BUILDING MATH SKILLS ONLINE
Instructor's Guide

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BASIC OPERATIONS, *Part 1* Instructor's Guide

Adding and Subtracting Whole Numbers

Addition and subtraction are inverse operations, which means one operation "undoes" the other. Show students how addition is used to check a subtraction problem and subtraction is used to check addition. Use Teaching Tip #1.

Add 14 + 23. Check the sum using subtraction.

14 + 23 = 37. Check: 37 – 23 = 14.

Subtract 89 – 44. Check the difference using addition.

89 – 44 = 45. Check: 45 + 44 = 89.

For a student to more fully understand what happens when the sum of a place value column is greater than 10, consider the numbers in expanded form.

$$\begin{array}{r} 275 \\ \underline{+948} \end{array}$$

Write the numbers in expanded form.

$$\begin{array}{r} \\ 200+70+5 \\ \underline{+900+40+8} \\ 13 \end{array} \rightarrow \begin{array}{r} +10 \\ 200+70+5 \\ \underline{+900+40+8} \\ 3 \end{array} \rightarrow \begin{array}{r} +10 \\ 200+70+5 \\ \underline{+900+40+8} \\ 120+3 \end{array} \rightarrow \begin{array}{r} 100+10 \\ 200+70+5 \\ \underline{+900+40+8} \\ 20+3 \end{array}$$

$$\begin{array}{r} 100+10 \\ 200+70+5 \\ \underline{+900+40+8} \\ 1200+20+3 \end{array} \rightarrow 1200 + 20 + 3 = 1{,}223$$

Use Teaching Tip #2.

Use a similar process with expanded form to show the borrowing (renaming) process to find a difference.

$$\begin{array}{r} 742 \\ \underline{-355} \end{array}$$

Write the numbers in expanded form.

Teaching Tip #1

Use proper terms when teaching addition and subtraction. The answer to an addition problem is a **sum**. The numbers being added are **addends**.

The answer to a subtraction problem is a **difference**. The terms for the numbers of a subtraction problem are not commonly used and are therefore not necessary to use. The number being subtracted is the **subtrahend**. The number from which a number is subtracted is the **minuend**.

Teaching Tip #2

The method shown at the left is for the purpose of better understanding the renaming process. Students do not need to use this method when finding a sum.

$$\begin{array}{r} 700+40+2 \\ \underline{-300+50+5} \end{array} \rightarrow \begin{array}{rrr} & 10 & \\ 700 & 30 & 2 \\ \hline -300 & 50 & 5 \end{array} \rightarrow \begin{array}{rrr} 700 & 30 & 12 \\ \hline -300 & 50 & 5 \\ & & 7 \end{array} \rightarrow \begin{array}{rrr} & 100 & \\ 600 & 30 & 12 \\ \hline -300 & 50 & 5 \\ & & 7 \end{array}$$

$$\begin{array}{rrr} 600 & 130 & 12 \\ \hline -300 & 50 & 5 \\ & & 7 \end{array} \rightarrow \begin{array}{rrr} 600 & 130 & 12 \\ \hline -300 & 50 & 5 \\ & 80 & 7 \end{array} \rightarrow \begin{array}{rrr} 600 & 130 & 12 \\ \hline -300 & 50 & 5 \\ 300 & 80 & 7 \end{array} \rightarrow 300 + 80 + 7 = 387$$

Use Teaching Tip #3.

Teaching Tip #3

The method shown at the left is for the purpose of better understanding the borrowing process. Students do not need to use this method after they understand how to find a difference.

Calculators are used by most all people to do most addition and subtraction problems beyond basic facts. Many people have calculators at their fingertips most of the time, but it is still reasonable to expect that people can add and subtract using paper and pencil.

Multiplying Whole Numbers

For many students, learning basic multiplication facts has not been a priority, and thus, they do not find consistent success multiplying multi-digit numbers. Even though most use calculators when multiplying, students should still commit basic multiplication facts for 1 to 12 to memory. Use Teaching Tip #4.

Teaching Tip #4

Multiplication facts are "shortcuts" for repeated addition. This means the facts for any multiplier follow a pattern. Guide students to understand the pattern for a multiplier. Students must practice in order to commit to memory multiplication facts, including writing lists of the facts repeatedly, taking drill tests, and making and using flash cards. Offer an incentive (reward) for students to learn the facts, and encourage them to spend time outside the classroom to achieve the goal that earns them the reward.

In addition to facts, understanding how place value plays a role in multiplying is also helpful. The exercises shown below are for demonstration and do not show the methods students would use when multiplying using paper and pencil. This strategy is good for students who struggle to understand the algorithm.

Multiply 143×5.

