} + \sqrt{a}\sqrt{b} - (\sqrt{b})^2 = a - b$   
*Or:* Use the formula for the difference of two squares to see that  $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$ .  
 (d)  $(2x+3)^2 = (2x+3)(2x+3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$ .  
*Note:* A quicker way to expand this binomial is to use the formula  $(a+b)^2 = a^2 + 2ab + b^2$  with  $a = 2x$  and  $b = 3$ :  
 $(2x+3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$   
 (e) See Reference Page 1 for the binomial formula  $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ . Using it, we get  
 $(x+2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3 = x^3 + 6x^2 + 12x + 8$ .
4. (a) Using the difference of two squares formula,  $a^2 - b^2 = (a+b)(a-b)$ , we have  
 $4x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5)$ .  
 (b) Factoring by trial and error, we get  $2x^2 + 5x - 12 = (2x-3)(x+4)$ .  
 (c) Using factoring by grouping and the difference of two squares formula, we have  
 $x^3 - 3x^2 - 4x + 12 = x^2(x-3) - 4(x-3) = (x^2-4)(x-3) = (x-2)(x+2)(x-3)$ .  
 (d)  $x^4 + 27x = x(x^3 + 27) = x(x+3)(x^2 - 3x + 9)$   
 This last expression was obtained using the sum of two cubes formula,  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$  with  $a = x$  and  $b = 3$ . [See Reference Page 1 in the textbook.]  
 (e) The smallest exponent on  $x$  is  $-\frac{1}{2}$ , so we will factor out  $x^{-1/2}$ .  
 $3x^{3/2} - 9x^{1/2} + 6x^{-1/2} = 3x^{-1/2}(x^2 - 3x + 2) = 3x^{-1/2}(x-1)(x-2)$   
 (f)  $x^3y - 4xy = xy(x^2 - 4) = xy(x-2)(x+2)$

2 □ DIAGNOSTIC TESTS

5. (a)  $\frac{x^2 + 3x + 2}{x^2 - x - 2} = \frac{(x+1)(x+2)}{(x+1)(x-2)} = \frac{x+2}{x-2}$

(b)  $\frac{2x^2 - x - 1}{x^2 - 9} \cdot \frac{x+3}{2x+1} = \frac{(2x+1)(x-1)}{(x-3)(x+3)} \cdot \frac{x+3}{2x+1} = \frac{x-1}{x-3}$

(c)  $\frac{x^2}{x^2 - 4} - \frac{x+1}{x+2} = \frac{x^2}{(x-2)(x+2)} - \frac{x+1}{x+2} = \frac{x^2}{(x-2)(x+2)} - \frac{x+1}{x+2} \cdot \frac{x-2}{x-2} = \frac{x^2 - (x+1)(x-2)}{(x-2)(x+2)}$   
 $= \frac{x^2 - (x^2 - x - 2)}{(x+2)(x-2)} = \frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2}$

(d)  $\frac{\frac{y}{1} - \frac{x}{1}}{\frac{y}{1} - \frac{x}{1}} = \frac{\frac{y}{1} - \frac{x}{1}}{\frac{y}{1} - \frac{x}{1}} \cdot \frac{xy}{xy} = \frac{y^2 - x^2}{x - y} = \frac{(y-x)(y+x)}{-(y-x)} = \frac{y+x}{-1} = -(x+y)$

6. (a)  $\frac{\sqrt{10}}{\sqrt{5}-2} = \frac{\sqrt{10}}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{50} + 2\sqrt{10}}{(\sqrt{5})^2 - 2^2} = \frac{5\sqrt{2} + 2\sqrt{10}}{5-4} = 5\sqrt{2} + 2\sqrt{10}$

(b)  $\frac{\sqrt{4+h}-2}{h} = \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \frac{4+h-4}{h(\sqrt{4+h}+2)} = \frac{h}{h(\sqrt{4+h}+2)} = \frac{1}{\sqrt{4+h}+2}$

7. (a)  $x^2 + x + 1 = (x^2 + x + \frac{1}{4}) + 1 - \frac{1}{4} = (x + \frac{1}{2})^2 + \frac{3}{4}$

(b)  $2x^2 - 12x + 11 = 2(x^2 - 6x) + 11 = 2(x^2 - 6x + 9 - 9) + 11 = 2(x^2 - 6x + 9) - 18 + 11 = 2(x-3)^2 - 7$

8. (a)  $x + 5 = 14 - \frac{1}{2}x \Leftrightarrow x + \frac{1}{2}x = 14 - 5 \Leftrightarrow \frac{3}{2}x = 9 \Leftrightarrow x = \frac{2}{3} \cdot 9 \Leftrightarrow x = 6$

(b)  $\frac{2x}{x+1} = \frac{2x-1}{x} \Rightarrow 2x^2 = (2x-1)(x+1) \Leftrightarrow 2x^2 = 2x^2 + x - 1 \Leftrightarrow x = 1$

(c)  $x^2 - x - 12 = 0 \Leftrightarrow (x+3)(x-4) = 0 \Leftrightarrow x+3 = 0 \text{ or } x-4 = 0 \Leftrightarrow x = -3 \text{ or } x = 4$

(d) By the quadratic formula,  $2x^2 + 4x + 1 = 0 \Leftrightarrow$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4 \pm 2\sqrt{2}}{4} = \frac{2(-2 \pm \sqrt{2})}{4} = \frac{-2 \pm \sqrt{2}}{2} = -1 \pm \frac{1}{2}\sqrt{2}.$$

(e)  $x^4 - 3x^2 + 2 = 0 \Leftrightarrow (x^2 - 1)(x^2 - 2) = 0 \Leftrightarrow x^2 - 1 = 0 \text{ or } x^2 - 2 = 0 \Leftrightarrow x^2 = 1 \text{ or } x^2 = 2 \Leftrightarrow$   
 $x = \pm 1 \text{ or } x = \pm\sqrt{2}$

(f)  $3|x-4| = 10 \Leftrightarrow |x-4| = \frac{10}{3} \Leftrightarrow x-4 = -\frac{10}{3} \text{ or } x-4 = \frac{10}{3} \Leftrightarrow x = \frac{2}{3} \text{ or } x = \frac{22}{3}$

(g) Multiplying through  $2x(4-x)^{-1/2} - 3\sqrt{4-x} = 0$  by  $(4-x)^{1/2}$  gives  $2x - 3(4-x) = 0 \Leftrightarrow$   
 $2x - 12 + 3x = 0 \Leftrightarrow 5x - 12 = 0 \Leftrightarrow 5x = 12 \Leftrightarrow x = \frac{12}{5}.$

9. (a)  $-4 < 5 - 3x \leq 17 \Leftrightarrow -9 < -3x \leq 12 \Leftrightarrow 3 > x \geq -4 \text{ or } -4 \leq x < 3.$

In interval notation, the answer is  $[-4, 3)$ .

(b)  $x^2 < 2x + 8 \Leftrightarrow x^2 - 2x - 8 < 0 \Leftrightarrow (x+2)(x-4) < 0.$  Now,  $(x+2)(x-4)$  will change sign at the critical values  $x = -2$  and  $x = 4$ . Thus the possible intervals of solution are  $(-\infty, -2)$ ,  $(-2, 4)$ , and  $(4, \infty)$ . By choosing a single test value from each interval, we see that  $(-2, 4)$  is the only interval that satisfies the inequality.

(c) The inequality  $x(x-1)(x+2) > 0$  has critical values of  $-2, 0,$  and  $1$ . The corresponding possible intervals of solution are  $(-\infty, -2), (-2, 0), (0, 1)$  and  $(1, \infty)$ . By choosing a single test value from each interval, we see that both intervals  $(-2, 0)$  and  $(1, \infty)$  satisfy the inequality. Thus, the solution is the union of these two intervals:  $(-2, 0) \cup (1, \infty)$ .

(d)  $|x-4| < 3 \Leftrightarrow -3 < x-4 < 3 \Leftrightarrow 1 < x < 7$ . In interval notation, the answer is  $(1, 7)$ .

(e)  $\frac{2x-3}{x+1} \leq 1 \Leftrightarrow \frac{2x-3}{x+1} - 1 \leq 0 \Leftrightarrow \frac{2x-3}{x+1} - \frac{x+1}{x+1} \leq 0 \Leftrightarrow \frac{2x-3-x-1}{x+1} \leq 0 \Leftrightarrow \frac{x-4}{x+1} \leq 0$ .

Now, the expression  $\frac{x-4}{x+1}$  may change signs at the critical values  $x = -1$  and  $x = 4$ , so the possible intervals of solution are  $(-\infty, -1), (-1, 4],$  and  $[4, \infty)$ . By choosing a single test value from each interval, we see that  $(-1, 4]$  is the only interval that satisfies the inequality.

10. (a) False. In order for the statement to be true, it must hold for all real numbers, so, to show that the statement is false, pick  $p = 1$  and  $q = 2$  and observe that  $(1+2)^2 \neq 1^2 + 2^2$ . In general,  $(p+q)^2 = p^2 + 2pq + q^2$ .

(b) True as long as  $a$  and  $b$  are nonnegative real numbers. To see this, think in terms of the laws of exponents:

$$\sqrt{ab} = (ab)^{1/2} = a^{1/2}b^{1/2} = \sqrt{a}\sqrt{b}.$$

(c) False. To see this, let  $p = 1$  and  $q = 2$ , then  $\sqrt{1^2+2^2} \neq 1+2$ .

(d) False. To see this, let  $T = 1$  and  $C = 2$ , then  $\frac{1+1(2)}{2} \neq 1+1$ .

(e) False. To see this, let  $x = 2$  and  $y = 3$ , then  $\frac{1}{2-3} \neq \frac{1}{2} - \frac{1}{3}$ .

(f) True since  $\frac{1/x}{a/x - b/x} \cdot \frac{x}{x} = \frac{1}{a-b}$ , as long as  $x \neq 0$  and  $a-b \neq 0$ .

## Test B Analytic Geometry

1. (a) Using the point  $(2, -5)$  and  $m = -3$  in the point-slope equation of a line,  $y - y_1 = m(x - x_1)$ , we get

$$y - (-5) = -3(x - 2) \Rightarrow y + 5 = -3x + 6 \Rightarrow y = -3x + 1.$$

(b) A line parallel to the  $x$ -axis must be horizontal and thus have a slope of 0. Since the line passes through the point  $(2, -5)$ , the  $y$ -coordinate of every point on the line is  $-5$ , so the equation is  $y = -5$ .

