Section 2.1: The Tangent and Velocity Problems

1. For the curve $f(x) = \sqrt{x+3}$, find the slope M_{PQ} of the secant line through the points $P = (1, f(1))$ and $Q = (6, f(6))$.

a. 2
b. 1
c. $\frac{2}{3}$
d. $\frac{3}{2}$
e. 5
f. $\frac{1}{3}$
g. $\frac{1}{2}$
h. 3

ANS: F PTS: 1

2. The displacement in meters of a particle moving in a straight line is given by $s = t^3$, where t is measured in seconds. Find the average velocity in meters per second over the time period [1, 2].

a. 5
b. 3
c. 8
d. 1
e. 7
f. 6
g. 2
h. 9

ANS: E PTS: 1

3. Suppose you drive for 60 miles at 60 miles per hour, then for 60 miles at 30 miles per hour. In miles per hour, what is your average velocity?

a. 45
b. 40
c. 36
d. 42
e. 52
f. 55
g. 50
h. 48

ANS: B PTS: 1

4. If a ball is thrown into the air with a velocity of 80 ft/s, its height in feet after t seconds is given by $s(t) = 80t - 16t^2$. It will be at maximum height when its instantaneous velocity is zero. Find its average velocity from the time it is thrown ($t = 0$) to the time it reaches its maximum height.

a. 50
b. 60
c. 100
d. 80
e. 40
f. 30
g. 48
h. 32

ANS: E PTS: 1

5. A weight is attached to a spring. Suppose the position (in meters) of the weight above the floor t seconds after it is released is given by $P(t) = 0.5\sin\left(\pi t + \frac{\pi}{2}\right) + 1.2$. What is the average rate of change of the position of the weight (in m/s) over the time interval [3, 5]?

a. –1.7
b. –1.0
c. –0.5
d. 0
e. 0.5
f. 1.0
g. 1.7
h. Cannot be determined

ANS: D PTS: 1

A car on a test track accelerates from 0 ft/s to 208 ft/s in 8 seconds. The car's velocity is given in the table below:

t (s)	0	1	2	3	4	5	6	7	8
v (t) (ft/s)	0	18	47	77	104	132	163	184	208

6. On what time interval does the car have the greatest average acceleration?
   a. [0, 1]
   b. [1, 2]
   c. [2, 3]
   d. [3, 4]
   e. [4, 5]
   f. [5, 6]
   g. [6, 7]
   h. [7, 8]

   ANS: F PTS: 1

7. On what time interval does the car have the lowest average acceleration?
   a. [0, 1]
   b. [1, 2]
   c. [2, 3]
   d. [3, 4]
   e. [4, 5]
   f. [5, 6]
   g. [6, 7]
   h. [7, 8]

   ANS: A PTS: 1

8. On what time interval does the car's average acceleration most closely approximate the average acceleration for the entire 8-second run?
   a. [0, 1]
   b. [1, 2]
   c. [2, 3]
   d. [3, 4]
   e. [4, 5]
   f. [5, 6]
   g. [6, 7]
   h. [7, 8]

   ANS: D PTS: 1

9. The point $P(1, \sqrt{3})$ lies on the curve $y = \sqrt{4 - x^2}$. Let Q be the point $\left(x, \sqrt{4 - x^2}\right)$.

   (a) What is the slope of the secant line PQ (correct to 6 decimal places) for the following values of x?

   (i) 2 (ii) 1.5 (iii) 1.1 (iv) 1.01 (v) 1.001
   (vi) 0 (vii) 0.5 (viii) 0.9 (ix) 0.99 (x) 0.999

   (b) Use your results from part (a) to estimate the slope of the tangent line to the graph of $y = \sqrt{4 - x^2}$ at $x = 1$.

   ANS:
   (a)

   (i) –1.732051 (ii) –0.818350 (iii) –0.617215 (iv) –0.581212 (v) –0.577735
   (vi) –0.267949 (vii) –0.408882 (viii) –0.540063 (ix) –0.573514 (x) –0.576965

   (b) The slope of the tangent line lies between –0:576965 and –0:577725, so it is approximately –0:577.

PTS: 1

10. A car on a test track accelerates from 0 ft/s to 208 ft/s in 8 seconds. The car's velocity is given in the table below:

t (s)	0	1	2	3	4	5	6	7	8
v (t) (ft/s)	0	18	47	77	104	132	163	184	208

(a) Find the car's average acceleration for the following time intervals:
(i) [4; 6] (ii) [4; 5] (iii) [3; 4]

(b) Estimate the car's acceleration at $t = 4$.

ANS:

(a) (i) $\frac{163-104}{6-4} = \frac{59}{2} = 29.5\ \text{ft/s}^2$

(ii) $\frac{132-104}{5-4} = \frac{28}{1} = 28\ \text{ft/s}^2$

(iii) $\frac{104-77}{4-3} = \frac{27}{1} = 27\ \text{ft/s}^2$

(b) Answer may vary. One possibility would be to average the average accelerations [3, 4] and [4; 5]
$\Rightarrow \frac{27+28}{2} = 27.5\ \text{ft/s}^2$

PTS: 1

11. A projectile is launched vertically upward from the surface of Mars. The table below gives the height of the object at the indicated time following launch.

Time (seconds):	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6
Height (feet):	0	18.2	34.4	48.4	60.4	70.4	78.3	84.2	88.1	90.0

Time (seconds):	4.0	4.4	4.8	5.2	5.6	6.0	6.4	6.8	7.2	7.6
Height (feet):	89.6	87.3	82.9	76.5	68.1	57.6	45.1	30.5	13.8	0

(a) Graph the data.

(b) Using the data, compute the average velocity of the projectile on the following time intervals:

(i) [0; 4.0] (ii) [0.8; 3.2]
(iii) [1.2; 2.8] (iv) [1.6; 2.4]
(v) [2.0; 2.4] (vi) [1.6; 2.0]

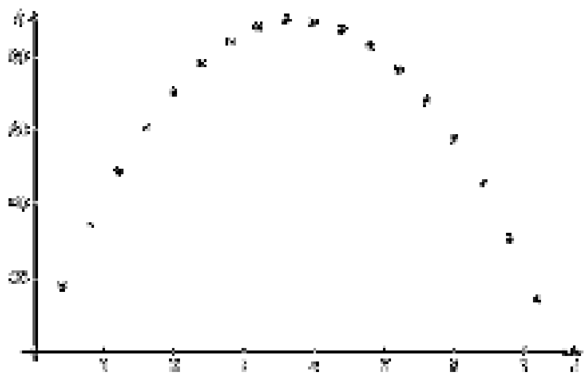
(c) Estimate the velocity of the projectile when $t = 2.0$. Justify your results.

(d) Using your graph and the table of values, determine when the projectile reaches its maximum height. Justify your answer.

(e) Using the graph and table of values, estimate the velocity of the projectile throughout the interval [0; 7.6] and sketch a graph of this velocity.

ANS:

(a)



(b) (i) $v = \dfrac{89.6 - 0}{4 - 0} = 22.4$

(ii) $v = \dfrac{88.1 - 34.4}{3.2 - 0.8} = \dfrac{53.7}{2.4} = 22.375$

(iii) $v = \dfrac{84.2 - 48.4}{2.8 - 1.2} = \dfrac{35.8}{1.6} = 22.375$

(iv) $v = \dfrac{78.3 - 60.4}{2.4 - 1.6} = \dfrac{17.9}{0.8} = 22.375$

(v) $v = \dfrac{78.3 - 70.4}{2.4 - 2.0} = \dfrac{7.9}{0.4} = 19.75$

(vi) $v = \dfrac{70.4 - 60.4}{2.0 - 1.6} = \dfrac{10}{0.4} = 25$

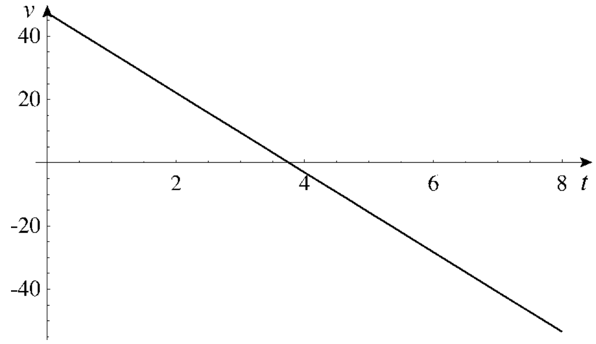
(c) Possible answer:

Time (seconds):	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4
Velocity (feet/sec):	42.4	37.3	32.3	27.2	22.2	17.1	12.1	7.1	2.0	–3

Time (seconds):	4.4	4.8	5.2	5.6	6.0	6.4	6.8	7.2	7.6
Velocity (feet/sec):	–8.0	–13.1	–18.1	–23.2	–28.2	–33.2	–38.3	–43.3	–48.4

(d) The largest height given in the table is $t = 3.6$, but the actual maximum height appears to occur for a somewhat larger value – perhaps $t = 3.7$ or 3.8.

(e)



PTS: 1

12. Suppose that the height of a projectile red vertically upward from a height of 80 feet with an initial velocity of 64 feet per second is given by $h\ (t) = -16t^2 + 64t + 80$.

(a) Compute the height of the object for $t = 0, 1, 2, 3, 4, 5$, and 6 seconds.

(b) What is the physical significance of h (6)? What does that suggest about the domain of h?

(c) What is the average velocity of the projectile for each of the following time intervals?

(i) [1; 3] (ii) [0; 2] (iii) [0; 4]

(d) What is the physical significance of an average velocity of 0?

(e) When does the projectile reach its maximum height?

(f) For what value(s) of t is $h\ (t) = 0$? Are all solutions to the equation valid? Explain.

ANS:
(a)

t	0	1	2	3	4	5	6
h	80	128	144	128	80	0	–112

(b) $h\ (6) = -112$ suggests that the object has already struck the ground. The domain of h should be [0; 5].

(c) (i) $v = \dfrac{h(3) - h(1)}{3 - 1} = \dfrac{128 - 128}{2} = 0$ ft/s

(ii) $v = \dfrac{h(2) - h(0)}{2 - 0} = \dfrac{144 - 80}{2} = 32$ ft/s

(iii) $v = \dfrac{h(4) - h(0)}{4 - 0} = \dfrac{80 - 80}{4} = 0$ ft/s

(d) An average velocity of 0 indicates that the object began and ended the interval at the same height.
(e) The maximum height occurs when $t = 2$ s.
(f) $h\ (t) = 0$ when $t = 5$ and $t = -1$. The negative value indicates a time before the object is red, and this solution is outside of the domain. Thus, the solution $t = -1$ is invalid.

PTS: 1

13. A weight is attached to a spring. Suppose the position (in meters) of the weight above the floor t seconds after it is released is given by $P(t) = 0.5\sin(\pi t + \frac{\pi}{2}) + 1.2$.

(a) What is the position of the weight when $t = 2$? When $t = 3$? When $t = 4$?

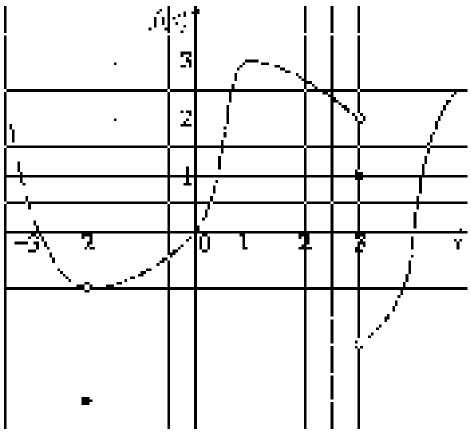
(b) What is the average rate of change of the position of the weight (in m/s) over the time interval [2, 4]? Over the time interval [2, 3]?

(c) The average rate of change of the position of the weight over the time period [2; 6] is 0. Does this mean that the weight has come to a stop? Why or why not?

ANS:
(a) 1.7 m, 0.7 m, 1.7 m
(b) $\frac{1.7-1.7}{4-2} = 0$; $\frac{0.7-1.7}{3-2} = -1.0$
(c) No, it simply means that the weight is in the same position at $t = 2$ and $t = 6$.

PTS: 1

Use the graph below for the following questions:



1. For the function whose graph is given above, determine $\lim_{x \to 3^+} f(x)$.

a. –3
b. –2
c. –1
d. 0
e. 1
f. 2
g. 3
h. Does not exist

ANS: B PTS: 1

2. For the function whose graph is given above, determine $\lim_{x \to 3} f(x)$.

a. –3
b. –2
c. –1
d. 0
e. 1
f. 2
g. 3
h. Does not exist

ANS: H PTS: 1

3. For the function whose graph is given above, determine $\lim_{x \to -2^-} f(x)$.

a. –3
b. –2
c. –1
d. 0
e. 1
f. 2
g. 3
h. Does not exist

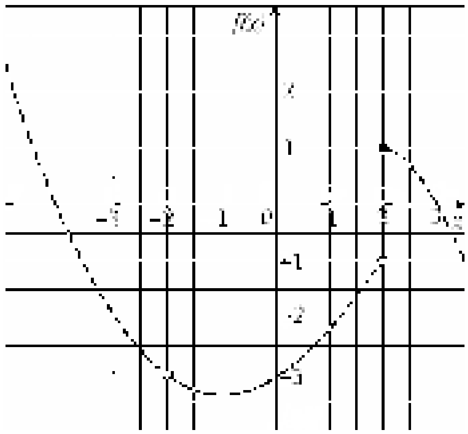
ANS: C PTS: 1

4. For the function whose graph is given above, determine $\lim_{x \to -2} f(x)$.

a. –3
b. –2
c. –1
d. 0
e. 1
f. 2
g. 3
h. Does not exist

ANS: C PTS: 1

Use the graph below for the following questions:



5. For the function whose graph is given above, determine $f(-2)$.
   a. –3
   b. –2
   c. –1
   d. 0
   e. 1
   f. 2
   g. 3
   h. Does not exist

   ANS: H PTS: 1

6. For the function whose graph is given above, determine $\lim_{x \to 2^-} f(x)$.
   a. –3
   b. –2
   c. –1
   d. 0
   e. 1
   f. 2
   g. 3
   h. Does not exist

   ANS: C PTS: 1

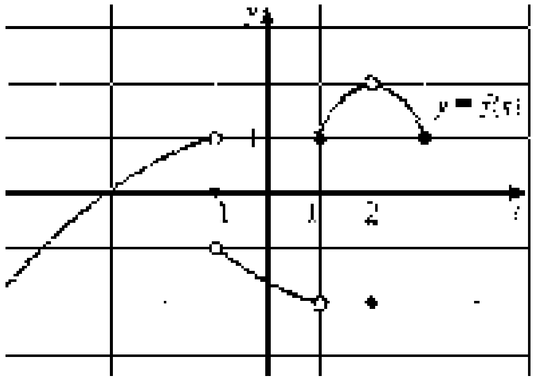
7. For the function whose graph is given above, determine $\lim_{x \to 2} f(x)$.
   a. –3
   b. –2
   c. –1
   d. 0
   e. 1
   f. 2
   g. 3
   h. Does not exist

   ANS: H PTS: 1

8. For the function whose graph is given above, determine $\lim_{x \to -2} f(x)$.
   a. –3
   b. –2
   c. –1
   d. 0
   e. 1
   f. 2
   g. 3
   h. Does not exist

   ANS: A PTS: 1

9. Use the given graph to find the indicated quantities:



(a) $\lim_{x \to -1^-} f(x)$ (b) $\lim_{x \to -1^+} f(x)$ (c) $\lim_{x \to -1} f(x)$

(d) $\lim_{x \to 1^-} f(x)$ (e) $\lim_{x \to 1^+} f(x)$ (f) $\lim_{x \to 1} f(x)$

(g) $\lim_{x \to 2^-} f(x)$ (h) $\lim_{x \to 2^+} f(x)$ (i) $\lim_{x \to 2} f(x)$

(j) $f(-1)$ (k) $f(0)$ (l) $f(1)$

(m) $f(2)$

ANS:

(a) $\lim_{x \to -1^-} f(x) = 1$ (b) $\lim_{x \to -1^+} f(x) = -1$ (c) $\lim_{x \to -1} f(x)$ does not exist

(d) $\lim_{x \to 1^-} f(x) = -2$ (e) $\lim_{x \to 1^+} f(x) = 1$ (f) $\lim_{x \to 1} f(x)$ does not exist

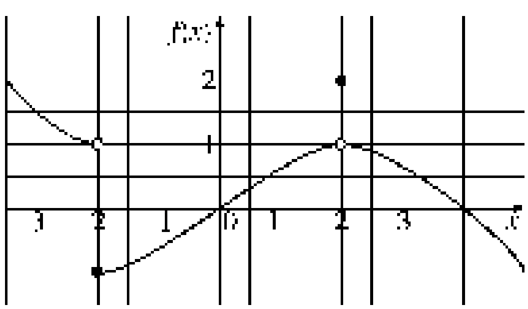
(g) $\lim_{x \to 2^-} f(x) = 2$ (h) $\lim_{x \to 2^+} f(x) = 2$ (i) $\lim_{x \to 2} f(x) = 2$

(j) $f(-1) = 0$ (k) $f(0) \approx -1.7$ (l) $f(1) = 1$

(m) $f(2) = -1$

PTS: 1

10. Use the graph of *f* below to determine the value of each of the following quantities, if it exists. If it does not exist, explain why.