Use expanded form of 143 to illustrate the work.

$$\begin{array}{r} 100 \\ \times\ 5 \\ \hline 500 \end{array} \quad \begin{array}{r} 40 \\ \times\ 5 \\ \hline 200 \end{array} \quad \begin{array}{r} 3 \\ \times\ 5 \\ \hline 15 \end{array}$$

Add the partial products. 500 + 200 + 15 = 715 Use Teaching Tip #5.

The same process can be used when the multiplier is a multi-digit number. Use the first factor in standard form and the second factor in expanded form.

Multiply 736 × 28.

736	736
× 8	× 20
5,888	14,720

Add the partial products. 5,888 + 14,720 = 20,608 See Common Student Error #1.

Until students are able to show they can successfully multiply using paper and pencil, you can restrict calculator use to checking their products.

Teaching Tip #5

A partial product is the result of multiplying one factor by one digit of the other factor. The actual product is the sum of the partial products.

Common Student Error #1

Students that make errors with renaming when a product for a place value column has two digits may get 5,648; 5,688, or 5,848 for the first partial product. They may get 14,620 for the second partial product.

Dividing Whole Numbers (with and without remainders)

A division problem that has a quotient with no remainder is a division problem whose quotient times the divisor equals the dividend. Use Teaching Tip #6.

Teach the divisibility rules so students will know when they divide by a single digit if there will be a remainder. When a number is *divisible* by another number, there is no remainder. Often people say, "It divides evenly."

Divisible by 1 – all numbers can be divided by 1. The quotient is the number itself.

Divisible by 2 – the digit in the ones place must be an even number: 0, 2, 4, 6, 8. Use Teaching Tip #7.

Divisible by 3 – the sum of the digits must be divisible by 3. Example: 123 → 1 + 2+ 3 = 6; 6 is divisible by 3, so 123 is divisible by 3; 123 ÷ 3 = 41

Teaching Tip #6

Division is the process of separating a quantity into equal-sized groups. The quantity being separated is the **dividend**. The desired number of groups is the **divisor**, and the **quotient** is the number of groups. In other words the quotient is the answer. The dividend is the first number when the problem is written in horizontal format and the number inside the box when written in long division format. The divisor is the second number in the horizontal format and the number outside the box in long division format.

Teaching Tip #7

Zero is an even number. Think of a number line. Zero is in an even position (every other number).

Divisible by 4 – the last two digits must be a multiple of 4.
Example: 2,044 → 44 is a multiple of 4 (11), so 2,044 is divisible by 4; 2,044 ÷ 4 = 511

Divisible by 5 – the digit in the ones place must be 0 or 5.

Divisible by 6 – the number must be even AND the sum of the digits is divisible by 3. Example: 2,352 → it is even; 2 + 3 + 5 + 2 = 12; 12 is divisible by 3, so 2,352 is divisible by 6; 2,352 ÷ 6 = 392

Divisible by 8 – the last three digits must be a multiple of 8.
Example: 7,408 → 408 is a multiple of 8 (51), so 7,408 is divisible by 8; 7,408 ÷ 8 = 926

Divisible by 9 – the sum of the digits must be divisible by 9.
Example: 855 → 8 + 5 + 5 = 18 is divisible by 9, so 855 is divisible by 9; 855 ÷ 9 = 95

Divisible by 10 – the digit in the ones place must be 0.

Divide 357 ÷ 3.

Add the digits of the dividend. 3 + 5 + 7 = 15. There will be no remainder.

```
   119
3)357   Use Teaching Tip #8.
  3
   5
   3
   27
   27
    0
```

When a division problem has a remainder, it can be handled three different ways: 1) Write the quotient, an uppercase R, followed by the remainder; 2) Write the quotient as a mixed number with the fractional part the remainder over the divisor; 3) Insert a decimal point and zeros at the end of the dividend and continue to divide until there is no remainder or to the desired place value.

Teaching Tip #8

A quotient can be checked using multiplication.

$$\begin{array}{r} 119 \\ \underline{\times\ 3} \\ 357 \end{array}$$

Divide 98 ÷ 5.

```
  19
5)98
  5
  48
  45
   3
```

Quotient can be written 19R3 or 19 3/5. See Common Student Error #2.

```
  19.6
5)98.0
  5
  48
  45
   30
   30
```

The quotient is 19.6.

Few people actually do long division using paper and pencil today. However, demonstrating and reviewing the process can be beneficial for some students.

Divide 36,178 ÷ 22. Round to the nearest hundredth.

Either carry the quotient out to three decimal places so that you can round to two decimal places, or compare the remainder to the divisor. Use Teaching Tip #9.

```
      1,644.45
22)36,178.00
   22
   141
   132
     97
     88
      98
      88
      100
       88
       12
```

Half of 22 is 11. 12 > 11. See Common Student Error #3.