(c) A line parallel to the  $y$ -axis is vertical with undefined slope. So the  $x$ -coordinate of every point on the line is 2 and so the equation is  $x = 2$ .

(d) Note that  $2x - 4y = 3 \Rightarrow -4y = -2x + 3 \Rightarrow y = \frac{1}{2}x - \frac{3}{4}$ . Thus the slope of the given line is  $m = \frac{1}{2}$ . Hence, the slope of the line we're looking for is also  $\frac{1}{2}$  (since the line we're looking for is required to be parallel to the given line).

$$\text{So the equation of the line is } y - (-5) = \frac{1}{2}(x - 2) \Rightarrow y + 5 = \frac{1}{2}x - 1 \Rightarrow y = \frac{1}{2}x - 6.$$

2. First we'll find the distance between the two given points in order to obtain the radius,  $r$ , of the circle:

$$r = \sqrt{[3 - (-1)]^2 + (-2 - 4)^2} = \sqrt{4^2 + (-6)^2} = \sqrt{52}.$$

Next use the standard equation of a circle,  $(x - h)^2 + (y - k)^2 = r^2$ , where  $(h, k)$  is the center, to get  $(x + 1)^2 + (y - 4)^2 = 52$ .

## 4 □ DIAGNOSTIC TESTS

3. We must rewrite the equation in standard form in order to identify the center and radius. Note that

$x^2 + y^2 - 6x + 10y + 9 = 0 \Rightarrow x^2 - 6x + 9 + y^2 + 10y = 0$ . For the left-hand side of the latter equation, we factor the first three terms and complete the square on the last two terms as follows:  $x^2 - 6x + 9 + y^2 + 10y = 0 \Rightarrow (x - 3)^2 + y^2 + 10y + 25 = 25 \Rightarrow (x - 3)^2 + (y + 5)^2 = 25$ . Thus, the center of the circle is  $(3, -5)$  and the radius is 5.

4. (a)  $A(-7, 4)$  and  $B(5, -12) \Rightarrow m_{AB} = \frac{-12 - 4}{5 - (-7)} = \frac{-16}{12} = -\frac{4}{3}$

(b)  $y - 4 = -\frac{4}{3}[x - (-7)] \Rightarrow y - 4 = -\frac{4}{3}x - \frac{28}{3} \Rightarrow 3y - 12 = -4x - 28 \Rightarrow 4x + 3y + 16 = 0$ . Putting  $y = 0$ , we get  $4x + 16 = 0$ , so the  $x$ -intercept is  $-4$ , and substituting 0 for  $x$  results in a  $y$ -intercept of  $-\frac{16}{3}$ .

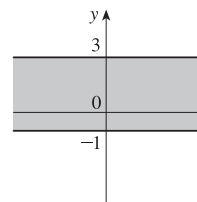
(c) The midpoint is obtained by averaging the corresponding coordinates of both points:  $(\frac{-7+5}{2}, \frac{4+(-12)}{2}) = (-1, -4)$ .

(d)  $d = \sqrt{[5 - (-7)]^2 + (-12 - 4)^2} = \sqrt{12^2 + (-16)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$

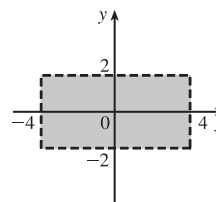
(e) The perpendicular bisector is the line that intersects the line segment  $\overline{AB}$  at a right angle through its midpoint. Thus the perpendicular bisector passes through  $(-1, -4)$  and has slope  $\frac{3}{4}$  [the slope is obtained by taking the negative reciprocal of the answer from part (a)]. So the perpendicular bisector is given by  $y + 4 = \frac{3}{4}[x - (-1)]$  or  $3x - 4y = 13$ .

(f) The center of the required circle is the midpoint of  $\overline{AB}$ , and the radius is half the length of  $\overline{AB}$ , which is 10. Thus, the equation is  $(x + 1)^2 + (y + 4)^2 = 100$ .

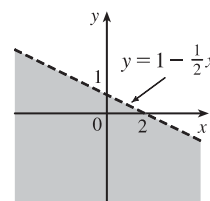
5. (a) Graph the corresponding horizontal lines (given by the equations  $y = -1$  and  $y = 3$ ) as solid lines. The inequality  $y \geq -1$  describes the points  $(x, y)$  that lie on or *above* the line  $y = -1$ . The inequality  $y \leq 3$  describes the points  $(x, y)$  that lie on or *below* the line  $y = 3$ . So the pair of inequalities  $-1 \leq y \leq 3$  describes the points that lie on or *between* the lines  $y = -1$  and  $y = 3$ .



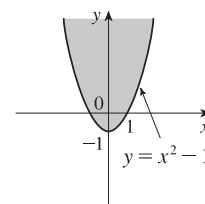
(b) Note that the given inequalities can be written as  $-4 < x < 4$  and  $-2 < y < 2$ , respectively. So the region lies between the vertical lines  $x = -4$  and  $x = 4$  and between the horizontal lines  $y = -2$  and  $y = 2$ . As shown in the graph, the region common to both graphs is a rectangle (minus its edges) centered at the origin.



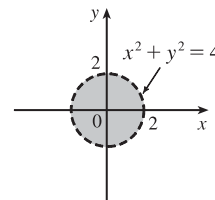
(c) We first graph  $y = 1 - \frac{1}{2}x$  as a dotted line. Since  $y < 1 - \frac{1}{2}x$ , the points in the region lie *below* this line.



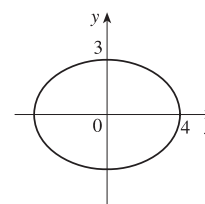
(d) We first graph the parabola  $y = x^2 - 1$  using a solid curve. Since  $y \geq x^2 - 1$ , the points in the region lie on or *above* the parabola.



(e) We graph the circle  $x^2 + y^2 = 4$  using a dotted curve. Since  $\sqrt{x^2 + y^2} < 2$ , the region consists of points whose distance from the origin is less than 2, that is, the points that lie *inside* the circle.



(f) The equation  $9x^2 + 16y^2 = 144$  is an ellipse centered at  $(0, 0)$ . We put it in standard form by dividing by 144 and get  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ . The  $x$ -intercepts are located at a distance of  $\sqrt{16} = 4$  from the center while the  $y$ -intercepts are a distance of  $\sqrt{9} = 3$  from the center (see the graph).



## Test C Functions

1. (a) Locate  $-1$  on the  $x$ -axis and then go down to the point on the graph with an  $x$ -coordinate of  $-1$ . The corresponding  $y$ -coordinate is the value of the function at  $x = -1$ , which is  $-2$ . So,  $f(-1) = -2$ .
 

(b) Using the same technique as in part (a), we get  $f(2) \approx 2.8$ .

(c) Locate  $2$  on the  $y$ -axis and then go left and right to find all points on the graph with a  $y$ -coordinate of  $2$ . The corresponding  $x$ -coordinates are the  $x$ -values we are searching for. So  $x = -3$  and  $x = 1$ .

(d) Using the same technique as in part (c), we get  $x \approx -2.5$  and  $x \approx 0.3$ .

(e) The domain is all the  $x$ -values for which the graph exists, and the range is all the  $y$ -values for which the graph exists. Thus, the domain is  $[-3, 3]$ , and the range is  $[-2, 3]$ .
2. Note that  $f(2 + h) = (2 + h)^3$  and  $f(2) = 2^3 = 8$ . So the difference quotient becomes
 
$$\frac{f(2 + h) - f(2)}{h} = \frac{(2 + h)^3 - 8}{h} = \frac{8 + 12h + 6h^2 + h^3 - 8}{h} = \frac{12h + 6h^2 + h^3}{h} = \frac{h(12 + 6h + h^2)}{h} = 12 + 6h + h^2.$$
3. (a) Set the denominator equal to 0 and solve to find restrictions on the domain:  $x^2 + x - 2 = 0 \Rightarrow (x - 1)(x + 2) = 0 \Rightarrow x = 1$  or  $x = -2$ . Thus, the domain is all real numbers except  $1$  or  $-2$  or, in interval notation,  $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$ .
 

(b) Note that the denominator is always greater than or equal to  $1$ , and the numerator is defined for all real numbers. Thus, the domain is  $(-\infty, \infty)$ .

(c) Note that the function  $h$  is the sum of two root functions. So  $h$  is defined on the intersection of the domains of these two root functions. The domain of a square root function is found by setting its radicand greater than or equal to  $0$ . Now,

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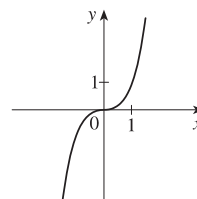
## 6 □ DIAGNOSTIC TESTS

$4 - x \geq 0 \Rightarrow x \leq 4$  and  $x^2 - 1 \geq 0 \Rightarrow (x - 1)(x + 1) \geq 0 \Rightarrow x \leq -1$  or  $x \geq 1$ . Thus, the domain of  $h$  is  $(-\infty, -1] \cup [1, 4]$ .

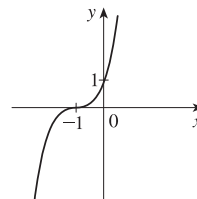
4. (a) Reflect the graph of  $f$  about the  $x$ -axis.  
(b) Stretch the graph of  $f$  vertically by a factor of 2, then shift 1 unit downward.  
(c) Shift the graph of  $f$  right 3 units, then up 2 units.

5. (a) Make a table and then connect the points with a smooth curve:

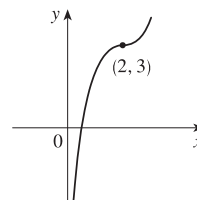
$x$	-2	-1	0	1	2
$y$	-8	-1	0	1	8



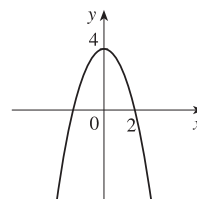
- (b) Shift the graph from part (a) left 1 unit.



- (c) Shift the graph from part (a) right 2 units and up 3 units.

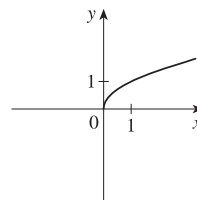


- (d) First plot  $y = x^2$ . Next, to get the graph of  $f(x) = 4 - x^2$ , reflect  $f$  about the  $x$ -axis and then shift it upward 4 units.

