

# RNUM: Real Numbers

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Workbook Pages .....	1-24
Building Blocks .....	1
Factor Pairings .....	2
Match Up on Fractions .....	3
Fractions Using a Calculator .....	4
Charting the Real Numbers .....	5
Venn Diagram of the Real Numbers .....	6
Linking Rational Numbers with Decimals .....	7
Using Addition Models .....	8
Match Up on Addition of Real Numbers .....	10
Scrambled Addition Tables .....	11
Signed Numbers Magic Puzzles .....	12
Language of Subtraction .....	13
Match Up on Subtraction of Real Numbers .....	15
Signed Numbers Using a Calculator .....	16
Pick Your Property .....	17
What's Wrong with Division by Zero? .....	18
Match Up on Multiplying and Dividing Real Numbers .....	19
The Exponent Trio .....	20
Order Operation .....	21
Assess Your Understanding: Real Numbers .....	22
Metacognitive Skills: Real Numbers .....	23
Teaching Guides .....	25-44
Fractions .....	25
The Real Numbers .....	30
Addition of Real Numbers .....	33
Subtraction of Real Numbers .....	35
Multiplication and Division of Real Numbers .....	37
Exponents and Order of Operations .....	40
Solutions to Workbook Pages .....	45-52



# Student Activity

## Building Blocks

**Fill In The Blanks:** Fill in the missing numerators with whole numbers to build equivalent fractions to the fraction in the "Goal" box. If there is not a whole number numerator that will work, then cross out the fraction.

**Example:**

Goal: $\frac{1}{2}$	$\frac{\boxed{15}}{30}$	<del><math>\frac{7}{14}</math></del>	$\frac{\boxed{5}}{10}$
------------------------	-------------------------	--------------------------------------	------------------------

Goal: $\frac{1}{4}$	$\frac{\quad}{24}$	$\frac{\quad}{36}$	$\frac{\quad}{16}$
------------------------	--------------------	--------------------	--------------------

Goal: $\frac{3}{16}$	$\frac{\quad}{32}$	$\frac{\quad}{4}$	$\frac{\quad}{80}$
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Goal: $\frac{3}{2}$	$\frac{\quad}{50}$	$\frac{\quad}{8}$	$\frac{\quad}{1000}$
------------------------	--------------------	-------------------	----------------------

Goal: $\frac{5}{8}$	$\frac{\quad}{48}$	$\frac{\quad}{32}$	$\frac{\quad}{60}$
------------------------	--------------------	--------------------	--------------------

Goal: $\frac{3}{3}$	$\frac{\quad}{9}$	$\frac{\quad}{15}$	$\frac{\quad}{99}$
------------------------	-------------------	--------------------	--------------------

Goal: $\frac{7}{9}$	$\frac{\quad}{99}$	$\frac{\quad}{81}$	$\frac{\quad}{54}$
------------------------	--------------------	--------------------	--------------------

Goal: $\frac{2}{3}$	$\frac{\quad}{18}$	$\frac{\quad}{15}$	$\frac{\quad}{1}$
------------------------	--------------------	--------------------	-------------------

Goal: $\frac{7}{5}$	$\frac{\quad}{25}$	$\frac{\quad}{35}$	$\frac{\quad}{14}$
------------------------	--------------------	--------------------	--------------------

# Student Activity

## Factor Pairings

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**Directions:** In each diagram, there is a number in the top box and exactly enough spaces beneath it to write all the possible factor-pairs involving whole numbers. See if you can find all the missing factor-pairs. The number 30 has been done for you.

<b>12</b>

<b>20</b>

<b>24</b>

<b>28</b>

<b>30</b>
1·30
2·15
3·10
5·6

<b>32</b>

<b>36</b>

<b>42</b>

<b>45</b>

<b>50</b>

<b>60</b>

<b>64</b>

1. What is the largest number that is a factor of 20 and 30? \_\_\_\_\_

Simplify:  $\frac{20}{30}$

2. What is the largest number that is a factor of 28 and 36? \_\_\_\_\_

Simplify:  $\frac{28}{36}$

3. What is the largest number that is a factor of 36 and 60? \_\_\_\_\_

Simplify:  $\frac{36}{60}$

4. What is the largest number that is a factor of 24 and 42? \_\_\_\_\_

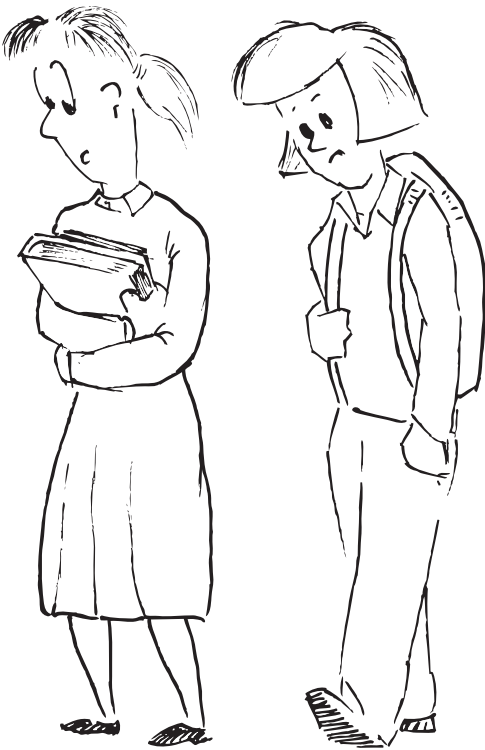
Simplify:  $\frac{24}{42}$

# Student Activity

## Match Up on Fractions

**Match-up:** Match each of the expressions in the squares of the table below with its simplified value at the top. If the solution is not found among the choices A through D, then choose E (none of these).

- A** 1      **B**  $\frac{3}{4}$       **C**  $\frac{7}{8}$       **D** 0      **E** None of these

$\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$	$\frac{7}{6} \div \frac{4}{3}$	$\frac{7}{8} \div 0$	$2\frac{1}{8} - \frac{5}{4}$
$\frac{15}{8} - 1$	$6\left(\frac{1}{6}\right)$	$\frac{2}{3} \div \frac{8}{9}$	$0 \div \frac{3}{4}$
$\frac{1}{2} + \frac{3}{8}$	$\frac{19}{12} - \frac{5}{6}$	 <p>I thought we shared a common denominator, but he was only a fraction of the person I thought he was.</p>	
$\frac{1}{5} + \frac{8}{10}$	$\frac{3}{4}(0)$		
$\frac{1}{2} + \frac{2}{2}$	$\frac{1}{3} \div \frac{1}{3}$		

# Student Activity

## Fractions Using a Calculator

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When you input fractions into a calculator, you must be careful to tell the calculator which parts are fractions. Each calculator has a set of algorithms that tell it what to do first (later on, we will learn the mathematical order of operations, which is similar). In order to ensure that fractions are treated as fractions, for now, you need to tell your calculator which parts ARE fractions.

1. For example, first show that  $\frac{3}{4} \div \frac{2}{5}$  is  $\frac{15}{8}$  by hand:

2. To get the decimal value of  $\frac{15}{8}$  on the calculator, we type  $15/8$  or  $15 \div 8$  (depending on the calculator). Practice by finding the decimal values for:

$$\frac{15}{8}$$

$$\frac{3}{20}$$

$$\frac{1}{4}$$

$$\frac{7}{8}$$

$$\frac{2}{3}$$

3. Now try using your calculator to evaluate  $\frac{3}{4} \div \frac{2}{5}$ , but do it without using any parentheses. Do you get the decimal value equal to  $15/8$ ?

4. Find the button(s) on your calculator that allow you to input parentheses and write down how to use them on your calculator.

5. Try it on your calculator like this now:  $\left(\frac{3}{4}\right) \div \left(\frac{2}{5}\right)$

On my calculators, I type  $(3/4)/(2/5)$  or  $(3 \div 4) \div (2 \div 5)$  to enter this expression. But each calculator is a little different. When you have done it correctly, you should get 1.875.

Write down how to do it on your calculator:

6. Now try these fraction problems *using parentheses* to tell your calculator which numbers represent fractions:

$$\frac{1}{2} \cdot \frac{4}{9}$$

$$\frac{3}{4} \div \frac{2}{15}$$

$$\frac{3}{8} + \frac{4}{5}$$

$$\frac{7}{12} - \frac{1}{5}$$

7. The operation in mixed numbers is **addition**, so when you input  $2\frac{3}{5}$  into your calculator, you must treat it like  $2 + (3/5)$ . What is  $2\frac{3}{5}$  as a decimal? \_\_\_\_\_

# Guided Learning Activity

## Charting the Real Numbers

The set of **natural numbers** is  $\{1, 2, 3, 4, 5, \dots\}$ .

The set of **whole numbers** is  $\{0, 1, 2, 3, 4, 5, \dots\}$ .

The set of **integers** is  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ .

The set of **rational numbers** consists of all numbers that can be expressed as a fraction (or *ratio*) of *integers* (except when zero is in the denominator). Note that all rational numbers can also be written as decimals that either terminate or repeat.

The set of **irrational numbers** consists of all *real* numbers that are *not* rational numbers.

The set of **positive numbers** consists of all the numbers *greater* than zero.

The set of **negative numbers** consists of all the numbers *less* than zero.

**Part I:** Using the definitions above, we will categorize each number below. For each of the numbers in the first column, place an “X” in any set to which that number belongs.

		Natural	Whole	Integer	Rational	Irrational	Positive	Negative
a.	5	X	X	X	X		X	
b.	1							
c.	0							
d.	-2							
e.	$\sqrt{3}$							
f.	$\frac{2}{3}$							
g.	$-1.\overline{4}$							
h.	0.75							
i.	$\frac{\pi}{3}$							

**Part II:** Now we’ll do it backwards. Given the checked properties, find a number (try to use one that is different from one of the numbers in the previous table) that fits the properties. If it is not possible to find a number with all these properties, write “impossible” instead.

