Chapter 2

1. Find the equation of the tangent line to $y = x^2 - 6x$ at $x = 3$.

   A) $y = -9$ B) $y = 3$ C) $y = -9x$ D) $y = 3x$
   Ans: A   Difficulty: Moderate   Section: 2.1

2. Find an equation of the tangent line to $y = f(x)$ at $x = 3$.
   $$f(x) = x^3 + x^2 + x$$

   A) $y = -12x - 36$ B) $y = 34x + 63$ C) $y = 12x - 36$ D) $y = 34x - 63$
   Ans: D   Difficulty: Moderate   Section: 2.1

3. Find an equation of the tangent line to $y = f(x)$ at $x = 2$.
   $$f(x) = 2x^3 + 5$$

   A) $y = 9x - 16$ B) $y = -24x - 27$ C) $y = 24x - 27$ D) $y = 24x + 27$
   Ans: C   Difficulty: Moderate   Section: 2.1

4. Find the equation of the tangent line to $y = \dfrac{2}{x+2}$ at $x = 3$.

   A) $y = \dfrac{2}{25}x + \dfrac{16}{25}$ C) $y = -\dfrac{2}{25}x + \dfrac{16}{25}$

   B) $y = -\dfrac{2}{25}x - \dfrac{16}{25}$ D) $y = \dfrac{2}{25}x - \dfrac{16}{25}$

   Ans: C   Difficulty: Moderate   Section: 2.1

5. Find the equation of the tangent line to $y = 6\sqrt{x-4}$ at $x = 5$.

   A) $y = 6x - 9$ B) $y = 3x - 9$ C) $y = 6x - 18$ D) $y = 3x - 18$
   Ans: B   Difficulty: Moderate   Section: 2.1

6. Compute the slope of the secant line between the points $x = -3.1$ and $x = -3$. Round your answer to the thousandths place.
   $$f(x) = \sin(2x)$$

   A) -0.995 B) 1.963 C) 5.963 D) -1.991
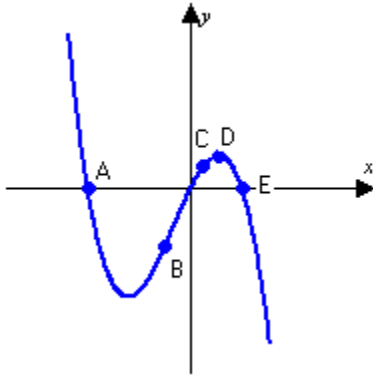   Ans: B   Difficulty: Easy   Section: 2.1

7. Compute the slope of the secant line between the points $x = 1$ and $x = 1.1$. Round your answer to the thousandths place.

$$f(x) = e^{0.5x}$$

A) 0.845   B) 5.529   C) 0.780   D) 1.691
Ans: A   Difficulty: Easy   Section: 2.1

8. List the points A, B, C, D, and E in order of increasing slope of the tangent line.

A) B, C, E, D, A   B) A, E, D, C, B   C) E, A, D, B, C   D) A, B, C, D, E
Ans: B   Difficulty: Easy   Section: 2.1

9. Use the position function $s(t) = -4.9t^2 + 1$ meters to find the velocity at time $t = 3$ seconds.

A) –43.1 m/sec   B) –29.4 m/sec   C) –28.4 m/sec   D) –44.1 m/sec
Ans: B   Difficulty: Moderate   Section: 2.1

10. Use the position function $s(t) = \sqrt{t + 5}$ meters to find the velocity at time $t = -1$ seconds.

A) 2 m/sec   B) 4 m/sec   C) $\dfrac{1}{2}$ m/sec   D) $\dfrac{1}{4}$ m/sec

Ans: D   Difficulty: Moderate   Section: 2.1

11. Find the average velocity for an object between $t = 3$ sec and $t = 3.1$ sec if $f(t) = -16t^2 + 100t + 10$ represents its position in feet.
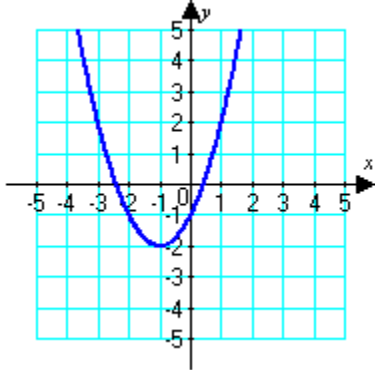
A) 2.4 ft/s   B) 4 ft/s   C) 0.8 ft/s   D) 166 ft/s
Ans: A   Difficulty: Moderate   Section: 2.1

12. Find the average velocity for an object between $t = 1$ sec and $t = 1.1$ sec if
    $f(t) = 5\sin(t) + 5$ represents its position in feet. (Round to the nearest thousandth.)
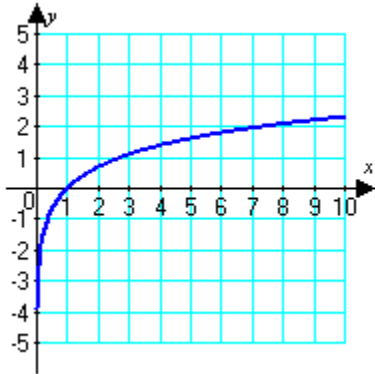
    A) 2.702   B) 2.268   C) 2.487   D) –2.487
    Ans: C   Difficulty: Moderate   Section: 2.1

13. Estimate the slope of the tangent line to the curve at $x = -2$.

    A) –1   B) –2   C) 2   D) 0
    Ans: B   Difficulty: Easy   Section: 2.1

14. Estimate the slope of the tangent line to the curve at $x = 3$.

    A) 3   B) –3   C) $\dfrac{1}{6}$ D) $\dfrac{1}{3}$
   Ans: D   Difficulty: Easy   Section: 2.1

15. The table shows the temperature in degrees Celsius at various distances, d in feet, from a specified point. Estimate the slope of the tangent line at $d = 2$ and interpret the result.

d	0	1	3	5	7
°C	13	20	14	7	1

A) $m \approx 4.67$; The temperature is increasing $4.67\,°C$ per foot at the point 2 feet from the specified point.

B) $m \approx -0.33$; The temperature is decreasing $0.33\,°C$ per foot at the point 2 feet from the specified point.

C) $m \approx -3$; The temperature is decreasing $3\,°C$ per foot at the point 2 feet from the specified point.

D) $m \approx 20$; The temperature is increasing $20\,°C$ per foot at the point 2 feet from the specified point.

Ans: C   Difficulty: Moderate   Section: 2.1

16. The graph below gives distance in miles from a starting point as a function of time in hours for a car on a trip. Find the fastest speed (magnitude of velocity) during the trip. Describe how the speed during the first 2 hours compares to the speed during the last 2 hours. Describe what is happening between 2 and 3 hours.
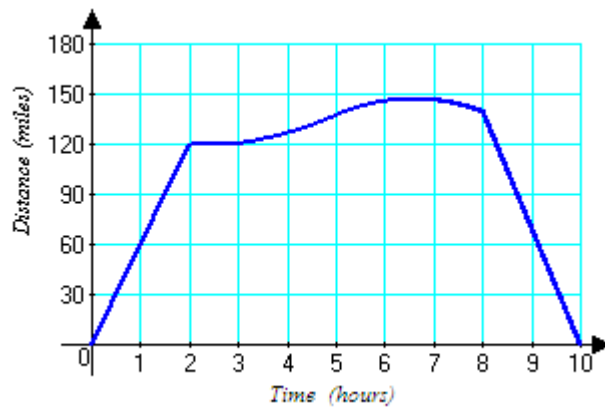
Ans: The fastest speed occurred during the last 2 hours of the trip when the car traveled at about 70 mph. The speed during the first 2 hours is 60 mph while the speed from 8 to 10 hours is about 70 mph. Between 2 and 3 hours the car was stopped.

Difficulty: Moderate   Section: 2.1

17. Compute $f'(3)$ for the function $f(x) = 5x^3 - 5x$.

A) 150   B) 130   C) 120   D) –130

Ans: B   Difficulty: Moderate   Section: 2.2

18.
Compute $f'(4)$ for the function $f(x) = \dfrac{2}{x^2 + 4}$.

A) $\dfrac{1}{4}$ B) $\dfrac{1}{25}$ C) $-\dfrac{2}{25}$ D) $-\dfrac{1}{25}$

Ans: D   Difficulty: Moderate   Section: 2.2

19.
Compute the derivative function $f'(x)$ of $f(x) = \dfrac{7}{3x-1}$.

A) $f'(x) = \dfrac{-21}{(3x-1)^2}$

C) $f'(x) = \dfrac{-7}{(3x-1)^2}$

B) $f'(x) = \dfrac{-3}{(3x-1)^2}$

D) $f'(x) = \dfrac{21}{(3x-1)^2}$

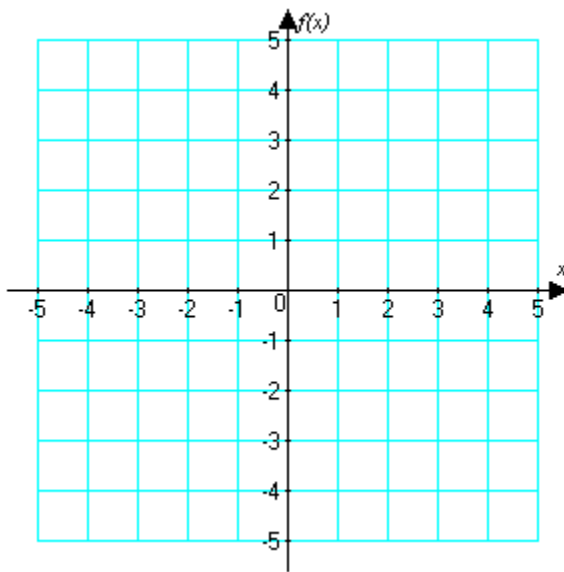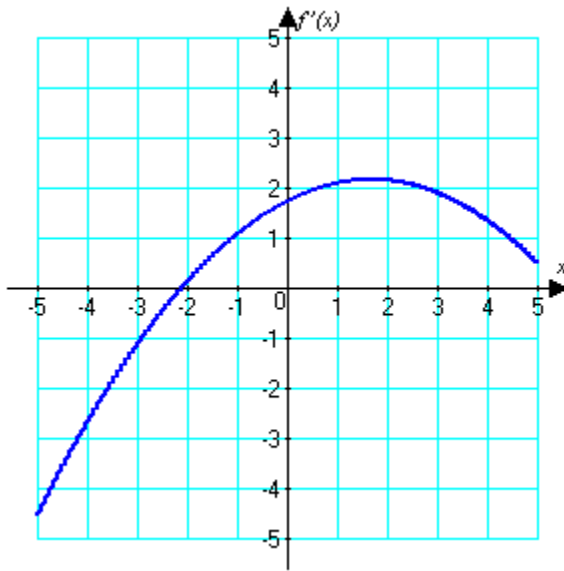Ans: A   Difficulty: Moderate   Section: 2.2