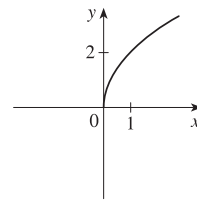


- (e) Make a table and then connect the points with a smooth curve:

$x$	0	1	4	9
$y$	0	1	2	3

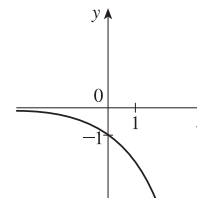


- (f) Stretch the graph from part (e) vertically by a factor of two.

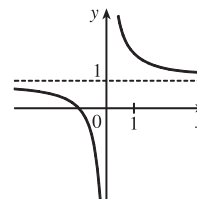




- (g) First plot  $y = 2^x$ . Next, get the graph of  $y = -2^x$  by reflecting the graph of  $y = 2^x$  about the  $x$ -axis.

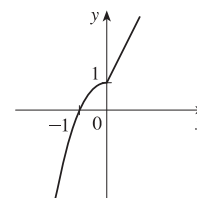


- (h) Note that  $y = 1 + x^{-1} = 1 + 1/x$ . So first plot  $y = 1/x$  and then shift it upward 1 unit.



6. (a)  $f(-2) = 1 - (-2)^2 = -3$  and  $f(1) = 2(1) + 1 = 3$

- (b) For  $x \leq 0$  plot  $f(x) = 1 - x^2$  and, on the same plane, for  $x > 0$  plot the graph of  $f(x) = 2x + 1$ .



7. (a)  $(f \circ g)(x) = f(g(x)) = f(2x - 3) = (2x - 3)^2 + 2(2x - 3) - 1 = 4x^2 - 12x + 9 + 4x - 6 - 1 = 4x^2 - 8x + 2$

(b)  $(g \circ f)(x) = g(f(x)) = g(x^2 + 2x - 1) = 2(x^2 + 2x - 1) - 3 = 2x^2 + 4x - 2 - 3 = 2x^2 + 4x - 5$

(c)  $(g \circ g \circ g)(x) = g(g(g(x))) = g(g(2x - 3)) = g(2(2x - 3) - 3) = g(4x - 9) = 2(4x - 9) - 3 = 8x - 18 - 3 = 8x - 21$

## Test D Trigonometry

1. (a)  $300^\circ = 300^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{300\pi}{180} = \frac{5\pi}{3}$

(b)  $-18^\circ = -18^\circ \left( \frac{\pi}{180^\circ} \right) = -\frac{18\pi}{180} = -\frac{\pi}{10}$

2. (a)  $\frac{5\pi}{6} = \frac{5\pi}{6} \left( \frac{180^\circ}{\pi} \right) = 150^\circ$

(b)  $2 = 2 \left( \frac{180^\circ}{\pi} \right) = \frac{360^\circ}{\pi} \approx 114.6^\circ$

3. We will use the arc length formula,  $s = r\theta$ , where  $s$  is arc length,  $r$  is the radius of the circle, and  $\theta$  is the measure of the central angle in radians. First, note that  $30^\circ = 30^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{\pi}{6}$ . So  $s = (12) \left( \frac{\pi}{6} \right) = 2\pi$  cm.

4. (a)  $\tan(\pi/3) = \sqrt{3}$  [You can read the value from a right triangle with sides 1, 2, and  $\sqrt{3}$ .]

- (b) Note that  $7\pi/6$  can be thought of as an angle in the third quadrant with reference angle  $\pi/6$ . Thus,  $\sin(7\pi/6) = -\frac{1}{2}$ , since the sine function is negative in the third quadrant.

- (c) Note that  $5\pi/3$  can be thought of as an angle in the fourth quadrant with reference angle  $\pi/3$ . Thus,

$$\sec(5\pi/3) = \frac{1}{\cos(5\pi/3)} = \frac{1}{1/2} = 2, \text{ since the cosine function is positive in the fourth quadrant.}$$

8 □ DIAGNOSTIC TESTS

5.  $\sin \theta = a/24 \Rightarrow a = 24 \sin \theta$  and  $\cos \theta = b/24 \Rightarrow b = 24 \cos \theta$

6.  $\sin x = \frac{1}{3}$  and  $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos x = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$ . Also,  $\cos y = \frac{4}{5} \Rightarrow \sin y = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$ .

So, using the sum identity for the sine, we have

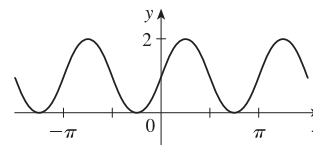
$$\sin(x + y) = \sin x \cos y + \cos x \sin y = \frac{1}{3} \cdot \frac{4}{5} + \frac{2\sqrt{2}}{3} \cdot \frac{3}{5} = \frac{4 + 6\sqrt{2}}{15} = \frac{1}{15}(4 + 6\sqrt{2})$$

7. (a)  $\tan \theta \sin \theta + \cos \theta = \frac{\sin \theta}{\cos \theta} \sin \theta + \cos \theta = \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta$

(b)  $\frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \sin x / (\cos x)}{\sec^2 x} = 2 \frac{\sin x}{\cos x} \cos^2 x = 2 \sin x \cos x = \sin 2x$

8.  $\sin 2x = \sin x \Leftrightarrow 2 \sin x \cos x = \sin x \Leftrightarrow 2 \sin x \cos x - \sin x = 0 \Leftrightarrow \sin x (2 \cos x - 1) = 0 \Leftrightarrow$   
 $\sin x = 0$  or  $\cos x = \frac{1}{2} \Rightarrow x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi.$

9. We first graph  $y = \sin 2x$  (by compressing the graph of  $\sin x$  by a factor of 2) and then shift it upward 1 unit.

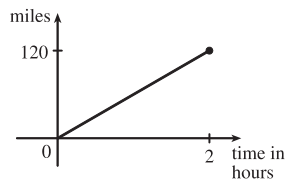


## 1 □ FUNCTIONS AND MODELS

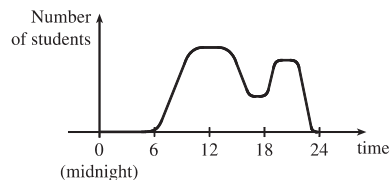
### 1.1 Four Ways to Represent a Function

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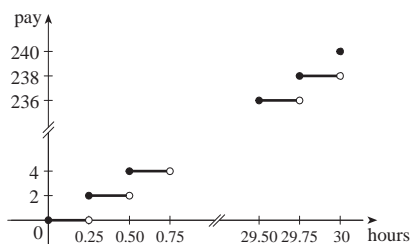
1. The functions  $f(x) = x + \sqrt{2-x}$  and  $g(u) = u + \sqrt{2-u}$  give exactly the same output values for every input value, so  $f$  and  $g$  are equal.
2.  $f(x) = \frac{x^2 - x}{x - 1} = \frac{x(x - 1)}{x - 1} = x$  for  $x - 1 \neq 0$ , so  $f$  and  $g$  [where  $g(x) = x$ ] are not equal because  $f(1)$  is undefined and  $g(1) = 1$ .
3. (a) The point  $(1, 3)$  is on the graph of  $f$ , so  $f(1) = 3$ .  
 (b) When  $x = -1$ ,  $y$  is about  $-0.2$ , so  $f(-1) \approx -0.2$ .  
 (c)  $f(x) = 1$  is equivalent to  $y = 1$ . When  $y = 1$ , we have  $x = 0$  and  $x = 3$ .  
 (d) A reasonable estimate for  $x$  when  $y = 0$  is  $x = -0.8$ .  
 (e) The domain of  $f$  consists of all  $x$ -values on the graph of  $f$ . For this function, the domain is  $-2 \leq x \leq 4$ , or  $[-2, 4]$ .  
 The range of  $f$  consists of all  $y$ -values on the graph of  $f$ . For this function, the range is  $-1 \leq y \leq 3$ , or  $[-1, 3]$ .  
 (f) As  $x$  increases from  $-2$  to  $1$ ,  $y$  increases from  $-1$  to  $3$ . Thus,  $f$  is increasing on the interval  $[-2, 1]$ .
4. (a) The point  $(-4, -2)$  is on the graph of  $f$ , so  $f(-4) = -2$ . The point  $(3, 4)$  is on the graph of  $g$ , so  $g(3) = 4$ .  
 (b) We are looking for the values of  $x$  for which the  $y$ -values are equal. The  $y$ -values for  $f$  and  $g$  are equal at the points  $(-2, 1)$  and  $(2, 2)$ , so the desired values of  $x$  are  $-2$  and  $2$ .  
 (c)  $f(x) = -1$  is equivalent to  $y = -1$ . When  $y = -1$ , we have  $x = -3$  and  $x = 4$ .  
 (d) As  $x$  increases from  $0$  to  $4$ ,  $y$  decreases from  $3$  to  $-1$ . Thus,  $f$  is decreasing on the interval  $[0, 4]$ .  
 (e) The domain of  $f$  consists of all  $x$ -values on the graph of  $f$ . For this function, the domain is  $-4 \leq x \leq 4$ , or  $[-4, 4]$ .  
 The range of  $f$  consists of all  $y$ -values on the graph of  $f$ . For this function, the range is  $-2 \leq y \leq 3$ , or  $[-2, 3]$ .  
 (f) The domain of  $g$  is  $[-4, 3]$  and the range is  $[0.5, 4]$ .
5. From Figure 1 in the text, the lowest point occurs at about  $(t, a) = (12, -85)$ . The highest point occurs at about  $(17, 115)$ . Thus, the range of the vertical ground acceleration is  $-85 \leq a \leq 115$ . Written in interval notation, we get  $[-85, 115]$ .
6. *Example 1:* A car is driven at 60 mi/h for 2 hours. The distance  $d$  traveled by the car is a function of the time  $t$ . The domain of the function is  $\{t \mid 0 \leq t \leq 2\}$ , where  $t$  is measured in hours. The range of the function is  $\{d \mid 0 \leq d \leq 120\}$ , where  $d$  is measured in miles.



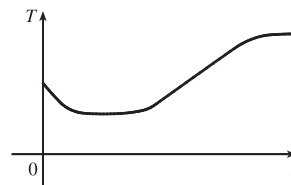
*Example 2:* At a certain university, the number of students  $N$  on campus at any time on a particular day is a function of the time  $t$  after midnight. The domain of the function is  $\{t \mid 0 \leq t \leq 24\}$ , where  $t$  is measured in hours. The range of the function is  $\{N \mid 0 \leq N \leq k\}$ , where  $N$  is an integer and  $k$  is the largest number of students on campus at once.