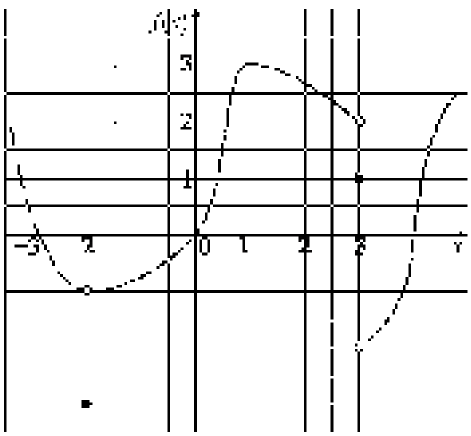


(a) $\lim_{x \to -2} f(x)$ (b) $\lim_{x \to 2} f(x)$ (c) $\lim_{x \to -2^-} f(x)$ (d) $f(-2)$ (e) $f(2)$

ANS:
(a) The limit does not exist because the left- and right-hand limits are different.
(b) $\lim_{x \to 2} f(x) = 1$ (c) $\lim_{x \to -2^-} f(x) = 1$
(d) $f(-2) = -1$ (e) $f(2) = 2$

PTS: 1

11. Use the given graph to find the indicated quantities:



(a) $\lim_{x \to 3^+} f(x)$ (b) $\lim_{x \to 3} f(x)$ (c) $f(3)$
(d) $\lim_{x \to -2^-} f(x)$ (e) $\lim_{x \to -2} f(x)$ (f) $f(-2)$

ANS:
(a) –2 (b) Does not exist (c) 1
(d) –1 (e) –1 (f) Undefined

PTS: 1

12. (a) Explain in your own words what is meant by $\lim_{x \to -2} f(x) = 3$

(b) Is it possible for this statement to be true yet for *f* –(2) = 5? Explain.

ANS:
(a) (Answers will vary.) $\lim_{x \to -2} f(x) = 3$ means that the values of *f* can be made as close as desired to 3 by taking values of *x* close enough to –2, but not equal to –2.
(b) Yes, it is possible for $\lim_{x \to -2} f(x) = 3$, but $f(-2) = 5$ The limit refers only to how the function behaves when *x* is close to –2. It does not tell us anything about the value of the function at *x* = –2.

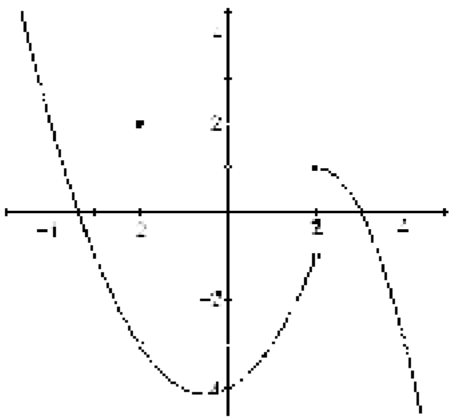
PTS: 1

13. Sketch the graph of a function f on $[-5, 5]$ that satisfies all of the following conditions:

$f(-4) = 2, f(-3) = -1, f(-2) = 2, f(1) = -3, f(2) = 1, f(3) = 0, f(4) = -3, \lim_{x \to -2} f(x) = -3, \lim_{x \to 2^-} f(x) = -1,$

and $\lim_{x \to 2^+} f(x) = 1$

ANS:
(Answers will vary)



PTS: 1

14. Consider the function $f(x) = \dfrac{x^2 - 8x + 15}{x^2 - 9}$. Make an appropriate table of values in order to determine the indicated limits:

(a) $\lim_{x \to -3^+} f(x)$ (b) $\lim_{x \to -3^-} f(x)$

(c) Does $\lim_{x \to -3} f(x)$ exist? If so, what is its value? If not, explain.

ANS:

x	$f(x)$
–2.9	–79
–2.99	–799
–2.999	–7999
–2.9999	–79999
–2.99999	–799999
–2.999999	–7999999
⋮	⋮

x	$f(x)$
⋮	⋮
–3.000001	8000001
–3.00001	800001
–3.0001	80001
–3.001	8001
–3.01	801
–3.1	81

Using the table values, the limits appear to be:

(a) $\lim_{x \to -3^+} f(x) = \infty$ (b) $\lim_{x \to -3^-} f(x) = \infty$

(c) Since the right-hand limit and the left-hand limit have different values, the limit does not exist at $x = -3$.

PTS: 1

15. Use a table of values to estimate the value of each of the following limits, to 4 decimal places.

(a) $\lim_{x \to 0} \frac{3^x - 1}{x}$ (b) $\lim_{x \to 0} \frac{\sin 5x}{\sin 3x}$ (c) $\lim_{x \to 0} (1 + x)^{1/x}$

ANS:
(a) 1.0986
(b) 1.6667
(c) 2.7183

PTS: 1

16. A cellular phone company has a roaming charge of 32 cents for every minute or fraction of a minute when you are out of your zone.

(a) Sketch a graph of the \out-of-your-zone" costs, C, of cellular phone usage as a function of the length of the call, t, for $0 \le t \le 5$..

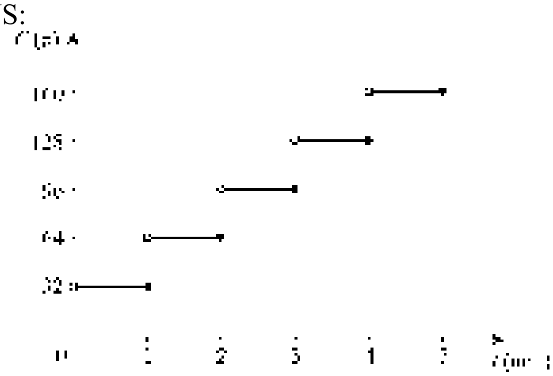
(b) Evaluate:

(i) $\lim_{t \to 2^-} C(t)$ (ii) $\lim_{t \to 2^+} C(t)$

(c) Explain the significance of the left limit (i) and the right limit (ii) to the cell phone user.

(d) For what values of t does $C(t)$ not have a limit? Justify your answer.

ANS:
(a)



(b) (i) 64 cents
(ii) 96 cents

(c) The fact that $\lim_{t \to 2^-} C(t) \ne \lim_{t \to 2^+} C(t)$ shows that there is an abrupt change in the cost of cellular phone usage at $t = 2$.

(d) For $t_0 = 1, 2, 3,$ and 4, $\lim_{t \to t_0} C(t)$ does not exist, since $\lim_{t \to t_0^-} C(t) \neq \lim_{t \to t_0^+} C(t)$.

PTS: 1

17. If $f(x) = 2^x$, how close to 3 does x have to be to ensure that $f(x)$ is within 0.1 of 8?

ANS:
Answers will vary. One reasonable answer is that $f(x)$ is within 0.1 of 8 when x is within 0.017 of 3, that is, when $2.983 < x < 3.017$.

PTS: 1

18. Determine $\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1}$ by producing an appropriate table.

ANS:

x	$f(x)$
1.001	1716.924
1.0001	1051.654
1.00001	1005.012
1.000001	1000.500
1.0000001	1000.049

x	$\frac{x^{1000} - 1}{x - 1}$
0.99	99.995683
0.999	632.304575
0.9999	951.671108
0.99999	995.021352
0.999999	999.499236
0.9999999	999.950052

From the tables, it appears that
$\lim_{x \to 1^+} \frac{x^{1000} - 1}{x - 1} = 1000$ and $\lim_{x \to 1^-} \frac{x^{1000} - 1}{x - 1} = 1000$ and therefore $\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1} = 1000$.

PTS: 1

Section 2.3: Calculating Limits Using the Limit Laws

1. Find the value of the limit $\lim_{x \to 1} (x^{17} - x + 3)$.

a. –4
b. 3
c. –2
d. 1
e. –8
f. 2
g. 0
h. –16

ANS: B PTS: 1

2. Find the value of the limit $\lim_{x \to 1} \frac{2x^2 + x - 3}{x^2 - x}$.

a. 5
b. 4
c. 3
d. 2
e. $\frac{7}{2}$
f. $\frac{3}{2}$
g. $\frac{1}{2}$
h. 0

ANS: A PTS: 1

3. Find the value of the limit $\lim_{x \to 4^+} \frac{|x - 4|}{x - 4}$.

a. 4
b. –4
c. $\frac{1}{2}$
d. –2
e. –1
f. 2
g. 1
h. $-\frac{1}{2}$

ANS: G PTS: 1

4. Find the value of the limit $\lim_{x \to 2^-} \frac{|x - 2|}{x - 2}$.

a. 1
b. –2
c. $\frac{1}{2}$
d. –4
e. –1
f. 4
g. $-\frac{1}{2}$
h. 2

ANS: E PTS: 1

5. Find the value of the limit $\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1}$.

a. $\sqrt{2}$
b. 1
c. –1
d. $-\sqrt{2}$
e. 2
f. –4
g. 4
h. –2

ANS: E PTS: 1

6. Find the value of the limit $\lim_{x \to -2} \left(\frac{1}{x+2} + \frac{4}{x^2 - 4} \right)$.

a. –4
b. –2
c. $-\frac{1}{2}$
d. $-\frac{1}{4}$
e. $\frac{1}{4}$
f. $\frac{1}{2}$
g. 2
h. 4

ANS: D PTS: 1

7. Find the value of the limit $\lim_{x \to 1} \frac{\frac{1}{x^2} - 1}{x - 1}$.

a. –3
b. –2
c. –1
d. 0
e. $-\frac{1}{2}$
f. $-\frac{1}{3}$
g. $-\frac{1}{4}$
h. $-\frac{1}{8}$

ANS: B PTS: 1

8. Find the value of the limit $\lim_{x \to 1} \frac{x - 1}{x^3 - 1}$.

a. 0
b. 2
c. $\frac{1}{3}$
d. 8
e. $\frac{1}{4}$
f. $\frac{1}{8}$
g. $\frac{1}{32}$
h. $\frac{1}{2}$

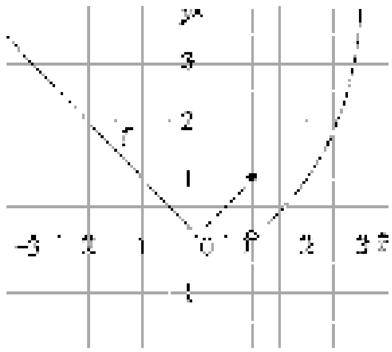
ANS: C PTS: 1

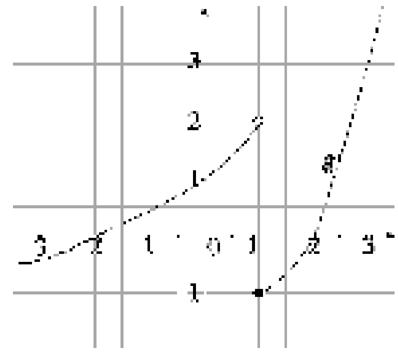
9. Find the value of the limit $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$.

a. –5
b. –3
c. –1
d. 0
e. 1
f. 3
g. 5
h. $\frac{1}{6}$

ANS: H PTS: 1

10. Use the graphs of *f* and *g* below to evaluate each limit, if it exists. If it does not exist, explain why.





(a) $\lim_{x \to -2} [f(x) + g(x)]$

(b) $\lim_{x \to 2} \left[\frac{g(x)}{f(x)}\right]$

(c) $\lim_{x \to 1} [f(x) \cdot g(x)]$

(d) $\lim_{x \to 0} \left[(x-3)^2 \cdot g(x)\right]$

ANS:

(a) $\lim_{x \to -2} [f(x) + g(x)] = 2 + 0 = 2$

(b) $\lim_{x \to 2} \left[\frac{g(x)}{f(x)}\right] = \frac{0}{1} = 0$

(c) $\lim_{x \to 1} [f(x) \cdot g(x)]$ does not exist because $\lim_{x \to 1^-} [f(x) \cdot g(x)] = 2 \neq 0 = \lim_{x \to 1^+} [f(x) \cdot g(x)]$

(d) $\lim_{x \to 0} \left[(x-3)^2 \cdot g(x)\right] = 9 \cdot 1 = 9$

PTS: 1

11. Given that $\lim_{x \to 3} f(x) = 5$, $\lim_{x \to 3} g(x) = 0$, and $\lim_{x \to 3} h(x) = -8$, find the following limits, if they exist. If a limit does not exist, explain why.

(a) $\lim_{x \to 3} (f(x) + h(x))$

(b) $\lim_{x \to 3} x^2 f(x)$

(c) $\lim_{x \to 3} f^2(x)$

(d) $\lim_{x \to 3} \frac{f(x)}{2h(x)}$

(e) $\lim_{x \to 3} \frac{g(x)}{f(x)}$

(f) $\lim_{x \to 3} \frac{f(x)}{g(x)}$

(g) $\lim_{x \to 3} \frac{2h(x)}{f(x) - h(x)}$

(h) $\lim_{x \to 3} \sqrt[3]{h(x)}$

ANS:

(a) 3 (b) 45 (c) 25 (d) $-\frac{5}{16}$ (e) 0

(f) Does not exist; the denominator is $\lim_{x \to 3} g(x) = 0$, and the numerator is $\lim_{x \to 3} f(x) = 5$.

(g) $-\frac{16}{13}$ (h) –2

PTS: 1

12. Evaluate the limit, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 2} (5x^3 - 3x^2 + 4)$ (b) $\lim_{x \to 9} \frac{x-9}{\sqrt{x} - 3}$ (c) $\lim_{x \to -1} \frac{x^4 - 1}{x^2 - 1}$

(d) $\lim_{x \to 3} \frac{\frac{2}{x} - \frac{2}{3}}{x - 3}$ (e) $\lim_{x \to -3} \frac{|x+3|}{2x+6}$ (f) $\lim_{x \to -3^-} \frac{|x+3|}{2x+6}$

ANS:

(a) $\lim_{x \to 2} (5x^3 - 3x^2 + 4) = 5 \cdot 2^3 - 3 \cdot 2^2 + 4 = 32$

(b) $\lim_{x \to 9} \frac{x-9}{\sqrt{x} - 3} = 6$

(c) $\lim_{x \to -1} \frac{x^4 - 1}{x^2 - 1} = \lim_{x \to -1} (x^2 + 1) = 2$

(d) $\lim_{x \to 3} \frac{\frac{2}{x} - \frac{2}{3}}{x - 3} = \lim_{x \to 3} \frac{6 - 2x}{(x-3) \cdot 3x} = \lim_{x \to 3} \frac{-2(x-3)}{(x-3) \cdot 3x} = -\frac{2}{9}$

(e) $\lim_{x \to -3} \frac{|x+3|}{2x+6}$ does not exist since the let- and right-hand limits are different.

(f) $\lim_{x \to -3^-} \frac{|x+3|}{2x+6} = \lim_{x \to -3^-} \frac{-(x+3)}{2(x+3)} = -\frac{1}{2}$

PTS: 1

13. Let $f(x) = \begin{cases} \sqrt{-2-x} & \text{if } x \le -2 \\ x & \text{if } -2 < x \le 2 \\ x^2 - 4x + 6 & \text{if } x > 2 \end{cases}$. Find the following limits. Justify your answers.

(a) $\lim_{x \to -2^-} f(x)$ (b) $\lim_{x \to -2^+} f(x)$ (c) $\lim_{x \to -2} f(x)$

(d) $\lim_{x \to 2^-} f(x)$ (e) $\lim_{x \to 2^+} f(x)$ (f) $\lim_{x \to 2} f(x)$

ANS:

(a) $\lim_{x \to -2^-} f(x) = \sqrt{-2-(-2)} = 0$

(b) $\lim_{x \to -2^+} f(x) = -2$

(c) $\lim_{x \to -2} f(x)$ does not exist since $\lim_{x \to -2^-} f(x) \ne \lim_{x \to -2^+} f(x)$.

(d) $\lim_{x \to 2^-} f(x) = 2$

(e) $\lim_{x \to 2^+} f(x) = 2^2 - 4(2) + 6 = 4 - 8 + 6 = 2$

(f) $\lim_{x \to 2} f(x) = 2$ since $\lim_{x \to 2^-} f(x) = 2 = \lim_{x \to 2^+} f(x)$.

PTS: 1

14. Explain why $\frac{(3x-2)(x-4)}{x-4} \neq 3x-2$, but $\lim_{x \to 4} \frac{(3x-2)(x-4)}{x-4} = \lim_{x \to 4} (3x-2)$.