20.
Compute the derivative function $f'(x)$ of $f(x) = \sqrt{4x^2 + 9}$.
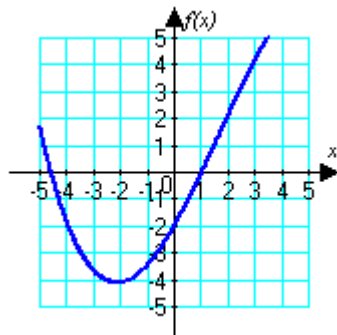
A) $f'(x) = \dfrac{-8x}{\sqrt{4x^2 + 9}}$

C) $f'(x) = \dfrac{-4x}{\sqrt{4x^2 + 9}}$

B) $f'(x) = \dfrac{4x}{\sqrt{4x^2 + 9}}$

D) $f'(x) = \dfrac{-4x}{\sqrt{8x + 9}}$

Ans: B   Difficulty: Moderate   Section: 2.2

21. Below is a graph of $f'(x)$. Sketch a plausible graph of a continuous function $f(x)$.
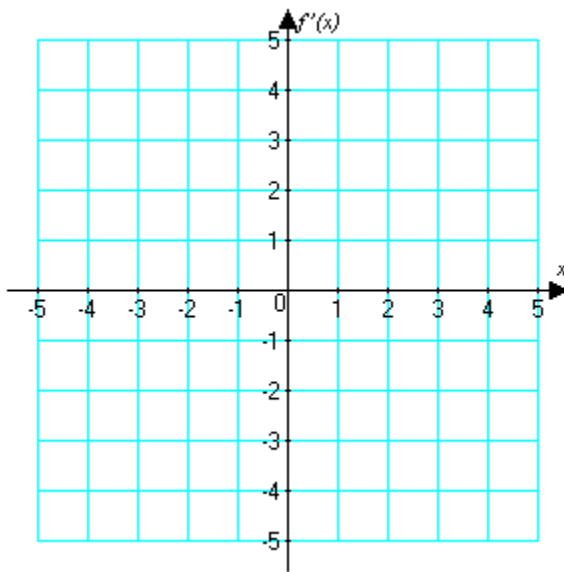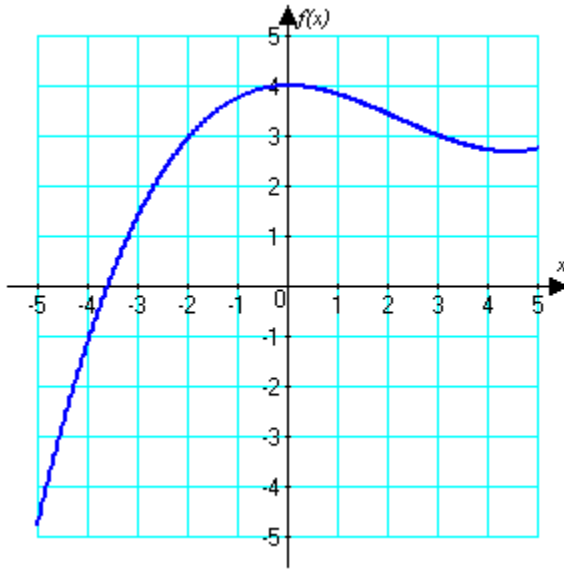

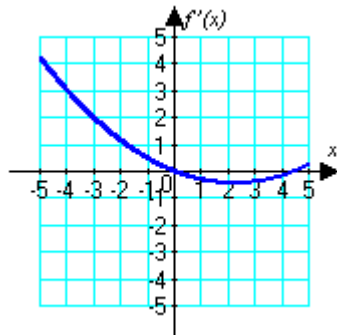Ans: Answers may vary. Below is one possible answer.

Difficulty: Moderate   Section: 2.2

22. Below is a graph of $f(x)$. Sketch a graph of $f'(x)$.

Ans:

9+

Difficulty: Moderate   Section: 2.2

23. Below is a graph of $f(x)$. Sketch a graph of $f'(x)$.
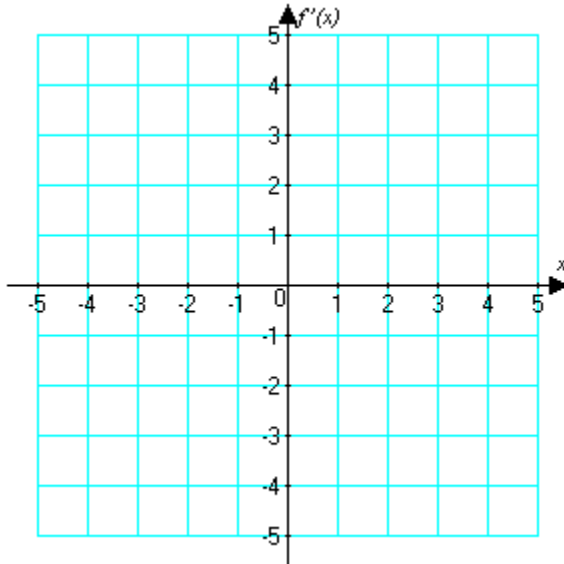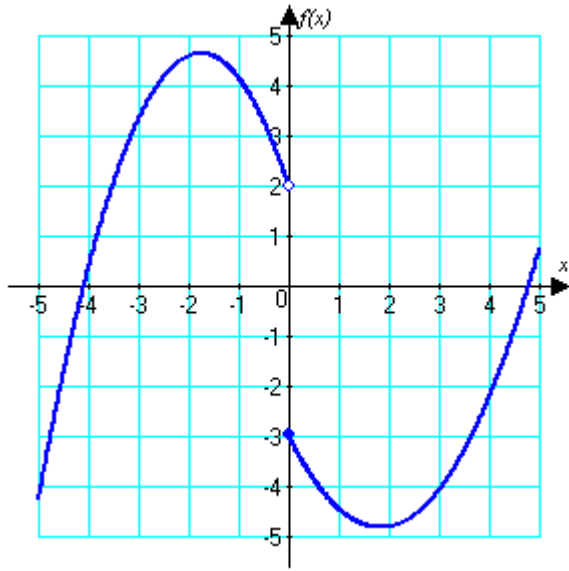
Ans:
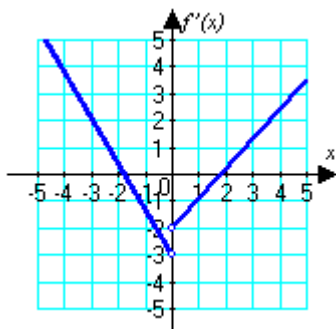
Difficulty: Difficult   Section: 2.2

24. Below is a graph of $f'(x)$. Sketch a plausible graph of a continuous function $f(x)$.
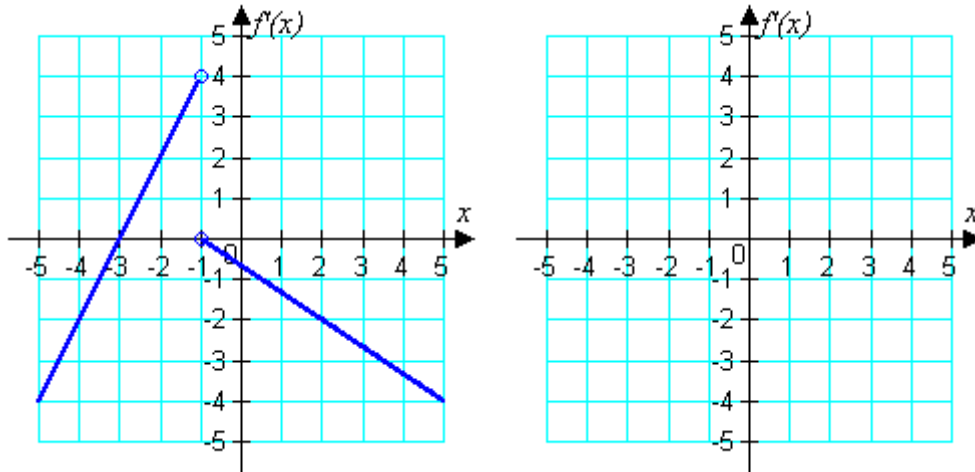
Ans: Answers may vary. Below is one possible answer.
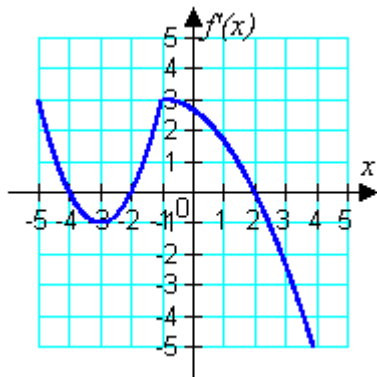
Difficulty: Difficult   Section: 2.2

25. Compute the right-hand derivative $D_+f(0) = \lim\limits_{h \to 0^+} \dfrac{f(h) - f(0)}{h}$ and the left-hand derivative $D_-f(0) = \lim\limits_{h \to 0^-} \dfrac{f(h) - f(0)}{h}$.

$$f(x) = \begin{cases} 4x + 8 & \text{if } x < 0 \\ -8x + 8 & \text{if } x \geq 0 \end{cases}$$

A) $D_+f(0) = -8$, $D_-f(0) = 4$
B) $D_+f(0) = 4$, $D_-f(0) = -8$
C) $D_+f(0) = 8$, $D_-f(0) = 8$
D) $D_+f(0) = -2$, $D_-f(0) = -2$

Ans: A   Difficulty: Moderate   Section: 2.2

26. Numerically estimate the derivative $f'(0)$ for $f(x) = 5xe^{3x}$.

    A) 0   B) 1   C) 3   D) 5
    Ans: D   Difficulty: Moderate   Section: 2.2

27. The table below gives the position $s(t)$ for a car beginning at a point and returning 5 hours later. Estimate the velocity $v(t)$ at two points around the third hour.

t (hours)	0	1	2	3	4	5
$s(t)$ (miles)	0	15	50	80	70	0

    Ans: The velocity is the change in distance traveled divided by the elapsed time. From hour 3 to 4 the average velocity is $(70 - 80)/(4 - 3) = -10$ mph. Likewise, the velocity between hour 2 and hour 3 is about 30 mph.
    Difficulty: Easy   Section: 2.2

28. Use the distances $f(t)$ to estimate the velocity at $t = 2.2$. (Round to 2 decimal places.)

t	1.6	1.8	2	2.2	2.4	2.6	2 8
$f(t)$	49	54	59.5	64	68.5	73.5	79

    A) –2250.00   B) 29.09   C) 22.50   D) 25.00
    Ans: C   Difficulty: Easy   Section: 2.2

29. For $f(x) = \begin{cases} 5x^2 - 6x & \text{if } x < 0 \\ ax + b & \text{if } x \geq 0 \end{cases}$ find all real numbers a and b such that $f'(0)$ exists.