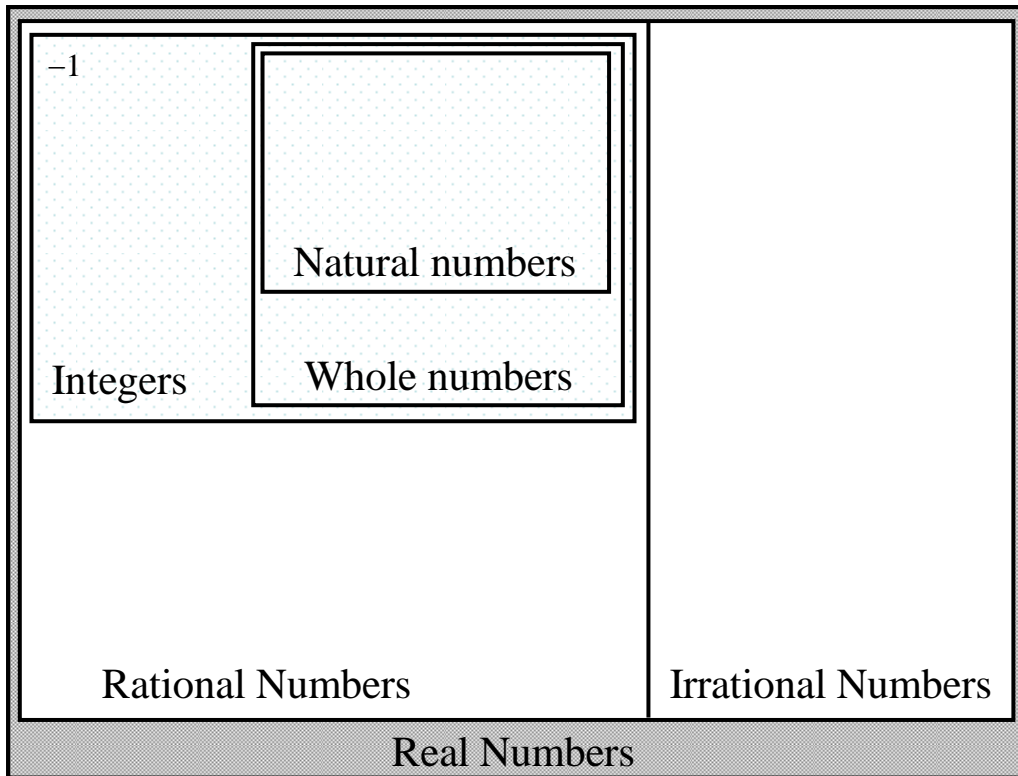
	Number	Natural	Whole	Integer	Rational	Irrational	Positive	Negative
a.				X	X			X
b.					X		X	
c.			X	X	X			
d.						X		X
e.					X			X
f.				X		X	X	
g.		X	X	X	X		X	
h.						X	X	

# Student Activity

## Venn Diagram of the Real Numbers

**Directions:** Place each number below in the *smallest* set in which it belongs. For example,  $-1$  is a real number, a rational number, and an integer, so we place it in the “Integers” box, but not inside the whole numbers or natural numbers.

8      $\frac{7}{3}$       $-1.3$       $\pi$       $2.175$       $0$       $-7$       $\sqrt{2}$       $1000$       $0.00005$



1. Given all possible real numbers, name at least one number that is a whole number, but not a natural number: \_\_\_\_\_
2. Can a number be both rational and irrational? \_\_\_\_\_ If yes, name one: \_\_\_\_\_
3. Can a number be both rational and an integer? \_\_\_\_\_ If yes, name one: \_\_\_\_\_
4. Given all possible real numbers, name at least one number that is an integer, but not a whole number: \_\_\_\_\_

**Side note:** Just for the record, this diagram in no way conveys the actual size of the sets. In mathematics, the number of elements that belong to a set is called the **cardinality** of the set. Technically (and with a lot more mathematics classes behind you) it can be proven that the cardinality of the irrational numbers (uncountable infinity) is actually larger than the cardinality of the rational numbers (countable infinity). Another interesting fact is that the cardinality (size) of the rational numbers, integers, whole numbers, and natural numbers are all equal. This type of mathematics is studied in a course called Real Analysis (that comes after the Calculus sequence).



# Student Activity

## Linking Rational Numbers with Decimals

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Let's investigate why we say that decimals that terminate and repeat are really rational numbers. You will need a calculator and some colored pencils for this activity.

**Rational numbers** consist of all numbers that can be expressed as a fraction (or *ratio*) of *integers* (except when zero is in the denominator).

In the grid below are a bunch of fractions of integers.

1. Work out the decimal equivalents using your calculator. If the decimals are repeating decimals, use an overbar to indicate the repeating sequence (like in the example that has been done for you).
2. Shade the grid squares in which fractions were equivalent to repeating decimals in one color and indicate the color here: \_\_\_\_\_.
3. Shade the grid squares in which fractions were equivalent to terminating decimals in another color and indicate the color here: \_\_\_\_\_.
4. In the last row of the grid, write some of your own fractions built using integers and repeat the steps above.


$\frac{2}{3} = 0.\overline{6}$	$\frac{1}{4} =$	$\frac{7}{8} =$	$\frac{5}{9} =$
$\frac{1}{2} =$	$\frac{4}{9} =$	$\frac{8}{5} =$	$\frac{17}{25} =$
$\frac{1}{1000} =$	$\frac{5}{12} =$	$\frac{3}{4} =$	$\frac{7}{27} =$
$\frac{1}{5} =$	$\frac{12}{5} =$	$\frac{4}{3} =$	$\frac{1}{6} =$


5. Are there any fractions in the grid that were not shaded as either terminating or repeating?
6. If you write one of these fractions as its decimal equivalent, what kind of decimal do you get?

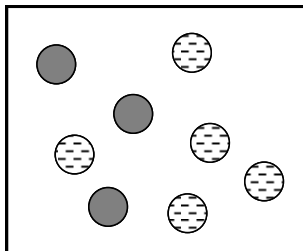
# Guided Learning Activity

## Using Addition Models

**Part I:** The first model for addition of real numbers that we look at is called the “colored counters” method. Traditionally, this is done with black and red counters, but we make a slight modification here to print in black and white.

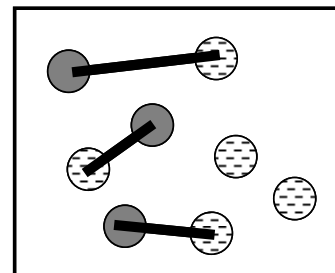
 Solid counters (black) represent positive integers, +1 for each counter.

 Dashed counters (red), represent negative integers, -1 for each counter.

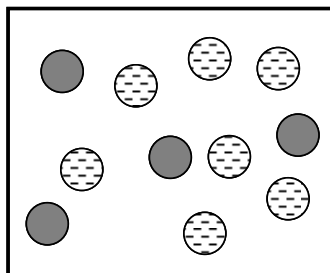


When we look at a collection of counters (inside each rectangle) we can write an addition problem to represent what we see. We do this by counting the number of solid counters (in this case 3) and counting the number of dashed counters (in this case 5). So the addition problem becomes  $3 + (-5) = \underline{\quad}$ .

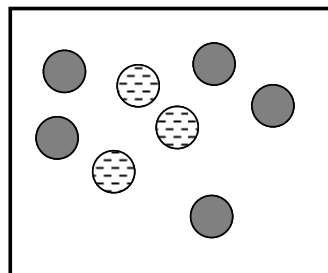
To perform the addition, we use the Additive Inverse Property, specifically, that  $1 + (-1) = 0$ . By matching up pairs of positive and negative counters until we run out of matched pairs, we can see the value of the remaining result. In this example, we are left with two dashed counters, representing the number -2. So the collection of counters represents the problem  $3 + (-5) = -2$ .



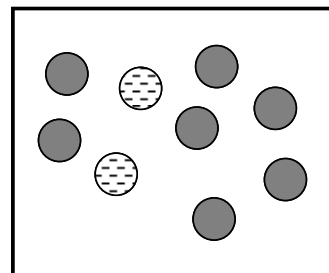
Now try to write the problems that represent the collections below.