ANS:
$\frac{(3x-2)(x-4)}{x-4} \neq 3x-2$ because the expression on the left does not permit $x = 4$, while there is no similar restriction on the right.
However, since $\frac{(3x-2)(x-4)}{x-4} = 3x-2$ whenever $x \neq 4$, and neither $\lim_{x \to 4} \frac{(3x-2)(x-4)}{x-4}$ nor $\lim_{x \to 4} (3x-2)$ involve x ever being equal to 4, the second equation is true.

PTS: 1

15. For each of the following problems, make an appropriate table to determine the limits.

(a) $\lim_{x \to 1} \frac{|x| - x}{x-1}$ (b) $\lim_{x \to 0} \frac{\cos x - 1}{x}$ (c) $\lim_{x \to 3} \frac{(1/x)-(1/3)}{x-3}$

ANS:
(a)

x	$\frac{\lvert x \rvert - x}{x-1}$
1.01	0
1.001	0
1.0001	0
1.0001	0

x	$\frac{\lvert x \rvert - x}{x-1}$
0.99	0
0.999	0
0.9999	0

From the tables, it appears that $\lim_{x \to 1} \frac{|x| - x}{x-1} = 0$.
Algebraically: Note that as $x \to 1$, we can assume that $x > 0$ and therefore $|x| = x$, so
$\lim_{x \to 1} \frac{|x| - x}{x-1} = \lim_{x \to 1} \frac{x - x}{x-1} = \lim_{x \to 1} 0 = 0$.

(b)

x (radians)	$\frac{\cos x - 1}{x}$
0.01	$-4{:}999958 \times 10^{-3}$
0.001	-5×10^{-4}
0.0002	-1×10^{-4}
0.00004	-2×10^{-5}

x (radians)	$\frac{\cos x - 1}{x}$
–0.01	$4{:}999958 \times 10^{-3}$
–0.001	5×10^{-4}
–0.0002	1×10^{-4}
–0.00004	2×10^{-5}

From the tables, it appears that $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$.

Algebraically:

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = \lim_{x \to 0} \frac{(\cos x - 1)(\cos x + 1)}{x(\cos x + 1)} = \lim_{x \to 0} \frac{-\sin^2 x}{x(\cos x + 1)}$$

$$= \lim_{x \to 0} \frac{-\sin x \cdot \sin x}{x} \cdot \frac{1}{\cos x + 1}$$

$$= \lim_{x \to 0} - \sin x \cdot \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{1}{\cos x + 1}$$

$$= 0 \cdot 1 \cdot \frac{1}{2} = 0$$

(c)

x	$\dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$
2.9	–0.1149425
2.98	–0.1118568
2.999	–0.1111481
2.9999	–0.1111148

x	$\dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$
3.01	–0.1107420
3.002	–0.111037
3.0005	–0.1110926
3.00001	–0.1111108

From the tables, it appears that $\lim_{x \to 3^-} f(x) = -0.\bar{1} = -\frac{1}{9}$ and $\lim_{x \to 3^+} f(x) = -0.\bar{1} = -\frac{1}{9}$, and so $\lim_{x \to 3} f(x) = -\frac{1}{9}$.

Algebraically: $\lim_{x \to 3} \dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3} = \lim_{x \to 3} \frac{3 - x}{3x} \cdot \frac{1}{x - 3} = \lim_{x \to 3} \left(-\frac{1}{3x} \right) = -\frac{1}{9}$

PTS: 1

16. Let $f(x) = \begin{cases} 2x - 1 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$

(a) Determine whether or not the following limits exist. Justify your answers.

(i) $\lim_{x \to 0} f(x)$ (ii) $\lim_{x \to -1} f(x)$ (iii) $\lim_{x \to \sqrt{2}} f(x)$

(b) Find a value b for which $\lim_{x \to b} f(x)$ exists.

ANS:

(a) (i) If we consider only rational values of x, then $\lim_{x \to 0} f(x) = \lim_{x \to 0} (2x - 1) = -1$. If we consider only irrational values of x, then $\lim_{x \to 0} f(x) = \lim_{x \to 0} 1 = 1$.

Because these two values are different, $\lim_{x \to 0} f(x)$ does not exist.

(ii) If we consider only rational values of x, then $\lim_{x \to -1} f(x) = (2x - 1) = -3$. If we consider only irrational values of x, then $\lim_{x \to -1} f(x) = \lim_{x \to -1} 1 = 1$. Because these two values are different, $\lim_{x \to -1} f(x)$ does not exist.

(iii) If we consider only rational values of x, then $\lim_{x \to \sqrt{2}} f(x) = \lim_{x \to \sqrt{2}} (2x - 1) = 2\sqrt{2 - 1}$. If we consider only irrational values of x, then $\lim_{x \to \sqrt{2}} f(x) = \lim_{x \to \sqrt{2}} 1 = 1$. Because these two values are different, $\lim_{x \to \sqrt{2}} f(x)$ does not exist.

(b) If we consider only rational values of x, $\lim_{x \to b} f(x) = \lim_{x \to b} (2x - 1) = 2b - 1$. If we consider only irrational values of x, then $\lim_{x \to b} f(x) = \lim_{x \to b} = 1$. In order for $\lim_{x \to b} f(x)$ to exist (and equal, say, L), the one-sided limits must be equal, so we must have $L = 2b - 1 = 1 \Rightarrow b = 1$. So the limit exists only for $b = 1$.

PTS: 1

17. Suppose we know that $-x^2 \le x^2 \sin\left(\frac{\pi}{x}\right) \le x^2$ for $x \in [-1, 1]$. Use the Squeeze Theorem to determine $\lim_{x \to 0} x^2 \sin\left(\frac{\pi}{x}\right)$.

ANS:

Since $-x^2 \le x^2 \sin\left(\frac{\pi}{x}\right) \le x^2$ for $x \in [-1, 1]$ and $\lim_{x \to 0} \left(-x^2\right) = 0 = \lim_{x \to 0} \left(x^2\right)$, by the Squeeze Theorem, $\lim_{x \to 0} x^2 \sin\left(\frac{\pi}{x}\right) = 0$.

PTS: 1

Section 2.4: Continuity

1. At what value(s) of x does the function $\frac{(x+1)^2}{x^2-1}$ have a removable discontinuity?

a. –3
b. 3
c. 2
d. –1
e. 1
f. –4
g. –2
h. 4

ANS: D PTS: 1

2. At what value(s) of x does the function $\frac{(x+2)^2}{x^2-4}$ have an infinite discontinuity?

a. 3
b. –4
c. –2
d. –3
e. –1
f. 1
g. 2
h. 4

ANS: C PTS: 1

3. Find the distance between the two values of x at which the function $\frac{1}{x^2-3x+2}$ is discontinuous.

a. 3
b. 2
c. 8
d. 1
e. 5
f. 4
g. 7
h. 6

ANS: D PTS: 1

4. At what value(s) of x is the function $f(x) = \begin{cases} x+2 & \text{if } x \le -1 \\ x^2 & \text{if } -1 < x < 1 \\ 3-x & \text{if } x \ge 1 \end{cases}$ discontinuous?

a. –1
b. 0
c. 1
d. –1, 0
e. 0, 1
f. –1, 1
g. –1, 0, 1
h. None

ANS: C PTS: 1

5. Find the constant(s) c that make(s) the function $f(x) = \begin{cases} c^2 - x^2 & \text{if } x < 2 \\ 2(c-x) & \text{if } x \ge 2 \end{cases}$ continuous on $(-\infty, \infty)$.

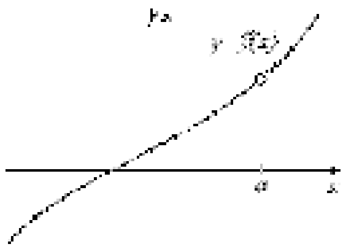
a. –4, –2
b. 2, 0
c. 2
d. 4
e. –2, 4
f. –2
g. 0
h. Does not exist

ANS: B PTS: 1

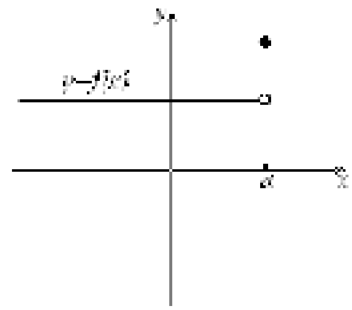
6. The definition of continuity of $f(x)$ at a point requires three things. List these three conditions, and in each case give an example (a graph or a formula) which illustrates how this condition can fail at $x = a$.

ANS:

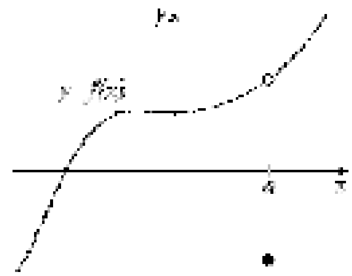
The example below fails the requirement that a must lie in the domain of $f(x)$.



The example below fails the requirement that $\lim_{x \to a} f(x)$ must exist.

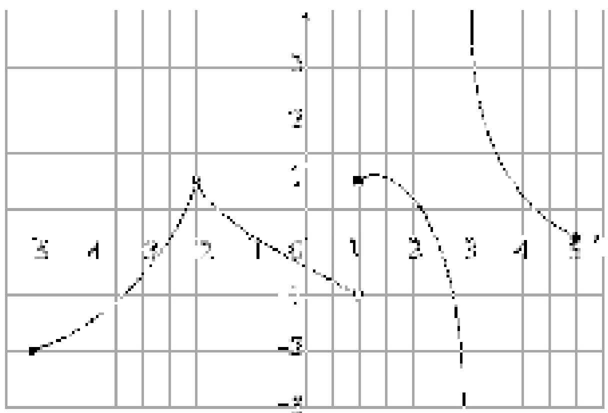


The example below fails the requirement that $\lim_{x \to a} f(x) = f(a)$.



PTS: 1

7. Given the graph of f below, state the intervals on which f is continuous.



ANS:
[–5, 1), (1, 3), and (3, 5]

PTS: 1

8. At what value(s) of x is the function $f(x) = \begin{cases} |x+1| - 1 & \text{if } x < 0 \\ x^2 + x & \text{if } 0 \le x < 1 \\ 3 - x & \text{if } 1 \le x \end{cases}$ discontinuous?

ANS:
It is obvious that $f(x)$ is continuous on $(-\infty, 0)$, $(0, 1)$, and $(1, \infty)$. We need only examine $x = 0$ and $x = 1$.

$f(x)$ is continuous at $x = 0$, since

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} |x+1| - 1 = 0 = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x^2 + x) \text{ so } \lim_{x\to 0} f(x) = 0 = f(0).$$

$f(x)$ is continuous at $x = 1$, since $\lim_{x\to 1^-} (x^2 + x) = 2 = \lim_{x\to 1^+} (3 - x)$ so $\lim_{x\to 1} f(x) = 2 = f(1)$.

So f is continuous everywhere.

PTS: 1

9. At what value(s) of x is the function $f(x) = \begin{cases} x^2 + 4x + 5 & \text{if } x < -2 \\ \frac{1}{2}x & \text{if } -2 \le x \le 2 \\ 1 + \sqrt{x-2} & \text{if } x > 2 \end{cases}$

ANS:
We need only examine $x = -2$ and $x = 2$ as the function is clearly continuous elsewhere.

f(x) is discontinuous at $x = -2$, because $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} (x^2 + 4x + 5) = 1$, but

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \frac{1}{2}x = -1$, so $\lim_{x\to -2} f(x)$ does not exist.

f(x) is continuous at $x = 2$, because $f(2)=1$ and both

$$\lim_{x\to -2^-} f(x) = \frac{1}{2}(2) = 1 \text{ and } \lim_{x\to -2^+} f(x) = \left(1 + \sqrt{2-2}\right) = 1, \text{ so } \lim_{x\to -2} f(x) = 1 = f(2).$$

Thus f is discontinuous at $x = -2$.

PTS: 1

10. Consider $f(x) = \dfrac{3x^2 - 7x + 2}{x^2 - 4}$.

(a) For what value(s) of x does f have an infinite discontinuity?

(b) For what value(s) of x does f have a removable discontinuity?

ANS:
We need only examine $x = \pm 2$ since f is continuous everywhere else.

(a) Note that $\lim_{x\to -2} f(x) = \lim_{x\to -2} \dfrac{3x^2 - 7x + 2}{x^2 - 4} = \lim_{x\to -2} \dfrac{(3x-1)(x-2)}{(x+2)(x-2)} = \lim_{x\to -2} \dfrac{3x-1}{x+2}$

Thus we have

$$\lim_{x \to -2^-} \frac{3x-1}{x+2} = -\infty \text{ and } \lim_{x \to -2^+} \frac{3x-1}{x+2} = \infty$$

so f has an infinite discontinuity at $x = -2$.

(b) $\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{3x-1}{x+2} = \frac{5}{4}$. Since this limit exists, f has a removable discontinuity at $x = 2$

because we can define $f(x) = \begin{cases} \dfrac{3x^2 - 7x + 2}{x^2 - 4} & x \neq 2 \\ \dfrac{5}{4} & x = 2 \end{cases}$ so that $\lim_{x \to 2} f(x) = \frac{4}{5} = f(2)$.

PTS: 1

11. Consider $f(x) = \begin{cases} 3x - 5 & \text{if } x < 2 \\ (5 - 2x)^2 & \text{if } x > 2 \end{cases}$ The function f is not defined for $x = 2$. How should we define $f(2)$ so that f is continuous at $x = 2$? Use the definition of continuity to verify your answer.

ANS:
Since $\lim_{x \to 2^-} f(x) = 1 = \lim_{x \to 2^+} f(x)$, if we define $f(2) = 1$, then f is continuous at $x = 2$, by definition.

PTS: 1

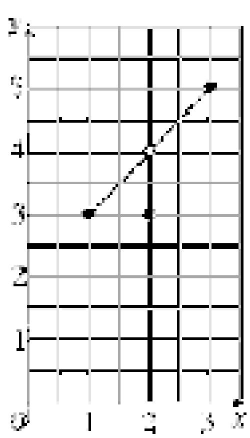
12. Use the Intermediate Value Theorem to show that $x^3 - 5x - 7 = 0$ for some value of x in $(2, 3)$.

ANS:
Answers will vary. One possible answer: Let $f(x) = x^3 - 5x - 7$. Since f is a polynomial, it is continuous on $\Re$ and thus continuous on $[2, 3]$. Since $f(2) = -9 < 0 < 5 = f(3)$, by the Intermediate Value Theorem, there must be some number $c \in (2, 3)$ such that $f(c) = c^3 - 5c - 7 = 0$.

PTS: 1

13. Suppose that $f(x)$ is defined on $[1, 3]$ and that $f(1) = 3$ and $f(3) = 5$. Sketch a possible graph of f that does not satisfy the conclusion of the Intermediate Value Theorem.

ANS:
Answers will vary. One possible answer:



PTS: 1

14. Let $f(x) = \dfrac{(x+1)^2}{x^2 - 1}$.

(a) At what value(s), if any, does f have a removable discontinuity? Justify your answer.

(b) At what value(s), if any, does f have an infinite discontinuity? Justify your answer.

ANS:

(a) $f(x)$ has a removable discontinuity at $x = -1$. We redefine $f(x) = \begin{cases} \dfrac{x+1}{x-1} & \text{if } x \neq -1 \\ 0 & \text{if } x = -1 \end{cases}$.

(b) $f(x)$ has a infinite discontinuity at $x = 1$, since $\lim\limits_{x \to 1^+} f(x) = \infty$ and $\lim\limits_{x \to 1^-} f(x) = -\infty$.

PTS: 1

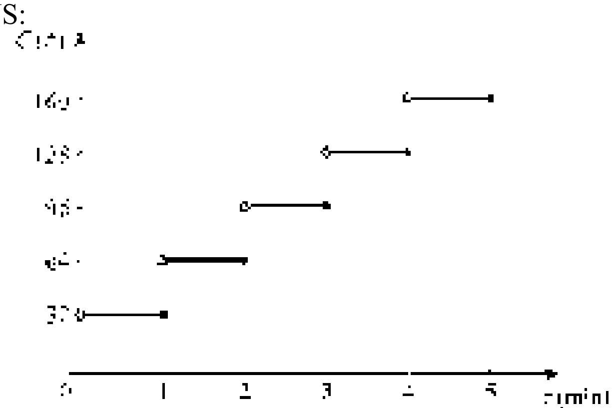
15. A cellular phone company has a roaming charge of 32 cents for every minute or fraction of a minute when you are out of your zone.

(a) Sketch a graph of the "out-of-your-zone" costs of cellular phone usage as a function of the length of the call.

(b) Discuss the discontinuities of this function and their significance to the cell phone user.

ANS:

(a)



(b) We consider $t = 1$: $\lim\limits_{t \to 1^-} f(t) = 32$ cents and $\lim\limits_{t \to 1^+} f(t) = 64$ cents. These limits show that there is an abrupt change in the cost of cellular phone usage at $t = 1$. The left limit represents the cost of a call lasting less than one minute, and the right limit represents the cost of a call lasting more than one minute. A similar situation exists at each positive integer n. Thus, the cellular phone user should try to avoid calls which are slightly over n minutes long, and instead hang up just before an integral number of minutes has passed.