*Example 3:* A certain employee is paid \$8.00 per hour and works a maximum of 30 hours per week. The number of hours worked is rounded down to the nearest quarter of an hour. This employee's gross weekly pay  $P$  is a function of the number of hours worked  $h$ . The domain of the function is  $[0, 30]$  and the range of the function is  $\{0, 2.00, 4.00, \dots, 238.00, 240.00\}$ .



7. No, the curve is not the graph of a function because a vertical line intersects the curve more than once. Hence, the curve fails the Vertical Line Test.
8. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is  $[-2, 2]$  and the range is  $[-1, 2]$ .
9. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is  $[-3, 2]$  and the range is  $[-3, -2) \cup [-1, 3]$ .
10. No, the curve is not the graph of a function since for  $x = 0, \pm 1$ , and  $\pm 2$ , there are infinitely many points on the curve.
11. The person's weight increased to about 160 pounds at age 20 and stayed fairly steady for 10 years. The person's weight dropped to about 120 pounds for the next 5 years, then increased rapidly to about 170 pounds. The next 30 years saw a gradual increase to 190 pounds. Possible reasons for the drop in weight at 30 years of age: diet, exercise, health problems.
12. First, the tub was filled with water to a height of 15 in. Then a person got into the tub, raising the water level to 20 in. At around 12 minutes, the person stood up in the tub but then immediately sat down. Finally, at around 17 minutes, the person got out of the tub, and then drained the water.
13. The water will cool down almost to freezing as the ice melts. Then, when the ice has melted, the water will slowly warm up to room temperature.



14. Runner A won the race, reaching the finish line at 100 meters in about 15 seconds, followed by runner B with a time of about 19 seconds, and then by runner C who finished in around 23 seconds. B initially led the race, followed by C, and then A. C then passed B to lead for a while. Then A passed first B, and then passed C to take the lead and finish first. Finally, B passed C to finish in second place. All three runners completed the race.

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ISBN-13: 978-0-8400-4947-6

ISBN-10: 0-8400-4947-1

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 **PREFACE**

This *Complete Solutions Manual* contains detailed solutions to all exercises in the text *Multivariable Calculus*, Seventh Edition (Chapters 10–17 of *Calculus*, Seventh Edition, and *Calculus: Early Transcendentals*, Seventh Edition) by James Stewart. A *Student Solutions Manual* is also available, which contains solutions to the odd-numbered exercises in each chapter section, review section, True-False Quiz, and Problems Plus section as well as all solutions to the Concept Check questions. (It does not, however, include solutions to any of the projects.)

Because of differences between the regular version and the *Early Transcendentals* version of the text, some references are given in a dual format. In these cases, users of the *Early Transcendentals* text should use the references denoted by “ET.”

While we have extended every effort to ensure the accuracy of the solutions presented, we would appreciate correspondence regarding any errors that may exist. Other suggestions or comments are also welcome, and can be sent to dan clegg at [dclegg@palomar.edu](mailto:dclegg@palomar.edu) or in care of the publisher: Brooks/Cole, Cengage Learning, 20 Davis Drive, Belmont CA 94002-3098.

We would like to thank James Stewart for entrusting us with the writing of this manual and offering suggestions and Kathi Townes of TECH-arts for typesetting and producing this manual as well as creating the illustrations. We also thank Richard Stratton, Liz Covello, and Elizabeth Neustaetter of Brooks/Cole, Cengage Learning, for their trust, assistance, and patience.

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## □ ABBREVIATIONS AND SYMBOLS

CD	concave downward
CU	concave upward
D	the domain of $f$
FDT	First Derivative Test
HA	horizontal asymptote(s)
I	interval of convergence
I/D	Increasing/Decreasing Test
IP	inflection point(s)
R	radius of convergence
VA	vertical asymptote(s)
$\underline{\underline{\text{CAS}}}$	indicates the use of a computer algebra system.
$\underline{\underline{\text{H}}}$	indicates the use of l'Hospital's Rule.
$\underline{\underline{j}}$	indicates the use of Formula $j$ in the Table of Integrals in the back endpapers.
$\underline{\underline{s}}$	indicates the use of the substitution $\{u = \sin x, du = \cos x dx\}$ .
$\underline{\underline{c}}$	indicates the use of the substitution $\{u = \cos x, du = -\sin x dx\}$ .

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# CONTENTS

## 10 PARAMETRIC EQUATIONS AND POLAR COORDINATES 1

---

- 10.1 Curves Defined by Parametric Equations 1  
*Laboratory Project* • *Running Circles Around Circles* 15
- 10.2 Calculus with Parametric Curves 18  
*Laboratory Project* • *Bézier Curves* 32
- 10.3 Polar Coordinates 33  
*Laboratory Project* • *Families of Polar Curves* 48
- 10.4 Areas and Lengths in Polar Coordinates 51
- 10.5 Conic Sections 63
- 10.6 Conic Sections in Polar Coordinates 74
- Review 80

### Problems Plus 93

## 11 INFINITE SEQUENCES AND SERIES 97

---

- 11.1 Sequences 97  
*Laboratory Project* • *Logistic Sequences* 110
- 11.2 Series 114
- 11.3 The Integral Test and Estimates of Sums 129
- 11.4 The Comparison Tests 138
- 11.5 Alternating Series 143
- 11.6 Absolute Convergence and the Ratio and Root Tests 149
- 11.7 Strategy for Testing Series 156
- 11.8 Power Series 160
- 11.9 Representations of Functions as Power Series 169
- 11.10 Taylor and Maclaurin Series 179  
*Laboratory Project* • *An Elusive Limit* 194
- 11.11 Applications of Taylor Polynomials 195  
*Applied Project* • *Radiation from the Stars* 209
- Review 210

### Problems Plus 223

INSTRUCTOR USE ONLY

NOT FOR SALE

## viii □ CONTENTS

□ **12 VECTORS AND THE GEOMETRY OF SPACE 235**

---

- 12.1 Three-Dimensional Coordinate Systems 235
- 12.2 Vectors 242
- 12.3 The Dot Product 251
- 12.4 The Cross Product 260
  - Discovery Project • The Geometry of a Tetrahedron 271*
- 12.5 Equations of Lines and Planes 273
  - Laboratory Project • Putting 3D in Perspective 285*
- 12.6 Cylinders and Quadric Surfaces 287
  - Review 297

**Problems Plus 307**□ **13 VECTOR FUNCTIONS 313**

---

- 13.1 Vector Functions and Space Curves 313
- 13.2 Derivatives and Integrals of Vector Functions 324
- 13.3 Arc Length and Curvature 333
- 13.4 Motion in Space: Velocity and Acceleration 348
  - Applied Project • Kepler's Laws 359*
  - Review 360

**Problems Plus 367**□ **14 PARTIAL DERIVATIVES 373**

---

- 14.1 Functions of Several Variables 373
- 14.2 Limits and Continuity 391
- 14.3 Partial Derivatives 398
- 14.4 Tangent Planes and Linear Approximations 416
- 14.5 The Chain Rule 425
- 14.6 Directional Derivatives and the Gradient Vector 437
- 14.7 Maximum and Minimum Values 449
  - Applied Project • Designing a Dumpster 469*
  - Discovery Project • Quadratic Approximations and Critical Points 471*

INSTRUCTOR USE ONLY

NOT FOR SALE

14.8	Lagrange Multipliers	474
	<i>Applied Project</i> ◦ <i>Rocket Science</i>	485
	<i>Applied Project</i> ◦ <i>Hydro-Turbine Optimization</i>	488
	Review	490

**Problems Plus 505**□ **15 MULTIPLE INTEGRALS 511**

15.1	Double Integrals over Rectangles	511
15.2	Iterated Integrals	516
15.3	Double Integrals over General Regions	521
15.4	Double Integrals in Polar Coordinates	534
15.5	Applications of Double Integrals	542
15.6	Surface Area	553
15.7	Triple Integrals	557
	<i>Discovery Project</i> ◦ <i>Volumes of Hyperspheres</i>	574
15.8	Triple Integrals in Cylindrical Coordinates	575
	<i>Discovery Project</i> ◦ <i>The Intersection of Three Cylinders</i>	582
15.9	Triple Integrals in Spherical Coordinates	584
	<i>Applied Project</i> ◦ <i>Roller Derby</i>	594
15.10	Change of Variables in Multiple Integrals	595
	Review	601

**Problems Plus 615**□ **16 VECTOR CALCULUS 623**

16.1	Vector Fields	623
16.2	Line Integrals	628
16.3	The Fundamental Theorem for Line Integrals	637
16.4	Green's Theorem	643
16.5	Curl and Divergence	650
16.6	Parametric Surfaces and Their Areas	659
16.7	Surface Integrals	673
16.8	Stokes' Theorem	684
16.9	The Divergence Theorem	689
	Review	694

**Problems Plus 705**

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x □ CONTENTS

□ **17 SECOND-ORDER DIFFERENTIAL EQUATIONS 711**

---

- 17.1 Second-Order Linear Equations 711
- 17.2 Nonhomogeneous Linear Equations 715
- 17.3 Applications of Second-Order Differential Equations 720
- 17.4 Series Solutions 725
- Review 729

□ **APPENDIX 735**

---

- H Complex Numbers 735

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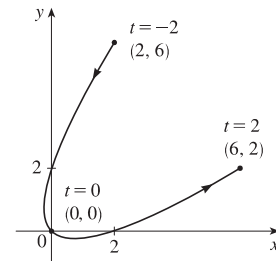
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## 10 □ PARAMETRIC EQUATIONS AND POLAR COORDINATES

### 10.1 Curves Defined by Parametric Equations

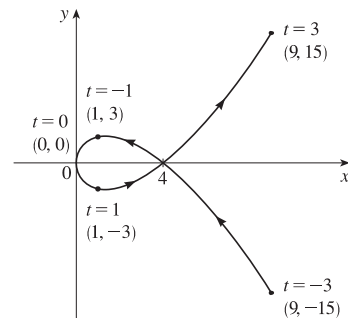
1.  $x = t^2 + t, y = t^2 - t, -2 \leq t \leq 2$