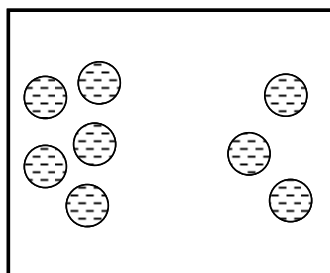
a.  $\underline{\quad} + \underline{\quad} = \underline{\quad}$



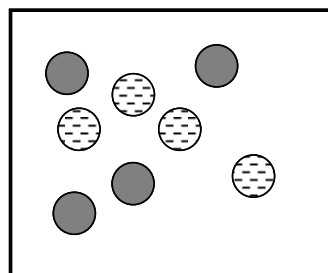
b.  $\underline{\quad} + \underline{\quad} = \underline{\quad}$



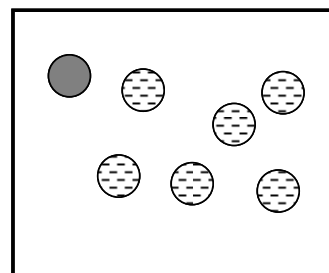
c.  $\underline{\quad} + \underline{\quad} = \underline{\quad}$



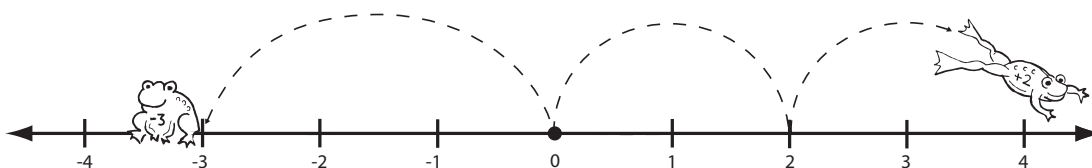
d.  $\underline{\quad} + \underline{\quad} = \underline{\quad}$



e.  $\underline{\quad} + \underline{\quad} = \underline{\quad}$

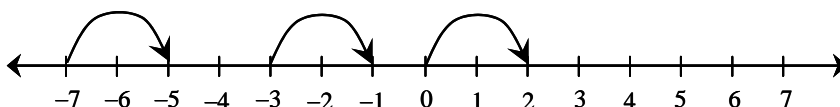


f.  $\underline{\quad} + \underline{\quad} = \underline{\quad}$

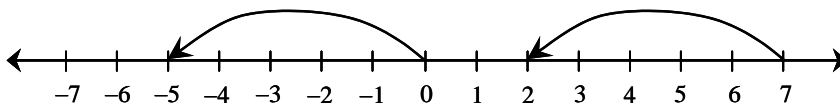


**Part II:** The second model for addition of real numbers that we look at is called the “number line” method. We use directional arcs to represent numbers that are positive and negative. The length that the arc represents corresponds to the magnitude of the number.

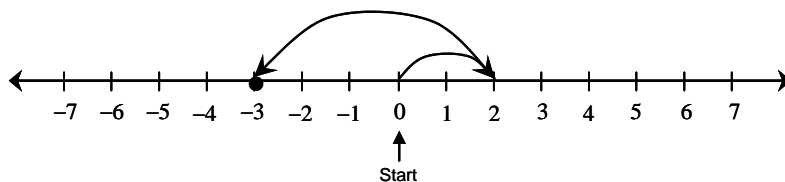
When a directional arc indicates a positive direction (to the right), it represents a positive number. In the diagram below, each arc represents the number 2, because each arc represents a length of two and each arrow points to the right.



In the next diagram, each directional arc represents the number  $-5$ , since each arc represents a length of five and each arrow points to the left.

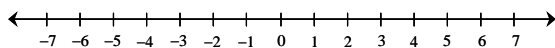


When we want to represent an addition problem, **we start at zero**, and travel from each number to the next using a new directional arc. Thus, the following number line diagram represents  $2 + (-5) = -3$ . The final landing point is the answer to the addition problem.

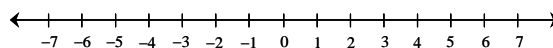


Now try to solve these addition problems on a number line using directional arcs.

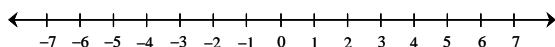
a.  $-3 + 7 = \underline{\hspace{2cm}}$



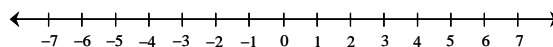
d.  $-6 + 2 = \underline{\hspace{2cm}}$



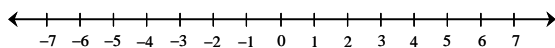
b.  $-2 + (-3) = \underline{\hspace{2cm}}$



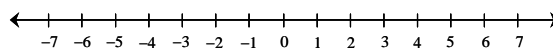
e.  $-2 + (-2) + (-1) = \underline{\hspace{2cm}}$



c.  $5 + (-5) = \underline{\hspace{2cm}}$



f.  $4 + (-8) + 3 = \underline{\hspace{2cm}}$



# Student Activity

## Match Up on Addition of Real Numbers

**Match-up:** Match each of the expressions in the squares of the table below with its simplified value at the top. If the solution is not found among the choices A through D, then choose E (none of these).

**A** 3      **B** -6      **C** -1      **D** 1      **E** None of these

$-8+2$	$-9+8$	$-7+10$	$-3+(-3)$
$-2+(-1)+6$	$3+(-3)$	$-10+(-4)+8$	$-\frac{1}{2}+\left(-\frac{1}{2}\right)$
$6+(-7)$	$5+(-6)$	$-1+(-1)+(-1)+4$	$-11+12$
$-4+\left(-\frac{1}{2}\right)+\frac{1}{2}$	$8+(-5)+(-9)$	$10+(-6)+(-1)$	$-6+12$
$0+(-6)$	$-1+7$	$30+(-27)$	$1+\frac{1}{4}+\frac{1}{2}+\left(-\frac{3}{4}\right)$

# Student Activity

## Scrambled Addition Tables

Here is a simple addition tables with natural number inputs.

**Addition:**

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

**Directions:** The first table that follows is an addition table involving integer inputs. The second table is a *scrambled* addition table with integer inputs (this means that the numbers in the first row and column do not increase nicely like 1, 2, 3, 4). Fill in the missing squares with the appropriate numbers.

+	-3	-2	-1	0	1	2	3
-3							
-2							
-1							
0							
1							
2							
3							

+	15	-5	20	-10	0	5	10
10							
5							
-5							
-15							
20							
0							
15							

**Directions:** The following tables are *scrambled* addition tables with the additional challenge of missing numbers in the shaded rows and columns. Fill in the missing squares with the appropriate numbers.

+	2			3	
-5					-5
-2			-3		
				7	
	8				
		-4		3	

+		-10	15		-5
-20	-15			-20	
			5		
				0	
10					
	10				

# Student Activity

## Signed Numbers Magic Puzzles

**Directions:** In these “magic” puzzles, each row and column adds to be the same “magic” number. Fill in the missing squares in each puzzle so that the rows and columns each add up to be the given magic number.

### Magic Puzzle #1

-2	8	
	-9	4

Magic Number = 5

### Magic Puzzle #2

		4
-5		
8		-6

Magic Number = 0

### Magic Puzzle #3

		$2\frac{1}{4}$
	$2\frac{1}{2}$	$-2\frac{1}{2}$
$\frac{1}{4}$		

Magic Number =  $\frac{1}{2}$

### Magic Puzzle #4

-2		9	-7
	-9		4
8	-4		-5
		-6	

Magic Number = 1

### Magic Puzzle #5

-8	-3		7
	-11	2	6
8	-2		-7

Magic Number = -2

# Guided Learning Activity

## Language of Subtraction

**How do you interpret the – sign?** It is a minus sign if it is **between** two numbers as a mathematical operation. Otherwise, it is a negative.

Other ways to signify minus: difference, less than, subtract ... from ...

Other ways to signify negative: opposite

**How do you tell if less means < or – ?** Look for the distinction between “is less than” and “less than.” See the two examples in the table below.

	<b>Expression? Equation? Or Inequality?</b>	<b>Equivalent statement or phrase in words</b>
$-(-4)$	Expression	the opposite of the opposite of 4 the opposite of negative 4
$8-3$	Expression	the difference of 8 and 3 subtract 3 from 8 8 minus 3 3 less than 8
$8-3=5$	Equation	The difference of 8 and 3 is 5. Subtract 3 from 8 to get 5. 8 minus 3 is 5. 3 less than 8 is 5.
$-2-4$	Expression	the difference of negative 2 and 4 subtract 4 from negative 2 negative 2 minus 4 4 less than negative 2
$-5 < -2$	Inequality	Negative 5 is less than negative 2.
$9-(-2)$	Expression	9 minus negative 2 subtract negative 2 from 9 the difference of 9 and negative 2

Note that expressions are represented in words by phrases (no verb) and equations and inequalities are represented by sentences (with verbs).

Now try these! For any problem with subtraction, find at least two ways to write it in words.

		<b>Expression? Equation? Or Inequality?</b>	<b>Equivalent statement or phrase in words</b>
<b>a.</b>			Zero is less than 8.
<b>b.</b>	$10 - 2 = 8$		
<b>c.</b>			the difference of 2 and negative 5
<b>d.</b>	$-(-10)$		
<b>e.</b>	$5 - (-10) = 15$		
<b>f.</b>			Negative 6 is less than negative 3.
<b>g.</b>			The opposite of negative 5 is 5.
<b>h.</b>	$3 < -(-6)$		
<b>i.</b>	$\frac{1}{2} - \frac{3}{4}$		
<b>j.</b>	$-19 < -18$		



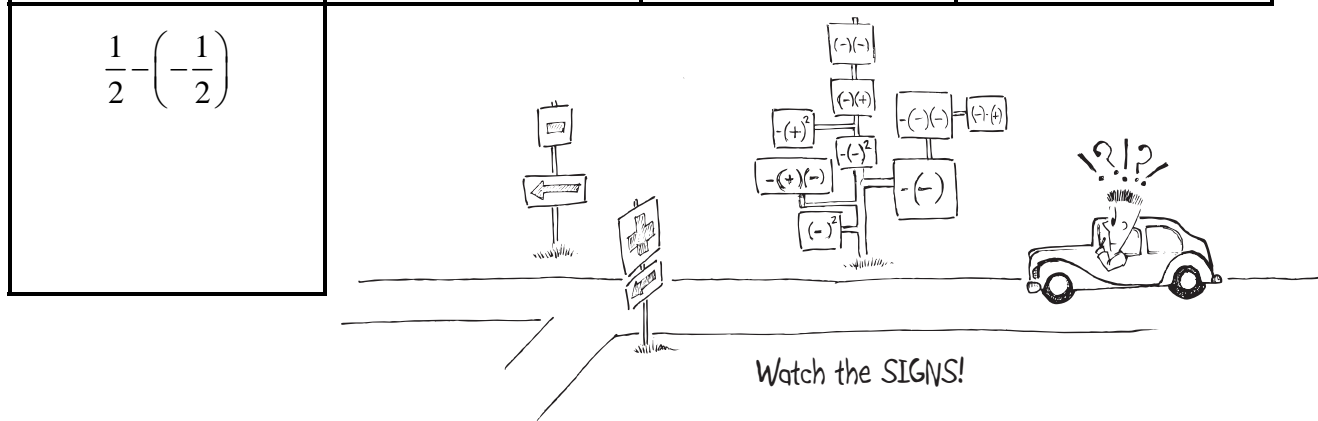
# Student Activity

## Match Up on Subtraction of Real Numbers

**Match-up:** Match each of the expressions in the squares of the table below with its simplified value at the top. If the solution is not found among the choices A through D, then choose E (none of these).