PTS: 1

16. Determine whether each function is continuous or discontinuous. Explain your choice.

(a) Postage charges to send a letter by first class mail.

(b) The altitude of an airplane as a function of the time it has been in the air.

(c) The temperature of an oven as it is run through its self cleaning cycle.

(d) The number of people waiting in a queue for a bank teller.

ANS:
(a) The cost of sending a letter changes abruptly at certain weights. Therefore, this is not a continuous function.

(b) The altitude of an airplane as a function of time is a continuous function, since a plane cannot jump from one altitude to another without passing through the intermediate altitudes.

(c) The temperature of an oven changes gradually, so this is a continuous function.

(d) At the point in time when a person either enters or leaves the queue, the number of people abruptly changes by one, therefore the function is not continuous.

PTS: 1

1. Find the value of the limit $\lim_{x \to \infty} \frac{7+3x}{4-x}$.

a. $\frac{3}{4}$
b. 3
c. 7
d. $-\frac{3}{4}$
e. –3
f. $-\frac{7}{4}$
g. –7
h. $\frac{7}{4}$

ANS: E PTS: 1

2. Find the value of the limit $\lim_{x \to \infty} \frac{4x^2-3x+5}{7+2x-2x^2}$.

a. $-\frac{1}{2}$
b. 1
c. 2
d. $\frac{1}{2}$
e. –1
f. $-\frac{4}{7}$
g. –2
h. $\frac{4}{7}$

ANS: G PTS: 1

3. Find the value of the limit $\lim_{x \to \infty} \sqrt{\frac{x+8x^2}{2x^2-1}}$.

a. –2
b. 1
c. $-\frac{1}{2}$
d. 4
e. $\frac{1}{2}$
f. 2
g. 0
h. –1

ANS: F PTS: 1

4. Find the value of the limit $\lim_{x \to \infty} \left(\sqrt{x^2+x} - x\right)$.

a. $-\frac{1}{2}$
b. $\sqrt{2}$
c. $-\sqrt{2}$
d. $\frac{1}{4}$
e. ∞
f. $-\frac{1}{4}$
g. 0
h. $\frac{1}{2}$

ANS: H PTS: 1

5. Find the value of the limit $\lim_{x \to -\infty} \left(\sqrt{x^2 + 2x} - 2x \right)$.

a. 3
b. $\frac{3}{2}$
c. 0
d. 1
e. 2
f. ∞
g. $-\infty$
h. –3

ANS: F PTS: 1

6. Find the value of the limit $\lim_{x \to \infty} \frac{e^{2x}}{e^{2x} + 1}$.

a. –2
b. 1
c. 2
d. 4
e. –1
f. –3
g. 0
h. 3

ANS: B PTS: 1

7. Find the value of x at which the curve $y = \frac{2x + 1}{2x^2 + 7x + 3}$ has a vertical asymptote.

a. –4
b. –3
c. –2
d. –1
e. $-\frac{1}{2}$
f. 1
g. 2
h. 3

ANS: B PTS: 1

8. Find the value of x at which the curve $y = \frac{x^2 - 16}{x^2 - 5x + 4}$ has a vertical asymptote.

a. –4
b. –3
c. –2
d. –1
e. 0
f. 1
g. 2
h. 3

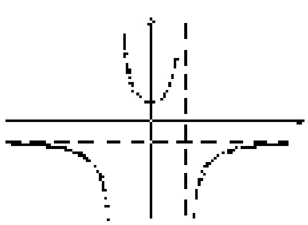
ANS: F PTS: 1

9. Given the following information about limits, select a graph which could be the graph of $y = f(x)$.

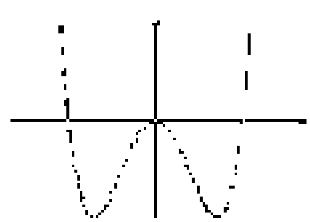
$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = -1$ $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x) = \infty$

$\lim_{x \to -1^-} f(x) = \lim_{x \to 1^+} f(x) = -\infty$

a.

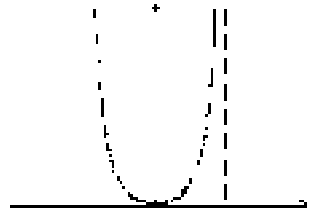


e.

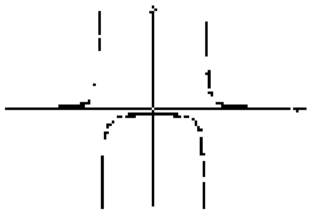


b.

f.

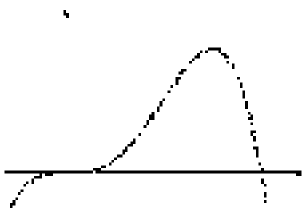


c.

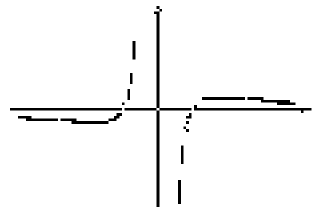


g.

d.



h.



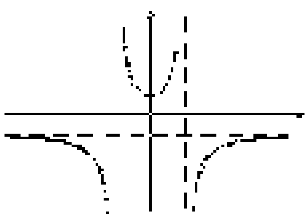
ANS: A PTS: 1

10. Given the following information about limits, select a graph which could be the graph of $y = f(x)$.

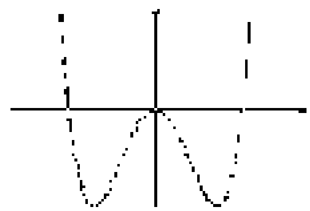
$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0 \qquad \lim_{x\to-1^-} f(x) = \lim_{x\to 1^+} f(x) = \infty$$

$$\lim_{x\to-1^+} f(x) = \lim_{x\to 1^-} f(x) = -\infty$$

a.

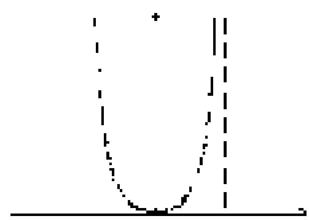


e.

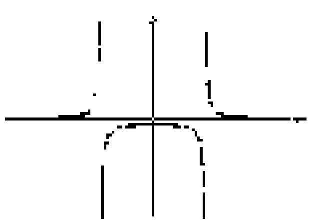


b.

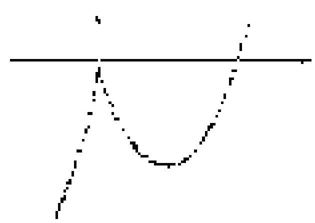
f.

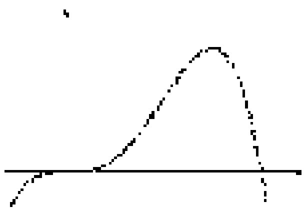


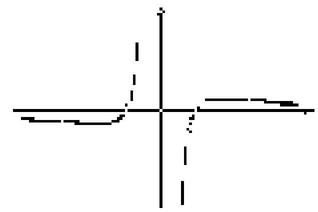
c.



g.



d. 

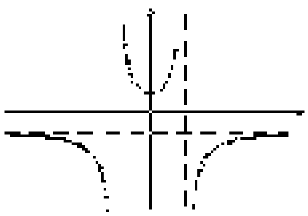
h. 

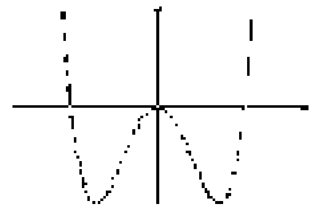
ANS: C PTS: 1

11. Given the following information about limits, select a graph which could be the graph of $y = f(x)$.

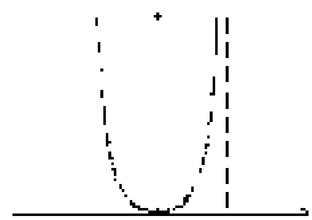
$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 0$ $\lim_{x \to -1^+} f(x) = \lim_{x \to 1^+} f(x) = \infty$

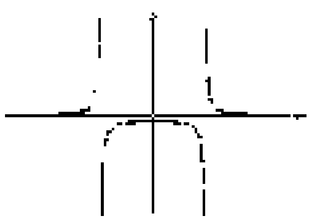
$\lim_{x \to -1^-} f(x) = \lim_{x \to 1^-} f(x) = -\infty$

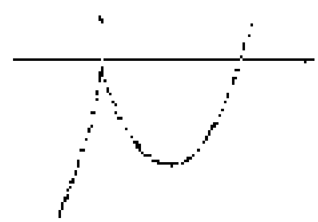
a. 

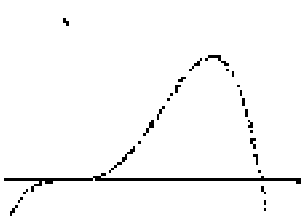
e. 

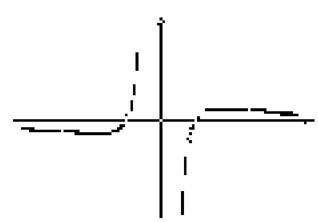
b. 

f. 

c. 

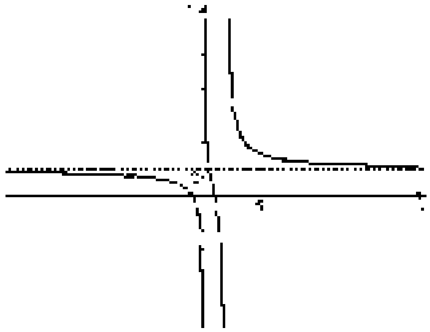
g. 

d. 

h. 

ANS: B PTS: 1

12. Using the graph below, determine the following:



(a) $\lim_{x \to \infty} f(x)$ (b) $\lim_{x \to 2^+} f(x)$ (c) $\lim_{x \to 2^-} f(x)$

(d) $\lim_{x \to -\infty} f(x)$ (e) $\lim_{x \to 0^+} f(x)$ (f) $\lim_{x \to 0^-} f(x)$

(g) Find the horizontal asymptote(s) of the graph of $y = f(x)$.

(h) Find the vertical asymptote(s) of the graph of $y = f(x)$.

ANS:
(a) 3 (b) ∞ (c) $-\infty$ (d) 2
(e) ∞ (f) $-\infty$ (g) $y = 3$ (h) $x = 0, x = 2$

PTS: 1

13. Suppose that $f(x) = \dfrac{x-3}{x^2-9}$ . . f is not defined for $x = \pm 3$. .

(a) For which of these two values does f have an infinite discontinuity? Explain.

(b) For which of these two values does f have a removable discontinuity? Explain.

ANS:
(a) Since $\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \dfrac{x-3}{x^2-9} = \lim_{x \to -3^-} = \dfrac{x-3}{(x-3)(x+3)} = \lim_{x \to -3^-} \dfrac{1}{x+3} = \infty$, f has an infinite discontinuity at $x = -3$.

(b) $\lim_{x \to -3} f(x) = \lim_{x \to 3} \dfrac{x-3}{x^2-9} = \lim_{x \to 3} = \dfrac{x-3}{(x-3)(x+3)} = \lim_{x \to 3} \dfrac{1}{x+3} = \dfrac{1}{6}$, f has a removable discontinuity at $x = 3$.

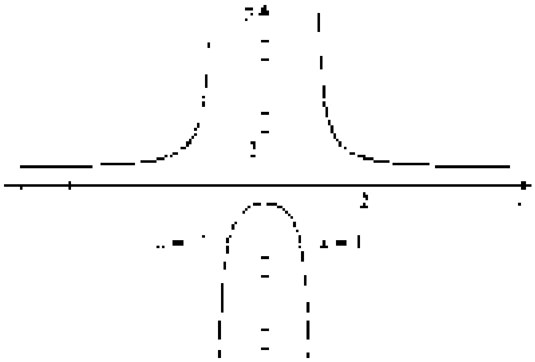
PTS: 1

14. Given the following information about limits, sketch a graph which could be the graph of $y = f(x)$. Label all horizontal and vertical asymptotes.

$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 1$ $\lim_{x \to -1^+} f(x) = \lim_{x \to 1^-} f(x) = -\infty$

$\lim_{x \to -1^-} f(x) = \lim_{x \to 1^+} f(x) = \infty$ $f = (0) = -1$

ANS:

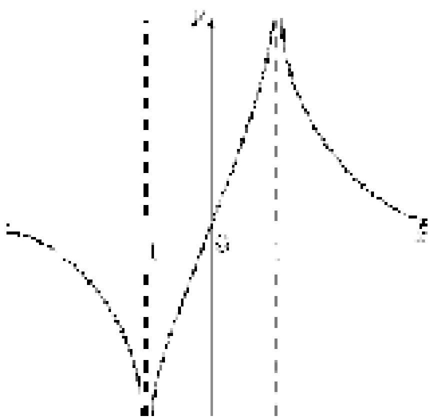


PTS: 1

15. Given the following information about limits, sketch a graph which could be the graph of $y = f(x)$. Label all horizontal and vertical asymptotes.

$$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 0, \quad \lim_{x \to 1} f(x) = \infty \ \lim_{x \to -1} f(x) = -\infty, \text{ and } f(0) = 0$$

ANS:



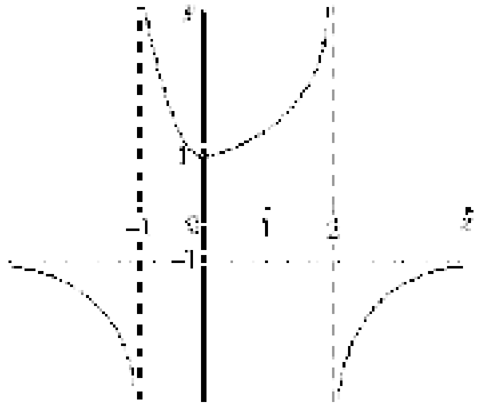
PTS: 1

16. Given the following information about limits, sketch a graph which could be the graph of $y = f(x)$. Label all horizontal and vertical asymptotes.

$$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = -1, \ \lim_{x \to 2^+} f(x) = \lim_{x \to -1^-} f(x) = -\infty$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to 2^-} f(x) = \infty, \text{ and } f = (0) = 1$$

ANS:



PTS: 1

17. Find the limit.

(a) $\lim\limits_{x \to 2^+} \dfrac{1}{x-2}$ (b) $\lim\limits_{x \to \infty} \dfrac{1-2x^3}{x^2+x^3}$ (c) $\lim\limits_{x \to \infty} \left(\sqrt{x^2+x} - x\right)$

ANS:

(a) ∞ (b) -2 (c) $\dfrac{1}{2}$

PTS: 1

18. Find the limit.

(a) $\lim\limits_{x \to \infty} \sin x$ (b) $\lim\limits_{x \to \infty} e^{-x}$ (c) $\lim\limits_{t \to \infty} \dfrac{t^3+3t}{1-t^4}$

ANS:

(a) Does not exist (b) 0 (c) 0

PTS: 1

19. Find the limit.

(a) $\lim\limits_{x \to \infty} \dfrac{1}{x^2}$ (b) $\lim\limits_{x \to \infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right)$ (c) $\lim\limits_{x \to -\infty} \left(x - 3x^2\right)$

ANS:

(a) 0 (b) 1 (c) $-\infty$

PTS: 1

20. Find the vertical asymptote(s) of the curve $y = \dfrac{x^2-4}{x^2-3x-10}$.

ANS:

$\lim_{x \to 5^+} \frac{x^2 - 4}{x^2 - 3x - 10} = \infty$ and $\lim_{x \to 5^-} \frac{x^2 - 4}{x^2 - 3x - 10} = -\infty$, so $x = 5$ is a vertical asymptote.

PTS: 1

Section 2.6: Tangents, Velocities, and Other Rates of Change

1. Find an equation of the line tangent to $f(x) = x^2 - 4x$ at the point (3, – 3).

a. $2x - y = 9$
b. $x - 2y = 9$
c. $y - 2x = 9$
d. $2y - x = 9$
e. $x - y = 9$
f. $y - x = 9$
g. $2x - y = 4$
h. $x - 2y = 4$

ANS: A PTS: 1

2. Find an equation of the line tangent to the curve $y = x + (1/x)$ at the point $\left(5, \frac{26}{25}\right)$.

a. $24x - 25y = 94$
b. $24x - 25y = -94$
c. $25x - 24y = 94$
d. $25x - 24y = -94$
e. $24x - 24y = 94$
f. $24x - 24y = -94$
g. $25x - 25y = 94$
h. $25x - 25y = -94$

ANS: B PTS: 1

The next questions refer to the following table:

Average Daily Temperatures in Moorhead, Minnesota						
Month	1 (Jan)	2 (Feb)	3 (Mar)	4 (Apr)	5 (May)	6 (Jun)
Temperature (°F)	6	14	26	43	57	66
Month	7 (Jul)	8 (Aug)	9 (Sep)	10 (Oct)	11 (Nov)	12 (Dec)
Temperature (°F)	71	70	58	46	27	13

3. During which of the following periods was the rate of change of the average daily temperature the greatest?

a. [2, 3]
b. [3, 4]
c. [4, 5]
d. [5, 6]
e. [6, 7]
f. [2, 4]
g. [3, 5]
h. [3, 6]

ANS: B PTS: 1

4. During which of the following periods was the rate of change of the average daily temperature the smallest?

a. [2, 6]
b. [4, 8]
c. [4, 9]
d. [6, 10]
e. [8, 10]
f. [9, 10]
g. [10, 11]
h. [11, 12]

ANS: G PTS: 1

5. The following table contains data from the historic flood of the Cedar River at Cedar Rapids, Iowa during June 2008. Readings were taken at 6:00 A.M. each day.