Increase the digit in the hundredths place by 1.

The quotient to the nearest hundredth is 1,644.46.

Common Student Error #2

Students often write the fraction incorrectly as the divisor over the remainder.

Teaching Tip #9

Compare the remainder to the divisor to decide if the digit in the hundredths place stays the same or increases by 1. If the remainder is more than half of the divisor, increase the quotient by 1.

Common Student Error #3

Many students insert the decimal point and two zeros to set the quotient up to be written to the nearest hundredth but forget the last step of comparing what is left to the divisor. If students forget this last step, the odds are 50:50 that the answer will be correct.

Order of Operations

To start a lesson on the order of operations, have students simplify a problem such as 5 + 20 ÷ 5 – 2 × 3 without listing the order of operations for the students to reference.

Without rules, students will get at least two different answers. This is an opportunity for students to learn that without agreed upon rules, no one would agree on the correct answer. For that matter, would there be a correct answer?

Students that work the problem above from left to right will get an answer of 9. The answer is 3 when the problem is simplified using the order of operations. Use Teaching Tip #1.

Provide the order of operations as the established set of rules for simplifying problems that include more than one operation.

1. Simplify any expression within grouping symbols. Grouping symbols include parentheses ( ), brackets [ ], and fraction bar —.
2. Simplify powers which are expressions with exponents.
3. Multiply and divide, in order from left to right in the expression.
4. Add and subtract, in order from left to right in the expression.

The more practice problems students do, the more comfortable they will be using these rules. In the beginning, it is best to work left to right for the first rule. Then work left to right for the second rule, and continue left to right for the third rule, followed by the fourth rule.

Teaching Tip #1

Shown is one way a student may complete the problem incorrectly.

5 + 20 ÷ 5 – 2 × 3

25 ÷ 5 – 2 × 3

5 – 2 × 3

3 × 3

9

Shown is another way a student may complete the problem incorrectly.

5 + 20 ÷ 5 – 2 × 3

25 ÷ 5 – 2 × 3

5 – 6

–1

There are other ways to get a wrong answer.

Shown is the correct way to solve the problem.

5 + 20 ÷ 5 – 2 × 3

5 + 4 – 2 × 3

5 + 4 – 6

9 – 6

4

Simplify $3 \times (2 + 8) \div 2 + 3$.

Add within the parentheses. $3 \times 10 \div 2 + 3$

Multiply. $30 \div 2 + 3$

Divide. $15 + 3$

Add. 18 See Common Student Error #1.

Simplify $(4^2 - 10) + 12 \div 3$.

Simplify the power. $(16 - 10) + 12 \div 3$

Subtract within the parentheses. $6 + 12 \div 3$

Divide. $6 + 4$

Add. 10 See Common Student Error #2.

Simplify $\frac{6 + 18 \div 0.5}{4 + 2}$.

In the numerator, divide. $\frac{6 + 36}{4 + 2}$

In the numerator, add. $\frac{42}{4 + 2}$

In the denominator, add. $\frac{42}{6}$

Divide. 7 See Common Student Error #3. Use Teaching Tip #2.

Simplify $\frac{6 - 4 \div 2 + 10}{2^3 + 4 \times 5}$.

In the numerator, divide. $\frac{6 - 2 + 10}{2^3 + 4 \times 5}$

In the numerator, subtract; then add. $\frac{14}{2^3 + 4 \times 5}$

In the denominator, simplify the power. $\frac{14}{8 + 4 \times 5}$

In the denominator, multiply. $\frac{14}{8 + 20}$

In the denominator, add. $\frac{14}{28}$

Write the fraction in lowest terms. $\frac{1}{2}$ See Common Student Error #4.

Use Teaching Tip #2.

Common Student Error #1

There are many ways to work the problem incorrectly. Several students may solve the problem as shown.
$3 \times 10 \div 2 + 3$; $30 \div 5 = 6$

Common Student Error #2

There are many ways to work the problem incorrectly. Several students may solve the problem as shown. $16 - 10 + 12 \div 3$; $6 + 12 \div 3$; $18 \div 3 = 6$

Common Student Error #3

There are many ways to work the problem incorrectly. Several students may solve the problem as shown. $(24 \div 0.5)/6$; $48/6 = 8$

Teaching Tip #2

As shown, the rules are used in the numerator. Then the rules are used in the denominator. Finally, the fraction is simplified. Another way to complete the work, yet get the same result, is to use rule 1 in the numerator and denominator, then rules 2, 3, and 4 before finally simplifying the fraction.