**A** 3      **B** -6      **C** -1      **D** 1      **E** None of these

$-1 - (-4)$	$0 - (-1)$	$-30 - (-25) + (-1)$	$-9 - (-8)$
$2 - (-1)$	$-3 + (-3)$	$3 - (-3)$	$-6 - (-3)$
$0 - (-3)$	$12 - (-6)$	$0 - (-1)$	$3 - 6$
$-\frac{9}{4} - \left(-\frac{1}{2}\right) + \frac{3}{4}$	$\frac{6}{4} - \frac{1}{2}$	$\frac{5}{2} - 4 - 2\frac{1}{2} + 3$	$5\frac{2}{3} - \left(-\frac{1}{3}\right) + (-3)$



# Student Activity

## Signed Numbers Using a Calculator

When you input expressions with signed numbers into a calculator, you must be careful to tell the calculator which “-” signs represent a minus, and which represent a negative.

1. The minus button on your calculator looks like this:  $\boxed{-}$ . It is found with the addition, multiplication, and division functions. The button on your calculator that is used to denote a negative may look like  $\boxed{+/-}$  or  $\boxed{(-)}$ , or it may be above one of the keys, accessed with a 2<sup>nd</sup> function,  $\boxed{2nd}$ . Locate where your calculator input for a negative is, and draw it here:

2. On some calculators, the negative is typed before the number, and on some it is typed after the number. We need to figure out which type you have. We'll calculate  $-2+5$  (which should be \_\_\_\_). Try it both ways. Write down exactly how to do  $-2+5$  on your calculator here:

3. Let's try something more complicated now. How would we write  $-8-3$  in words using the word minus? \_\_\_\_\_  
What should the answer be? \_\_\_\_ Now write down the keystrokes for inputting this expression into your calculator here:

4. Work out each of these expressions by hand, then write down how to express them in words, and finally, write down how to input the keystrokes properly into your calculator.

Expression	Answer	In words (using negative and/or minus)	Keystrokes
$-6+3$			
$5-(-4)$			
$-19-6$			
$-1.25-0.25$			
$10-(-8)$			
$-3-(-3)$			

# Student Activity

## Pick Your Property

**Match-up:** Match each of the equations in the squares of the table below with the proper property of the real numbers.

- A** Associative Property of Addition
- B** Associative Property of Multiplication
- C** Commutative Property of Addition
- D** Commutative Property of Multiplication

- E** Inverse Property of Addition
- F** Inverse Property of Multiplication
- G** Identity Property of Addition
- H** Identity Property of Multiplication
- I** Multiplication Property of Zero

$0 + (-2) = -2$	$77(-2) = (-2)77$	$\left(\frac{1}{4} + \frac{2}{3}\right) + \frac{1}{3} = \frac{1}{4} + \left(\frac{2}{3} + \frac{1}{3}\right)$
$2\left(\frac{3}{100}\right)\left(\frac{100}{3}\right) = 2(1)$	$8(-9 + 9) = 8(0)$	$\left(\frac{1}{4} + \frac{2}{3}\right) + \frac{1}{3} = \left(\frac{2}{3} + \frac{1}{4}\right) + \frac{1}{3}$
$\frac{4}{5}\left(\frac{5}{4}\right) = \frac{5}{4}\left(\frac{4}{5}\right)$	$5 + (6 + (-6)) = (5 + 6) + (-6)$	$\left(2 \cdot \frac{3}{100}\right)\left(\frac{100}{3}\right) = 2 \cdot \left(\frac{3}{100} \cdot \frac{100}{3}\right)$
$5\left(\frac{1}{5} \cdot 1\right) = 5\left(\frac{1}{5}\right)$	$-5 + (-8 + 2) = (-8 + 2) + (-5)$	$5 + (6 + (-6)) = 5 + 0$
$\frac{4}{5}\left(\frac{5}{4}\right) = 1$	$5 + 9(0) = 5 + 0$	$24 + 6(1) = 24 + 6$

# Student Activity

## What's Wrong with Division by Zero?

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Let's spend some time investigating why division by zero is undefined. You will need a calculator for this activity.

1. First, let's see what your calculator thinks. Try the following division problems on your calculator and write down the results:

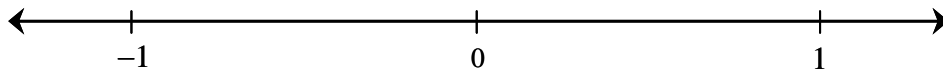
$0 \div 5$

$5 \div 0$

$\frac{12}{0}$

$\frac{0}{12}$

2. Even your calculator will reject the idea of division by zero, so let's try dividing by numbers *close* to zero. Looking at the number line below, name and label some numbers that are really close to zero (on both sides of zero).



3. Find the following quotients using your calculator.

$5 \div 0.1$

$5 \div 0.01$

$5 \div 0.001$

$5 \div 0.0001$

4. What is happening to the quotients in #3 as the divisor gets closer to zero?

5. Find the following quotients using your calculator.

$5 \div (-0.1)$

$5 \div (-0.01)$

$5 \div (-0.001)$

$5 \div (-0.0001)$

6. What is happening to the quotients in problem 5 as the divisor gets closer to zero?

7. Using the results from problems 4 and 6, why do you think that division by zero is undefined?

# Student Activity

RNUM-19

## Match Up on Multiplying and Dividing Real Numbers

**Match-up:** Match each of the expressions in the squares of the table below with its simplified value at the top. If the solution is not found among the choices A through D, then choose E (none of these). Note that some of the expressions involve other operations besides multiplication and division, so be careful!

**A** 2      **B** 0      **C** -8      **D** 12      **E** None of these

$\frac{-6}{-3}$	$-\frac{4}{3}\left(-\frac{3}{2}\right)$	$(-4)(-2)(-1)$	$-2 \div 0$
$(-0.5)(-24)$	$(-0.25)(-8)$	$(0)(-2)(-1)$	$32 \div (-4)$
$36 \div (-3)$	$-3\left(\frac{4}{6}\right)$	$2 \div \left(-\frac{1}{2}\right)$	$-36\left(-\frac{2}{6}\right)$
$4 - (-3) + 5$	$5 \div \left(-\frac{5}{8}\right)$	$\left(-\frac{4}{3}\right)(6)$	$0(-8) - (-2)$
$-2(-2)(-3)(-1)$	$-\frac{1}{3}(-6)$	$-\frac{3}{2} \div \left(-\frac{1}{8}\right)$	$\frac{0}{-5}$

# Student Activity

## The Exponent Trio

**Directions:** In each of the “trios” below, place three equivalent expressions of the following format:

Expanded Expression using multiplication	
Compact exponential expression	Simplified expression

The first one has been done for you. Sometimes there are two possible trios for a simplified exponential expression, so you will see some of these listed twice.

$(-3)(-3)$
$(-3)^2$   9

3·3

2·2·2·2

$-3^2$

$(-2)^4$

$-(2·2·2·2)$

8

$(-2)(-2)(-2)$

$-2^3$

$-(4·4)$

$(-4)^2$

$-10^3$

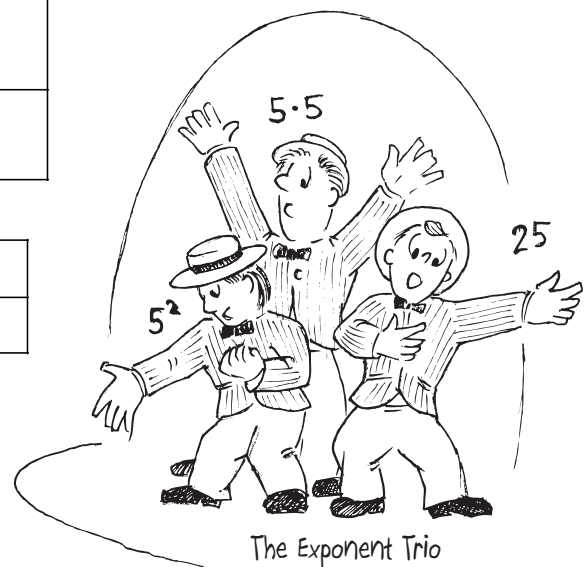
$\left(-\frac{1}{2}\right)^2$

$\frac{1}{4}$

2·2·2·2·2·2·2·2

$(-9)^2$

125



# Student Activity

## Order Operation

**Directions:** With a highlighter, shade the numbers and operation that comes first in the order of operations in each problem. For example, for  $7 - 3 \cdot 2$ , you would highlight  $3 \cdot 2$ . If more than one operation could be done first (at the same time), shade both sets. Once you are certain that you have chosen the **first** steps correctly, then simplify each expression.