Day	Stage (ft)
08	10.46
09	13.79
10	17.54
11	19.62
12	25.88
13	31.03
14	29.23
15	24.93
16	21.95
17	20.13
18	18.85
19	16.25
20	13.63
21	12.12
22	10.94

Source: US Army Corps of Engineers - Rock Island District - Water Control Center

(a) Find the average daily rate of change of the river level over each of the following periods:

(i) [08, 13] (ii) [13, 22] (iii) [12, 13] (iv) [13, 14]

(b) Estimate the daily rate of change of the stage level on June 14th.

ANS:
(a)
(i) $\frac{31.03 - 10.46}{13 - 8} \approx 4.11$ ft/day (ii) $\frac{10.94 - 31.03}{22 - 13} \approx -2.23$ ft/day

(iii) $\frac{31.03 - 25.88}{13 - 12} \approx 5.15$ ft/day (iv) $\frac{29.23 - 31.03}{14 - 13} \approx -1.80$ ft/day

(b) Answers may vary. One possible response would be $\frac{24.93 - 31.03}{51 - 13} \approx -3.05$ ft/day

PTS: 1

6. The following table shows the relationship between pressure (in atmospheres) and volume (in liters) of hydrogen gas at 0° C.

Pressure (atm)	1	2	3	4	5	6
Volume (L)	22.4	11.2	7.5	5.6	4.5	3.7

(a) Find the average rate of change of volume with respect to pressure for the following pressure intervals:

(i) [3, 5] (ii) [3, 4] (iii) [2, 3]

(b) Estimate the rate of change of volume with respect to pressure if the pressure is 2 atmospheres.

ANS:
(a)
(i) $\frac{4.5-7.5}{2} = -1.5\frac{\text{liters}}{\text{atm}}$ (ii) $\frac{5.6-7.5}{1} = -1.9\frac{\text{liters}}{\text{atm}}$ (iii) $\frac{11.2-7.5}{-1} = -3.7\frac{\text{liters}}{\text{atm}}$

(b) About –2.8 liters=atm (answers may vary).

PTS: 1

7. Below are the sunrise and sunset times (in standard time) for Moorhead, Minnesota on the 21st of each month in 2007.

Sunrise and Sunset in Moorhead, Minnesota on the 21st of each month in 2007				
Date	**Day of Year**	**Sunrise**	**Sunset**	**Day Length (h)**
Jan 21	21	08:03	17:14	9.18
Feb 21	52	07:21	18:01	10.67
Mar 21	80	06:28	18:41	12.22
Apr 21	111	05:29	19:24	13.92
May 21	141	04:45	20:02	15.28
Jun 21	172	04:32	20:25	15.88
Jul 21	202	04:54	20:12	15.30
Aug 21	233	05:32	19:27	13.92
Sep 21	264	06:12	18:27	12.25
Oct 21	294	06:53	17:29	10.60
Nov 21	325	07:39	16:47	9.13
Dec 21	355	08:09	16:41	8.53

(a) Find the rate of change in day length with respect to time (in hours per day)
(i) from March 21 through July 21.
(ii) from March 21 through May 21.
(iii) from March 21 through April 21.

(b) Estimate the instantaneous rate of change in day length per day (in hours per day) for March 21.

(c) During which one-month period did the day length change the greatest amount?

(d) During which one month period did the average change in day length change the greatest amount? Is this the same time period as in part (c)? Explain.

ANS:
(a) (i) 0.025 hr/day (ii) 0.050 hr/day (iii) 0.055 hr/day

(b) Answers will vary. Using February through April, we get approximately 0;055 hr/day.

(c) The one month period with the greatest change in day length was March 21 to April 21, 1.7 hours in one month.

(d) The period from February 21 to March 21 had an average daily rate of change of 0.0553 hr/day. This is not the same time period we found in part (c). The reason for this is that while from February 21 to March 21 there was a gain of 1.49 hours over the period, slightly less than the 1.55 hours for the March 21 to April 21 period, it was over a period of only 28 days compared to 31 days for the March to April interval.

PTS: 1

8. The following table shows the concentration (in mol/l) of a certain chemical in terms of reaction time (in hours) during a decomposition reaction.

Time (hours)	0	5	10	20	30	50
Concentration (mol/L)	2.32	1.86	1.49	0.98	0.62	0.25

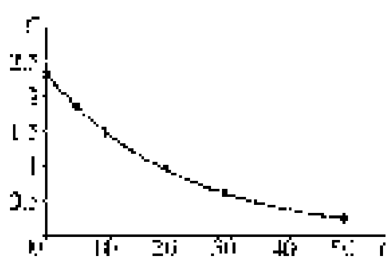
(a) Find the average rate of change of concentration with respect to time for the following time intervals:
(i) [0; 5] (ii) [10; 20] (iii) [30; 50]

(b) Plot the points from the table and fit an appropriate exponential model to the data.

(c) From your model in part (b), determine the instantaneous rate of change of concentration with respect to time.

(d) Is the rate of change of concentration increasing or decreasing with respect to time? Justify your answer.

ANS:
(a)
(i) –0.092 mol/L/h
(ii) –0.051 mol/L/h
(iii) –0.0185 mol/L/h

(b)


(c) $C(t) = 2.3357e^{-0.0445t}$ mol/L, $\frac{dC}{dt} = -0.1039e^{-0.0445t}$ mol/L/h

(d) Since $\frac{dC}{dt} < 0$, the rate of change of concentration is decreasing with respect to time.

PTS: 1

9. (a) Find the slope of the tangent line to the curve $y = x - \frac{1}{x}$ at $(1, 0)$.

(i) using Definition 1 from Section 2.6.
(ii) using Definition 2 from Section 2.6.

(b) Using your results from part (a), find an equation of the tangent line at (1, 0).

ANS:

(a) (i) $m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h) - \frac{1}{1+h} - 0}{h} = \lim_{h \to 0} \frac{\left[(1+h) - \frac{1}{1+h}\right](1+h)}{h(1+h)}$

$= \lim_{h \to 0} \frac{(1+h)^2 - 1}{h(1+h)} = \lim_{h \to 0} \frac{(1+2h+h^2) - 1}{h(1+h)} = \lim_{h \to 0} \frac{h(2+h)}{h(1+h)} = 2$

(ii) $m = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{x - \frac{1}{x} - 0}{x - 1} = \lim_{x \to 1} \frac{\left(x - \frac{1}{x}\right)x}{(x-1)x} = \lim_{x \to 1} \frac{x^2 - 1}{(x-1)x}$

$= \lim_{x \to 1} \frac{(x-1)(x+1)}{(x-1)x} = 2$

(b) $m = 2$; $y - 0 = 2(x - 1)$ or $y = 2x - 2$

PTS: 1

Section 2.7 Derivatives

1. If $f(x) = \sqrt{x}$, which of the following represents $f'(4)$?

a. $\lim_{x \to 4} \frac{\sqrt{x} - 4}{x - 4}$

b. $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

c. $\lim_{x \to 4} \frac{\sqrt{x} - 4}{x - 2}$

d. $\frac{\sqrt{x} - 2}{x - 4}$

e. $\lim_{x \to 4} \frac{\sqrt{x} - 4}{x}$

f. $\lim_{x \to 4} \frac{\sqrt{x} + 2}{x - 4}$

g. Does not exist

h. None of the above

ANS: B PTS: 1

2. If $f(x) = x^2 - 2x$, which of the following represents $f'(1)$?

a. $\lim_{h \to 0} \frac{(1+h)^2 - 2(1+h) - 1}{h}$

b. $\lim_{h \to 0} \frac{(1+h)^2 - 2 + h - 1}{h}$

c. $\lim_{h \to 0} \frac{(1+h)^2 - 2(1+h) + 1}{h}$

d. $\frac{(1+h)^2 - 2(1+h) - 1}{h}$

e. $\lim_{h \to 0} \frac{h^2 + 1 - 2(1+h) + 1}{h}$

f. $\lim_{h \to 0} \frac{(1+h)^2 - 2 + 2h + 1}{h}$

g. Does not exist

h. None of the above

ANS: C PTS: 1

3. If $f(x) = \frac{x+1}{2x-1}$, which of the following represents $f'(0)$?

a. $\lim_{h \to 0} \frac{\frac{h+1}{h-1} + 1}{h}$

b. $\lim_{h \to 0} \frac{\frac{h+1}{2h-1} - 1}{h}$

c. $\lim_{h \to 0} \frac{\frac{h+1}{2h-1} + 1}{h}$

d. $\lim_{h \to 0} \frac{\frac{h+1}{2h-1} + 1}{h}$

e. $\frac{\frac{h+1}{h-1} + 1}{h}$

f. $\lim_{h \to 0} \frac{\frac{h}{2h} + 1}{h}$

g. Does not exist

h. None of the above

ANS: D PTS: 1

4. If $f(x) = 5$, which of the following represents $f'(3)$?

a. $\lim_{\Delta x \to 0} \frac{5}{\Delta x}$

e. $\lim_{\Delta x \to 0} \frac{5 - 5}{\Delta x}$

b. 3

c. $\lim_{\Delta x \to 0} \frac{5-3}{\Delta x}$

d. 5

f. $\lim_{\Delta x \to 0} \frac{3}{\Delta x}$

g. Does not exist

h. None of the above

ANS: E PTS: 1

5. If $f(1) = 5$ and $f'(1) = -3$, find an equation of the tangent line at $x = 1$.

a. $y = -3x + 5$

b. $y = 5x - 3$

c. $y = -3x + 8$

d. $y = -3x - 8$

e. $y = 3x + 8$

f. $y = 3x - 8$

g. Does not exist

h. None of the above

ANS: C PTS: 1

6. If $f(-1) = 3$ and $f'(-1) = 2$, find an equation of the tangent line at $x = -1$.

a. $y = -5 + 2x$

b. $y = 1 + 2x$

c. $y = -2x + 1$

d. $y = 5 - 2x$

e. $y = -1 + 2x$

f. $y = 5 + 2x$

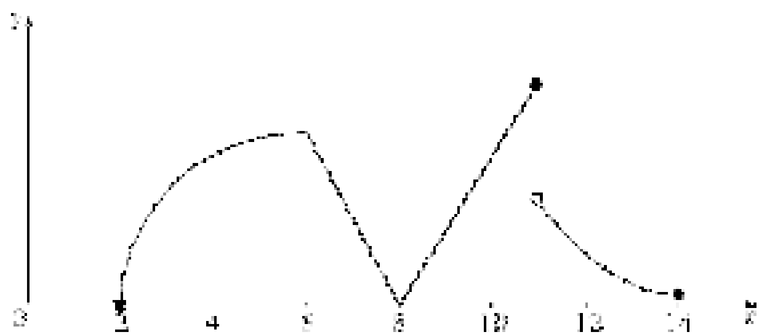
g. Does not exist

h. None of the above

ANS: F PTS: 1

7. The graph of f is given below. State, with reasons, the number(s) at which

(a) f is not differentiable. (b) f is not continuous.



ANS:

(a) f is not differentiable at $x = 6$ or at $x = 8$, because the graph has a corner there; and at $x = 11$, because there is a discontinuity there.

(b) f is not continuous at $x = 11$ because $\lim_{x \to 11} f(x)$ does not exist.

PTS: 1

8. Consider the function $f(x) = |x - 2|$.

(a) What is the domain of f?

(b) At what number(s), if any, is f discontinuous? Explain your answer.

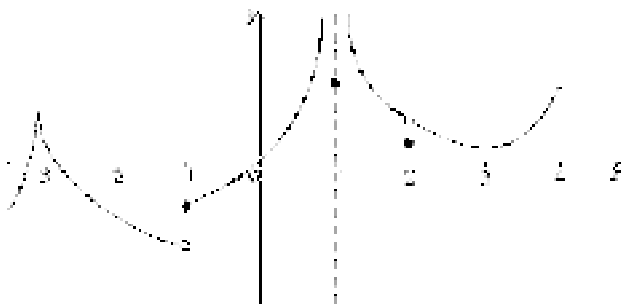
(c) At what number(s), if any, is f not differentiable? Explain your answer.

ANS:
(a) Domain is $(-\infty, \infty)$.
(b) f is continuous on $(-\infty, \infty)$.
(c) f is not differentiable at 2 because the graph of f has a corner there.

PTS: 1

9. The graph of f is given below. State, with reasons, the number(s) at which

(a) f is not differentiable. (b) f is not continuous.

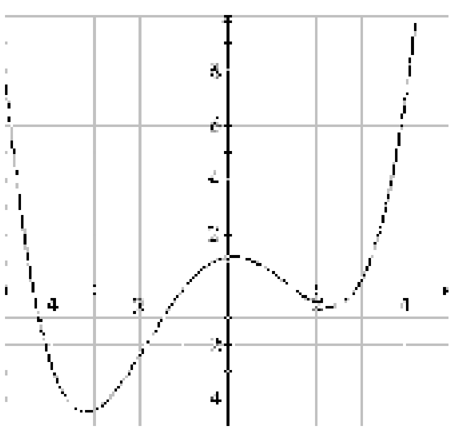


ANS:
(a) f is not differentiable at $x = -3$ or at $x = 4$, because the graph has a corner there; at $x = -1$ or $x = 1$, because the limit does not exist there; and at $x = 2$, because $\lim_{x \to 2} f(x) \neq f(2)$.
(b) f is not continuous at $x = -1$ or $x = 1$ because the limit does not exist there; and at $x = 2$, because $\lim_{x \to 2} f(x) \neq f(2)$.

PTS: 1

10. For the function f whose graph is given, arrange the following values in increasing order and explain your reasoning.

$f'(-4), f'(-3), f'(-1), f'(0), f'(1), f'(2), f'(4)$



ANS:
$f'(-4) < f'(1) < f'(2) < f'(0) < f'(-3) < f'(-1) < f'(4)$ by inspection of slopes.

PTS: 1

11. Determine a function f and a number a where the given limit represents the derivative of f at a.

(a) $\lim\limits_{h \to 0} \dfrac{\sqrt{(3+h)^2 - 5} - 2}{h}$

(b) $\lim\limits_{h \to 0} \dfrac{(6+h)^2 - 5(6+h) - 11 - 55}{h}$

(c) $\lim\limits_{x \to 0} \dfrac{3e^x - 3}{x}$

(d) $\lim\limits_{h \to -4} \dfrac{x^3 + 64}{x + 4}$

ANS:

(a) $f(x) = \sqrt{x^2 - 5}, a = 3$

(b) $f(x) = x^2 - 5x - 11,\ a = 6$

(c) $f(x) = 3e^x,\ a = 0$

(d) $f(x) = x^3,\ a = 4$

PTS: 1

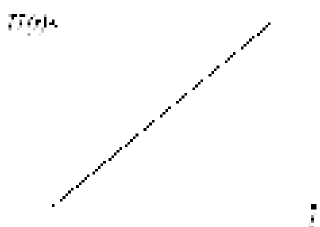
12. Water is flowing into a large cylindrical tank at a constant rate. Let $H(t)$ represent the height of the water level at time t.

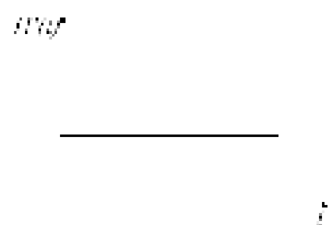
(a) Sketch a possible graph of $H(t)$.

(b) Describe how the rate of change of H with respect to t varies as t increases.

(c) Sketch a graph of $H'(t)$.

ANS:

(a)


(c)


(b) $H'(t)$ is constant.

PTS: 1

13. A tank is in the shape of a large, inverted (point-down) cone. Water is owing into the tank at a constant rate. Let $H(t)$ represent the height of the water level at time t.

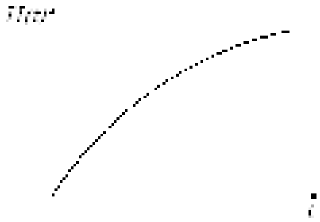
(a) Sketch a possible graph of $H(t)$.

(b) Describe how the rate of change of H with respect to t varies as t increases.