Common Student Error #4

There are many ways to work the problem incorrectly. Several students may solve the problem as shown. $(2 \div 2 + 10)/(8 + 4 \times 5)$;
$(1 + 10)/(12 \times 5)$; $11/60$

Place Value and Rounding

Place value is a topic that is essential to understanding the meaning of a number; yet most students think its only value is rounding.

A useful exercise to learn place value is to write numbers in expanded form. Such an exercise makes students become aware of the value of each digit in a number.

Write 4,712 in expanded form.

Write the number as a sum of the product of each digit and its place value.

(4 × 1,000) + (7 × 100) + (1 × 10) + (2 × 1)

Write 805 in expanded form.

You can include a product for the tens place or you can omit it.

(8 × 100) + (0 × 10) + (5 × 1) Use Teaching Tip #1.

Although rounding is taught in elementary school, many students do not master the skill. The place value chart should be reviewed for students to find success. Below is an informal way to explain the rounding process.

6	4	2	,	5	1	9	.	3	0	7	8
hundred thousand	ten thousand	thousand		hundred	ten	one		tenth	hundredth	thousandth	ten thousandth

Round the number above to the nearest ten.

Draw a line right of the digit in the tens place. Use Teaching Tip #2.

642,51|9.3078

Teaching Tip #1

The product for the tens place need not be included.
(8 × 100) + (5 × 1)

Teaching Tip #2

Students can use an index card to make the bookmark with a place value chart so that they have a reference at their fingertips.

The digit right of the line tells you to increase the digit left of the line to 2. All digits right of the line are replaced with 0s. To the nearest ten, the rounded number is 642,520.0000, which can be written 642,520. Use Teaching Tip #3.

Round 642,519.3078 to the nearest whole number.

Whole number is the same as rounding to the nearest one. Draw a line right of the digit in the ones place.

642,519|.3078

The 3 tells you 9 stays as a 9. Replace all digits right of the line with 0s. There is no need to include those 0s. To the nearest whole number, the rounded number is 642,519.

Round 642,519.3078 to the nearest hundredth.

Draw a line right of the digit in the tenths place. See Common Student Error #1.

642,519.30|78

The 7 tells you 0 becomes 1. Replace all digits right of the line with 0s or in this case just do not write any digits after the 1. To the nearest hundredth, the rounded number is 642,519.31

When a problem involves money amounts, an answer can be rounded to a whole dollar or to the nearest cent. If no rounding instruction is given, it is assumed the amount needs to be rounded to the nearest cent.

Multiply 2.5 × $7.75. Round to the nearest cent.

Multiply. The product has three decimals. The nearest cent means the same thing as nearest hundredth. Round $19.375 to the hundredth, or two decimal places. The rounded amount is $19.38.

Often, rounding is the last step to solving a problem. But, rounding can be a first step. When a problem involves estimating, rounding is the first step.

Teaching Tip #3

Make the distinction between the 0 in the ones place and the 0s in places right of the decimal point. The 0s right of the decimal point do not change the value if they are dropped. However, if the 0 in the tens place is dropped, the number changes to 64,252.

Common Student Error #1

Students commonly identify the digit in the hundredths place incorrectly. Because the hundreds place is three places left of the decimal point, they think the third place right of the decimal point is the hundredths place.

Suggestion

Guide students to realize the place value chart is not "symmetrical" at the decimal point. Because there is no *oneths* place, the hundreds place and the hundredths place are not the same number of places on opposites side of the decimal point.

Estimating

Estimation is a value skill. As the teacher, you should be flexible with the strategies students use and accept answers within a reasonable range of the exact answer.

Students will get the most from practicing a variety of problems, some requiring a single operation and some requiring multiple operations and steps.

The examples presented in the Estimating didactic page provide a good sample. Use Teaching Tip #4.

A good habit to get into for any estimation problem is to determine if the estimate is an overestimate or an underestimate.

Teaching Tip #4

Work at least one example together as a class. Then have students work a couple examples in pairs or small groups. If students need more practice problems, have the students change the numbers given in the examples and trade their problems with another student. This activity provides students with problems that are solved using the same steps they just used, but with different numbers and therefore different answers.

Measures of Central Tendency

Measures of central tendency are median, mean, and mode. These measures of center are used to describe a set of the numbers. A good way to teach measures of center is to use one set of numbers and find all three measures.

Find the median of the set of quiz scores.

35, 42, 18, 28, 35, 12, 49, 25, 50

The median is the number in the middle position when the set is written in sequential order. Rearrange the numbers.

12, 18, 25, 28, 35, 35, 42, 49, 50

Count from the left and right into the number in the middle.

, 35,

The median is 35. Use Teaching Tip #5.