    A) $a = 10$, b any real number   C) $a = -6$, b any real number
    B) $a = 4$, $b = 0$ D) $a = -6$, $b = 0$
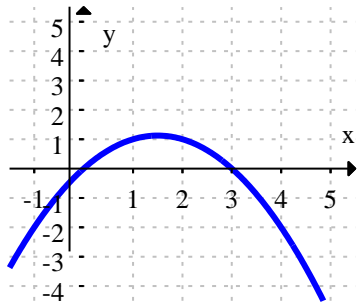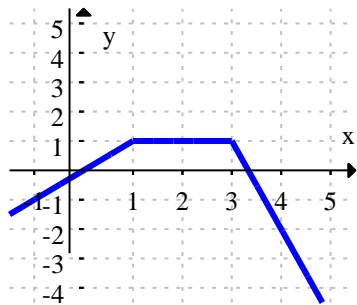   Ans: D   Difficulty: Moderate   Section: 2.2

30. Sketch the graph of a function with the following properties: $f(0) = 0$, $f(2) = 1$, $f(4) = -2$, $f'(0) = 1$, $f'(2) = 0$, and $f'(4) = -3$.

A)

B)

C)

D)
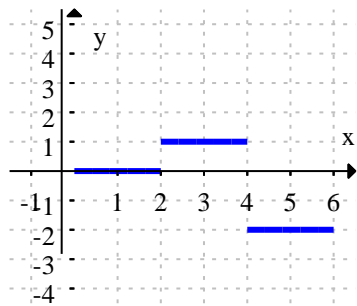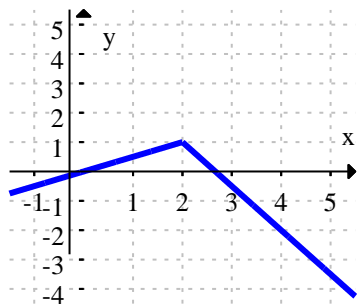
Ans: B   Difficulty: Moderate   Section: 2.2

31. Suppose a sprinter reaches the following distances in the given times. Estimate the velocity of the sprinter at the 6 second mark. Round to the nearest integer.

t sec	5	5.5	6	6.5	7
$f(t)$ ft	120.7	142.1	158.3	174.5	193.5

A) 32 ft/sec   B) 36 ft/sec   C) 26 ft/sec   D) 28 ft/sec
Ans: A   Difficulty: Moderate   Section: 2.2

32. $\lim\limits_{h \to 0} \dfrac{(1+h)^3 + (1+h) - 2}{h}$ equals $f'(a)$ for some function $f(x)$ and some constant a.
Determine which of the following could be the function $f(x)$ and the constant a.

A) $f(x) = x^3 - x$ and $a = -1$ C) $f(x) = x^3 + x - 20$ and $a = 0$

B) $f(x) = x^3 + x^2$ and $a = 0$ D) $f(x) = x^3 + x$ and $a = 1$

Ans: D   Difficulty: Moderate   Section: 2.2

33. $\lim\limits_{h \to 0} \dfrac{\dfrac{1}{(h+3)^2} - \dfrac{1}{9}}{h}$ equals $f'(a)$ for some function $f(x)$ and some constant a. Determine which of the following could be the function $f(x)$ and the constant a.

A) $f(x) = \dfrac{1}{x^2}$ and $a = 3$ C) $f(x) = -\dfrac{1}{x^2}$ and $a = 4$

B) $f(x) = \dfrac{3}{x^2}$ and $a = 3$ D) $f(x) = -\dfrac{1}{x^2}$ and $a = -3$

Ans: A   Difficulty: Moderate   Section: 2.2

34. Find the derivative of $f(x) = x^2 + 3x + 2$.

A) $x + 3$ B) $2x^2 + 2$ C) $2x + 3$ D) $-2x - 3$
Ans: C   Difficulty: Easy   Section: 2.3

35. Differentiate the function.
$$f(t) = 5t^3 - 2\sqrt{t}$$

A) $f'(t) = 15t^2 - 4\sqrt{t}$

C) $f'(t) = \dfrac{15t^{5/2} - 1}{\sqrt{t}}$

B) $f'(t) = 15t^2 - 4$

D) $f'(t) = \dfrac{15t^2 - 1}{\sqrt{t}}$

Ans: C   Difficulty: Moderate   Section: 2.3

36. Find the derivative of $f(x) = \dfrac{4}{x} + 4x - 3$.

A) $f'(x) = \dfrac{4}{x^2} + 4$

C) $f'(x) = -\dfrac{4}{x} + 4$

B) $f'(x) = -\dfrac{4}{x^2} + 4$

D) $f'(x) = -\dfrac{4}{x^2} + 8x^2$

Ans: B   Difficulty: Easy   Section: 2.3

37. Differentiate the function.
$$f(s) = 5s^{3/2} - 7s^{-1/3}$$

A) $f'(s) = \dfrac{45s^{5/3} + 2}{6s^{2/3}}$

C) $f'(s) = \dfrac{45s^{1/2} + 2s^{2/3}}{6}$

B) $f'(s) = \dfrac{45s^{1/2} + 2s^{1/3}}{6}$

D) $f'(s) = \dfrac{45s^{11/6} + 14}{6s^{4/3}}$

Ans: D   Difficulty: Moderate   Section: 2.3

38. Find the derivative of $f(x) = \dfrac{x^2 + 5x - 2}{4x}$.

A) $f'(x) = \dfrac{2x + 5}{4}$

C) $f'(x) = \dfrac{1}{4} + \dfrac{1}{2x^2}$

B) $f'(x) = -\dfrac{x}{2} - \dfrac{5}{4}$

D) $f'(x) = \dfrac{x^2}{4} + \dfrac{5x}{4} - \dfrac{1}{2x}$

Ans: C   Difficulty: Moderate   Section: 2.3

39. Find the derivative of $f(x) = \dfrac{-5x^2 - 7x - 7}{\sqrt{x}}$.

A) $f'(x) = -\dfrac{15\sqrt{x}}{2} - \dfrac{7}{2\sqrt{x}} + \dfrac{7}{2\sqrt{x^3}}$

C) $f'(x) = -\dfrac{15\sqrt{x}}{2} + \dfrac{7}{2\sqrt{x}} - \dfrac{7}{2\sqrt{x^3}}$

B) $f'(x) = -\dfrac{20x + 14}{x}$

D) $f'(x) = -15\sqrt{x} - \dfrac{7}{\sqrt{x}} - \dfrac{7}{\sqrt{x^3}}$

Ans: A   Difficulty: Moderate   Section: 2.3

40. Differentiate the function.
$$f(x) = x\left(3x^2 - 6\sqrt{x}\right)$$

A) $f'(x) = 9x^2 - 9\sqrt{x}$

C) $f'(x) = 6x^2 - 3\sqrt{x}$

B) $f'(x) = \dfrac{6x^{3/2} - 3}{\sqrt{x}}$

D) $f'(x) = 6x - 3\sqrt{x}$

Ans: A   Difficulty: Moderate   Section: 2.3

41. Find the third derivative of $f(x) = 2x^5 + 8x + \dfrac{3}{x}$.

A) $f'''(x) = 120x^2 + \dfrac{18}{x^4}$

C) $f'''(x) = 40x^3 + \dfrac{6}{x^3}$

B) $f'''(x) = 120x^2 + 8 - \dfrac{18}{x^4}$

D) $f'''(x) = 120x^2 - \dfrac{18}{x^4}$

Ans: D   Difficulty: Moderate   Section: 2.3

42. Find the second derivative of $y = -4x - \dfrac{6}{\sqrt{x}}$.

A) $\dfrac{d^2y}{dx^2} = -4 - \dfrac{9}{2\sqrt{x^5}}$

C) $\dfrac{d^2y}{dx^2} = \dfrac{9}{2\sqrt{x^5}}$

B) $\dfrac{d^2y}{dx^2} = -\dfrac{9}{2\sqrt{x^5}}$

D) $\dfrac{d^2y}{dx^2} = -\dfrac{9}{2\sqrt{x^3}}$

Ans: B   Difficulty: Moderate   Section: 2.3

43. Using the position function $s(t) = 3t^4 - 4t^3 + \dfrac{2}{t}$, find the velocity function.

A) $v(t) = 12t^3 - 12t^2 - \dfrac{2}{t^2}$

C) $v(t) = 12t^3 - 12t^2 + \dfrac{2}{t^2}$

B) $v(t) = 9t^3 - 8t^2 - \dfrac{2}{t^2}$

D) $v(t) = -12t^3 + 12t^2 - \dfrac{2}{t^2}$

Ans: A   Difficulty: Moderate   Section: 2.3

44. Using the position function $s(t) = -7t^3 - 6t - 8$, find the acceleration function.

A) $a(t) = -21t$ B) $a(t) = -14t$ C) $a(t) = -42t$ D) $a(t) = -42t - 6$
Ans: C   Difficulty: Moderate   Section: 2.3

45. Using the position function $s(t) = -\sqrt{t} + \dfrac{3}{t}$, find the velocity function.

A) $v(t) = \dfrac{1}{2\sqrt{t}} + \dfrac{3}{t^2}$

C) $v(t) = \dfrac{1}{2\sqrt{t}} - \dfrac{3}{t^2}$

B) $v(t) = -\dfrac{1}{2\sqrt{t}} - \dfrac{3}{t^2}$

D) $v(t) = -\dfrac{1}{2\sqrt{t}} - \dfrac{6}{t^2}$

Ans: B   Difficulty: Moderate   Section: 2.3

46. Using the position function $s(t) = -\dfrac{8}{\sqrt{t}} + 1$, find the acceleration function.

A) $a(t) = \dfrac{6}{\sqrt{t^5}}$ B) $a(t) = -\dfrac{2}{\sqrt{t^5}}$ C) $a(t) = \dfrac{4}{\sqrt{t^3}}$ D) $a(t) = -\dfrac{6}{\sqrt{t^5}}$