1.  $5 + 3 \cdot 5 - 2$

2.  $12 \div 2 \cdot 6 + 1$

3.  $4 - 3^2 + 6$

4.  $6 \cdot 2 - 8 \div 2 + 4$

5.  $-12 - (4 + 3)$

6.  $5 - 2(4 \cdot 3) - 5$

7.  $9 \cdot 2 \div 6 - 5(2)^2$

8.  $\frac{2 - 4}{(-3)^2 + 1}$

9.  $[4 + 2(2 - 5)^2] - 3$

10.  $\frac{3}{4} - \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)^2$

# Assess Your Understanding

RNUM-22

## Real Numbers

For each of the following, describe the strategies or key steps that will help you **start** the problem. You do **not** have to complete the problems.

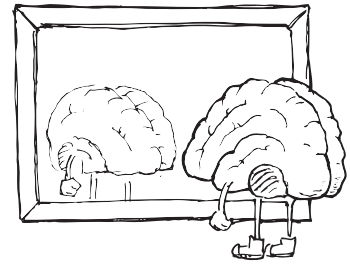
		What will help you to start this problem?
1.	Add: $-3 + (-7)$	
2.	Use $<$ or $>$ to make this statement true: $-3 \square -4$	
3.	Divide: $\frac{3}{4} \div \frac{1}{6}$	
4.	Simplify: $7 - 2 \cdot 3 + 4$	
5.	Subtract: $-2 - (-4)$	
6.	Is 1.5 rational or irrational?	
7.	Add: $\frac{1}{3} + \frac{4}{5}$	
8.	Fill in the missing value to make the statement true: $\frac{2}{5} = \frac{\square}{20}$	
9.	Is $-3$ a natural number? A whole number? An Integer?	
10.	Simplify: $(-4)^4$	



# Metacognitive Skills

## Real Numbers

Metacognitive skills refer to the ability to judge how well you have learned something and to effectively direct your own learning and studying. This is a self-evaluation tool designed to help you focus your studying and to improve your metacognitive skills with regards to this math class.



Fill the 1<sup>st</sup> column out **before** you begin studying. Fill the 2<sup>nd</sup> column out after you study for your test.

Go back to this assessment after your test and circle any of the ratings that you would change – this identifies the “disconnects” between what you **thought** you knew well and what you **actually** knew well.

**Use the scale below to assign a number to each topic.**

- 5 *I am confident I can do any problems in this category correctly.*
- 4 *I am confident I can do most of the problems in this category correctly.*
- 3 *I understand how to do the problems in this category, but I still make a lot of mistakes.*
- 2 *I feel unsure about how to do these problems.*
- 1 *I know I don't understand how to do these problems.*

Topic or Skill	Before Studying	After Studying
Finding the prime-factored form of a number; finding a factor-pair for a number.		
Simplifying a fraction to write it in lowest terms.		
Multiplying or dividing (signed) fractions and simplifying the result.		
Building equivalent fractions.		
Adding and subtracting (signed) fractions and simplifying the result.		
Working with mixed numbers in mathematical expressions.		
Knowing what opposite, inverse and reciprocal mean in terms of real numbers.		
Categorizing numbers in different number sets (Real, rational, natural, etc.)		
Evaluating expressions involving absolute value.		
Using an inequality symbol (< or >) to determine the order of real numbers (like they would be found on a number line).		
Adding and subtracting real numbers (signed numbers).		
Identifying which addition or multiplication property has been used in a statement (commutative, associative, identity, inverse, etc.).		
Solving application problems that involve addition or subtraction of signed numbers.		
Multiplying or dividing real numbers (signed numbers).		
Identifying division by zero and understanding why division by zero is undefined.		
Evaluating or rewriting exponential expressions.		
Distinguishing between exponential expressions that involve parentheses, for example: $(-3)^2$ and $-3^2$ .		
Knowing the rules of the order of operations.		
Applying the order of operations to a numerical expression.		
Applying the order of operations when it involves absolute value or a fraction bar.		
Finding the average of a set of data.		



### Preparing for Your Class

#### Common Vocabulary

- Whole numbers, prime number, composite number
- Prime factorization, prime-factored form, factor (as a noun and as a verb)
- Numerator, denominator, fraction bar, equivalent fractions, building fractions
- Reciprocal, multiplicative identity
- Simplest form, lowest terms, simplify a fraction, mixed number
- Common denominator, least common denominator (LCD)

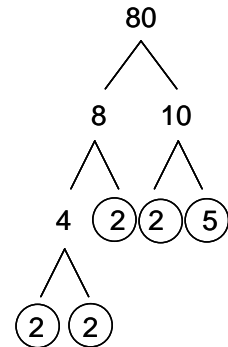
#### Instruction Tips

- If students do not know their multiplication tables, teaching fractions (and later, factoring) will be very difficult. There are numerous resources on the internet (games, flash cards, tutorials) to help students re-learn multiplication tables. Ensuring that students know their multiplication tables may be worth the time up front in your class, because it will save *you* time and frustration later on.
- For the vocabulary of numerator and denominator, this may help your students to remember which is which: The “D”enominator is “D”own below the fraction bar.
- With division of fractions, students often have the preconceived notion that it doesn’t matter which fraction you take the reciprocal of when you change to multiplication; for example, they think that  $\frac{2}{3} \div \frac{4}{5}$  is the same as  $\frac{2}{3} \cdot \frac{5}{4}$  and  $\frac{3}{2} \cdot \frac{4}{5}$ . I suspect that these students are trying to apply “commutative” thinking to an inappropriate situation. Continue to stress that it is the *second* fraction that must be inverted.
- When you build fractions with common denominators, perform the step where you multiply by an equivalent form of 1 *in color* and give the students colored pencils so that they can do the same in their notes. For example,
 
$$\frac{2}{3} + \frac{3}{5} = \frac{2}{3} \cdot \frac{5}{5} + \frac{3}{5} \cdot \frac{3}{3} = \dots$$
 (do the  $\cdot \frac{5}{5}$  and the  $\cdot \frac{3}{3}$  *in color* to show this crucial step)
- In algebraic notation, multiplication becomes an “invisible” operation in many of our expressions. For example, consider that  $3 \cdot x$  is always written as  $3x$  and  $(4) \cdot (5)$  can be written as  $(4)(5)$ . So it should not be surprising that students often believe that a mixed number like  $3\frac{1}{2}$  is really  $3 \cdot \frac{1}{2}$ . You will have to carefully address (and re-address) this issue to affirm that the mixed number notation really represents addition:  $3 + \frac{1}{2}$ .
- You may want to replace the word “reduce” with the word “simplify” when fractions are simplified to their lowest terms. This is because the word *reduce* connotes a reduction in value (for example, to reduce the amount of cholesterol in your diet). We want to emphasize that equivalent fractions have the **same** value (not a lesser value).

## Teaching Your Class

### Primes, composites, and prime factorizations

- A *prime* number is a whole number greater than 1 that has only itself and 1 as factors.
- A *composite* number is a whole number greater than 1 that is *not* prime.
- Give the students a list of numbers (for example: 5, 7, 8, 9, 10, 13, 14, 17...) and ask them to circle the ones that are prime. Note that the smallest prime number is 2.
- It may be helpful to teach factor trees as a method of finding the prime factorization (see diagram). When a prime number is reached in one of the branches, circle it in red to indicate that it is one of the “apples” on the tree. To write the prime-factored form, collect all the “apples” on the tree.



**Examples:** Write the prime-factored form for each number.

- $30 = 2 \cdot 3 \cdot 5$
- $80 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$
- $18 = 2 \cdot 3 \cdot 3$

(be careful, if you have not discussed exponents yet at this point in the term, you may not want to write  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$  as  $2^4 \cdot 5$  just yet)

### Multiplying and dividing fractions

- Go over fraction notation and vocabulary, including the special fraction forms:

$$\frac{a}{a} = 1 \text{ for any non-zero number } a \text{ and } \frac{a}{1} = a$$

- To multiply two fractions, multiply the numerators and multiply the denominators.
- To divide two fractions, multiply the first fraction by the reciprocal of the second.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \text{ and } \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c} \text{ (assuming no division by zero)}$$

**Examples:** Multiply.

- $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$
- $\frac{3}{7} \cdot \frac{2}{5} = \frac{6}{35}$
- $2 \cdot \frac{2}{15} = \frac{2}{1} \cdot \frac{2}{15} = \frac{4}{15}$

**Examples:** Divide.

- $\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \cdot \frac{3}{2} = \frac{9}{10}$
- $\frac{3}{8} \div 2 = \frac{3}{8} \cdot \frac{1}{2} = \frac{3}{16}$
- $\frac{5}{6} \div 3 = \frac{5}{6} \cdot \frac{1}{3} = \frac{5}{18}$

### Building equivalent fractions

- Now that your students know how to multiply fractions, have them practice building equivalent fractions with specific denominators, for example:

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{30} \qquad \frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{30} = \frac{10}{100}$$

- Then make a game of creating new equivalent fractions with any denominator, for example:

$$\frac{3}{4} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad} \quad \text{You may want to use this as a point for a discussion about whether it is possible to make } \frac{3}{4} \text{ have any denominator (can it be written in tenths?).}$$

**Student Activity:** *Building Blocks.* This activity will give students practice building an equivalent fraction and deciding when they might be trying to create an inappropriate equivalent. (RNUM-1)

### Simplest form or lowest terms

Here are several instructional methods for simplifying fractions

- Use prime-factored form and slashes to cross out prime numbers that simplify to be 1:

$$\frac{18}{60} = \frac{2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 5} = \frac{\cancel{2}^1 \cdot 3 \cdot \cancel{3}^1}{\cancel{2}^1 \cdot 2 \cdot \cancel{3}^1 \cdot 5} = \frac{3}{2 \cdot 5} = \frac{3}{10}$$

- Use the GCF and write both the numerator and denominator as a factor-pair involving the GCF.