When a set has an even number of numbers, there is no number in the middle. In these cases, use the two numbers in the middle. To find the median, add the two numbers in the middle and divide by 2. Use Teaching Tip #6.

Teaching Tip #5

If you live an area that has medians in the roads, tell students to use the knowledge that a median is in the middle of a road to remember that median is the measure of center that describes the number in the middle.

Teaching Tip #6

An example:

Find the median of 3, 5, 13, 19. The numbers in the middle are 5 and 13. Add and divide by 2.

5 + 13 = 18; 18 ÷ 2 = 9
The median is 9.

Find the mean of the set of quiz scores. Round to the nearest tenth.

35, 42, 18, 28, 35, 12, 49, 25, 50

To find the mean add the numbers in the set and divide by 9, the number of numbers in the set.

35 + 42 +18 + 28 + 35 + 12 + 49 + 25 + 50 = 294;

294 ÷ 9 = 32.7

The mean is 32.7. See Common Student Error #2.

Find the mode of the set of quiz scores.

35, 42, 18, 28, 35, 12, 49, 25, 50

The mode is the number(s) that appear in the set the most number of times. The mode is 35.

Have students compare the three measures. The measures are relatively close. Any of these measures can be used to describe the set.

median: 35; mean: 32.7; mode: 35

Review the following tips about measures of center. Use Teaching Tip #7.

A median is not skewed by an "extreme value" in the set.

If an "extreme value" is a mode, none of the measures of center will be a good representative of the set.

Common Student Error #2

An answer of 32.6 has been rounded incorrectly. An answer of $32.\overline{6}$ *has not been rounded.*

Teaching Tip #7

An "extreme value" is a number whose value is significantly higher or lower than the other numbers in the set. For the set used throughout this section, 1 is an example of an extreme value on the lower end and 89 is an example of an extreme value on the high end.

Powers of 10

Students like to learn shortcuts. Teach the powers of 10 as shortcuts. When the powers of 10 are written as a power, the **exponent** represents the number of places the decimal point is to move.

When the powers of 10 are written in standard form, the **number of zeros** represents the number of places the decimal point is to move.

Use the same base factor when you first teach multiplying by powers of 10. This allows students to (easily) see how the value changes when the location of the decimal point changes. Use Teaching Tip #1:

With powers:

$3.2 \times 10^4 = 32{,}000$
$3.2 \times 10^1 = 3.2$
$3.2 \times 10^{-3} = 0.0032$

With standard form:

$3.2 \times 10{,}000 = 32{,}000$
$3.2 \times 10 = 3.2$
$3.2 \times 0.001 = 0.0032$ See Common Student Error #1.

It works well for understanding when you teach dividing by powers of 10 right after multiplying by powers of 10.

With powers:

$3.2 \div 10^4 = 0.00032$
$3.2 \div 10^1 = 0.32$
$3.2 \div 10^{-3} = 3{,}200$

With standard form:

$3.2 \div 10{,}000 = 0.00032$
$3.2 \div 10 = 0.32$
$3.2 \div 0.001 = 3{,}200$ See Common Student Error #2.

Teaching Tip #1

Teaching powers of 10 can be a lesson of its own or can be integrated into a lesson on powers that covers different bases. By teaching powers of 10 before powers with different bases, students can simplify expressions without the use of a calculator. Students can focus on the shortcuts to use when multiplying and dividing by multiples of 10.

Common Student Error #1

Problems like this are counterintuitive because students have learned that multiplication moves the decimal to the right, but in this case the decimal actually moves to the left.

Suggestion

If a student needs to be convinced this answer is correct, have them verify the answer using a calculator.

Common Student Error #2

Problems like this are also counterintuitive because students have learned that division moves the decimal to the left, but in this case the decimal actually moves to the right.

Students should only use a calculator to verify their answers when they have first simplified the expressions using paper and pencil or mental math.

Powers

After students learn the meaning of a power, allow students to use a calculator. When a base is greater than 5 or the exponent is greater than 3, the computation can be cumbersome.

One way to explain the power 4^3 is to say, "Use the base 4 as a factor 3 times, which is written 4 × 4 × 4." Use Teaching Tip #2.

For the general form of a power B^x, say: "Multiply the number that is the base by itself *x* times."

Simplify 8^4.

Write the multiplication expression 8 × 8 × 8 × 8. 8^4 = 4,096 See Common Student Error #3. Use Teaching Tip #2.

The base of a power can also be a decimal.

Simplify 1.2^2.

Write the multiplication expression 1.2 × 1.2. 1.2^2 = 1.44 See Common Student Error #4. Use Teaching Tip #3.