Ans: D   Difficulty: Moderate   Section: 2.3

47. The height of an object at time t is given by $h(t) = -16t^2 + 4t - 1$. Determine the object's velocity at $t = 2$.

A) 60   B) –59   C) –60   D) –28
Ans: C   Difficulty: Easy   Section: 2.3

48. The height of an object at time t is given by $h(t) = 8t^2 - 4t$. Determine the object's acceleration at $t = 3$.

A) 60   B) 16   C) 44   D) –16
Ans: B   Difficulty: Easy   Section: 2.3

49. Find an equation of the line tangent to $f(x) = x^2 + 5x - 8$ at $x = 2$.

A) $g(x) = 9x - 12$ C) $g(x) = 9x - 10$
B) $g(x) = 4x - 12$ D) $g(x) = 4x - 10$

Ans: A   Difficulty: Easy   Section: 2.3

50. Find an equation of the line tangent to $f(x) = 7\sqrt{x} - 2x - 4$ at $x = 3$.

A) $g(x) = \left(\dfrac{-7\sqrt{3}+12}{6}\right)x - \dfrac{7}{2}\sqrt{3} + 4$

C) $g(x) = \left(\dfrac{7\sqrt{3}-6}{6}\right)x + \dfrac{7}{2}\sqrt{3}$

B) $g(x) = \left(\dfrac{7\sqrt{3}-4}{3}\right)x + \dfrac{7}{2}\sqrt{3} + 4$

D) $g(x) = \left(\dfrac{7\sqrt{3}-12}{6}\right)x + \dfrac{7}{2}\sqrt{3} - 4$

Ans: D   Difficulty: Moderate   Section: 2.3

51. Use the graph of $f(x)$ below to sketch the graph of $f''(x)$ on the same axes. (Hint: sketch $f'(x)$ first.)
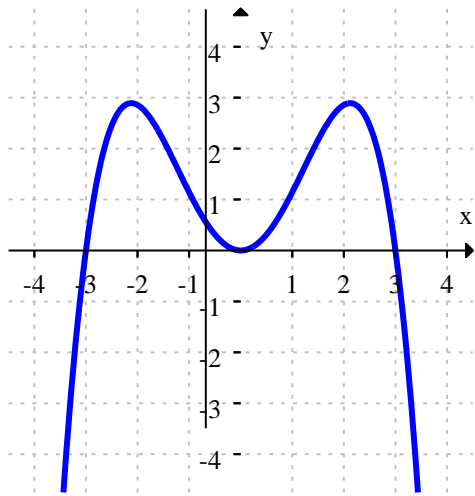
A)
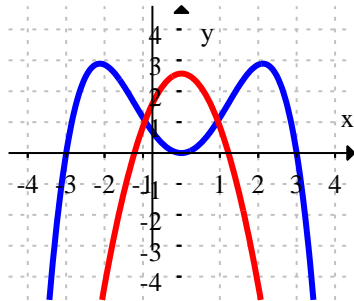
B)
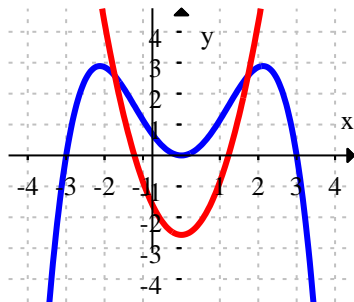
C)
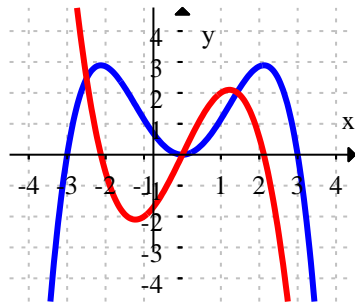
D)
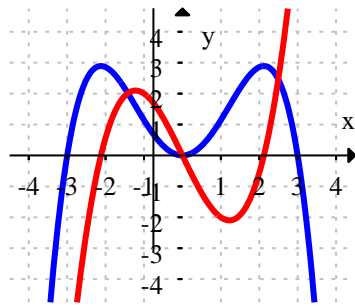
Ans: A   Difficulty: Difficult   Section: 2.3

52. Determine the real value(s) of x for which the line tangent to $f(x) = 7x^2 + 9x - 4$ is horizontal.

A) $x = -\dfrac{9}{14}, x = 0$ B) $x = \dfrac{-9 \pm \sqrt{193}}{14}$ C) $x = -\dfrac{9}{14}$ D) $x = 0$

Ans: C   Difficulty: Easy   Section: 2.3

53. Determine the real value(s) of x for which the line tangent to $f(x) = 2x^4 - 4x^2 - 1$ is horizontal.

A) $x = -1, x = 1$ B) $x = 0, x = -1, x = 1$ C) $x = 0$ D) $x = 0, x = 1$

Ans: B   Difficulty: Easy   Section: 2.3

54. Determine the value(s) of x, if there are any, for which the slope of the tangent line to $f(x) = |x^2 + 3x - 54|$ does not exist.

A) $x = -1.5$ C) $x = -9, x = 6$
B) $x = -6, x = 9$ D)   The slope exists for all values of x.

Ans: C   Difficulty: Moderate   Section: 2.3

55. Find the second-degree polynomial (of the form $ax^2 + bx + c$) such that $f(0) = 0, f'(0) = 5$, and $f''(0) = 1$.

A) $\dfrac{x^2}{2} + 5x$ B) $-\dfrac{x^2}{2} + 5x$ C) $\dfrac{x^2}{2} - 5x + 1$ D) $-\dfrac{x^2}{2} + 5x + 1$

Ans: A   Difficulty: Moderate   Section: 2.3

56. Find a formula for the nth derivative $f^{(n)}(x)$ of $f(x) = \dfrac{4}{x+8}$.

A) $f^{(n)}(x) = (-1)^{n+1} \dfrac{32n!}{(x+8)^{n+1}}$

C) $f^{(n)}(x) = (-1)^n \dfrac{32n!}{(x+8)^n}$

B) $f^{(n)}(x) = (-1)^{n+1} \dfrac{4n!}{(x+8)^n}$

D) $f^{(n)}(x) = (-1)^n \dfrac{4n!}{(x+8)^{n+1}}$

Ans: D   Difficulty: Difficult   Section: 2.3

57. Find a function with the given derivative.
$$f'(x) = 20x^4$$

A) $f(x) = 20x^5$ B) $f(x) = 4x^5$ C) $f(x) = 20x^3$ D) $f(x) = 80x^3$

Ans: B   Difficulty: Moderate   Section: 2.3

58. Let $f(t)$ equal the average monthly salary of families in a certain city in year t. Several values are given in the table below. Estimate and interpret $f''(2010)$.

t	1995	2000	2005	2010
$f(t)$	$1700	$2000	$2100	$2250

A) $f''(2010) \approx 2$; The rate at which the average monthly salary is increasing each year in 2010 is increasing by $2 per year.

B) $f''(2010) \approx 2$; The average monthly salary is increasing by $2 per year in 2010.

C) $f''(2010) \approx 30$; The rate at which the average monthly salary is increasing each year in 2010 is increasing by $30 per year.

D) $f''(2010) \approx 30$; The average monthly salary is increasing by $30 per year in 2010.

Ans: A   Difficulty: Moderate   Section: 2.3

59.

Find the derivative of $f(x) = \left(9\sqrt{x} + 5x\right)\left(-3x^2 - \dfrac{1}{x}\right)$.

A) $\quad f'(x) = -45x^2 + \dfrac{135}{2}x^{3/2} + \dfrac{9}{2x^{3/2}}$

B) $\quad f'(x) = -45x^2 - \dfrac{135}{2}x^{3/2} + \dfrac{9}{2x^{3/2}}$

C) $\quad f'(x) = 45x^2 - \dfrac{135}{2}x^{3/2} - \dfrac{9}{2x^{3/2}}$

D) $\quad f'(x) = -45x^2 - \dfrac{135}{2}x^{3/2} - \dfrac{10}{x} + \dfrac{9}{2x^{3/2}}$

Ans: B   Difficulty: Moderate   Section: 2.4

60.

Find the derivative of $f(x) = \dfrac{2x+2}{-3x+2}$.

A) $\dfrac{-10}{(-3x+2)^2}$ B) $-\dfrac{2}{3}$ C) $\dfrac{2}{3}$ D) $\dfrac{10}{(-3x+2)^2}$

Ans: D   Difficulty: Moderate   Section: 2.4

61.

Find the derivative of $f(x) = \dfrac{4x}{-8x^2 - 3}$.

A) $\dfrac{32x^2 - 12}{(-8x^2 - 3)^2}$ B) $\dfrac{1}{2x^2}$ C) $\dfrac{-32x^2 + 12}{(-8x^2 - 3)^2}$ D) $-\dfrac{1}{2x^2}$

Ans: A   Difficulty: Moderate   Section: 2.4

62. Find the derivative of $f(x) = \left(-5\sqrt[3]{x} + 6\right)x$.

A) $\quad f'(x) = \dfrac{20}{3}\sqrt[3]{x} + 6$

C) $\quad f'(x) = -\dfrac{20}{3}\sqrt[3]{x} + 6$

B) $\quad f'(x) = -\dfrac{5}{3}\sqrt[3]{x} - 6$

D) $\quad f'(x) = -\dfrac{10}{3}\sqrt[3]{x} + 12$

Ans: C   Difficulty: Moderate   Section: 2.4

63. Find an equation of the line tangent to $h(x) = f(x)g(x)$ at $x = -3$ if $f(-3) = 2$, $f'(-3) = 1$, $g(-3) = 3$, and $g'(-3) = 3$.

A) $y = 3x - 3$ B) $y = 3x + 33$ C) $y = 9x + 33$ D) $y = 9x - 21$

Ans: C   Difficulty: Moderate   Section: 2.4

64.
Find an equation of the line tangent to $h(x) = \dfrac{f(x)}{g(x)}$ at $x = 3$ if

$f(3) = 1$, $f'(3) = -1$, $g(3) = 1$, and $g'(3) = -2$.

A) $y = -3x - 2$ B) $y = x - 2$ C) $y = -3x + 10$ D) $y = x + 4$
Ans: B   Difficulty: Moderate   Section: 2.4

65. A small company sold 1500 widgets this year at a price of $12 each. If the price increases at rate of $1.75 per year and the quantity sold increases at a rate of 200 widgets per year, at what rate will revenue increase?

A) $350/year   B) $5025/year   C) $225/year   D) $5375/year
Ans: B   Difficulty: Moderate   Section: 2.4

66. The Dieterici equation of state, $Pe^{an/VRT}(V - nb) = nRT$, gives the relationship between pressure P, volume V, and temperature T for a liquid or gas. At the critical point, $P'(V) = 0$ and $P''(V) = 0$ with T constant. Using the result of the first derivative and substituting it into the second derivative, find the critical volume V_c in terms of the constants n, a, b, and R.