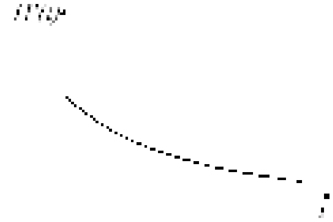
(c) Sketch a graph of $H'(t)$.

ANS:

(a)



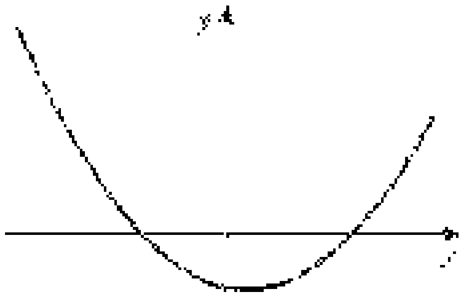
(c)



(b) $H'(t)$ is always positive and decreasing.

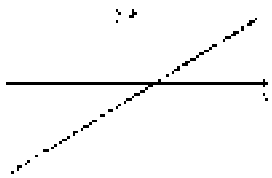
PTS: 1

1. Given the graph of $y = f(x)$ below, select a graph which best represents the graph of $y = f'(x)$.



a.

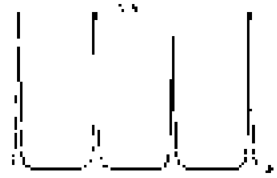
b.



c.

d.

e.



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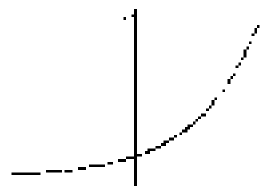
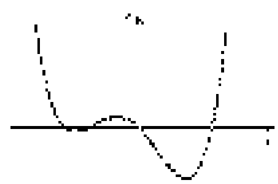


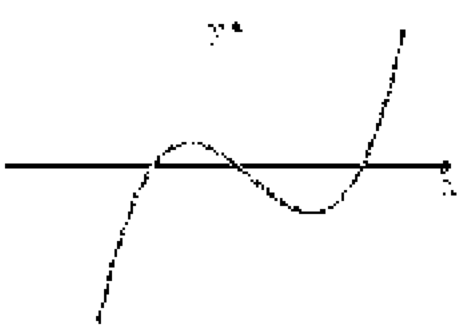
g.

h.



ANS: B PTS: 1

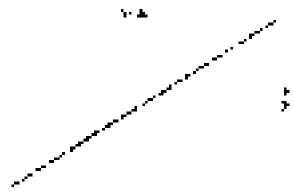
2. Given the graph of $y = f(x)$ below, select a graph which best represents the graph of $y = f'(x)$.



a.

e.

b.



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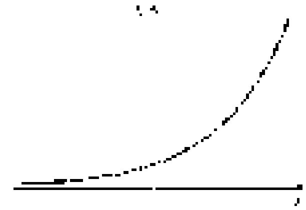

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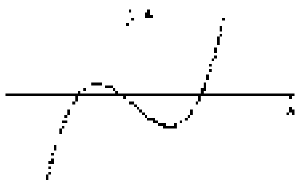

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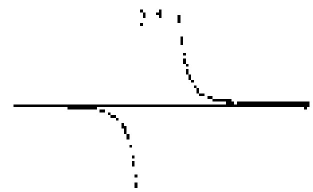

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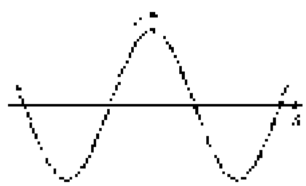

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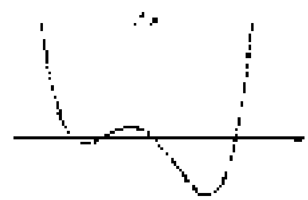
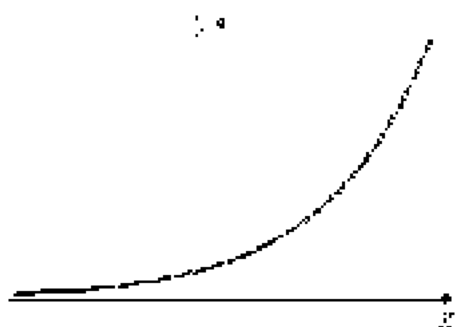


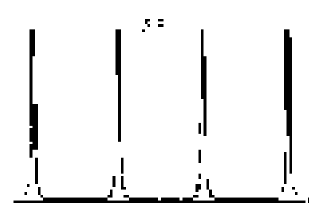
ANS: A PTS: 1

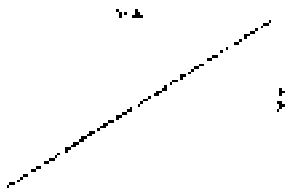
3. Given the graph of $y = f(x)$ below, select a graph which best represents the graph of $y = f'(x)$.

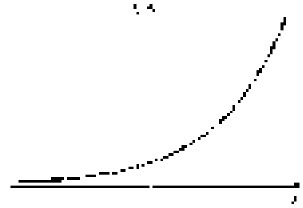


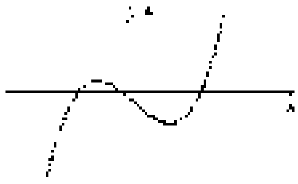
a.

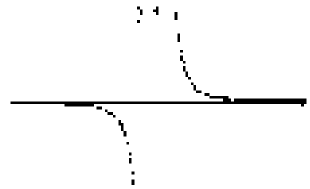
e.

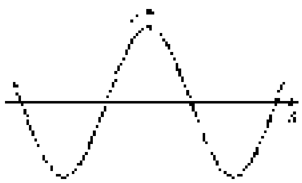


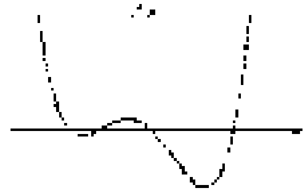
b. 

f. 

c. 

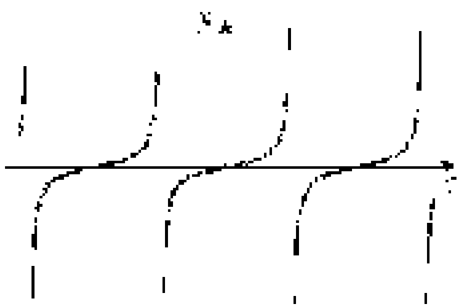
g. 

d. 

h. 

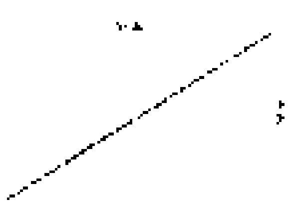
ANS: F PTS: 1

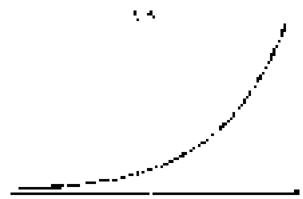
4. Given the graph of $y = f(x)$ below, select a graph which best represents the graph of $y = f'(x)$.



a. 

e. 

b. 

f. 

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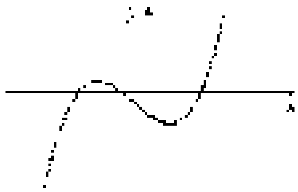
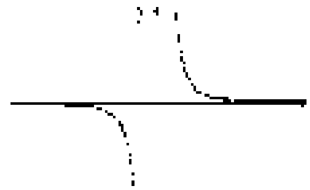


g.



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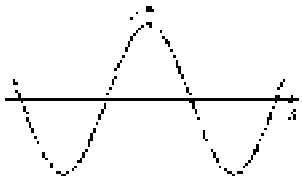

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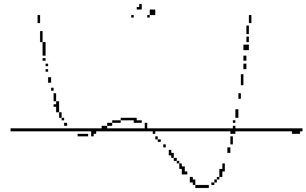
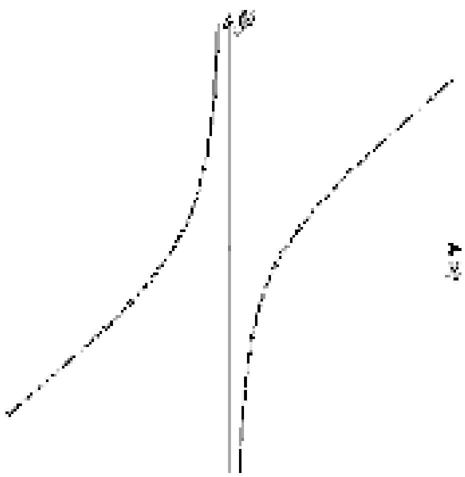
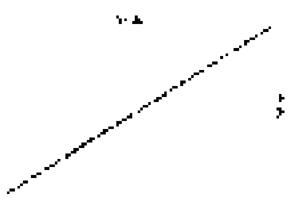


ANS: E PTS: 1

5. Given the graph of $y = f(x)$ below, select a graph which best represents the graph of $y = f'(x)$.



a.



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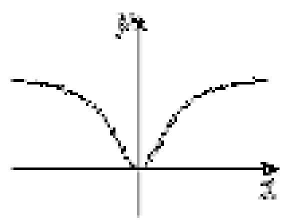

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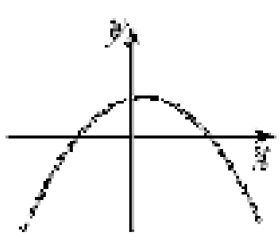

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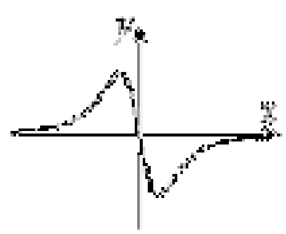

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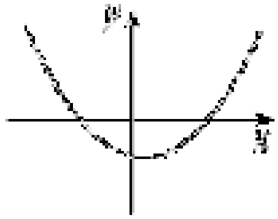

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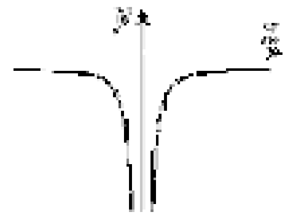

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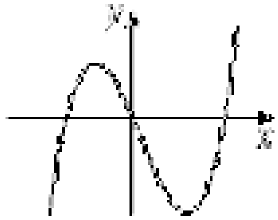

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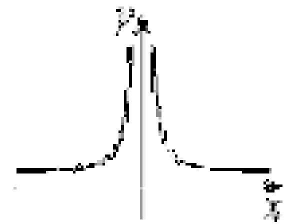
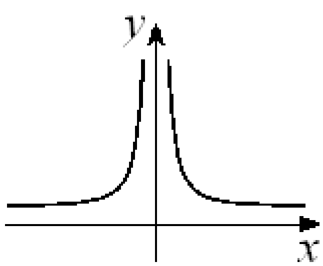


ANS: H PTS: 1

6. Given the graph of $y = f(x)$ below, select a graph which best represents the graph of $y = f'(x)$.



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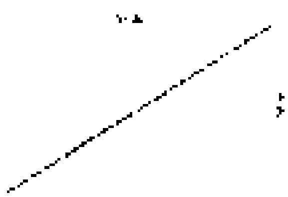

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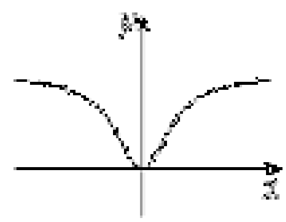

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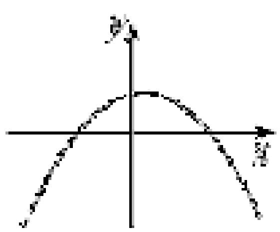

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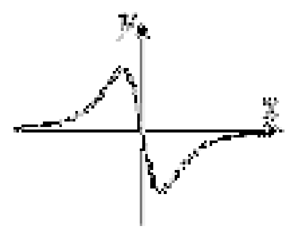

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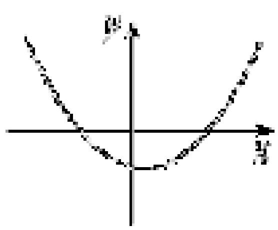

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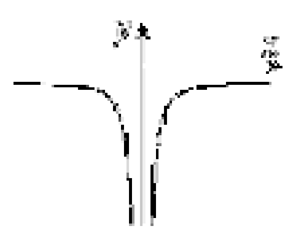

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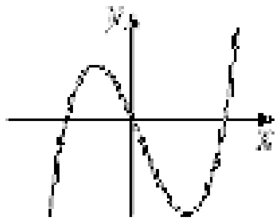

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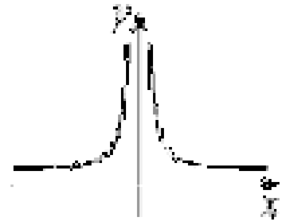
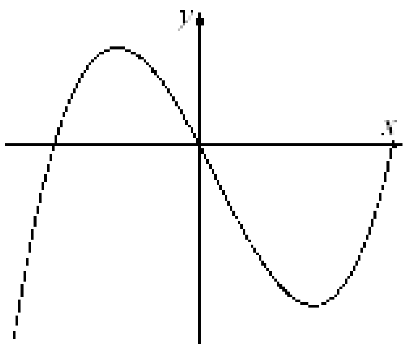
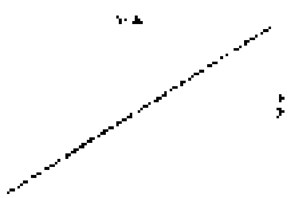


ANS: F PTS: 1

7. Given the graph of $y = f(x)$ below, select a graph which best represents the graph of $y = f'(x)$.



a.



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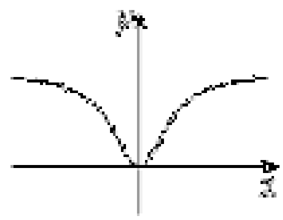

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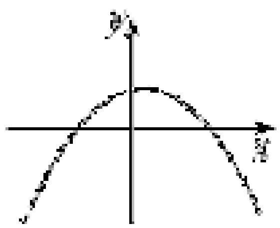

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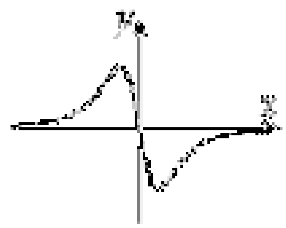
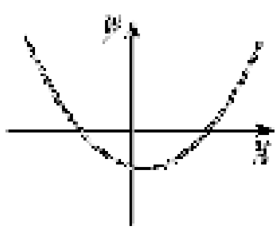
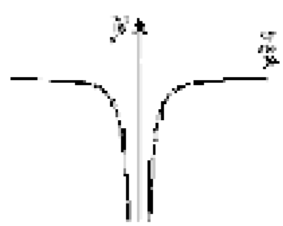


c.



g.



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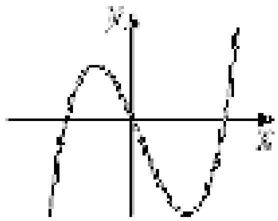

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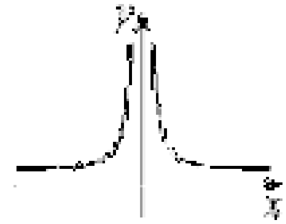
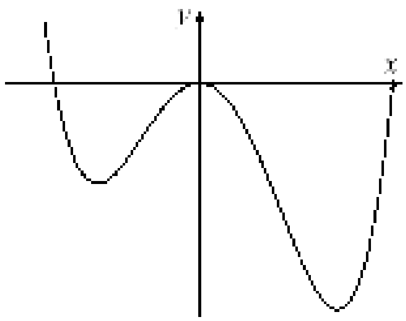
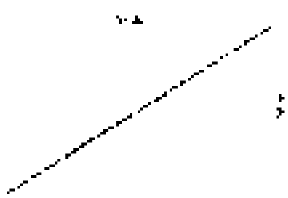


ANS: C PTS: 1

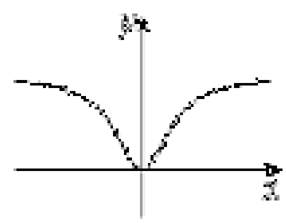
8. Given the graph of $y = f(x)$ below, select a graph which best represents the graph of $y = f'(x)$.



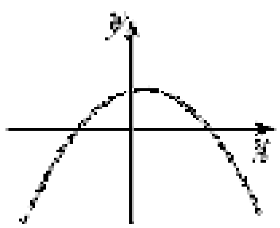
a.



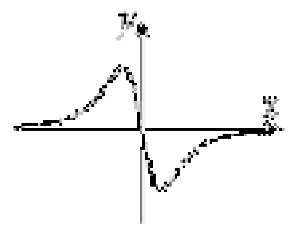
e.



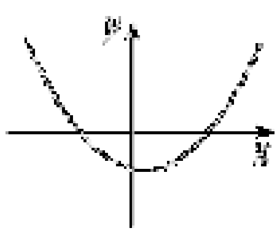
b.



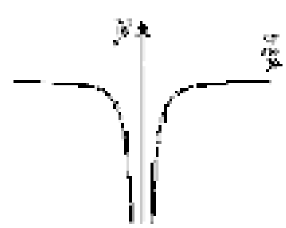
f.