When a problem includes greater numbers, students will certainly use a calculator. It is easy to lose count when entering the base into the calculator multiple times. Demonstrate ways to help students manage the computation. Use Teaching Tip #4.

$$\text{Simplify } 11^7 = 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$$
$$= (11 \times 11) \times (11 \times 11) \times (11 \times 11) \times 11$$
$$= 121 \times 121 \times 121 \times 11 = 19{,}487{,}171$$

A student that knows 11 × 11 = 121 without a calculator, can find the answer using the third line of the work shown.

Common Student Error #3

A student that answers 32 has made the most common mistake. The student simply multiplied the base by the exponent.

Teaching Tip #2

Do not use 2^2 as an example when you first start to teach powers. This is not a good example because if students make the common mistake named above, they will not know that they did not solve the problem incorrectly.

Common Student Error #4

A common mistake is to place the decimal point in the same location as it is in the base. A student making this mistake answers 14.4.

Teaching Tip #3

Because 12 × 12 = 144 is a common fact most students have memorized, this problem can be simplified using mental math. Consider the base as 12. Then place the decimal point in 144 knowing there are two decimal places in the problem.

Teaching Tip #4

The x^y key on a calculator is the key used to simplify powers.

Roots

Although the square root is the most commonly used root, students need to be aware that there are other roots, such as cube roots, fourth roots, and so on.

Another key point students need to understand is that if a radicand is not a perfect number of the root, its value can only be approximated. Use Teaching Tip #5.

Unless a student recognizes a number as a perfect number, he/she will use a calculator. To recognize if 324 is a perfect number, ask is there a number multiplied by itself that equals 324

Evaluate $\sqrt{324}$.

In any basic calculator, enter 324. Press the radical key. The display shows 18. There is no need to press the = key. Because 18 is a whole number, you know 324 is a perfect square number.

Evaluate $\sqrt{650}$ to the nearest tenth.

In any basic calculator, enter 650. Press the radical key. The display shows 25.495…. To the nearest tenth, the square root of 650 is 25.5. See Common Student Error #5.

To find a root other than a square root, you must use a calculator than has a $\sqrt[y]{x}$ key. Enter the value of *x* into the calculator; press the root key followed by the value of y.

Evaluate $\sqrt[5]{32}$.

Enter 32. Press the $\sqrt[y]{x}$ key. Press 5. Press the = key. The display shows 2. See Common Student Error #6.

Evaluate $\sqrt[4]{12}$ to the nearest tenth.

Enter 12. Press the $\sqrt[y]{x}$ key. Press 4. Press the = key. The display shows 1.8612097.... Round to 1.9. See Common Student Error #7.

Teaching Tip #5

Students can use a basic calculator that has a key with a radical symbol to evaluate any square root. However, not all basic calculators have a square root key. To evaluate another root (not square) students have to use a scientific or graphing calculator.

Common Student #5

When evaluating a square root, the answer does not include a radical symbol. Many students want to include the symbol as part of their answers. Be certain students understand the difference between 25.5 and $\sqrt{25.5}$*, and the first is the answer.*

Common Student #6

If the numbers are entered in the reverse order, the display shows the non-terminating decimal 1.05158119….

Suggestion

Round the decimal to the nearest tenth. Use that value to check the answer. Multiply 1.1 by itself five times to see if the product is 32.

$1.1 \times 1.1 \times 1.1 \times 1.1 \times 1.1 = 1.61051$

Common Student #7

Students not paying attention can interpret the problem as 12 ÷ 4.

Combined Operations with Powers and Roots

Problems that involve both powers and roots are best solved by writing each step of work on paper, even though a calculator can be used for computations.

Until students recognize a square root squared equals the number under the radical, have them show their work.

Simplify $\left(\sqrt{9}\right)^2$.

$\sqrt{9} - \sqrt{9} = \sqrt{81} = 9$

The expression above can be simplified differently, but yields the same result.

$\left(\sqrt{9}\right)^2 = (3)^2 = 9$ Use Teaching Tip #6.

Show the same type of work even when the problem involves a fraction.

Simplify $\left(\sqrt{\frac{121}{9}}\right)^2$. See Common Student Error#8.

Using the first method shown: $\sqrt{\frac{121}{9}} - \sqrt{\frac{121}{9}} = \sqrt{\frac{14,641}{81}} = \frac{121}{9}$.

Using the second method shown: $\left(\sqrt{\frac{121}{9}}\right)^2 = \left(\frac{11}{3}\right)^2 = \frac{11^2}{3^2} = \frac{121}{9}$.

Use Teaching Tip #7.

Use the order of operations when an expression includes multiple operations.

Simplify $\sqrt{23 - 5^2 + 1}$.

Evaluate the expression under the radical and then take the square root of that number.