Ans:
$$P'(V) = \left(\frac{an^2}{V^2} - \frac{nRT}{(V - nb)} \right)\left(\frac{1}{V - nb} \right)e^{-an/VRT} = 0 \text{ gives the result that}$$

$$RT = \frac{an(V - nb)}{V^2}.$$

$$P''(V) = \left(\frac{-2an^2}{V^3(V - nb)} + \frac{2nRT}{(V - nb)^3} - \frac{2an^2}{V^2(V - nb)^2} + \frac{a^2n^3}{V^4(V - nb)^2 RT} \right)e^{-an/VRT} = 0.$$

When the result of the first derivative is substituted for RT in the parentheses, the result is that $V_c = 2nb$.
Difficulty: Difficult   Section: 2.4

67.
Find the derivative of $f(x) = \dfrac{(x^2 + 2)^4}{6}$.

A) $f'(x) = \dfrac{2}{3}x(x^2 + 2)^3$

C) $f'(x) = \dfrac{4}{3}x(x^2 + 2)^3$

B) $f'(x) = \dfrac{1}{3}x(x^2 + 2)^3$

D) $f'(x) = \dfrac{1}{6}x(x^2 + 2)^3$

Ans: C   Difficulty: Moderate   Section: 2.5

68. Find the derivative of $f(x) = \sqrt{x^2 - 2}$.

A) $f'(x) = \dfrac{2x}{\sqrt{x^2 - 2}}$

C) $f'(x) = \dfrac{-x}{\sqrt{x^2 - 2}}$

B) $f'(x) = \dfrac{4x}{\sqrt{x^2 - 2}}$

D) $f'(x) = \dfrac{x}{\sqrt{x^2 - 2}}$

Ans: D   Difficulty: Moderate   Section: 2.5

69. Differentiate the function.

$$f(t) = t^6 \sqrt{t^3 - 5}$$

A) $f'(t) = \dfrac{13t^6 - 60t^5}{2\sqrt{t^3 - 5}}$

C) $f'(t) = \dfrac{15t^8 - 60t^5}{2\sqrt{t^3 - 5}}$

B) $f'(t) = \dfrac{6t^5}{2\sqrt{t^3 - 5}}$

D) $f'(t) = \dfrac{9t^7}{\sqrt{t^3 - 5}}$

Ans: C   Difficulty: Difficult   Section: 2.5

70. Find the derivative of $f(x) = \sqrt{\dfrac{x}{x^2 + 9}}$.

A) $\dfrac{1}{2}\left[\dfrac{1}{\sqrt{x(x^2 + 9)}} - \sqrt{\dfrac{x}{(x^2 + 9)^3}} \right]$

C) $\dfrac{1}{2}\left[\dfrac{1}{\sqrt{x(x^2 + 9)}} - \sqrt{x(x^2 + 9)} \right]$

B) $\dfrac{1}{2\sqrt{x(x^2 + 9)}} - \sqrt{\left(\dfrac{x}{x^2 + 9}\right)^3}$

D) $\sqrt{\dfrac{1}{x^2 + 9} - \dfrac{2x^2}{(x^2 + 9)^2}}$

Ans: B   Difficulty: Moderate   Section: 2.5

71. Find the derivative of $f(x) = \dfrac{-3}{\sqrt{8x^2 - 9}}$.

A) $f'(x) = \dfrac{24x}{\sqrt{(8x^2 - 9)^3}}$

C) $f'(x) = \dfrac{-24x}{\sqrt{(8x^2 - 9)^3}}$

B) $f'(x) = \dfrac{-48x}{\sqrt{(8x^2 - 9)^3}}$

D) $f'(x) = \dfrac{-6x}{\sqrt{(8x^2 - 9)^3}}$

Ans: A   Difficulty: Moderate   Section: 2.5

72. Differentiate the function.

$$f(x) = \left(\sqrt{x^3 - 4} + 3x \right)^{-2}$$

A)
$$f'(x) = -\frac{6\sqrt{x^3 - 4} + 3x^2}{\left(\sqrt{x^3 - 4} \right)\left(\sqrt{x^3 - 4} + 3x \right)^3}$$

B)
$$f'(x) = -\frac{12\sqrt{x^3 - 4} + 3x^2}{\left(2\sqrt{x^3 - 4} \right)\left(\sqrt{x^3 - 4} + 3x \right)^2}$$

C)
$$f'(x) = -\frac{2\sqrt{x^3 - 4} + 6}{\left(\sqrt{x^3 - 4} + 3x \right)^3}$$

D)
$$f'(x) = -\frac{2\sqrt{x^3 - 4} + 6}{\left(\sqrt{x^3 - 4} + 3x \right)^2}$$

Ans: A   Difficulty: Difficult   Section: 2.5

73. $f(x) = -5x^3 - 6x + 6$ has an inverse $g(x)$. Compute $g'(17)$.

A) $g'(17) = \frac{1}{21}$ B) $g'(17) = -\frac{1}{9}$ C) $g'(17) = -\frac{1}{21}$ D) $g'(17) = \frac{1}{9}$

Ans: C   Difficulty: Moderate   Section: 2.5

74. $f(x) = 2x^5 + 3x^3 + 2x$ has an inverse $g(x)$. Compute $g'(7)$.

A) $g'(7) = \frac{1}{24453}$ B) $g'(7) = \frac{1}{21}$ C) $g'(7) = -\frac{1}{7}$ D) $g'(7) = \frac{1}{7}$

Ans: B   Difficulty: Moderate   Section: 2.5

75. The function $f(x) = \sqrt{x^3 + 5x + 36}$ has an inverse $g(x)$. Find $g'(6)$.

A) $g'(6) = \frac{12}{5}$ B) $g'(6) = \frac{5}{12}$ C) $g'(6) = 6$ D) $g'(6) = \frac{1}{6}$

Ans: A   Difficulty: Moderate   Section: 2.5

76. Find an equation of the line tangent to $f(x) = \dfrac{1}{\sqrt{x^2 - 24}}$ at $x = 5$.

A) $y = -5x + 24$ B) $y = -5x$ C) $y = 5x + 6$ D) $y = -5x + 26$

Ans: D   Difficulty: Moderate   Section: 2.5

77. Use the position function $s(t) = \sqrt{t^2 + 48}$ meters to find the velocity at $t = 4$ seconds.

A) 8 m/s   B) $\dfrac{1}{2}$ m/s   C) $\dfrac{1}{8}$ m/s   D) $\dfrac{1}{4}$ m/s

Ans: B   Difficulty: Moderate   Section: 2.5

78. Compute the derivative of $h(x) = f(g(x))$ at $x = 9$ where
$f(9) = -5$, $g(9) = -8$, $f'(9) = -2$, $f'(-8) = -4$, $g'(9) = 6$, and $g'(-8) = -7$.

A) $h'(9) = -12$ B) $h'(9) = -30$ C) $h'(9) = -24$ D) $h'(9) = 40$

Ans: C   Difficulty: Moderate   Section: 2.5

79. Find the derivative where f is an unspecified differentiable function.
$$f(3x^7)$$

A) $21x^6 f'(3x^7)$ B) $(21x^6 + 3x^7)f'(3x^7)$ C) $f'(21x^6)$ D) $f'(21x^6 + 3x^7)$

Ans: A   Difficulty: Moderate   Section: 2.5

80. Find the second derivative of the function.
$$f(x) = \sqrt{9 - x^2}$$

A) $f''(x) = \dfrac{9x}{(9-x^2)^{3/2}}$ C) $f''(x) = -\dfrac{9}{(9-x^2)^{3/2}}$

B) $f''(x) = \dfrac{x^2 + 9}{(9-x^2)^{3/2}}$ D) $f''(x) = -\dfrac{9x}{(9-x^2)^{3/2}}$

Ans: C   Difficulty: Moderate   Section: 2.5

81. Find a function $g(x)$ such that $g'(x) = f(x)$.
$$f(x) = (x^2 - 9)^8 (2x)$$

A) $\left(\dfrac{x^3}{3} - 9x\right)^9 \dfrac{x^2}{9}$ C) $g(x) = (x^2 - 9)^9$

B) $g(x) = (x^2 - 9)^7 (32x)$ D) $g(x) = \dfrac{(x^2 - 9)^9}{9}$

Ans: D   Difficulty: Moderate   Section: 2.5

82. Use the table of values to estimate the derivative of $h(x) = f(g(x))$ at $x = 6$.

x	−1	0	1	2	3	4	5	6	7
f(x)	−5	−4	−3	−4	−5	−6	−5	−3	−1
g(x)	6	4	2	2	4	6	4	2	1

A) $h'(6) \approx 2$ B) $h'(6) \approx -3$ C) $h'(6) \approx -2$ D) $h'(6) \approx 3$

Ans: A Difficulty: Moderate Section: 2.5

83. Find the derivative of $f(x) = -4\sin(x) + 9\cos(3x) - x$.

A) $f'(x) = -4\cos x - 27\sin 3x - 1$
B) $f'(x) = -4\cos x - 9\sin 3x - 1$
C) $f'(x) = 4\cos x + 27\sin 3x - 1$
D) $f'(x) = \cos x - 3\sin 3x - 1$

Ans: A Difficulty: Easy Section: 2.6

84. Find the derivative of $f(x) = 4\sin^2 x - 3x^2$.

A) $f'(x) = -8\sin x \cos x - 6x$
B) $f'(x) = 8\sin x \cos x - 3x$
C) $f'(x) = 8\sin x - 6x$
D) $f'(x) = 8\sin x \cos x - 6x$

Ans: D Difficulty: Easy Section: 2.6

85. Find the derivative of $f(x) = \dfrac{-6\cos x^2}{x^2}$.

A) $f'(x) = \dfrac{-12(x^2 \sin x^2 + \cos x^2)}{x^3}$

B) $f'(x) = \dfrac{12(x\sin x^2 + \cos x^2)}{x^3}$

C) $f'(x) = \dfrac{12(x^2 \sin x^2 + \cos x^2)}{x^3}$

D) $f'(x) = \dfrac{12(x^2 \sin x^2 + \cos x^2)}{x^4}$

Ans: C Difficulty: Moderate Section: 2.6

86. Find the derivative of $f(x) = \sqrt{-\sin x \sec x}$.

A) $f'(x) = -\dfrac{\sec x}{2\sqrt{-\tan x}}$

B) $f'(x) = -\dfrac{\sec^2 x}{2\sqrt{-\tan x}}$

C) $f'(x) = -\dfrac{\sec^2 x}{\sqrt{-\tan x}}$

D) $f'(x) = -\dfrac{\sec x \tan x}{2\sqrt{-\tan x}}$

Ans: B Difficulty: Moderate Section: 2.6

87. Find the derivative of the function.
$$f(w) = w^2 \sec^2 10w$$

   A) $f'(w) = 20w \sec^2(10w) \tan(10w)$

   B) $f'(w) = 2w \sec^2(10w) + 20w^2 \sec^2(10w) \tan(10w)$

   C) $f'(w) = 2w \sec^2(10w) + 20w^2 \sec(10w)$

   D) $f'(w) = 2w \sec^2(10w) + 20w^2 \sec^2(10w) \tan^2(10w)$

   Ans: B   Difficulty: Moderate   Section: 2.6

88. Find the derivative of the function.
$$f(x) = \cos^3\left(\sin\left(\left(x^5 + 7x^4\right)^2\right)\right)$$

   Ans: $f'(x) = -6\cos^2\left(\sin\left(\left(x^5 + 7x^4\right)^2\right)\right) \cdot \sin\left(\sin\left(\left(x^5 + 7x^4\right)^2\right)\right) \cdot \cos\left(\left(x^5 + 7x^4\right)^2\right) \cdot \left(x^5 + 7x^4\right) \cdot \left(5x^4 + 28x^3\right)$

   Difficulty: Difficult   Section: 2.6

89. Find an equation of the line tangent to $f(x) = x \sin 10x$ at $x = \pi$.

   A) $y = -10(x - \pi)$ C) $y = -10\pi(x - \pi)$

   B) $y = 10(x - \pi)$ D) $y = 10\pi(x - \pi)$

   Ans: D   Difficulty: Moderate   Section: 2.6

90. Find an equation of the line tangent to $f(x) = \tan 4x$ at $x = -1$. (Round coefficients to 3 decimal places.)

   A) $y = -6.12x + 8.204$ C) $y = 9.362x + 8.204$

   B) $y = -9.362x - 13.993$ D) $y = 9.362x - 10.751$

   Ans: C   Difficulty: Moderate   Section: 2.6

91. Find an equation of the line tangent to $f(x) = x \cos x$ at $x = -4$. (Round coefficients to 3 decimal places.)

   A) $y = 3.681x + 12.109$ C) $y = 2.374x - 12.109$

   B) $y = 2.374x + 12.109$ D) $y = 3.681x - 12.109$

   Ans: B   Difficulty: Moderate   Section: 2.6

92. Use the position function $s(t) = \cos 2t - t^2$ feet to find the velocity at $t = 3$ seconds. (Round answer to 2 decimal places.)