$$\frac{18}{60} = \frac{6 \cdot 3}{6 \cdot 10} = \frac{\cancel{6}^1 \cdot 3}{\cancel{6}^1 \cdot 10}$$

- Use the any common factor and write both the numerator and denominator as a factor-pair involving the common factor. This may require multiple iterations, but is easier for students with really poor multiplication skills.

$$\frac{18}{60} = \frac{2 \cdot 9}{2 \cdot 30} = \frac{\cancel{2}^1 \cdot 9}{\cancel{2}^1 \cdot 30} = \frac{9}{30} = \frac{3 \cdot 3}{3 \cdot 10} = \frac{\cancel{3}^1 \cdot 3}{\cancel{3}^1 \cdot 10} = \frac{3}{10}$$

### Examples:

- Simplify:  $\frac{10}{16}, \frac{14}{21}, \frac{10}{10}, \frac{15}{18}, \frac{6}{24}, \frac{15}{56}, \frac{18}{45}, \frac{720}{810}$  (note that 15/56 does not simplify)
- Simplify to create equivalent fractions:  $\frac{24}{64} = \frac{\quad}{32} = \frac{\quad}{16} = \frac{\quad}{8}$

**Student Activity:** *Factor Pairings.* This activity will give students practice remembering the factor-pairs for commonly used numbers and show them the connection between these necessary multiplication facts and reducing fractions. (RNUM-2)

## Adding and Subtracting Fractions

- Adding and subtracting in algebra (no matter what the topic) boils down to the ability to identify like terms.
- Consider: 3 dogs + 2 dogs = 5 dogs, but 3 dogs + 2 cats cannot be simplified unless we change the language; for example we could write this as 3 animals + 2 animals and then add to get 5 animals.
- Likewise, with fractions,  $\frac{2}{5} + \frac{1}{2}$  can be written as 2 fifths + 1 half, which does not have consistent units (fifth and half are not like terms). But, using equivalent fractions:  $\frac{4}{10} + \frac{5}{10}$  is like 4 tenths + 5 tenths, which is 9 tenths.
- When you use a whole number, you are counting ones, so  $\frac{2}{3} + 1$  is like 2 thirds + 1 whole, we need to rewrite it so that we have 2 thirds + 3 thirds = 5 thirds.
- Here are two instructional algorithms for finding the least common denominator:

- Find the smallest number that each denominator will divide exactly.

$$\frac{3}{10} + \frac{1}{4} \dots 4 \text{ and } 10 \text{ divide exactly into } 20, 40, 60, \dots \text{ So the LCD is } 20.$$

- Find the prime-factored form for both denominators and then construct the least common denominator from those. This is a useful method later on when you teach rational expressions and need to find an LCD for something like  $5x^3$  and  $25x^2$

$$\frac{3}{10} + \frac{1}{4} \dots \begin{array}{l} 10 = 2 \cdot 5 \\ 4 = 2 \cdot 2 \end{array} \quad \begin{array}{l} 10 = 2 \cdot 5 \\ 4 = 2 \cdot 2 \end{array} \quad \text{LCD: } 2 \cdot$$

$$\begin{array}{l} 10 = 2 \cdot 5 \\ 4 = 2 \cdot 2 \end{array} \quad \text{LCD: } 2 \cdot 2 \cdot$$

$$\begin{array}{l} 10 = 2 \cdot 5 \\ 4 = 2 \cdot 2 \end{array} \quad \text{LCD: } 2 \cdot 2 \cdot 5$$

- It is also good to mention that, although an LCD is preferred, *any* common denominator will work to add or subtract fractions.

**Examples:** Add or subtract as appropriate.

$$\bullet \quad \frac{7}{12} + \frac{5}{12} = \frac{12}{12} = 1$$

$$\bullet \quad \frac{5}{7} - \frac{1}{6} = \frac{30}{42} - \frac{7}{42} = \frac{23}{42}$$

$$\bullet \quad \frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15} = \frac{13}{15}$$

$$\bullet \quad \frac{7}{12} - \frac{1}{2} = \frac{7}{12} - \frac{6}{12} = \frac{1}{12}$$

$$\bullet \quad \frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$$

$$\bullet \quad \frac{3}{4} + 2 = \frac{3}{4} + \frac{8}{4} = \frac{11}{4}$$

$$\bullet \quad \frac{5}{6} + \frac{5}{12} = \frac{10}{12} + \frac{5}{12} = \frac{15}{12}$$

$$\bullet \quad 5 - \frac{4}{3} = \frac{15}{3} - \frac{4}{3} = \frac{11}{3}$$

## Mixed numbers

- When we write a number like  $2\frac{3}{5}$ , the unwritten operation is addition:  $2 + \frac{3}{5}$ .
- Most students will know the final algorithm for converting from a mixed number to an improper fraction. For example in  $2\frac{3}{5}$ , they multiply  $2 \cdot 5$  then add 3 to get  $\frac{13}{5}$ . However, few understand the actual mathematics behind it. Since you have just discussed addition with common denominators, now is an appropriate time to bring it up again.

**Examples:** Write each mixed number as an improper fraction.

- $2\frac{3}{5} = 2 + \frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$
- $5\frac{1}{3} = 5 + \frac{1}{3} = \frac{15}{3} + \frac{1}{3} = \frac{16}{3}$
- Remind students how to go from an improper fraction to a mixed number (divide and use the remainder to build the new proper fraction part). **Examples:**  $\frac{12}{7}$ ,  $\frac{16}{3}$ ,  $\frac{25}{4}$
- If a problem with mathematical operations involves a mixed number, it is usually best to write it as an improper fraction first (but this is not *always* the case if common sense dictates otherwise, consider  $3\frac{1}{2} + 5$ ).

**Student Activity:** *Match Up on Fractions.* The fraction problems in this activity involve all four of the standard mathematical operations. Make sure you have also covered a few examples involving mixed numbers before assigning this activity. These “Match Up” activities work well in groups or with students working in pairs at a whiteboard. (RNUM-3)

**Student Activity:** *Fractions Using a Calculator.* If you allow calculator use, this activity will help your students figure out how to use their calculator to simplify fractions. If you plan to restrict calculator usage until later chapters, you can always come back to this worksheet later on. (RNUM-4)

### Preparing for Your Class

#### Common Vocabulary

- Set, members, elements, braces  $\{ \}$ , set-builder notation, inequality symbol
- Natural number, whole number, integer, rational number, irrational number, real number
- Positive numbers, negative numbers, signed numbers, positive sign, negative sign
- Number line, opposites, graph a number, absolute value
- Terminating decimals, repeating decimals, overbar

#### Instruction Tips

- The *natural* numbers are the ones you naturally use when you count on your fingers (starting at one). These are also called the *counting* numbers.
- A number is either rational or irrational. A *real* number is either positive, negative, or zero.
- Be sure to stress that rational numbers are fractions of *integers*. Otherwise, students will get the mistaken idea that fractions like  $\pi/3$  and  $\sqrt{2}/2$  are rational.
- If you have never taken a course in Real Analysis or Number Theory, be careful if you mention the size of different sets. Technically, we call the number of elements that belong to a set the *cardinality* of the set. It can be proven that the cardinality of the irrational numbers (uncountable infinity) is actually larger than the cardinality of the rational numbers (countable infinity). Another interesting fact is that the cardinality (size) of the rational numbers, integers, whole numbers, and natural numbers are all equal. So, if you say that the set of natural numbers is smaller than the set of integers, it would be *incorrect*. You *can* say that the set of natural numbers is a *subset* of the integers.
- It is common for students to have a preconceived notion that all fractions must fall between 0 and 1 on the number line. While these students *will* be sure that  $2\frac{1}{3}$  must fall between 2 and 3, they will also be sure that  $\frac{7}{3}$  must fall between 0 and 1.
- Because we also use the wording “less than” and “greater than” when we describe addition and subtraction (3 less than  $x$  is written  $x-3$ ), be sure to use “is less than” and “is greater than” when you describe the two inequality symbols,  $<$  and  $>$ .
- If you tip the written  $<$  symbol slightly, a capital L is made (L = Less). It helps to make the “L” with your index finger and thumb (left hand), and then tip it to show the  $<$  symbol.
- There is another common student misconception in this section, about inequalities and negative numbers. For example, students often believe that  $-8$  is greater than  $-4$ . Consider that for the majority of their education, students were probably taught something like the inequality “eats” the *bigger* number. Well, 8 still seems bigger than 4, and so, to these students,  $-8$  does seem bigger than  $-4$  (of course, the *magnitude* of  $-8$  is greater than the magnitude of  $-4$ ). If you continue to emphasize that the “lesser number is on the left” you may be successful at eventually overcoming this mistaken student preconception.
- Try to avoid using a “blank” between pairs of numbers in missing inequalities, like this one:  $-3 \underline{\hspace{1cm}} -4$ , because that blank begins to look an awful lot like part of a  $\leq$  or  $\geq$ . You can just use an empty box, like in this one:  $-3 \square -4$ .
- Students may incorrectly develop the idea that an absolute value means to change the sign. If you *only* show examples with negative numbers inside of absolute value symbols, it will be very easy for students to learn the concept of absolute value incorrectly.