$23 \times 5^2 + 1 = 23 \times 25 + 1 = 575 + 1 = 576$

Evaluate $\sqrt{576} = 24$. See Common Student Error #9.

Teaching Tip #6

Show students both methods. Allow students to use the method that makes the most sense to them. Remind students that one method may be better for one problem and not the next. Students should notice that the number under the radical is the answer. Ask students to simplify similar problems: $\left(\sqrt{6}\right)^2$, $\left(\sqrt{15}\right)^2$, $\left(\sqrt{32}\right)^2$, and $\left(\sqrt{2,089}\right)^2$.

(Answers: 6; 15; 32; 2,089)

Common Student #8

It is common for students to answer 11/3 because they take the square root of the numerator and denominator. The student simply ignored the exponent 2.

Teaching Tip #7

A student who has learned the square of a square root equals the number of the radical can immediately identify the answer as 121/9 without showing any work.

Common Student #9

If the order of operations is used incorrectly, a student may get an answer of 24.5 ($\sqrt{598}$ rounded to the nearest tenth).

Equivalent Fractions

When first beginning to teach or review fractions, remind students that fractions can be written in a vertical (stacked) format or a horizontal format. When initially learning and practicing with fractions, the fractions are usually presented in the vertical format, such as $\frac{1}{2}$, because students find it easier to keep track of the numerators and denominators. When fractions are used in construction documents and for other technical documentations, the fractions are often shown horizontally, such as 1/2, because it requires less vertical space.

Equivalent fractions are a fundamental skill students need to find success when performing operations with fractions and when solving application problems that involve fractions. Before you begin teaching equivalent fractions, review the Multiplication Property of 1. Ask students what they can multiply any number by that does not change the number's value. You want students to understand the power of multiplying by 1. Use Teaching Tip #1.

Using fraction models, demonstrate to students any number over itself equals 1. Have students write eight fractions for 1 using the digits 2 to 9. Use Teaching Tip #2.

The reason 1 is so powerful is because you can write 1 using whatever you need to achieve a desired denominator.

If you want to write the value 2/3 as a fraction with 6 in the denominator, then you need to know how to write 1 so that you can get 6 in the denominator but not change the value of 2/3.

$\frac{2}{3}\times\frac{2}{2}=\frac{4}{6}$ Use Teaching Tip #3.

Teaching Tip #1

When students learn about the Multiplication Property of 1, most think it is a "useless" property. It seems obvious to most students that know the multiplication facts that multiplying by 1 does not change the value of a number.

Teaching Tip #2

A fraction model is usually a circle with sector pieces. A model for 2/2 is two semi-circles. A model for 3/3 has three sectors (or pie pieces) of the same size that make a whole circle. A model of 4/4 has four quarter sections of a circle. This pattern is used to make fraction models using any given denominator.

Teaching Tip #3

Many mathematics instructors believe it is best to teach multiplying fractions as the first operation.

The purpose of equivalent fractions is to make fractions look different (have different denominators), yet still represent the same value. You can generate an endless number of fractions that equal the same value.

Write four fractions equivalent to 1/5.

When no specific instructions are given for a desired denominator, write 1 however you choose.

$1/5 \times 2/2 = 2/10$ $1/5 \times 5/5 = 5/25$

$1/5 \times 10/10 = 10/50$ $1/5 \times 20/20 = 20/100$

Write a fraction equivalent to 4/9 that has 72 in the denominator.

What times 9 equals 72? 8

Multiply 4/9 by 8/8. $4/9 \times 8/8 = 32/72$

Write a fraction equivalent to 15/24 with 8 in the denominator.

Because 8 is less than 24, decide what you can divide 24 by to get 8. 3

Divide both the numerator and the denominator in 15/24 by 3. $(15 \div 3)/(24 \div 3) = 5/8$ Use Teaching Tip #4.

If the instruction for the above problem had been to have 6 in the denominator, the problem could not be solved. To get 6 in the denominator, divide 24 by 4. But, when you divide 15 by 4, the answer is the decimal 3.75. It is not acceptable to have a decimal as a numerator or denominator of a fraction.

Teaching Tip #4

Dividing by 1 does not change a number, the same as multiplying by 1 does not. In some cases, division is needed to generate an equivalent fraction to meet the desired attribute.

Fractions in Simplest Form

Simplest form is also known as simplified form or lowest terms. Simplest form means a numerator and denominator of a fraction do not have a common factor.

The numerator and denominator of a fraction must be divided by the greatest common factor for it to be in simplest form.

Write 45/120 in simplest form.