A) $v(3) = -5.44$ ft/s   C) $v(3) = 6.56$ ft/s
B) $v(3) = -6.56$ ft/s   D) $v(3) = -7.92$ ft/s
Ans: A   Difficulty: Moderate   Section: 2.6

93. Use the position function $s(t) = 7\sin(2t) + 6$ meters to find the velocity at $t = 4$ seconds. (Round answer to 2 decimal places.)

A) $v(4) = 13.85$ m/s   C) $v(4) = -1.02$ m/s
B) $v(4) = -9.15$ m/s   D) $v(4) = -2.04$ m/s
Ans: D   Difficulty: Moderate   Section: 2.6

94. Use the position function to find the velocity at time $t = t_0$. Assume units of feet and seconds.

$$s(t) = \frac{\sin 10t}{t}, \quad t = \pi$$

A) $v(\pi) = 0$ ft/sec   C) $v(\pi) = \dfrac{10}{\pi}$ ft/sec

B) $v(\pi) = -\dfrac{10}{\pi^2}$ ft/sec   D) $v(\pi) = \dfrac{1}{\pi^2}$ ft/sec

Ans: C   Difficulty: Moderate   Section: 2.6

95. A weight hanging by a spring from the ceiling vibrates up and down. Its vertical position is given by $s(t) = 9\sin(7t)$. Find the maximum speed of the weight and its position when it reaches maximum speed.

A) speed = 9, position = 63   C) speed = 7, position = 9
B) speed = 63, position = 0   D) speed = 63, position = 7
Ans: B   Difficulty: Moderate   Section: 2.6

96. Given that $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$, find $\lim\limits_{t \to 0} \dfrac{\sin(7t)}{-8t}$.

A) $-\dfrac{1}{8}$ B) -56 C) $-\dfrac{7}{8}$ D) $\dfrac{1}{7}$
Ans: C   Difficulty: Easy   Section: 2.6

97. Given that $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x} = 0$, find $\lim\limits_{t \to 0} \dfrac{\cos t - 1}{2t}$.

A) 0   B) $\dfrac{1}{2}$ C) 2   D) $\dfrac{1}{2}$

Ans: A   Difficulty: Easy   Section: 2.6

98. Given that $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$, find $\lim\limits_{t \to 0} \dfrac{6t}{\sin(7t)}$.

A) 42   B) $\dfrac{1}{6}$ C) $\dfrac{7}{6}$ D) $\dfrac{6}{7}$

Ans: D   Difficulty: Easy   Section: 2.6

99. Given that $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$, find $\lim\limits_{t \to 0} \dfrac{\tan(7t)}{8t}$.

A) $\dfrac{1}{7}$ B) $\dfrac{7}{8}$ C) $\dfrac{8}{7}$ D) $\dfrac{1}{8}$

Ans: B   Difficulty: Moderate   Section: 2.6

100. For $f(x) = \sin x$, find $f^{(22)}(x)$.

A) $\cos x$ B) $-\cos x$ C) $\sin x$ D) $-\sin x$

Ans: D   Difficulty: Easy   Section: 2.6

101. The total charge in an electrical circuit is given by $Q(t) = 3\sin(3t) + t + 2$. The current is the rate of change of the charge, $i(t) = \dfrac{dQ}{dt}$. Determine the current at $t = 0$ (Round answer to 2 decimal places.)

A) $i(0) = 4$ B) $i(0) = 10$ C) $i(0) = 12$ D) $i(0) = 1$

Ans: B   Difficulty: Moderate   Section: 2.6

102. Find the derivative of $f(x) = x^{-9}e^{-2x}$.

A) $f'(x) = \left(-9x^{-8} + 2x^{-9}\right)e^{-2x}$

B) $f'(x) = -9x^{-10}e^{-2} - 2x^{-9}e^{-2x-1}$

C) $f'(x) = \left(-9x^{-10} - 2x^{-9}\right)e^{-2x}$

D) $f'(x) = -9x^{-10} - 2e^{-2x}$

Ans: C   Difficulty: Easy   Section: 2.7

103. Differentiate the function.
$$f(x) = e^{3x} \cos 4x$$

A) $f'(x) = -12e^{3x} \sin 4x$

C) $f'(x) = 12e^{3x} \sin 4x$

B) $f'(x) = 3e^{3x} \cos 4x + 4e^{3x} \sin 4x$

D) $f'(x) = 3e^{3x} \cos 4x - 4e^{3x} \sin 4x$

Ans: D   Difficulty: Moderate   Section: 2.7

104. Find the derivative of $f(x) = 9^{3x+8}$.

A) $f'(x) = 9^{3x+8}(3 \ln 9)$

C) $f'(x) = 9^{3x+8} \ln 9$

B) $f'(x) = (3)9^{3x+8}$

D) $f'(x) = 9^{3x+8}(3x+8) \ln 9$

Ans: A   Difficulty: Easy   Section: 2.7

105. Differentiate the function.
$$f(w) = \frac{w}{e^{3w}}$$

A) $f'(w) = \dfrac{1-3w}{e^{3w}}$ B) $f'(w) = \dfrac{1}{3e^{3w}}$ C) $f'(w) = \dfrac{3}{e^{3w}}$ D) $f'(w) = \dfrac{3w-1}{e^{3w}}$

Ans: A   Difficulty: Moderate   Section: 2.7

106. Find the derivative of $f(x) = \ln(2x)$.

A) $f'(x) = \dfrac{1}{x} + \dfrac{1}{2}$ B) $f'(x) = \dfrac{2}{x}$ C) $f'(x) = \dfrac{1}{2x}$ D) $f'(x) = \dfrac{1}{x}$

Ans: D   Difficulty: Easy   Section: 2.7

107. Find the derivative of $f(x) = \ln\left(\sqrt{3x}\right)$.

A) $f'(x) = \dfrac{1}{6x}$ B) $f'(x) = \dfrac{2}{3x}$ C) $f'(x) = \dfrac{1}{2x}$ D) $f'(x) = \dfrac{1}{2}\left[\dfrac{1}{x} + \dfrac{1}{3}\right]$

Ans: C   Difficulty: Easy   Section: 2.7

108. Differentiate the function.
$$f(t) = \ln(t^5 + 8t)$$

A) $f'(t) = \dfrac{1}{t^5 + 8t}$

C) $f'(t) = (5t^4 + 8)\ln(t^5 + 8t)$

B) $f'(t) = \dfrac{1}{5t^4 + 8}$

D) $f'(t) = \dfrac{5t^4 + 8}{t^5 + 8t}$

Ans: D   Difficulty: Moderate   Section: 2.7

109. Differentiate the function.
$$g(x) = \sin x \, \ln(x^5 + 3)$$

A) $g'(x) = \cos x \, \ln(x^5 + 3) + \dfrac{5x^4 \sin x}{x^5 + 3}$

C) $g'(x) = \dfrac{5x^4 \cos x}{x^5 + 3}$

B) $g'(x) = -\cos x \, \ln(x^5 + 3) + \dfrac{\sin x}{x^5 + 3}$

D) $g'(x) = \dfrac{\cos x}{x^5 + 3}$

Ans: A   Difficulty: Moderate   Section: 2.7

110. Differentiate the function.
$$h(x) = 7^{e^x}$$

A) $h'(x) = 7^{e^x}$ B) $h'(x) = 7^{e^x}\ln 7$ C) $h'(x) = e^x 7^{e^x}\ln 7$ D) $h'(x) = e^x 7^{e^x}$

Ans: C   Difficulty: Moderate   Section: 2.7

111. Find an equation of the line tangent to $f(x) = 3^x$ at $x = 3$.

A) $y = 27(x\ln 3 - (1 + 3\ln 3))$

C) $y = 27(x\ln 3 + (1 - 3\ln 3))$

B) $y = x\ln 3 + (1 - 3\ln 3)$

D) $y = x\ln 3 + (3\ln 3 - 1)$

Ans: C   Difficulty: Moderate   Section: 2.7

112. Find an equation of the line tangent to $f(x) = 3\ln(x^4)$ at $x = 2$.

A) $y = \dfrac{x}{2} + (\ln 2 - 1)$

C) $y = 12\left(\dfrac{x}{2} + (1 - \ln 2)\right)$

B) $y = 12\left(\dfrac{x}{2} + (\ln 2 - 1)\right)$

D) $y = \dfrac{x}{2} + (1 - \ln 2)$

Ans: B   Difficulty: Moderate   Section: 2.7

113. Find all values of x for which the tangent line to $f(x) = x^2 e^{-4x}$ is horizontal.

A) $x = 0$ B) $x = 0$, $x = -4$ C) $x = 0$, $x = 8$ D) $x = 0$, $x = \dfrac{1}{2}$

Ans: D   Difficulty: Moderate   Section: 2.7

114. The value of an investment is given by $v(t) = (600)4^t$. Find the instantaneous percentage rate of change. (Round to 2 decimal places.)

A)   1.39 % per year
B)   33.27 % per year
C)   138.63 % per year
D)   17.31 % per year

Ans: C   Difficulty: Moderate   Section: 2.7

115. A bacterial population starts at 300 and quadruples every day. Calculate the percent rate of change rounded to 2 decimal places.