$t$	-2	-1	0	1	2
$x$	2	0	0	2	6
$y$	6	2	0	0	2



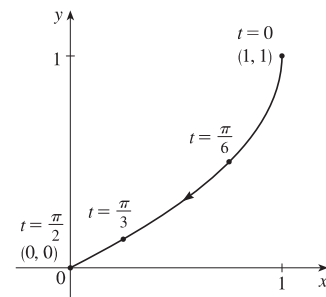
2.  $x = t^2, y = t^3 - 4t, -3 \leq t \leq 3$

$t$	±3	±2	±1	0
$x$	9	4	1	0
$y$	±15	0	∓3	0



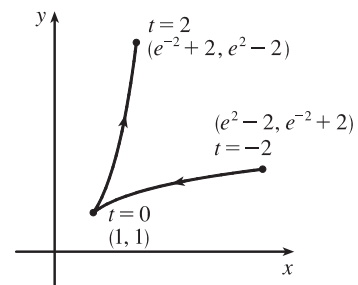
3.  $x = \cos^2 t, y = 1 - \sin t, 0 \leq t \leq \pi/2$

$t$	0	$\pi/6$	$\pi/3$	$\pi/2$
$x$	1	$3/4$	$1/4$	0
$y$	1	$1/2$	$1 - \frac{\sqrt{3}}{2} \approx 0.13$	0



4.  $x = e^{-t} + t, y = e^t - t, -2 \leq t \leq 2$

$t$	-2	-1	0	1	2
$x$	$e^2 - 2$ 5.39	$e - 1$ 1.72	1	$e^{-1} + 1$ 1.37	$e^{-2} + 2$ 2.14
$y$	$e^{-2} + 2$ 2.14	$e^{-1} + 1$ 1.37	1	$e - 1$ 1.72	$e^2 - 2$ 5.39



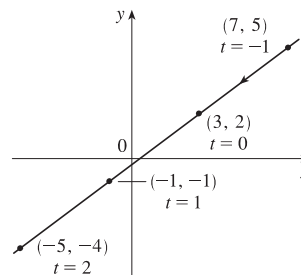
5.  $x = 3 - 4t$ ,  $y = 2 - 3t$

(a)

$t$	-1	0	1	2
$x$	7	3	-1	-5
$y$	5	2	-1	-4

(b)  $x = 3 - 4t \Rightarrow 4t = -x + 3 \Rightarrow t = -\frac{1}{4}x + \frac{3}{4}$ , so

$$y = 2 - 3t = 2 - 3\left(-\frac{1}{4}x + \frac{3}{4}\right) = 2 + \frac{3}{4}x - \frac{9}{4} \Rightarrow y = \frac{3}{4}x - \frac{1}{4}$$



6.  $x = 1 - 2t$ ,  $y = \frac{1}{2}t - 1$ ,  $-2 \leq t \leq 4$

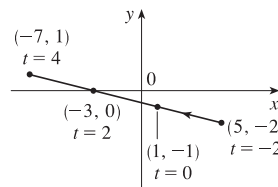
(a)

$t$	-2	0	2	4
$x$	5	1	-3	-7
$y$	-2	-1	0	1

(b)  $x = 1 - 2t \Rightarrow 2t = -x + 1 \Rightarrow t = -\frac{1}{2}x + \frac{1}{2}$ , so

$$y = \frac{1}{2}t - 1 = \frac{1}{2}\left(-\frac{1}{2}x + \frac{1}{2}\right) - 1 = -\frac{1}{4}x + \frac{1}{4} - 1 \Rightarrow y = -\frac{1}{4}x - \frac{3}{4}$$

with  $-7 \leq x \leq 5$



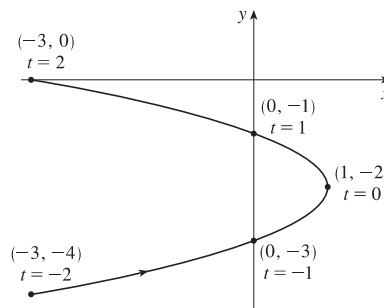
7.  $x = 1 - t^2$ ,  $y = t - 2$ ,  $-2 \leq t \leq 2$

(a)

$t$	-2	-1	0	1	2
$x$	-3	0	1	0	-3
$y$	-4	-3	-2	-1	0

(b)  $y = t - 2 \Rightarrow t = y + 2$ , so  $x = 1 - t^2 = 1 - (y + 2)^2 \Rightarrow$

$$x = -(y + 2)^2 + 1, \text{ or } x = -y^2 - 4y - 3, \text{ with } -4 \leq y \leq 0$$



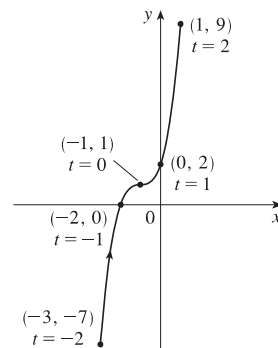
8.  $x = t - 1$ ,  $y = t^3 + 1$ ,  $-2 \leq t \leq 2$

(a)

$t$	-2	-1	0	1	2
$x$	-3	-2	-1	0	1
$y$	-7	0	1	2	9

(b)  $x = t - 1 \Rightarrow t = x + 1$ , so  $y = t^3 + 1 \Rightarrow y = (x + 1)^3 + 1$ ,

or  $y = x^3 + 3x^2 + 3x + 2$ , with  $-3 \leq x \leq 1$





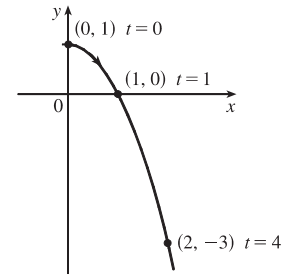
9.  $x = \sqrt{t}$ ,  $y = 1 - t$

(a)

$t$	0	1	2	3	4
$x$	0	1	1.414	1.732	2
$y$	1	0	-1	-2	-3

(b)  $x = \sqrt{t} \Rightarrow t = x^2 \Rightarrow y = 1 - t = 1 - x^2$ . Since  $t \geq 0$ ,  $x \geq 0$ .

So the curve is the right half of the parabola  $y = 1 - x^2$ .

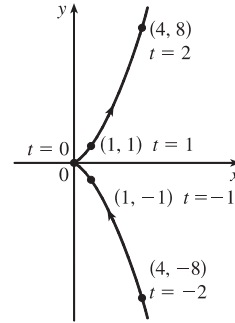


10.  $x = t^2$ ,  $y = t^3$

(a)

$t$	-2	-1	0	1	2
$x$	4	1	0	1	4
$y$	-8	-1	0	1	8

(b)  $y = t^3 \Rightarrow t = \sqrt[3]{y} \Rightarrow x = t^2 = (\sqrt[3]{y})^2 = y^{2/3}$ .  $t \in \mathbb{R}$ ,  $y \in \mathbb{R}$ ,  $x \geq 0$ .



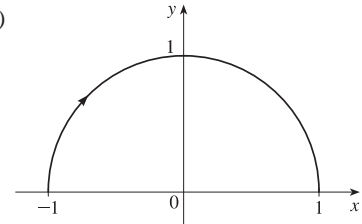
11. (a)  $x = \sin \frac{1}{2}\theta$ ,  $y = \cos \frac{1}{2}\theta$ ,  $-\pi \leq \theta \leq \pi$ .

$x^2 + y^2 = \sin^2 \frac{1}{2}\theta + \cos^2 \frac{1}{2}\theta = 1$ . For  $-\pi \leq \theta \leq 0$ , we have

$-1 \leq x \leq 0$  and  $0 \leq y \leq 1$ . For  $0 < \theta \leq \pi$ , we have  $0 < x \leq 1$

and  $1 > y \geq 0$ . The graph is a semicircle.

(b)



12. (a)  $x = \frac{1}{2} \cos \theta$ ,  $y = 2 \sin \theta$ ,  $0 \leq \theta \leq \pi$ .

$(2x)^2 + (\frac{1}{2}y)^2 = \cos^2 \theta + \sin^2 \theta = 1 \Rightarrow 4x^2 + \frac{1}{4}y^2 = 1 \Rightarrow$

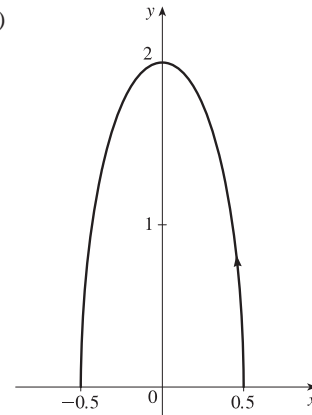
$\frac{x^2}{(1/2)^2} + \frac{y^2}{2^2} = 1$ , which is an equation of an ellipse with

$x$ -intercepts  $\pm \frac{1}{2}$  and  $y$ -intercepts  $\pm 2$ . For  $0 \leq \theta \leq \pi/2$ , we have

$\frac{1}{2} \geq x \geq 0$  and  $0 \leq y \leq 2$ . For  $\pi/2 < \theta \leq \pi$ , we have  $0 > x \geq -\frac{1}{2}$

and  $2 > y \geq 0$ . So the graph is the top half of the ellipse.

(b)

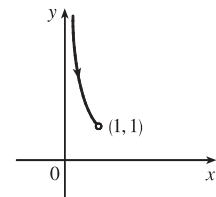


13. (a)  $x = \sin t$ ,  $y = \csc t$ ,  $0 < t < \frac{\pi}{2}$ .  $y = \csc t = \frac{1}{\sin t} = \frac{1}{x}$ .

For  $0 < t < \frac{\pi}{2}$ , we have  $0 < x < 1$  and  $y > 1$ . Thus, the curve is the

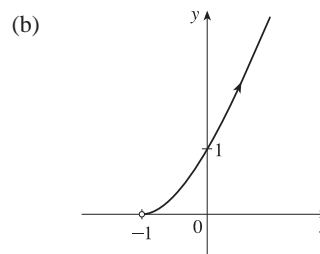
portion of the hyperbola  $y = 1/x$  with  $y > 1$ .

(b)



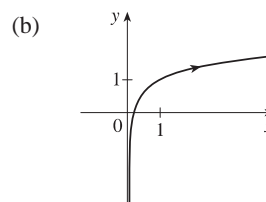
14. (a)  $x = e^t - 1, y = e^{2t}$ .