## Teaching Your Class

**Guided Learning Activity:** *Charting the Real Numbers.* This is a guided lecture activity to help students learn about the various sets of numbers. You and the students should work through this together. For Part II, you could ask students to put all their answers on the board so that you and the class can discuss which student responses are correct. (RNUM-5)

### Set-builder Notation

- A set is denoted with braces:  $\{ \}$
- The notation  $x | \dots$  is read “ $x$  such that...”
- $\{x | x \text{ is a positive integer}\}$  would be the set  $\{1, 2, 3, \dots\}$  (or the natural numbers)
- $\{a | a \text{ is a multiple of } 5\}$  would be the set  $\{5, 10, 15, \dots\}$

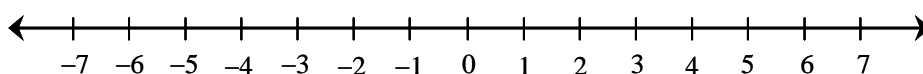
**Negative Fractions:**  $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$  (there can only be one negative sign, but it doesn't matter where you see it, as long as it is *before* a number or the fraction)

**Student Activity:** *Venn Diagram of the Real Numbers.* In this activity, students are given a large Venn diagram showing the nested sets of numbers. Students must place numbers into their appropriate set in the Venn diagram. Emphasize that the students should *read the directions* on this one. (RNUM-6)

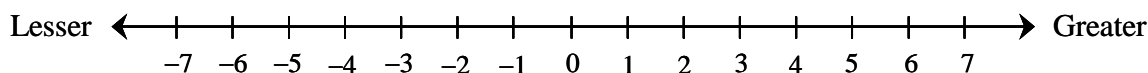
**Student Activity:** *Linking Rational Numbers with Decimals.* Students often do not make the connection that all fractions of integers are either terminating or repeating decimals. This activity will hopefully cement this concept. You will need calculators for this activity. (RNUM-7)

### Number Lines

- Draw a large number line on the board to use with your discussion/lecture



- Ask the class to tell you where on the number line to **graph** these numbers with a solid dot:  
 $3, -4, 4.2, \frac{3}{4}, -5\frac{1}{2}, \pi, \sqrt{2}, \frac{7}{3}, -6.8$
- One number is less than another if it is to the *left* on the number line. You might modify your number line like this:



### Inequalities

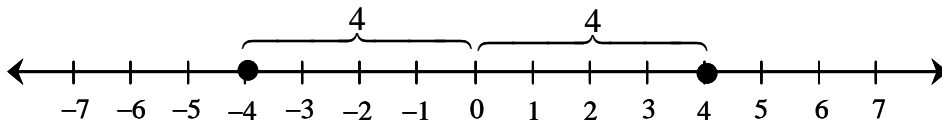
- The inequality symbol  $<$  means “is less than” and  $>$  means “is greater than.”
- Emphasize that the “lesser number is on the left.”

**Examples:** Use  $<$  or  $>$  to make each statement true.

- $5 \square 2 \quad >$
- $-3 \square -1 \quad <$
- $0 \square -4 \quad >$
- $3 \square \pi \quad <$
- $\frac{2}{3} \square \frac{3}{4} \quad <$
- $-\frac{1}{3} \square -\frac{4}{3} \quad >$

### Absolute Value

- The absolute value of a number is its distance from zero on the number line. Distances are always positive numbers. Here is an example that shows  $|-4| = 4$  and  $|4| = 4$ .



- Numbers like  $-4$  and  $4$  are called **opposites** because they are the same distance from zero, but on opposite sides of it.

**Examples:** Evaluate.

- $|-1| = 1$
- $|8.3| = 8.3$
- $|\frac{-3}{4}| = \frac{3}{4}$
- $|0| = 0$

**Examples:** Use  $=$ ,  $<$ , or  $>$  to make each statement true.

- $|-7| \square 7 \quad =$
- $|-4| \square |-5| \quad <$
- $0 \square |-2| \quad <$
- $-2 \square |-2| \quad <$

### Preparing for Your Class

#### Common Vocabulary

- Commutative Property of Addition
- Associative Property of Addition
- Addition Property of Zero, identity element
- Inverse Property of Addition, opposites, additive inverses

#### Instruction Tips

- When you are teaching about properties of addition, be careful not to use generalizations like “if you see parentheses, you know the associative property is being used” – consider an example like  $-5 + (3 + 0) = -5 + 3$ , which shows the Addition Property of Zero, but uses parentheses.
- Parentheses are always used around a trailing negative number in sums, like  $2 + (-3)$  or  $-2 + (-2)$ . Consider that with many operations, it would not make sense to allow two adjacent operations, for example, we would not write  $8 \div \div 2$  or  $2 \cdot \div 4$ . So in the instance of addition, it is not proper notation to have two adjacent operations unless they are separated by parentheses.
- Don't be surprised if students think that the opposite of  $a$  is  $1/a$  instead of  $-a$ . You may have to review the difference between an opposite and a reciprocal
- An inverse reverses the mathematical operation. For example, addition and subtraction are inverses, as are multiplication and division. For the process of addition, another word for the additive inverse is the “opposite.”
- While it might seem helpful to write down a list of “rules” for students to follow for addition of signed numbers, it is important that students construct a working set of “rules” in their own minds. Use the activity *Using Addition Models* to provide students with at least two ways they can visualize the process of addition and of signed numbers.

### Teaching Your Class

**Guided Learning Activity:** *Using Addition Models*. This is a guided lecture activity to help the students understand the concepts of adding signed numbers. The activity uses the colored-counter model and the number-line model to help the student develop their conceptual understanding of the addition of signed numbers. (RNUM-8)

**Properties of Addition:**

- Commutative Property of Addition:  $a + b = b + a$   
(the word commute means to go back and forth, like *commuting* between work and home)
- Associative Property of Addition:  $a + (b + c) = (a + b) + c$   
(the word associate means to join a group, like the friends you *associate* with)
- Addition Property of Zero:  $a + 0 = a$   
(0 is the identity element for addition, think of *identical* twins)
- Inverse Property of Addition:  $a + (-a) = 0$  and  $-a + a = 0$

NOTE: Be careful to not use subtraction in your examples, or you could get yourself into trouble having to explain the distributive property or subtraction of negatives a bit early.

**Examples:** Identify the property of addition that is used.

- $(3 + \frac{1}{4}) + (-\frac{3}{4}) = 3 + (\frac{1}{4} + (-\frac{3}{4}))$  (associative)
- $500 + 57 + 300 = 500 + 300 + 57$  (commutative)
- $-4 + 4 = 0$  (additive inverse)
- $(-2 + 3) + 10 = 10 + (-2 + 3)$  (commutative)
- $-5 + (3 + 0) = -5 + 3$  (property of zero)
- $6 + (-2 + 2) = 6 + 0$  (additive inverses)
- $-4 + (-2 + 5) + 8 = (-4 + (-2)) + (5 + 8)$  (associative)

**Student Activity:** *Match Up on Addition of Real Numbers.* This activity works well in groups or with students working in pairs at a whiteboard (have students draw an empty grid on the board for their answers). This activity also involves fractions, so students will get more practice with this topic. (RNUM-10)

**Student Activity:** *Scrambled Addition Tables.* This activity is based on the standard “addition tables” learned in elementary school. The scrambled tables can be quite challenging, students may find it helpful to write out the mathematical sentence for each empty box. (RNUM-11)

**Student Activity:** *Signed Numbers Magic Puzzles.* In this activity, students get practice with addition of signed numbers in an interesting set of puzzles. Because they also have to work with finding the “missing” numbers for a sum, they will also begin to lay the foundation for subtraction of signed numbers. Note: If you’re familiar with Magic Squares, these puzzles are basically the same, except that the diagonals do not sum to the same value as the rows and columns. (RNUM-12)

### Preparing for Your Class

#### Common Vocabulary

- Opposite of an opposite
- Range (as in the difference between a maximum and minimum value)

#### Instruction Tips

- Students may have trouble distinguishing between when a  $-$  sign is identified as a minus, and when it is identified as a negative. If the  $-$  is **between** two numbers as an operation, then it is minus. Otherwise, it is a negative. This may seem trivial, but on most calculators, a student must be able to distinguish between when to use the negative keys:  $\boxed{+/-}$  or  $\boxed{(-)}$  and the minus key:  $\boxed{-}$ .
- Be careful about saying something like “a double-negative can always be replaced by a plus sign.” Consider these counterexamples:  $-(-2+5)$  and  $-|-5|$ .
- Often students are confused about why an expression like  $3-(-2)$  is not equal to  $3(2)$ . This is not surprising, as we write:  $-(-a) = a$ , so  $-(-2)$  would seem to equal 2 with no plus sign. If you approach this example from a language point-of-view, it is 3 **minus** negative 2. In no way can we change subtraction into multiplication; we can only change between subtraction and addition algebraically.

### Teaching Your Class

**Guided Learning Activity:** *Language of Subtraction.* There are so many ways to interpret a  $-$  sign and to translate it into words. This classroom activity helps students begin to make proper translations between natural language and mathematical language. (RNUM-13)

**Opposite of an opposite:**  $-(-a) = a$

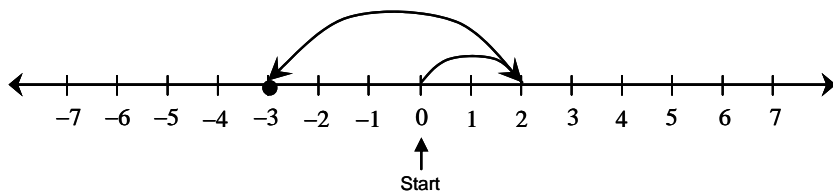
- In English, we use double-negatives quite a bit, here's an example:  
*I passed the test.* (a positive statement)  
*I failed the test.* (a negative statement)  
*I didn't fail the test.* (a double negative; the same meaning as the positive statement)

**Examples:** Simplify or subtract.

- $-(-4) = 4$
- $-|-2| = -2$
- $6+(-1) = 5$
- $1-(-6) = 7$
- $0-(-5) = 5$
- $-(-x) = x$

**Subtraction as Addition:**  $a - b = a + (-b)$ 

- If you used number line models when showing addition of real numbers, this would be a good way to show this property. If addition is represented by a directional arc pointing to the right, then it follows that subtraction is represented by a directional arc that points to the left. Thus,  $2 - 5 = -3$  would look like the diagram below. As would  $2 + (-5) = -3$ .