A) 160.94 %   B) 138.63 %   C) 1.39 %   D) 88.63 %

Ans: B   Difficulty: Moderate   Section: 2.7

116. Use logarithmic differentiation to find the derivative of $f(x) = x^{\cos 2x}$.

A) $f'(x) = x^{\cos 2x}\left[\dfrac{\cos 2x}{x} - 2(\sin 2x)\ln x\right]$

B) $f'(x) = (-2\sin 2x)x^{\cos 2x}$

C) $f'(x) = (\cos 2x)x^{\cos 2x - 1}$

D) $f'(x) = x^{\cos 2x}(\ln x - 2\sin 2x)$

Ans: A   Difficulty: Moderate   Section: 2.7

117. Find the derivative of $f(x) = (x^3)^{3x}$.

A) $f'(x) = x^{9x}(\ln x + 9)$

B) $f'(x) = 9x^{9x - 1}$

C) $f'(x) = 9x^{9x}$

D) $f'(x) = 9x^{9x}(\ln x + 1)$

Ans: D   Difficulty: Easy   Section: 2.7

118. The position of a weight attached to a spring is described by $s(t) = e^{-2t} \sin 3t$. Determine and graph the velocity function for positive values of t and find the approximate first time when the velocity is zero. Find the approximate position of the weight the first time the velocity is zero. Round answers to tenths.
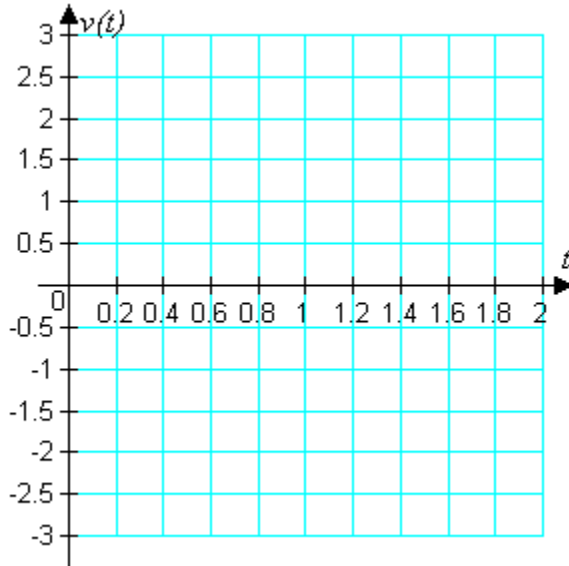
Ans: $v(t) = e^{-2t}(3\cos 3t - 2\sin 3t)$. The velocity is first zero at about 0.3 and its position is about 0.4.
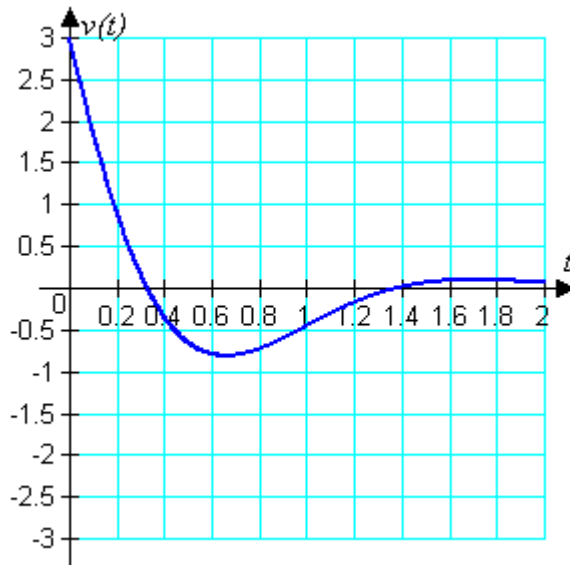
Difficulty: Moderate   Section: 2.7

119. An investment compounded continuously will be worth $f(t) = Ae^{rt}$, where A is the investment in dollars, r is the annual interest rate, and t is the time in years. APY can be defined as $(f(1) - A)/A$, the relative increase of worth in one year. Find the APY for an interest rate of 5%. Express the APY as a percent rounded to 2 decimal places.

A) $APY = 105.13\%$ C) $APY = 5.13\%$
B) $APY = 4.13\%$ D) $APY = 6.13\%$
Ans: C   Difficulty: Moderate   Section: 2.7

120. Compute the slope of the line tangent to $3x^2 + 3xy + 7y^2 = 34$ at $(2, -1)$.

A) slope $= \dfrac{15}{8}$ B) slope $= \dfrac{9}{8}$ C) slope $= \dfrac{8}{9}$ D) slope $= \dfrac{15}{14}$
Ans: B   Difficulty: Moderate   Section: 2.8

121. Find the derivative $y'(x)$ implicitly.
$$x^2 y^2 - 7y = 5x$$

A) $y'(x) = \dfrac{5}{4xy + 7}$ C) $y'(x) = -\dfrac{4xy - 5}{7}$

B) $y'(x) = \dfrac{5 - 2xy^2}{2x^2 y - 7}$ D) $y'(x) = \dfrac{2xy^2 + 12}{2x^2 y}$

Ans: B   Difficulty: Moderate   Section: 2.8

122. Find the derivative $y'(x)$ implicitly if $2y^2 - \sqrt{xy} = -6$.

A) $y'(x) = -\dfrac{y}{4y\sqrt{xy} + x}$ C) $y'(x) = \dfrac{y}{8y\sqrt{xy} - x}$

B) $y'(x) = \dfrac{y\sqrt{xy}}{8y - x}$ D) $y'(x) = \dfrac{y}{8y - x\sqrt{xy}}$

Ans: C   Difficulty: Moderate   Section: 2.8

123. Find the derivative $y'(x)$ implicitly if $4\sin xy + 5x = -5$.

A) $y'(x) = \dfrac{5}{4x\cos xy} + \dfrac{y}{x}$ C) $y'(x) = -\dfrac{5\cos xy}{4x} - \dfrac{y}{x}$

B) $y'(x) = -\dfrac{5}{4x} - \dfrac{y}{x\cos xy}$ D) $y'(x) = -\dfrac{5}{4x\cos xy} - \dfrac{y}{x}$

Ans: D   Difficulty: Moderate   Section: 2.8

124. Find the derivative $y'(x)$ implicitly.

$$xe^y - 9y\cos x = 2$$

A)
$$y'(x) = -\frac{e^y}{9\sin x + xe^y}$$

C)
$$y'(x) = -\frac{9\sin x}{e^y}$$

B)
$$y'(x) = -\frac{e^y}{9\sin x}$$

D)
$$y'(x) = \frac{e^y + 9y\sin x}{9\cos x - xe^y}$$

Ans: D   Difficulty: Difficult   Section: 2.8

125. Find the derivative $y'(x)$ implicitly.

$$e^{5y} - \ln(y^2 - 1) = 3x$$

A)
$$y'(x) = \frac{3(y^2 - 1)}{5(y^2 - 1)e^{5y} - 2y}$$

C)
$$y'(x) = \frac{3(y^2 - 1)}{5(y^2 - 1)e^{5y} - 1}$$

B)
$$y'(x) = \frac{(3 - 5e^{5x})(y^2 - 1)}{2y}$$

D)
$$y'(x) = \frac{3(y^2 - 1) + 2y}{5(y^2 - 1)e^{5y}}$$

Ans: A   Difficulty: Difficult   Section: 2.8

126. Find an equation of the tangent line at the given point.

$$x^2 - 16y^3 = 0 \text{ at } (4, 1)$$

A) $y = -\dfrac{1}{6}x + \dfrac{4}{3}$ B) $y = -\dfrac{1}{12}x + \dfrac{4}{3}$ C) $y = \dfrac{1}{6}x + \dfrac{1}{3}$ D) $y = \dfrac{1}{12}x + \dfrac{1}{3}$

Ans: C   Difficulty: Moderate   Section: 2.8

127. Find an equation of the tangent line at the given point.

$$x^2 y^2 = 3y + 1 \text{ at } (2, 1)$$

Ans:
$$y = -\frac{4}{5}x + \frac{13}{5}$$

Difficulty: Moderate   Section: 2.8

128. Find the second derivative, $y''(x)$, of $-2\sqrt{x^3} + 4\sqrt{y^3} = -3$.

A)
$$y''(x) = \frac{1}{4\sqrt{xy}} - \frac{y'}{2y}$$

C)
$$y''(x) = -\frac{1}{2\sqrt{xy}} - \frac{(y')^2}{2y}$$

B)
$$y''(x) = \frac{1}{4\sqrt{xy}} - \frac{(y')^2}{2y}$$

D)
$$y''(x) = -\frac{1}{4\sqrt{xy}} + \frac{(y')^2}{2y}$$

Ans: B   Difficulty: Moderate   Section: 2.8

129. Find the second derivative, $y''(x)$, of $-3y^2 = -2x^3 + x - \cos y$.

A) $$y''(x) = \frac{-4x + (-\cos y - 3)(y')^2}{-3y + \sin y}$$

C) $$y''(x) = \frac{-12x + (\cos y - 3)y^2}{-6y^2 - \sin y}$$

B) $$y''(x) = \frac{-2x + (\cos y - 6)y'}{-6y - \cos y}$$

D) $$y''(x) = \frac{-12x + (\cos y + 6)(y')^2}{-6y - \sin y}$$

Ans: D   Difficulty: Moderate   Section: 2.8

130. Find the derivative of $f(x) = \cos^{-1}(x^5 - 2)$.

A) $$f'(x) = \frac{5x^4 \sin(x^5 - 2)}{\cos^2(x^5 - 2)}$$

C) $$f'(x) = \frac{5x^4}{\sqrt{1 - (x^5 - 2)^2}}$$

B) $$f'(x) = \frac{5x^4}{\cos^2(x^5 - 2)}$$

D) $$f'(x) = -\frac{5x^4}{\sqrt{1 - (x^5 - 2)^2}}$$

Ans: D   Difficulty: Moderate   Section: 2.8

131. Find the derivative of $f(x) = \tan^{-1}(3/x)$.

A) $$f'(x) = -\frac{3}{9 + x^2}$$

C) $$f'(x) = -\frac{3}{1 + 9x^2}$$

B) $$f'(x) = -\frac{3}{3 + x^2}$$

D) $$f'(x) = -\frac{3}{1 + 3x^2}$$

Ans: A   Difficulty: Moderate   Section: 2.8

132. Find the derivative of $f(x) = 5e^{3\tan^{-1}x}$.

A) $$f'(x) = \frac{30}{1 - x^2} e^{3\tan^{-1}x}$$

C) $$f'(x) = \frac{15}{1 + x^2} e^{3\tan^{-1}x}$$

B) $$f'(x) = \frac{5}{1 + x^2} e^{3\tan^{-1}x}$$

D) $$f'(x) = \frac{3}{1 - x^2} e^{3\tan^{-1}x}$$

Ans: C   Difficulty: Moderate   Section: 2.8

133. Find the derivative of $f(x) = 4\sec^{-1}(x^5)$.

A) $f'(x) = \dfrac{20x^4}{|x^5|\sqrt{x^{10}-1}}$

C) $f'(x) = \dfrac{4x^4}{|x|\sqrt{x^2+1}}$

B) $f'(x) = \dfrac{-20x^5}{|x|\sqrt{x^2-1}}$

D) $f'(x) = \dfrac{5x^4}{|x^4|\sqrt{x^8-1}}$

Ans: A   Difficulty: Moderate   Section: 2.8

134. Find the location of all horizontal and vertical tangents for $x^2 - xy^2 = 49$.

A) horizontal: none; vertical: (–7, 0), (7, 0)
B) horizontal: (7, 0); vertical: (–7, 0), (7, 0)
C) horizontal: (–7, 0), (7, 0); vertical: none
D) horizontal: none; vertical: (7, 0)
Ans: A   Difficulty: Moderate   Section: 2.8