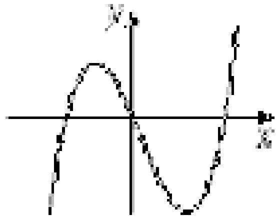
c.



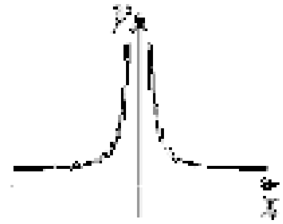
g.



d.



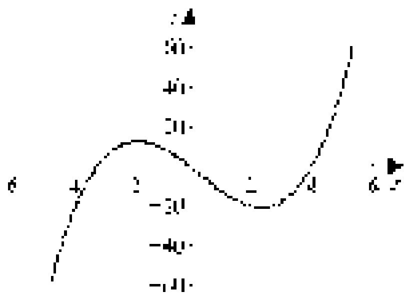
h.



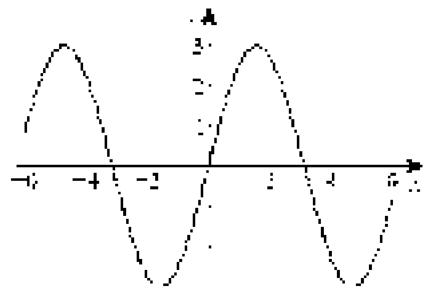
ANS: D PTS: 1

9. Given the graph of $y = f(x)$, sketch the graph of $y = f'(x)$.

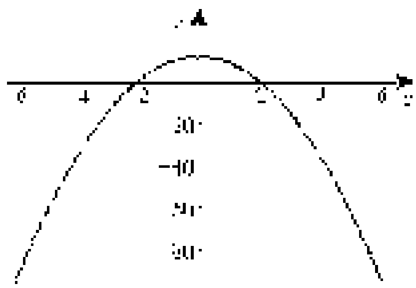
(a)



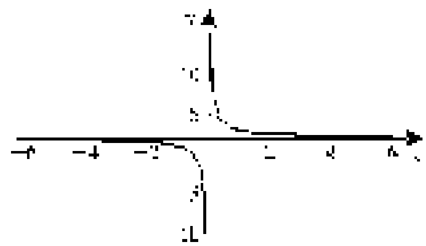
(b)



(c)



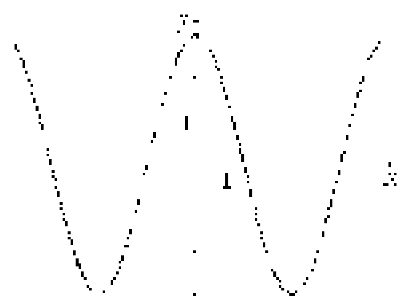
(d)



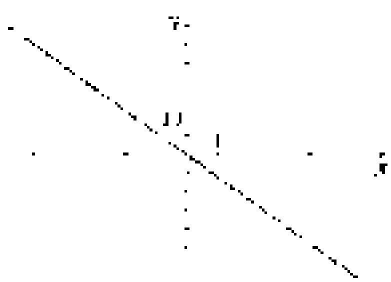
ANS:

(a)

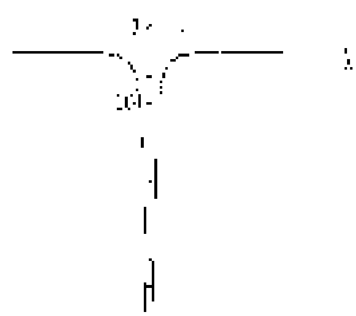
(b)



(c)



(d)



PTS: 1

10. In order to determine an appropriate delivery schedule to a group of rural homes in North Dakota, a fuel oil distributor monitors fuel oil consumption and the daily outdoor temperature (in degrees Fahrenheit). A table was constructed for a function $F(T)$ of fuel oil consumption (in gallons per day) as a function of the temperature T.

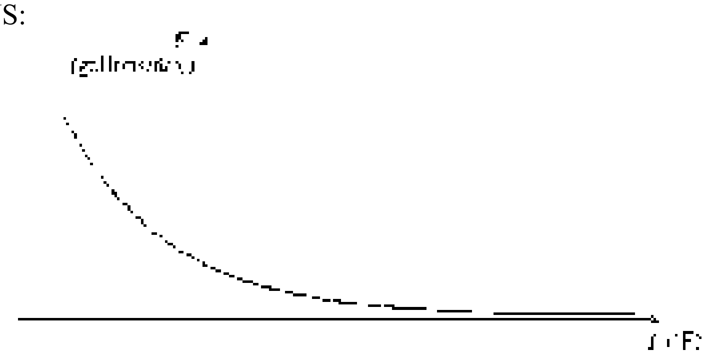
(a) Sketch a graph which you believe would approximate the graph of $y = F(T)$.

(b) What is the meaning of $F'(T)$? What are its units?

(c) Write a sentence that would explain to an intelligent layperson the meaning of $F'(0) = -0.4$.

ANS:

(a)



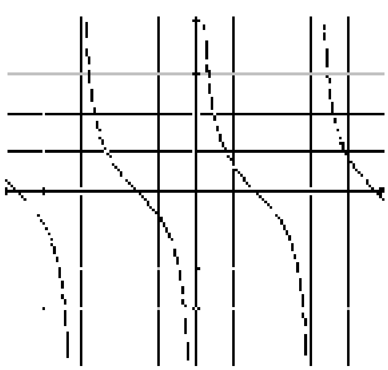
(There are many possible graphs, though presumably they are all decreasing!)

(b) $F'(T)$ represents the rate of change of fuel oil consumption with respect to temperature. Its units are (gallons/day) / (°F).

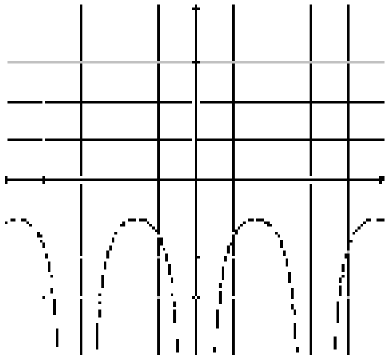
(c) $F'(0) = -0.4$ means that as the temperature increases past 0° F, the fuel consumption is decreasing by 0.4 (gallons/day) / (F).

PTS: 1

11. Given the graph of $y = f(x)$ below, sketch the graph of $y = f'(x)$.

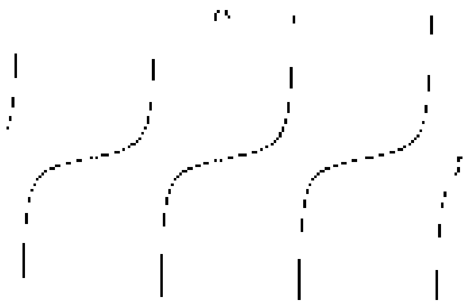


ANS:

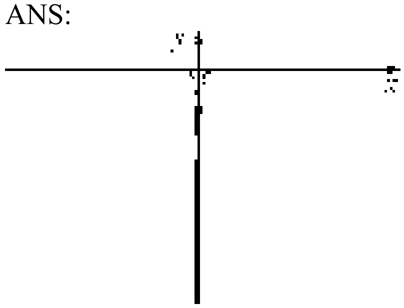


PTS: 1

12. Given the graph of $y = f(x)$ below, sketch the graph of $y = f'(x)$.

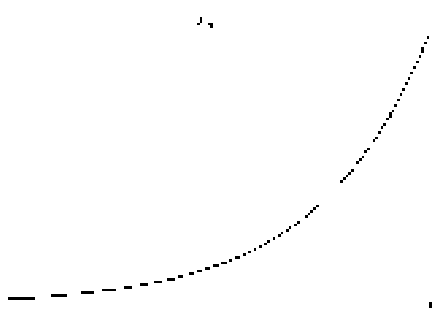


ANS:

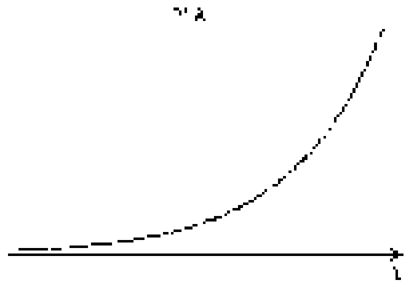


PTS: 1

13. Given the graph of $y = f(x)$ below, sketch the graph of $y = f'(x)$.

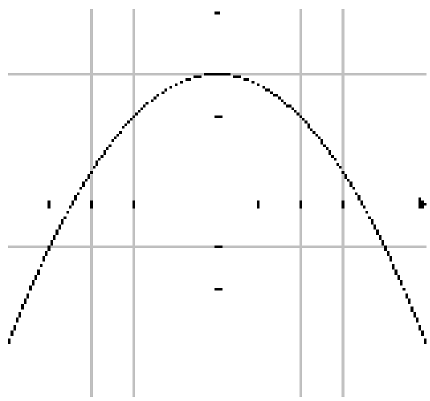


ANS:

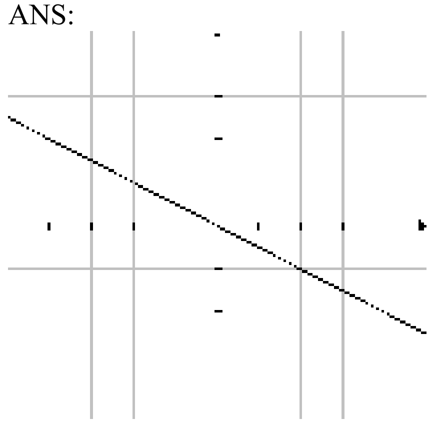


PTS: 1

14. Given the graph of $y = f(x)$ below, sketch the graph of $y = f'(x)$.

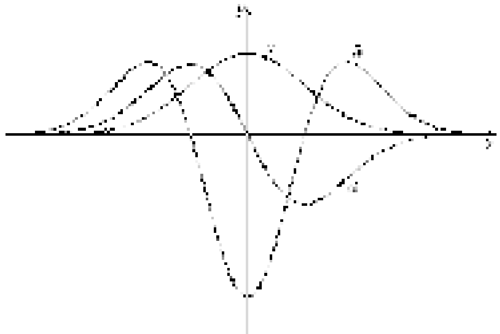


ANS:



PTS: 1

15. Below are the graphs of a function and its first and second derivatives. Identify which of the following graphs (a, b, and c) is $f(x)$, which is $f'(x)$, and which is $f''(x)$. Justify your choices.

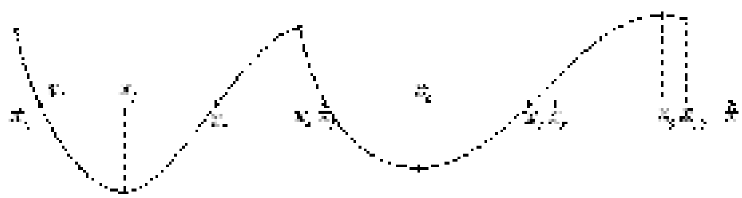


ANS:
a must be the derivative of *c* since it is above the *x*-axis where *c* increases, below the *x*-axis where *c* decreases, and is 0 where *c* has a horizontal tangent line. Similarly, *b* must be the derivative of graph *a*. So $f(x) = c, f'(x) = a,$ and $f''(x) = b$.

PTS: 1

16. Given the graph of $y = f(x)$, find all values of x for which

(a) $f'(x) > 0$ (b) $f'(x) < 0$ (c) $f'(x) = 0$ (d) $f''(x) > 0$



ANS:
(a) $f'(x) > 0$ on (x_2, x_4) and (x_6, x_9).

(b) $f'(x) < 0$ on (x_0, x_2) and (x_4, x_6), and (x_9, x_{10}).

(c) $f'(x) = 0$ at x_2, x_6 and x_9.

(d) $f''(x) > 0$ on (x_0, x_3) and (x_4, x_8).

PTS: 1

17. Find the derivative of $f(x) = 3x - 5$ using the definition of the derivative.

ANS:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h) - 5 - (3x - 5)}{h} = \lim_{h \to 0} \frac{3x + 3h - 5 - 3x + 5}{h}$$

$$= \lim_{h \to 0} \frac{3h}{h} = 3$$

PTS: 1

18. Find the derivative of $f(x) = \sqrt{4-x}$ using the definition of the derivative.

ANS:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{4-(x+h)} - \sqrt{4-x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{4-(x+h)} - \sqrt{4-x})}{h} \frac{(\sqrt{4-(x+h)} - \sqrt{4-x})}{(\sqrt{4-(x+h)} + \sqrt{4-x})} = \lim_{h \to 0} \frac{4-(x+h)-(4-x)}{h(\sqrt{4-(x+h)} + \sqrt{4-x})}$$

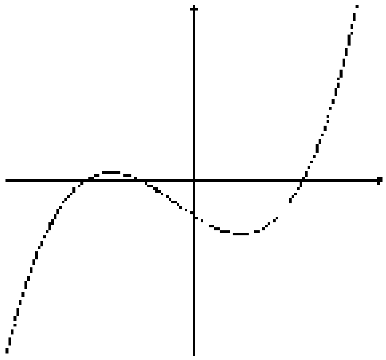
$$= \lim_{h \to 0} \frac{4-x-h-4+x}{(\sqrt{4-(x+h)} + \sqrt{4-x})} = \frac{-h}{(\sqrt{4-(x+h)} + \sqrt{4-x})}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{4-(x+h)} + \sqrt{4-x}} = \frac{-1}{2\sqrt{4-x}}$$

PTS: 1

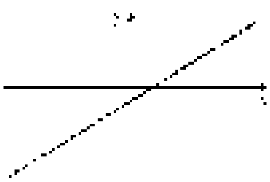
Section 2.9: What Does f′ Say About f?

1. Given the graph of $y = f'(x)$ below, select a graph which could be that of $y = f(x)$.

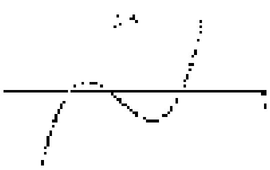


a.

b.

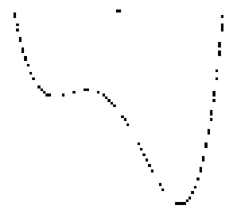


c.

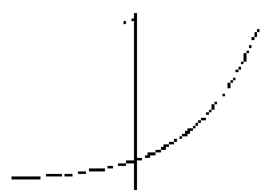


d.

e.

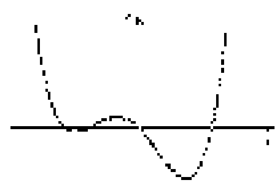


f.



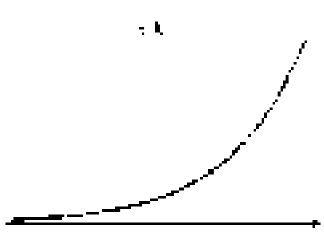
g.

h.



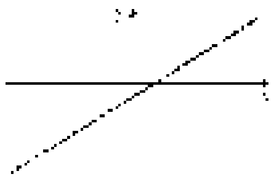
ANS: E PTS: 1

2. Given the graph of $y = f'(x)$ below, select a graph which could be that of $y = f(x)$.



a.

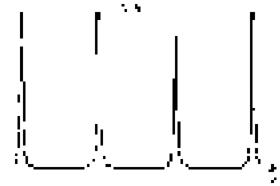
b.



c.

d.

e.



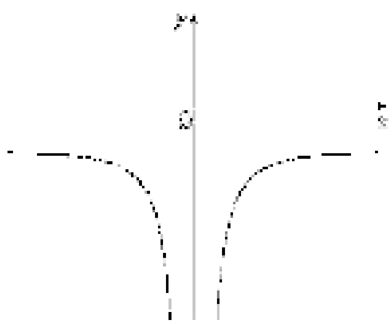
f.

g.

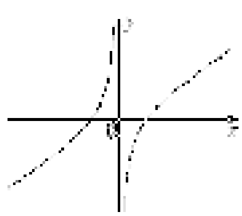
h.

ANS: F PTS: 1

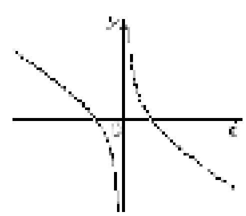
3. Given the graph of $y = f'(x)$ below, select a graph which could be that of $y = f(x)$.



a.



e.



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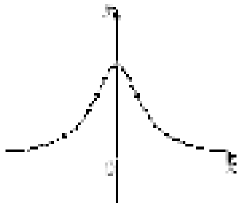
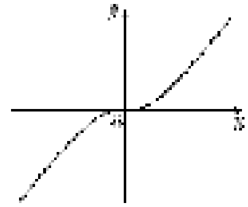
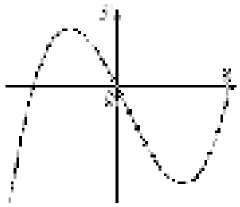


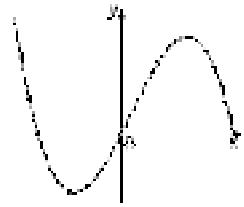
f.