$y = (e^t)^2 = (x + 1)^2$  and since  $x > -1$ , we have the right side of the parabola  $y = (x + 1)^2$ .



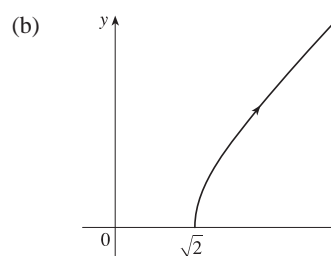
15. (a)  $x = e^{2t} \Rightarrow 2t = \ln x \Rightarrow t = \frac{1}{2} \ln x$ .

$y = t + 1 = \frac{1}{2} \ln x + 1$ .

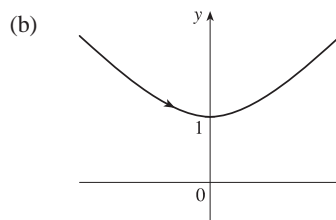


16. (a)  $x = \sqrt{t+1} \Rightarrow x^2 = t+1 \Rightarrow t = x^2 - 1$ .

$y = \sqrt{t-1} = \sqrt{(x^2 - 1) - 1} = \sqrt{x^2 - 2}$ . The curve is the part of the hyperbola  $x^2 - y^2 = 2$  with  $x \geq \sqrt{2}$  and  $y \geq 0$ .

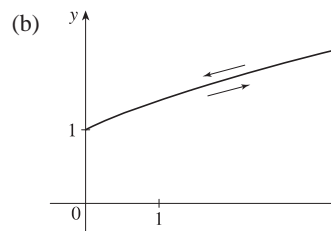


17. (a)  $x = \sinh t, y = \cosh t \Rightarrow y^2 - x^2 = \cosh^2 t - \sinh^2 t = 1$ . Since  $y = \cosh t \geq 1$ , we have the upper branch of the hyperbola  $y^2 - x^2 = 1$ .



18. (a)  $x = \tan^2 \theta, y = \sec \theta, -\pi/2 < \theta < \pi/2$ .

$1 + \tan^2 \theta = \sec^2 \theta \Rightarrow 1 + x = y^2 \Rightarrow x = y^2 - 1$ . For  $-\pi/2 < \theta \leq 0$ , we have  $x \geq 0$  and  $y \geq 1$ . For  $0 < \theta < \pi/2$ , we have  $0 < x$  and  $1 < y$ . Thus, the curve is the portion of the parabola  $x = y^2 - 1$  in the first quadrant. As  $\theta$  increases from  $-\pi/2$  to 0, the point  $(x, y)$  approaches  $(0, 1)$  along the parabola. As  $\theta$  increases from 0 to  $\pi/2$ , the point  $(x, y)$  retreats from  $(0, 1)$  along the parabola.

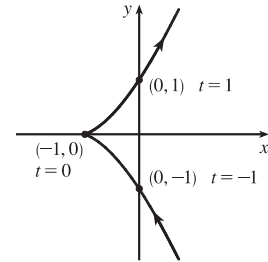


19.  $x = 3 + 2 \cos t, y = 1 + 2 \sin t, \pi/2 \leq t \leq 3\pi/2$ . By Example 4 with  $r = 2, h = 3$ , and  $k = 1$ , the motion of the particle takes place on a circle centered at  $(3, 1)$  with a radius of 2. As  $t$  goes from  $\pi/2$  to  $3\pi/2$ , the particle starts at the point  $(3, 3)$  and moves counterclockwise along the circle  $(x - 3)^2 + (y - 1)^2 = 4$  to  $(3, -1)$  [one-half of a circle].

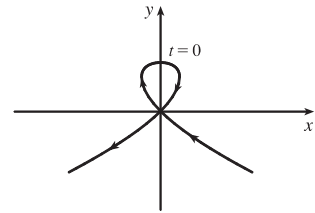
20.  $x = 2 \sin t, y = 4 + \cos t \Rightarrow \sin t = \frac{x}{2}, \cos t = y - 4, \sin^2 t + \cos^2 t = 1 \Rightarrow \left(\frac{x}{2}\right)^2 + (y - 4)^2 = 1$ . The motion of the particle takes place on an ellipse centered at  $(0, 4)$ . As  $t$  goes from 0 to  $3\pi/2$ , the particle starts at the point  $(0, 5)$  and moves clockwise to  $(-2, 4)$  [three-quarters of an ellipse].

21.  $x = 5 \sin t, y = 2 \cos t \Rightarrow \sin t = \frac{x}{5}, \cos t = \frac{y}{2}. \sin^2 t + \cos^2 t = 1 \Rightarrow \left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ . The motion of the particle takes place on an ellipse centered at  $(0, 0)$ . As  $t$  goes from  $-\pi$  to  $5\pi$ , the particle starts at the point  $(0, -2)$  and moves clockwise around the ellipse 3 times.
22.  $y = \cos^2 t = 1 - \sin^2 t = 1 - x^2$ . The motion of the particle takes place on the parabola  $y = 1 - x^2$ . As  $t$  goes from  $-2\pi$  to  $-\pi$ , the particle starts at the point  $(0, 1)$ , moves to  $(1, 0)$ , and goes back to  $(0, 1)$ . As  $t$  goes from  $-\pi$  to  $0$ , the particle moves to  $(-1, 0)$  and goes back to  $(0, 1)$ . The particle repeats this motion as  $t$  goes from  $0$  to  $2\pi$ .
23. We must have  $1 \leq x \leq 4$  and  $2 \leq y \leq 3$ . So the graph of the curve must be contained in the rectangle  $[1, 4]$  by  $[2, 3]$ .
24. (a) From the first graph, we have  $1 \leq x \leq 2$ . From the second graph, we have  $-1 \leq y \leq 1$ . The only choice that satisfies either of those conditions is III.
- (b) From the first graph, the values of  $x$  cycle through the values from  $-2$  to  $2$  four times. From the second graph, the values of  $y$  cycle through the values from  $-2$  to  $2$  six times. Choice I satisfies these conditions.
- (c) From the first graph, the values of  $x$  cycle through the values from  $-2$  to  $2$  three times. From the second graph, we have  $0 \leq y \leq 2$ . Choice IV satisfies these conditions.
- (d) From the first graph, the values of  $x$  cycle through the values from  $-2$  to  $2$  two times. From the second graph, the values of  $y$  do the same thing. Choice II satisfies these conditions.

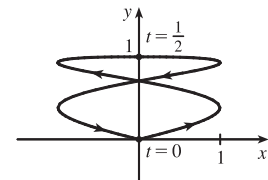
25. When  $t = -1$ ,  $(x, y) = (0, -1)$ . As  $t$  increases to  $0$ ,  $x$  decreases to  $-1$  and  $y$  increases to  $0$ . As  $t$  increases from  $0$  to  $1$ ,  $x$  increases to  $0$  and  $y$  increases to  $1$ . As  $t$  increases beyond  $1$ , both  $x$  and  $y$  increase. For  $t < -1$ ,  $x$  is positive and decreasing and  $y$  is negative and increasing. We could achieve greater accuracy by estimating  $x$ - and  $y$ -values for selected values of  $t$  from the given graphs and plotting the corresponding points.



26. For  $t < -1$ ,  $x$  is positive and decreasing, while  $y$  is negative and increasing (these points are in Quadrant IV). When  $t = -1$ ,  $(x, y) = (0, 0)$  and, as  $t$  increases from  $-1$  to  $0$ ,  $x$  becomes negative and  $y$  increases from  $0$  to  $1$ . At  $t = 0$ ,  $(x, y) = (0, 1)$  and, as  $t$  increases from  $0$  to  $1$ ,  $y$  decreases from  $1$  to  $0$  and  $x$  is positive. At  $t = 1$ ,  $(x, y) = (0, 0)$  again, so the loop is completed. For  $t > 1$ ,  $x$  and  $y$  both become large negative. This enables us to draw a rough sketch. We could achieve greater accuracy by estimating  $x$ - and  $y$ -values for selected values of  $t$  from the given graphs and plotting the corresponding points.

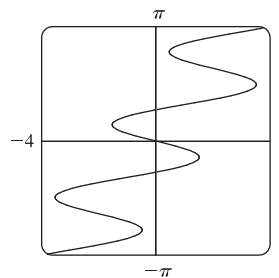


27. When  $t = 0$  we see that  $x = 0$  and  $y = 0$ , so the curve starts at the origin. As  $t$  increases from  $0$  to  $\frac{1}{2}$ , the graphs show that  $y$  increases from  $0$  to  $1$  while  $x$  increases from  $0$  to  $1$ , decreases to  $0$  and to  $-1$ , then increases back to  $0$ , so we arrive at the point  $(0, 1)$ . Similarly, as  $t$  increases from  $\frac{1}{2}$  to  $1$ ,  $y$  decreases from  $1$  to  $0$  while  $x$  repeats its pattern, and we arrive back at the origin. We could achieve greater accuracy by estimating  $x$ - and  $y$ -values for selected values of  $t$  from the given graphs and plotting the corresponding points.

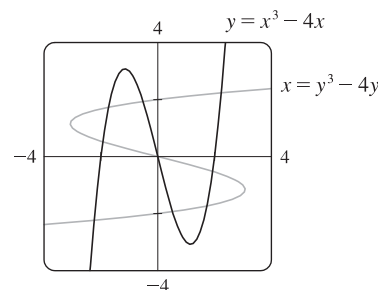


28. (a)  $x = t^4 - t + 1 = (t^4 + 1) - t > 0$  [think of the graphs of  $y = t^4 + 1$  and  $y = t$ ] and  $y = t^2 \geq 0$ , so these equations are matched with graph V.
- (b)  $y = \sqrt{t} \geq 0$ .  $x = t^2 - 2t = t(t - 2)$  is negative for  $0 < t < 2$ , so these equations are matched with graph I.
- (c)  $x = \sin 2t$  has period  $2\pi/2 = \pi$ . Note that  
 $y(t + 2\pi) = \sin[t + 2\pi + \sin 2(t + 2\pi)] = \sin(t + 2\pi + \sin 2t) = \sin(t + \sin 2t) = y(t)$ , so  $y$  has period  $2\pi$ .  
 These equations match graph II since  $x$  cycles through the values  $-1$  to  $1$  twice as  $y$  cycles through those values once.
- (d)  $x = \cos 5t$  has period  $2\pi/5$  and  $y = \sin 2t$  has period  $\pi$ , so  $x$  will take on the values  $-1$  to  $1$ , and then  $1$  to  $-1$ , before  $y$  takes on the values  $-1$  to  $1$ . Note that when  $t = 0$ ,  $(x, y) = (1, 0)$ . These equations are matched with graph VI.
- (e)  $x = t + \sin 4t$ ,  $y = t^2 + \cos 3t$ . As  $t$  becomes large,  $t$  and  $t^2$  become the dominant terms in the expressions for  $x$  and  $y$ , so the graph will look like the graph of  $y = x^2$ , but with oscillations. These equations are matched with graph IV.
- (f)  $x = \frac{\sin 2t}{4 + t^2}$ ,  $y = \frac{\cos 2t}{4 + t^2}$ . As  $t \rightarrow \infty$ ,  $x$  and  $y$  both approach 0. These equations are matched with graph III.