135. Find the location of all horizontal and vertical tangents for $x^2 + xy^2 + 81 = 0$.

A) horizontal: $\left(-9,-3\sqrt{2}\right),\left(-9,3\sqrt{2}\right)$; vertical: (–81, 0)

B) horizontal: $\left(-9,-3\sqrt{2}\right),\left(-9,3\sqrt{2}\right)$; vertical: (0, 0)

C) horizontal: $\left(-9,-3\sqrt{2}\right),\left(-9,3\sqrt{2}\right)$; vertical: none

D) horizontal: $\left(9,-3\sqrt{2}\right),\left(9,3\sqrt{2}\right)$; vertical: (–81, 0)
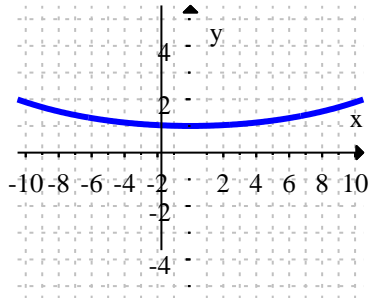
Ans: C   Difficulty: Moderate   Section: 2.8

136. Sketch the graph of the function.
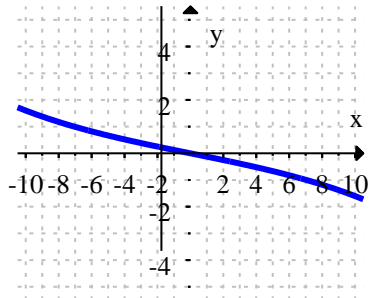$$f(x) = \cosh(x/8)$$

A)
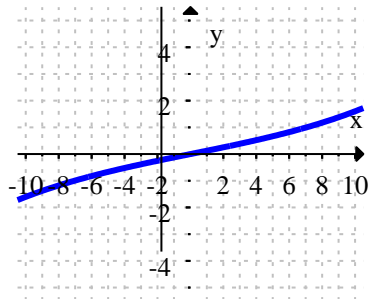

B)

C)

D)
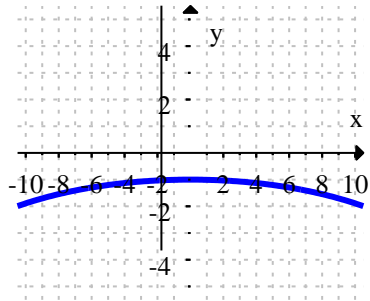
Ans: B   Difficulty: Moderate   Section: 2.9

137. Find the derivative of $f(x) = \cosh \sqrt{2x}$.

A) $f'(x) = -\dfrac{\sqrt{2}\cosh\sqrt{2x}}{2\sqrt{x}}$

C) $f'(x) = -\dfrac{\sqrt{2}\sinh\sqrt{2x}}{2\sqrt{x}}$

B) $f'(x) = \dfrac{\sqrt{2}\cosh\sqrt{2x}}{2\sqrt{x}}$

D) $f'(x) = \dfrac{\sqrt{2}\sinh\sqrt{2x}}{2\sqrt{x}}$

Ans: D Difficulty: Moderate Section: 2.9

138. Find the derivative of $f(x) = (\tanh x)^3$.

A) $f'(x) = 3(\tanh x)^2$

C) $f'(x) = \operatorname{sech}^6 x$

B) $f'(x) = 3(\tanh x)^2 \operatorname{sech}^2 x$

D) $f'(x) = 3\operatorname{sech}^5 x$

Ans: B Difficulty: Moderate Section: 2.9

139. Find the derivative of $f(x) = \operatorname{sech} 4x$.

A) $f'(x) = -4\operatorname{sech} 4x \tanh 4x$

C) $f'(x) = 4\operatorname{sech}^2 4x$

B) $f'(x) = 4\operatorname{sech} 4x \tanh 4x$

D) $f'(x) = \operatorname{sech}^2 4x$

Ans: A Difficulty: Moderate Section: 2.9

140. Find the derivative of $f(x) = x^4 \sinh 10x$.

A) $f'(x) = 40x^3 \cosh 10x$

B) $f'(x) = 4x^3 \cosh 10x$

C) $f'(x) = 4x^3 \sinh 10x + 10x^4 \cosh 10x$

D) $f'(x) = 4x^3 \sinh 10x + x^4 \cosh 10x$

Ans: C Difficulty: Moderate Section: 2.9

141. Find the derivative of $f(x) = \dfrac{\cosh 4x}{x-2}$.

A) $f'(x) = \dfrac{4(x-2)\sinh 4x - \cosh 4x}{(x-2)^2}$

C) $f'(x) = \dfrac{4\sinh 4x}{x-2}$

B) $f'(x) = \dfrac{(x-2)\sinh 4x - 4\cosh 4x}{(x-2)^2}$

D) $f'(x) = \dfrac{4\sinh 4x}{(x-2)^2}$

Ans: A Difficulty: Moderate Section: 2.9

142. Find the derivative of $f(x) = \cosh^{-1} 8x$.

A) $f'(x) = \dfrac{8}{\sqrt{64 - x^2}}$

C) $f'(x) = \dfrac{8}{\sqrt{64x^2 - 1}}$

B) $f'(x) = \dfrac{8}{\sqrt{x^2 - 64}}$

D) $f'(x) = \dfrac{8}{\sqrt{1 - 64x^2}}$

Ans: C   Difficulty: Moderate   Section: 2.9

143. A general equation for a catenary is $y = a\cosh(x/b)$. Find a and b to match the following characteristics of a hanging cable. The ends are 20 m apart and have a height of $y = 20$ m. The height in the middle is $y = 10$ m.

Ans:
$$a = 10, \quad b = \frac{10}{\ln(\sqrt{3} + 2)}, \quad y = 10\cosh\left(\frac{\ln(\sqrt{3} + 2)}{10} x\right)$$

Difficulty: Moderate   Section: 2.9

144. Suppose that the vertical velocity $v(t)$ of a falling object of mass $m = 30$ kg subject to gravity and air drag is given by
$$v(t) = -\sqrt{\frac{9.8m}{k}} \tanh\left(\sqrt{\frac{9.8k}{m}} t\right)$$
for some positive constant k. Suppose $k = 0.5$ and find the terminal velocity v_T by computing $\lim_{t \to \infty} v(t)$.

A) $v_T \approx -96.8$ m/sec

C) $v_T \approx -24.2$ m/sec

B) $v_T \approx -48.4$ m/sec

D) $v_T \approx -12.1$ m/sec

Ans: C   Difficulty: Moderate   Section: 2.9

145. Determine if the function satisfies Rolle's Theorem on the given interval. If so, find all values of c that make the conclusion of the theorem true.
$$f(x) = 36 - x^2, \quad [-9, 9]$$

A) $x = 0$ B) $x = 36$ C) $x = -6, \ x = 6$ D) Rolle's Theorem not satisfied

Ans: A   Difficulty: Easy   Section: 2.10

146. Using the Mean Value Theorem, find a value of c that makes the conclusion true for $f(x) = 4x^3 + 5x^2$, in the interval $[-1, 1]$.

A) $c \approx -1.129$ B) One or more hypotheses fail   C) $c \approx 0.295$ D) $c = 0$

Ans: C   Difficulty: Easy   Section: 2.10

147. Using the Mean Value Theorem, find a value of c that makes the conclusion true for

$f(x) = \cos x, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right]$

A) One or more hypotheses fail   B) $c = 0$ C) $c = \dfrac{\pi}{4}$ D) $c \approx .881$

Ans: B   Difficulty: Easy   Section: 2.10

148. Prove that $9x^3 + 9x - 9 = 0$ has exactly one solution.

Ans: Let $f(x) = 9x^3 + 9x - 9$. The function $f(x)$ is continuous and differentiable everywhere. Since $f(0) < 0$ and $f(1) > 0$, $f(x)$ must have at least one zero. The derivative of $f(x) = 9x^3 + 9x - 9$ is $f'(x) = 27x^2 + 9$, which is always greater than zero. Therefore $f(x)$ can only have one zero.

Difficulty: Moderate   Section: 2.10

149. Find all functions g such that $g'(x) = f(x)$.

$f(x) = 6x^4$

A) $g(x) = 24x^3$

B) $g(x) = \dfrac{6}{5} x^5$

C) $g(x) = 24x^3 + C$, for some constant C

D) $g(x) = \dfrac{6}{5} x^5 + C$, for some constant C

Ans: D   Difficulty: Easy   Section: 2.10

150. Find all the functions $g(x)$ such that $g'(x) = \dfrac{6}{x^9}$.

A) $g(x) = -\dfrac{3}{4x^8}$ C) $g(x) = \dfrac{12}{25x^8}$

B) $g(x) = -\dfrac{3}{5x^{10}} + c$ D) $g(x) = -\dfrac{3}{4x^8} + c$

Ans: D   Difficulty: Moderate   Section: 2.10

151. Find all the functions $g(x)$ such that $g'(x) = -\sin x$.

A) $g(x) = -\cos x + c$ C) $g(x) = \cos x$

B) $g(x) = \cos x + c$ D) $g(x) = \sin x + c$

Ans: B   Difficulty: Moderate   Section: 2.10

152. Determine if the function $f(x) = 4x^3 + 5x + 2$ is increasing, decreasing, or neither.

    A) Increasing   B) Decreasing   C) Neither
    Ans: A   Difficulty: Easy   Section: 2.10

153. Determine if the function $f(x) = -5x^4 - 4x^2 + 9$ is increasing, decreasing, or neither.

    A) Increasing   B) Decreasing   C) Neither
    Ans: C   Difficulty: Easy   Section: 2.10

154. Explain why it is not valid to use the Mean Value Theorem for the given function on the specified interval. Show that there is no value of c that makes the conclusion of the theorem true.

$$f(x) = \frac{1}{x-4}, \ [3,5]$$

    Ans: The function is not continuous on the specified interval, so the Mean Value
         Theorem does not apply. Note that $f(3) = -1$ and $f(5) = 1$, so that
$$\frac{f(5) - f(3)}{(5)-(3)} = \frac{1-(-1)}{2} = 1.$$
    Also, $f'(x) = -\dfrac{1}{(x-4)^2}$.
    Since $f'(x) < 0$ for all x in the domain of f, there is no value of c such that
    $f'(c) = 1$. That is, there is no value of c such that $f'(c) = \dfrac{f(5) - f(3)}{(5)-(3)}$.

    Difficulty: Moderate   Section: 2.10