Factors of 45 to consider are 3, 5, 9, and 15. Both 45 and 120 are divisible by 3, 5, and 15. Use 15 as the greatest common factor. Divide both the numerator and the denominator by 15. See Common Student Error #1.

$$\frac{45 \div 15}{120 \div 15} = \frac{3}{8}$$

Write 16,650/24,900 in simplest form.

The numbers of this fraction are quite large and determining the greatest common factor may be time consuming. Begin by dividing by any common factor. Then divide by another common factor.

$$\frac{16{,}650 \div 25}{24{,}900 \div 25} = \frac{666}{996}$$

$$\frac{666 \div 6}{996 \div 6} = \frac{111}{166}$$

With the second reduction, the numerator and denominator have no more common factors. The simplest form is 111/166.

Write 1,040,000/5,200,000 in simplest form.

Begin by dividing by 10,000. This means marking off four zeros from the numerator and denominator.

$$\frac{1{,}04\cancel{0}{,}\cancel{0}\cancel{0}\cancel{0}}{5{,}20\cancel{0}{,}\cancel{0}\cancel{0}\cancel{0}}$$

Now reduce 104/520.

$$\frac{104}{520} = \frac{104 \div 8}{520 \div 8} = \frac{13}{65}$$

$$\frac{13 \div 13}{65 \div 13} = \frac{1}{5}$$

The simplest form is 1/5. See Common Student Error #2.

Common Student Error #1

If a student divides the numerator and denominator by 5, he/she will get 9/24. But 9 and 24 have a common factor of 3. So divide the numerator and denominator by 3 to get 3/8. If a common factor is used, but not the greatest common factor, the student will need to divide the fraction more than one time.

Common Student Error #2

Many students will stop at 13/65. They recognize 13 as a prime number and think the fraction cannot be reduced any further because 13 only has factors of 1 and 13. Students do not recognize that 13 is a factor of 65.

Rounding Fractions

Rounding fractions is a new idea for many students. Review the rules for rounding; if a digit to the right of the place being rounded is 5 or greater, increase the place being rounded by 1. Five is used because it is halfway. When rounding fractions, it must be determined how the fraction compares to 1/2.

To determine if a fraction is equal to or greater than 1/2, multiply the numerator by 2. Then compare that number to the denominator.

- If the numerator doubled is less than the denominator, the fraction is less than 1/2.
- If the numerator doubled is equal to the denominator, the fraction equals 1/2.
- If the numerator doubled is greater than the denominator, the fraction is greater than 1/2.

Have students compare these fractions to 1/2.

5/12	$5 \times 2 = 10; 10 < 12$	$5/12 < 1/2$
3/5	$3 \times 2 = 6; 6 > 5$	$3/5 > 1/2$
11/18	$11 \times 2 = 22; 22 > 18$	$11/18 > 1/2$
32/64	$32 \times 2 = 64; 64 = 64$	$32/64 = 1/2$

Use this technique when asked to round a mixed number to a whole number.

Round 5 16/45 to a whole number.

$16 \times 2 = 32; 32 < 45; 16/45 < 1/2$

Drop the fraction part of the mixed number. Rounded, 5 16/45 decreases to 5. See Common Student Error #3.

Common Student Error #3

Often students make the mistake of decreasing the whole number by 1. These students will think the mixed number rounds to 4.

Round 24 41/78 to a whole number.

41 × 2 = 82; 82 > 78; 41/72 > 1/2

Drop the fraction part of the mixed number and increase the whole number by 1. Rounded, 24 41/72 increases to 25.

Round 511 180/360 to a whole number.

180 × 2 = 360; 360= 360; 180/360 = 1/2

Drop the fraction part of the mixed number and increase the whole number by 1. Rounded, 511 180/360 increases to 512.

Mixed Numbers and Improper Fractions

Mixed numbers and improper fractions are always greater than 1.

The procedure for changing a mixed number to an improper fraction requires multiplication and addition. An improper fraction is a fraction that has a numerator that is greater than its denominator. Improper is not a "good" term for this type of fraction. The word "improper" implies something is wrong with the fraction.

The procedure: Multiply the whole number by the denominator and add the numerator to that product. Write the product as the numerator, and the denominator remains the same.

Write 6 3/4 as an improper fraction.

$$6\frac{3}{4} = \frac{6 \times 4 + 3}{4} = \frac{27}{4}$$ Use Teaching Tip #5.

Write 12 5/8 as an improper fraction.

$$12\frac{5}{8} = \frac{12 \times 8 + 5}{8} = \frac{101}{8}$$

Write 1 1/18 as an improper fraction.

$$1\frac{1}{18} = \frac{1 \times 18 + 1}{18} = \frac{19}{18}$$

Teaching Tip #5

It is good practice to state the procedure as you or the students are doing the computation.

Six times four plus 3, all over 4.