29. Use  $y = t$  and  $x = t - 2 \sin \pi t$  with a  $t$ -interval of  $[-\pi, \pi]$ .



30. Use  $x_1 = t$ ,  $y_1 = t^3 - 4t$  and  $x_2 = t^3 - 4t$ ,  $y_2 = t$  with a  $t$ -interval of  $[-3, 3]$ . There are 9 points of intersection;  $(0, 0)$  is fairly obvious. The point in quadrant I is approximately  $(2.2, 2.2)$ , and by symmetry, the point in quadrant III is approximately  $(-2.2, -2.2)$ . The other six points are approximately  $(\mp 1.9, \pm 0.5)$ ,  $(\mp 1.7, \pm 1.7)$ , and  $(\mp 0.5, \pm 1.9)$ .



31. (a)  $x = x_1 + (x_2 - x_1)t$ ,  $y = y_1 + (y_2 - y_1)t$ ,  $0 \leq t \leq 1$ . Clearly the curve passes through  $P_1(x_1, y_1)$  when  $t = 0$  and through  $P_2(x_2, y_2)$  when  $t = 1$ . For  $0 < t < 1$ ,  $x$  is strictly between  $x_1$  and  $x_2$  and  $y$  is strictly between  $y_1$  and  $y_2$ . For every value of  $t$ ,  $x$  and  $y$  satisfy the relation  $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ , which is the equation of the line through  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$ .
- Finally, any point  $(x, y)$  on that line satisfies  $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$ ; if we call that common value  $t$ , then the given parametric equations yield the point  $(x, y)$ ; and any  $(x, y)$  on the line between  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$  yields a value of  $t$  in  $[0, 1]$ . So the given parametric equations exactly specify the line segment from  $P_1(x_1, y_1)$  to  $P_2(x_2, y_2)$ .
- (b)  $x = -2 + [3 - (-2)]t = -2 + 5t$  and  $y = 7 + (-1 - 7)t = 7 - 8t$  for  $0 \leq t \leq 1$ .

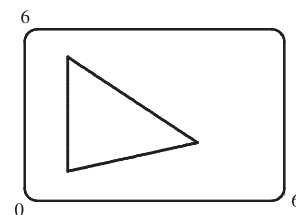
32. For the side of the triangle from  $A$  to  $B$ , use  $(x_1, y_1) = (1, 1)$  and  $(x_2, y_2) = (4, 2)$ .

Hence, the equations are

$$\begin{aligned} x &= x_1 + (x_2 - x_1)t = 1 + (4 - 1)t = 1 + 3t, \\ y &= y_1 + (y_2 - y_1)t = 1 + (2 - 1)t = 1 + t. \end{aligned}$$

Graphing  $x = 1 + 3t$  and  $y = 1 + t$  with  $0 \leq t \leq 1$  gives us the side of the

triangle from  $A$  to  $B$ . Similarly, for the side  $BC$  we use  $x = 4 - 3t$  and  $y = 2 + 3t$ , and for the side  $AC$  we use  $x = 1$  and  $y = 1 + 4t$ .



33. The circle  $x^2 + (y - 1)^2 = 4$  has center  $(0, 1)$  and radius 2, so by Example 4 it can be represented by  $x = 2 \cos t$ ,  $y = 1 + 2 \sin t$ ,  $0 \leq t \leq 2\pi$ . This representation gives us the circle with a counterclockwise orientation starting at  $(2, 1)$ .

(a) To get a clockwise orientation, we could change the equations to  $x = 2 \cos t$ ,  $y = 1 - 2 \sin t$ ,  $0 \leq t \leq 2\pi$ .

(b) To get three times around in the counterclockwise direction, we use the original equations  $x = 2 \cos t$ ,  $y = 1 + 2 \sin t$  with the domain expanded to  $0 \leq t \leq 6\pi$ .

(c) To start at  $(0, 3)$  using the original equations, we must have  $x_1 = 0$ ; that is,  $2 \cos t = 0$ . Hence,  $t = \frac{\pi}{2}$ . So we use

$$x = 2 \cos t, \quad y = 1 + 2 \sin t, \quad \frac{\pi}{2} \leq t \leq \frac{3\pi}{2}.$$

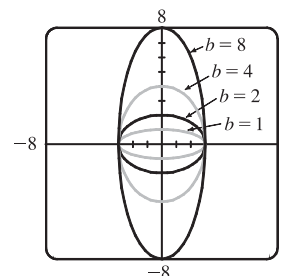
Alternatively, if we want  $t$  to start at 0, we could change the equations of the curve. For example, we could use

$$x = -2 \sin t, \quad y = 1 + 2 \cos t, \quad 0 \leq t \leq \pi.$$

34. (a) Let  $x^2/a^2 = \sin^2 t$  and  $y^2/b^2 = \cos^2 t$  to obtain  $x = a \sin t$  and  $y = b \cos t$  with  $0 \leq t \leq 2\pi$  as possible parametric equations for the ellipse  $x^2/a^2 + y^2/b^2 = 1$ .

(b) The equations are  $x = 3 \sin t$  and  $y = b \cos t$  for  $b \in \{1, 2, 4, 8\}$ .

(c) As  $b$  increases, the ellipse stretches vertically.



35. *Big circle*: It's centered at  $(2, 2)$  with a radius of 2, so by Example 4, parametric equations are

$$x = 2 + 2 \cos t, \quad y = 2 + 2 \sin t, \quad 0 \leq t \leq 2\pi$$

*Small circles*: They are centered at  $(1, 3)$  and  $(3, 3)$  with a radius of 0.1. By Example 4, parametric equations are

$$\text{(left)} \quad x = 1 + 0.1 \cos t, \quad y = 3 + 0.1 \sin t, \quad 0 \leq t \leq 2\pi$$

and

$$\text{(right)} \quad x = 3 + 0.1 \cos t, \quad y = 3 + 0.1 \sin t, \quad 0 \leq t \leq 2\pi$$

*Semicircle*: It's the lower half of a circle centered at  $(2, 2)$  with radius 1. By Example 4, parametric equations are

$$x = 2 + 1 \cos t, \quad y = 2 + 1 \sin t, \quad \pi \leq t \leq 2\pi$$

To get all four graphs on the same screen with a typical graphing calculator, we need to change the last  $t$ -interval to  $[0, 2\pi]$  in order to match the others. We can do this by changing  $t$  to  $0.5t$ . This change gives us the upper half. There are several ways to get the lower half—one is to change the “+” to a “-” in the  $y$ -assignment, giving us

$$x = 2 + 1 \cos(0.5t), \quad y = 2 - 1 \sin(0.5t), \quad 0 \leq t \leq 2\pi$$

36. If you are using a calculator or computer that can overlay graphs (using multiple  $t$ -intervals), the following is appropriate.

*Left side:*  $x = 1$  and  $y$  goes from 1.5 to 4, so use

$$x = 1, \quad y = t, \quad 1.5 \leq t \leq 4$$

*Right side:*  $x = 10$  and  $y$  goes from 1.5 to 4, so use

$$x = 10, \quad y = t, \quad 1.5 \leq t \leq 4$$

*Bottom:*  $x$  goes from 1 to 10 and  $y = 1.5$ , so use

$$x = t, \quad y = 1.5, \quad 1 \leq t \leq 10$$

*Handle:* It starts at (10, 4) and ends at (13, 7), so use

$$x = 10 + t, \quad y = 4 + t, \quad 0 \leq t \leq 3$$

*Left wheel:* It's centered at (3, 1), has a radius of 1, and appears to go about  $30^\circ$  above the horizontal, so use

$$x = 3 + 1 \cos t, \quad y = 1 + 1 \sin t, \quad \frac{5\pi}{6} \leq t \leq \frac{13\pi}{6}$$

*Right wheel:* Similar to the left wheel with center (8, 1), so use

$$x = 8 + 1 \cos t, \quad y = 1 + 1 \sin t, \quad \frac{5\pi}{6} \leq t \leq \frac{13\pi}{6}$$

If you are using a calculator or computer that cannot overlay graphs (using one  $t$ -interval), the following is appropriate.

We'll start by picking the  $t$ -interval  $[0, 2.5]$  since it easily matches the  $t$ -values for the two sides. We now need to find parametric equations for all graphs with  $0 \leq t \leq 2.5$ .