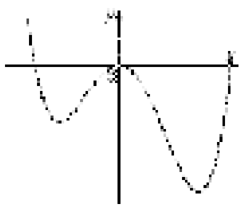
c.



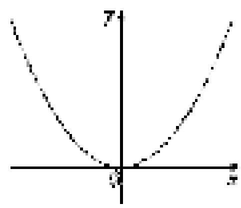
g.



d.

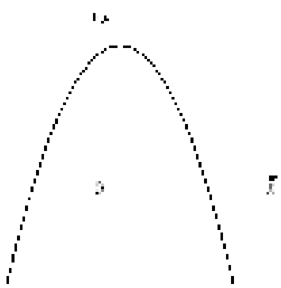


h.

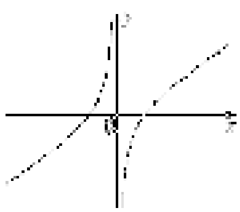


ANS: E PTS: 1

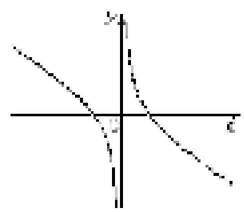
4. Given the graph of $y = f'(x)$ below, select a graph which could be that of $y = f(x)$.



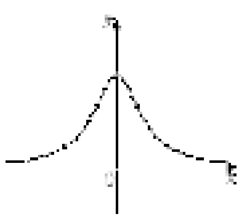
a.



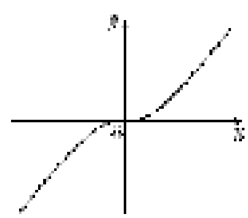
e.



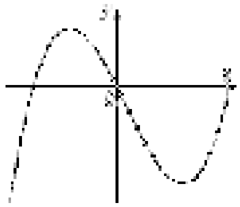
b.



f.



c.



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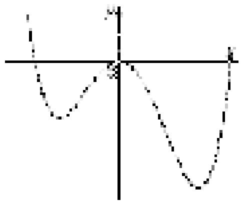
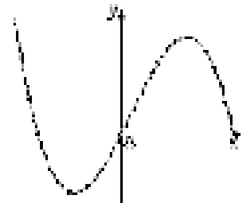
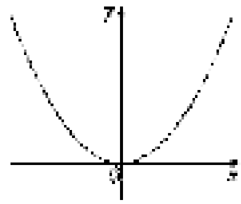


g.

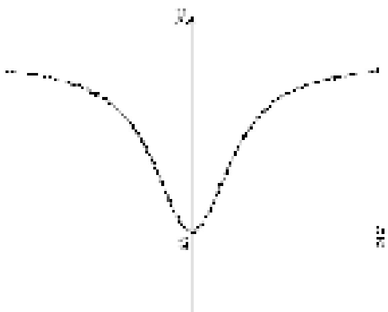


h.

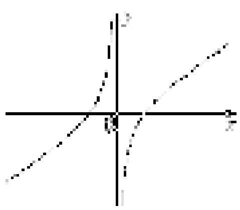


ANS: G PTS: 1

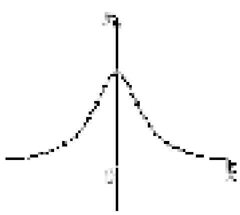
5. Given the graph of $y = f'(x)$ below, select a graph which could be that of $y = f(x)$.



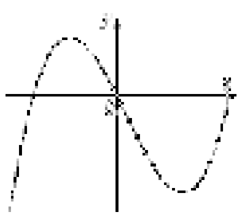
a.



b.



c.



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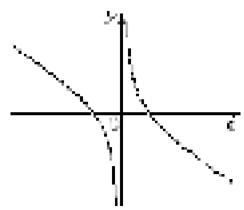
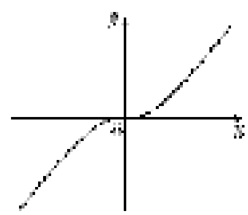
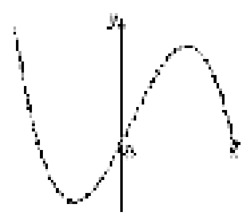


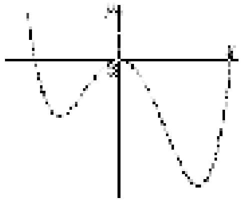
f.



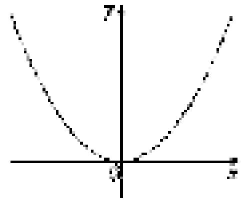
g.



d.

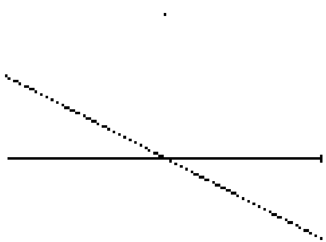


h.

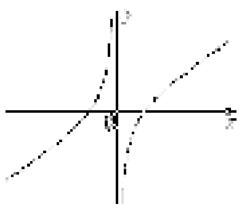


ANS: F PTS: 1

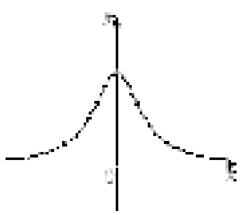
6. Given the graph of $y = f'(x)$ below, select a graph which could be that of $y = f(x)$.



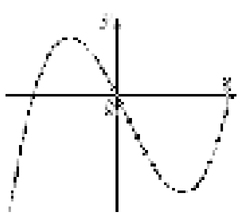
a.



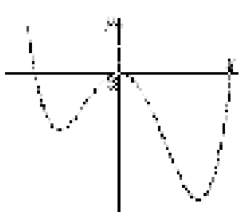
b.



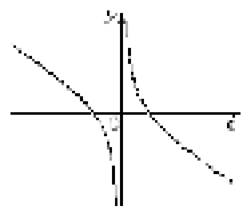
c.



d.



e.



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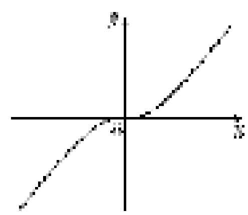
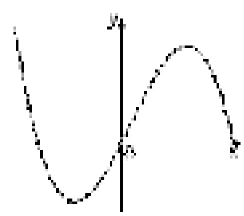
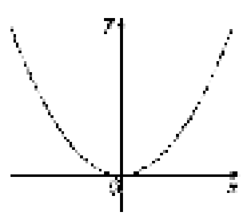


g.

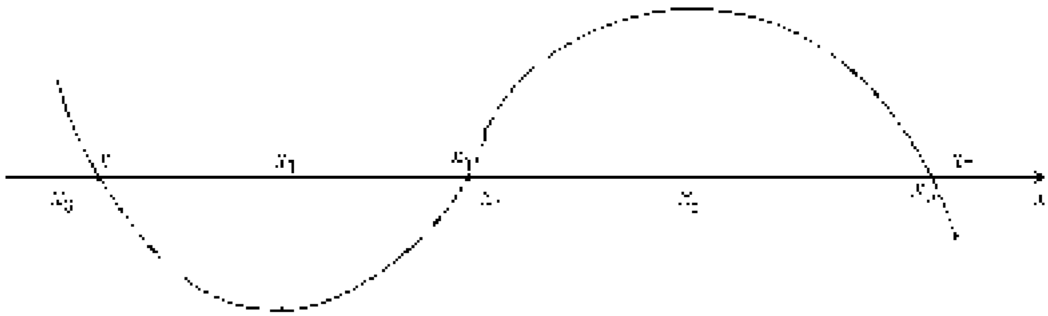


h.



ANS: H PTS: 1

7. Given the graph of $y = f(x)$, answer the following questions.



Find all values of x at which

(a) $f'(x) > 0$.
(b) $f'(x) < 0$.
(c) $f'(x) = 0$.
(d) f is increasing.
(e) f is decreasing.
(f) $f''(x) > 0$.
(g) $f''(x) < 0$.
(h) f has an inflection point.
(i) f' is increasing.
(j) f' is decreasing.

ANS:
(a) $f'(x) > 0$ on (x_2, x_5).

(b) $f'(x) < 0$ on (x_0, x_2) and (x_5, x_7).

(c) $f'(x) = 0$ at x_2 and $x = x_5$.

(d) f is increasing on (x_2, x_5).

(e) f is decreasing on (x_0, x_2) and (x_5, x_7).

(f) $f''(x) > 0$ on (x_0, x_4).

(g) $f''(x) < 0$ on (x_4, x_7).

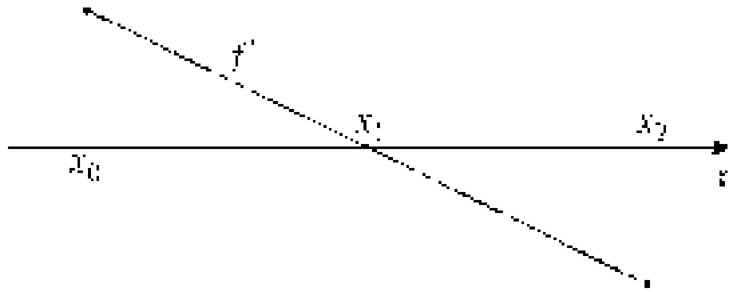
(h) f has an inflection point at $x = x_4$.

(i) f' is increasing on (x_0, x_4).

(j) f' is decreasing on (x_4, x_7).

PTS: 1

8. Given the graph of $y = f'(x)$, answer the questions that follow.



(a) Find all values of x at which

(i) f is increasing.

(ii) f is decreasing.

(iii) $f''(x) > 0$.

(iv) $f''(x) < 0$.

(v) $f''(x) = 0$.

(vi) f' is increasing.

(vii) f' is decreasing.

(viii) f has a local maximum.

(ix) f has a local minimum.

(b) Sketch a graph which could represent $y = f(x)$.

ANS:
(a)

(i) f is increasing on $[x_0, x_1]$.

(ii) f is decreasing on $[x_1, x_2]$.

(iii) $f''(x) > 0$ nowhere.

(iv) $f''(x) < 0$ on (x_0, x_2).

(v) $f''(x) = 0$ nowhere.

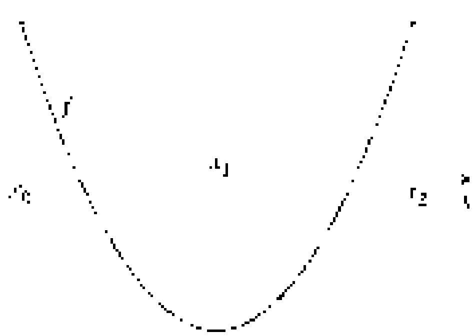
(vi) f' is increasing nowhere.

(vii) f' is decreasing on $[x_0, x_2]$.

(viii) f has a local maximum at $x = x_1$.

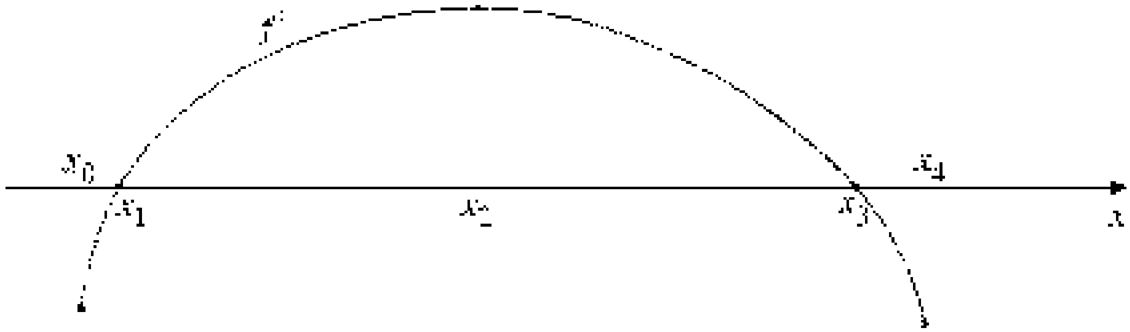
(ix) f has no local minimum.

(b)



PTS: 1

9. Given the graph of $y = f'(x)$, answer the questions that follow.



(a) Find all values of x at which

(i) f is increasing.
(ii) f is decreasing.
(iii) $f''(x) > 0$.
(iv) $f''(x) < 0$.
(v) f has an inflection point.
(vii) f' is decreasing.
(viii) f has a local maximum.

(b) Sketch a graph which could represent $y = f(x)$.

ANS:
(a)

(i) f is increasing on $[x_1, x_3]$.

(ii) f is decreasing on $[x_0, x_1]$ and $[x_3, x_4]$.

(iii) $f''(x) > 0$ on $[x_0, x_2]$.

(iv) $f''(x) < 0$ on (x_2, x_4).

(v) f has an inflection point at $x = x_2$.

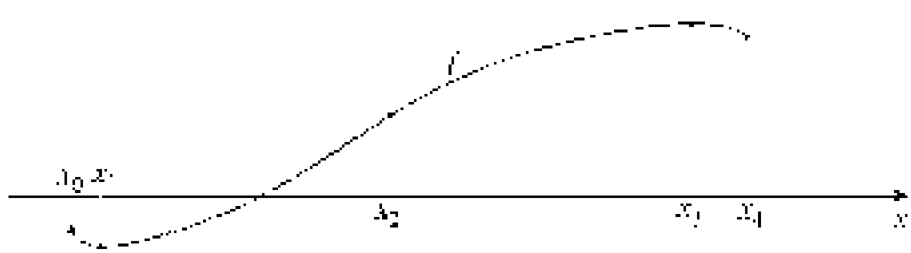
(vi) f' is increasing on $[x_0, x_2]$.

(vii) f' is decreasing on $[x_2, x_4]$.

(viii) f has a local maximum at $x = x_3$.

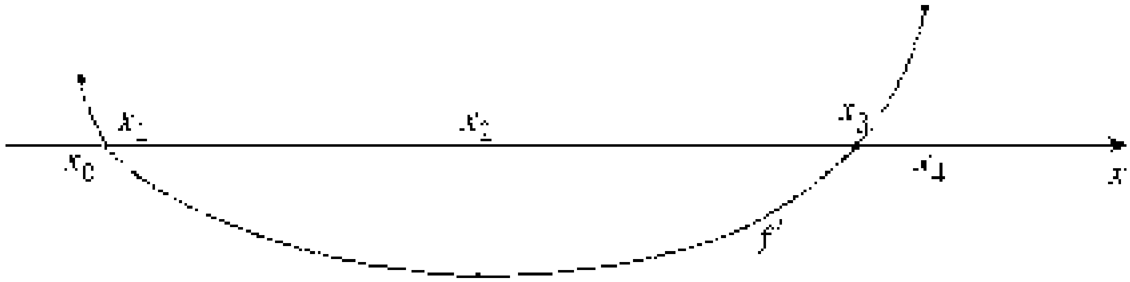
(ix) f has a local minimum at $x = x_2$.

(b)



PTS: 1

10. Given the graph of $y = f'(x)$, answer the questions that follow.



(a) Find all values of x at which

(i) f is increasing.
(ii) f is decreasing.
(iii) $f''(x) > 0$.
(iv) $f''(x) < 0$.
(v) f has an inflection point.
(vi) f' is increasing.
(vii) f' is decreasing.
(viii) f has a local maximum.
(ix) f has a local minimum.

(b) Sketch a graph which could represent $y = f(x)$.

ANS:
(a)

(i) f is increasing on $[x_0, x_1]$ and $[x_3, x_4]$.

(ii) f is decreasing on $[x_1, x_3]$.

(iii) $f''(x) > 0$ on $[x_2, x_4]$.

(iv) $f''(x) < 0$ on (x_0, x_2).

(v) f has an inflection point at $x = x_2$.

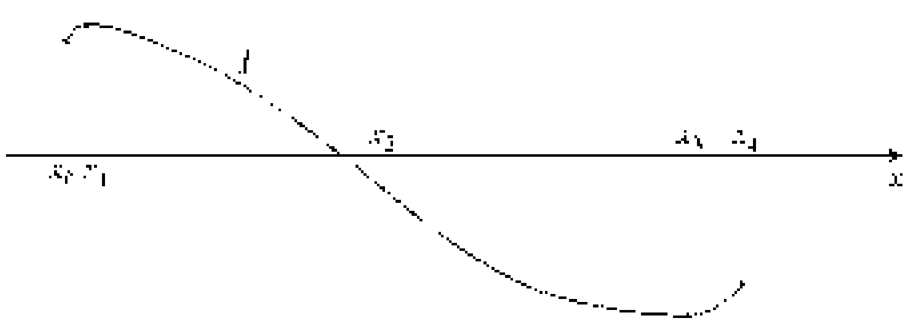
(vi) f' is increasing on $[x_2, x_4]$.

(vii) f' is decreasing on $[x_0, x_2]$.

(viii) f has a local maximum at $x = x_1$.

(ix) f has a local maximum at $x = x_3$.

(b)



PTS: 1