*Left side:*  $x = 1$  and  $y$  goes from 1.5 to 4, so use

$$x = 1, \quad y = 1.5 + t, \quad 0 \leq t \leq 2.5$$

*Right side:*  $x = 10$  and  $y$  goes from 1.5 to 4, so use

$$x = 10, \quad y = 1.5 + t, \quad 0 \leq t \leq 2.5$$

*Bottom:*  $x$  goes from 1 to 10 and  $y = 1.5$ , so use

$$x = 1 + 3.6t, \quad y = 1.5, \quad 0 \leq t \leq 2.5$$

To get the  $x$ -assignment, think of creating a linear function such that when  $t = 0$ ,  $x = 1$  and when  $t = 2.5$ ,  $x = 10$ . We can use the point-slope form of a line with  $(t_1, x_1) = (0, 1)$  and  $(t_2, x_2) = (2.5, 10)$ .

$$x - 1 = \frac{10 - 1}{2.5 - 0}(t - 0) \Rightarrow x = 1 + 3.6t.$$

*Handle:* It starts at (10, 4) and ends at (13, 7), so use

$$x = 10 + 1.2t, \quad y = 4 + 1.2t, \quad 0 \leq t \leq 2.5$$

$$(t_1, x_1) = (0, 10) \text{ and } (t_2, x_2) = (2.5, 13) \text{ gives us } x - 10 = \frac{13 - 10}{2.5 - 0}(t - 0) \Rightarrow x = 10 + 1.2t.$$

$$(t_1, y_1) = (0, 4) \text{ and } (t_2, y_2) = (2.5, 7) \text{ gives us } y - 4 = \frac{7 - 4}{2.5 - 0}(t - 0) \Rightarrow y = 4 + 1.2t.$$

*Left wheel:* It's centered at  $(3, 1)$ , has a radius of 1, and appears to go about  $30^\circ$  above the horizontal, so use

$$x = 3 + 1 \cos\left(\frac{8\pi}{15}t + \frac{5\pi}{6}\right), \quad y = 1 + 1 \sin\left(\frac{8\pi}{15}t + \frac{5\pi}{6}\right), \quad 0 \leq t \leq 2.5$$

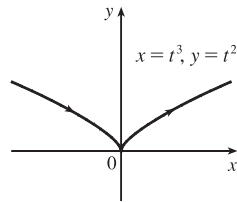
$$(t_1, \theta_1) = \left(0, \frac{5\pi}{6}\right) \text{ and } (t_2, \theta_2) = \left(\frac{5}{2}, \frac{13\pi}{6}\right) \text{ gives us } \theta - \frac{5\pi}{6} = \frac{\frac{13\pi}{6} - \frac{5\pi}{6}}{\frac{5}{2} - 0}(t - 0) \Rightarrow \theta = \frac{5\pi}{6} + \frac{8\pi}{15}t.$$

*Right wheel:* Similar to the left wheel with center  $(8, 1)$ , so use

$$x = 8 + 1 \cos\left(\frac{8\pi}{15}t + \frac{5\pi}{6}\right), \quad y = 1 + 1 \sin\left(\frac{8\pi}{15}t + \frac{5\pi}{6}\right), \quad 0 \leq t \leq 2.5$$

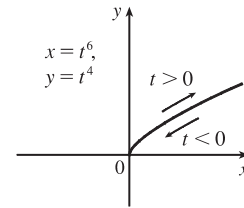
37. (a)  $x = t^3 \Rightarrow t = x^{1/3}$ , so  $y = t^2 = x^{2/3}$ .

We get the entire curve  $y = x^{2/3}$  traversed in a left to right direction.



(b)  $x = t^6 \Rightarrow t = x^{1/6}$ , so  $y = t^4 = x^{4/6} = x^{2/3}$ .

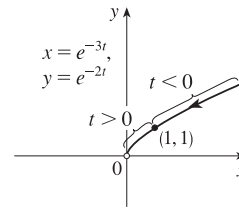
Since  $x = t^6 \geq 0$ , we only get the right half of the curve  $y = x^{2/3}$ .



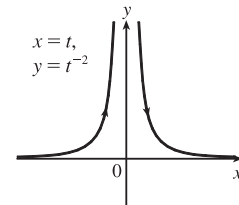
(c)  $x = e^{-3t} = (e^{-t})^3$  [so  $e^{-t} = x^{1/3}$ ],

$$y = e^{-2t} = (e^{-t})^2 = (x^{1/3})^2 = x^{2/3}.$$

If  $t < 0$ , then  $x$  and  $y$  are both larger than 1. If  $t > 0$ , then  $x$  and  $y$  are between 0 and 1. Since  $x > 0$  and  $y > 0$ , the curve never quite reaches the origin.

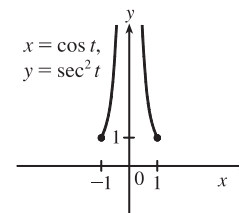


38. (a)  $x = t$ , so  $y = t^{-2} = x^{-2}$ . We get the entire curve  $y = 1/x^2$  traversed in a left-to-right direction.

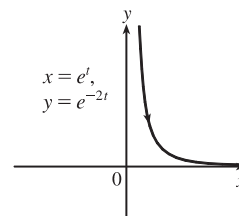


(b)  $x = \cos t$ ,  $y = \sec^2 t = \frac{1}{\cos^2 t} = \frac{1}{x^2}$ . Since  $\sec t \geq 1$ , we only get the

parts of the curve  $y = 1/x^2$  with  $y \geq 1$ . We get the first quadrant portion of the curve when  $x > 0$ , that is,  $\cos t > 0$ , and we get the second quadrant portion of the curve when  $x < 0$ , that is,  $\cos t < 0$ .



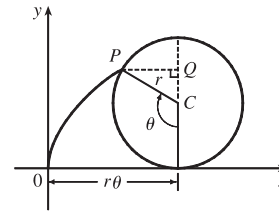
(c)  $x = e^t$ ,  $y = e^{-2t} = (e^t)^{-2} = x^{-2}$ . Since  $e^t$  and  $e^{-2t}$  are both positive, we only get the first quadrant portion of the curve  $y = 1/x^2$ .



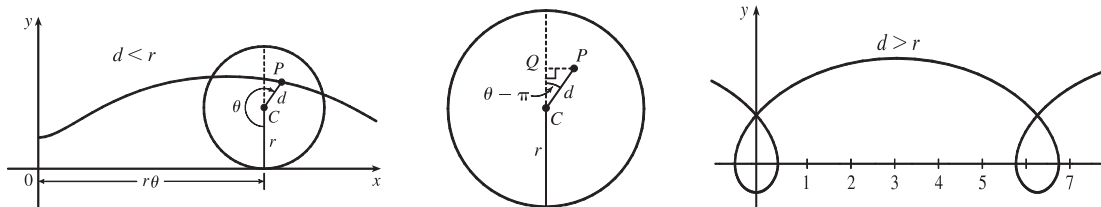
# NOT FOR SALE

10 □ CHAPTER 10 PARAMETRIC EQUATIONS AND POLAR COORDINATES

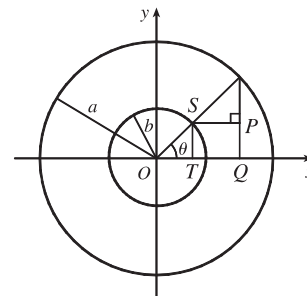
39. The case  $\frac{\pi}{2} < \theta < \pi$  is illustrated.  $C$  has coordinates  $(r\theta, r)$  as in Example 7, and  $Q$  has coordinates  $(r\theta, r + r \cos(\pi - \theta)) = (r\theta, r(1 - \cos \theta))$  [since  $\cos(\pi - \alpha) = \cos \pi \cos \alpha + \sin \pi \sin \alpha = -\cos \alpha$ ], so  $P$  has coordinates  $(r\theta - r \sin(\pi - \theta), r(1 - \cos \theta)) = (r(\theta - \sin \theta), r(1 - \cos \theta))$  [since  $\sin(\pi - \alpha) = \sin \pi \cos \alpha - \cos \pi \sin \alpha = \sin \alpha$ ]. Again we have the parametric equations  $x = r(\theta - \sin \theta)$ ,  $y = r(1 - \cos \theta)$ .



40. The first two diagrams depict the case  $\pi < \theta < \frac{3\pi}{2}$ ,  $d < r$ . As in Example 7,  $C$  has coordinates  $(r\theta, r)$ . Now  $Q$  (in the second diagram) has coordinates  $(r\theta, r + d \cos(\theta - \pi)) = (r\theta, r - d \cos \theta)$ , so a typical point  $P$  of the trochoid has coordinates  $(r\theta + d \sin(\theta - \pi), r - d \cos \theta)$ . That is,  $P$  has coordinates  $(x, y)$ , where  $x = r\theta - d \sin \theta$  and  $y = r - d \cos \theta$ . When  $d = r$ , these equations agree with those of the cycloid.

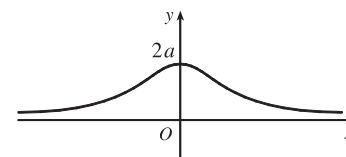


41. It is apparent that  $x = |OQ|$  and  $y = |QP| = |ST|$ . From the diagram,  $x = |OQ| = a \cos \theta$  and  $y = |ST| = b \sin \theta$ . Thus, the parametric equations are  $x = a \cos \theta$  and  $y = b \sin \theta$ . To eliminate  $\theta$  we rearrange:  $\sin \theta = y/b \Rightarrow \sin^2 \theta = (y/b)^2$  and  $\cos \theta = x/a \Rightarrow \cos^2 \theta = (x/a)^2$ . Adding the two equations:  $\sin^2 \theta + \cos^2 \theta = 1 = x^2/a^2 + y^2/b^2$ . Thus, we have an ellipse.



42.  $A$  has coordinates  $(a \cos \theta, a \sin \theta)$ . Since  $OA$  is perpendicular to  $AB$ ,  $\triangle OAB$  is a right triangle and  $B$  has coordinates  $(a \sec \theta, 0)$ . It follows that  $P$  has coordinates  $(a \sec \theta, b \sin \theta)$ . Thus, the parametric equations are  $x = a \sec \theta$ ,  $y = b \sin \theta$ .

43.  $C = (2a \cot \theta, 2a)$ , so the  $x$ -coordinate of  $P$  is  $x = 2a \cot \theta$ . Let  $B = (0, 2a)$ . Then  $\angle OAB$  is a right angle and  $\angle OBA = \theta$ , so  $|OA| = 2a \sin \theta$  and  $A = ((2a \sin \theta) \cos \theta, (2a \sin \theta) \sin \theta)$ . Thus, the  $y$ -coordinate of  $P$  is  $y = 2a \sin^2 \theta$ .



44. (a) Let  $\theta$  be the angle of inclination of segment  $OP$ . Then  $|OB| = \frac{2a}{\cos \theta}$ . Let  $C = (2a, 0)$ . Then by use of right triangle  $OAC$  we see that  $|OA| = 2a \cos \theta$ . Now

$$\begin{aligned} |OP| &= |AB| = |OB| - |OA| \\ &= 2a \left( \frac{1}{\cos \theta} - \cos \theta \right) = 2a \frac{1 - \cos^2 \theta}{\cos \theta} = 2a \frac{\sin^2 \theta}{\cos \theta} = 2a \sin \theta \tan \theta \end{aligned}$$

So  $P$  has coordinates  $x = 2a \sin \theta \tan \theta \cdot \cos \theta = 2a \sin^2 \theta$  and  $y = 2a \sin \theta \tan \theta \cdot \sin \theta = 2a \sin^2 \theta \tan \theta$ .