**Examples:** Subtract.

- $3 - 8 = -5$
- $7 - (-2)$  (write as  $7 + 2$ )  $= 9$
- $-\frac{1}{3} - \frac{2}{3} = -\frac{3}{3} = -1$
- $-3 - 9 = -12$
- $0 - |-5|$  (write as  $0 - 5$ )  $= -5$
- $-\frac{1}{3} - \left(-\frac{1}{3}\right)$  (write as  $-\frac{1}{3} + \frac{1}{3}$ )  $= 0$
- $-6 - 8 = -14$

**Student Activity:** *Match up on Subtraction of Real Numbers.* This activity works well in groups or with students working in pairs at a whiteboard. This activity also involves fractions and addition, so students will get more practice with these topics. (RNUM-15)

**Applications of Addition and Subtraction of Real Numbers:**

- Changes in temperature (maximum minus minimum to get temperature range)
- Changes to checkbook balance (current balance minus withdrawals plus deposits)
- Changes to the water level in a reservoir or storage tank (old level plus or minus change)

**Student Activity:** *Signed Numbers Using a Calculator.* If you are going to allow calculator use in this section, this will help your students figure out how to use their calculator to perform calculations with signed numbers. If you plan to restrict calculator usage until a later section, you can always come back to this activity later. (RNUM-16)

### Preparing for Your Class

#### Common Vocabulary

- Commutative Property of Multiplication
- Associative Property of Multiplication
- Multiplication Property of Zero
- Inverse Property of Multiplication, reciprocal, multiplicative inverses
- Identity Property of Multiplication

#### Instruction Tips

- Students get quite used to seeing parentheses used for the second term of addition or subtraction problems that involve negative numbers. If students are not careful readers, they easily confuse expressions like  $2 + (-3)$ ,  $2(-3)$ , and  $2 - (3)$ . One way to help them to see the proper operations is to ask students to read these problems out loud first before they attempt working the problem.
- Students often have a difficult time remembering whether it's  $\frac{a}{0}$  or  $\frac{0}{a}$  that is undefined. A silly way to remember it is that in  $\frac{a}{0}$ , there is a "hole" on the bottom; the number on top falls into the hole, never to be seen again... so it no longer exists, and is undefined. Using the student activity, *What's Wrong with Division by Zero?* should help to provide a more concept-oriented way for students to remember why it is that  $\frac{a}{0}$  is undefined.
- When you are teaching about properties, be careful not to use generalizations like "if you see parentheses, you know the associative property is being used" – consider an example like  $(3 \cdot x)2 = 2(3 \cdot x)$ , which is commutative, but uses parentheses.

### Teaching Your Class

You may find it helpful to review the Properties of Addition and then show the corresponding Properties of Multiplication. The Multiplication Property of Zero has no corresponding property in addition.

#### Properties of Multiplication:

- Commutative Property of Multiplication:  $a \cdot b = b \cdot a$   
(the word *commute* means to go back and forth)
- Associative Property of Multiplication:  $a \cdot (b \cdot c) = (a \cdot b) \cdot c$   
(the word *associate* means to join a group)
- Multiplication Property of Zero:  $a \cdot 0 = 0$
- Identity Property of Multiplication:  $a \cdot 1 = a$  (1 is the identity element)
- Inverse Property of Multiplication:  $a \cdot \frac{1}{a} = 1$  (provided  $a \neq 0$ )

**Examples:** Identify the property of multiplication.

- $3(-2) = -2(3)$  (commutative)
- $2 + 3\left(\frac{1}{3}\right) = 2 + 1$  (inverse property)
- $\left(-4 \cdot \frac{1}{5}\right) \cdot 5 = -4\left(\frac{1}{5} \cdot 5\right)$  (associative)
- $-6 + 4(1) = -6 + 4$  (identity)
- $(3 \cdot x)2 = 2(3 \cdot x)$  (commutative)
- $6 \cdot 0 + (-3) = 0 + (-3)$  (property of 0)

**Student Activity:** *Pick Your Property.* In this activity, students practice identifying both the properties of addition and multiplication (associative, commutative, inverse, identity, and the zero property). (RNUM-17)

**Student Activity:** *What's Wrong with Division by Zero?* This activity will require a calculator, but should help students understand why division by zero has to be undefined without going into asymptotes and calculus. (RNUM-18)

**Properties of Division:**

- Division Properties:  $\frac{a}{1} = a$  and  $\frac{a}{a} = 1$  (where  $a \neq 0$ )
- Division Involving 0:  $\frac{0}{a} = 0$  and  $\frac{a}{0}$  is undefined (where  $a \neq 0$ )

**Examples:** Simplify.

- $\frac{-3}{-3} = 1$
- $\frac{-5}{1} = -5$
- $\frac{0}{-2} = 0$
- $\frac{1}{7}(7) = 1$
- $\frac{-4}{0}$  undefined

**Multiplication or Division of Real Numbers:**



Note: The above chart is for non-zero real numbers.



**Examples:** Simplify.

- $(-3)(-7) = 21$
- $-12(5) = -60$
- $(-2)(-2)(-3) = -12$
- $\frac{-16}{-8} = 2$
- $-\frac{7}{9}\left(\frac{3}{2}\right) = -\frac{7}{6}$
- $\frac{-56}{8} = -7$
- $-\frac{2}{3} \div \left(\frac{4}{9}\right) = -\frac{3}{2}$
- $\frac{18}{-3} = -6$
- $(3)(-9) = -27$

**Student Activity:** Match up on *Multiplying and Dividing Real Numbers*.

This activity works well in groups or with students working in pairs at a whiteboard. This activity also involves fractions, and a few problems with addition or subtraction, so students will get more practice with these topics.

(RNUM-19)

**Examples with expressions that look alike:** Simplify.

- $\frac{8-2}{6} \quad \frac{8(-2)}{-16} \quad \frac{8+(-2)}{6} \quad \frac{8 \div (-2)}{-4}$
- $\frac{-6(-3)}{18} \quad \frac{-6-3}{-9} \quad \frac{-6+(-3)}{-9} \quad \frac{-6-(-3)}{-3} \quad \frac{-6(-3)}{18}$
- $\frac{0+(-2)}{-2} \quad \frac{0-(-2)}{2} \quad \frac{0 \div (-2)}{0} \quad \frac{0(-2)}{0}$

### Preparing for Your Class

#### Common Vocabulary

- Exponential notation, exponent, base, exponential expression
- Order of operations
- Grouping symbols: ( ), [ ], | |, ——— (fraction bar)
- Innermost pair, outermost pair (as in parentheses or grouping symbols)
- Arithmetic mean, average

#### Instruction Tips

- A common student misconception is that  $(-3)^2$  and  $-3^2$  are the same. Likely this is because at this point in the course, the students don't think the parentheses in math really do much of anything. For example, we rewrite  $3-8$  as  $3+(-8)$ , but practically, the result does not change. And so, students get the idea that parentheses don't have much meaning, which is, of course, wrong. If you continue to stress the expanded notation for these expressions:  $(-3)^2 = (-3)(-3)$  and  $-3^2 = -(3)(3)$  or  $-(3 \cdot 3)$ , your students will eventually understand the difference. You should prepare yourself to re-explain this concept *every* time it comes up. In fact, rather than explaining it yourself, ask the student who is confused to try to explain the difference to you instead.
- Often instructors teach the order of operations with “**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally” to stand for **P**arentheses, **E**xponents, **M**ultiplication, **D**ivision, **A**ddition, and **S**ubtraction. Unfortunately, this leaves students with the mistaken impression that multiplication always comes before division, which is not true. Multiplications and divisions are evaluated from left to right. So perhaps it is better to say “**P**lease **E**xcuse **M**y **D**ear **L**ittle **R**elative, **A**unt **S**ally” to remind students about the left-to-right part.
- Some expressions are more “tempting” to incorrectly evaluate than others. These expressions give strong visual cues for the student to perform an incorrect operation first. For example, in  $5-2 \cdot 3+8$ , the  $5-2$  and the  $3+8$  are very tempting visually. Consider that the students may be much more comfortable with addition and subtraction symbols than they are with the raised dot symbol for multiplication, and so their eyes are drawn into the wrong symbols first. Also, the  $-$  and  $+$  symbols print much larger than the  $\cdot$  symbol, and are naturally more noticeable. Here's a few more “tempting” examples and why they tempt students:
  - $6-2|3-8|$  (doesn't this just look like two side-by-side subtraction problems?)
  - $\frac{2+6}{2}$  (students really want to “cancel” the 2's here)
  - $8 \div 2 \cdot 16$  (if students have a misconception of multiplication before division, they'll do this one wrong)
  - $2 \cdot 3^2$  (now the  $2 \cdot 3$  is more familiar than the  $3^2$  part, so they want to multiply first)

## Teaching Your Class

### Language of Exponents:

- Use  $5^2$  and  $3^4$  to discuss the vocabulary *base* and *exponent*.
- The notation  $5^2$  means the factor 5 is repeated 2 times:  $5 \cdot 5$ , the notation  $3^4$  means that the factor 3 is repeated 4 times:  $3 \cdot 3 \cdot 3 \cdot 3$ .
- Discuss the base and exponent for  $(-5)^2$  and  $-5^2$

### Examples:

- $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$
- $(-3)^4 = (-3)(-3)(-3)(-3) = 81$
- $(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$
- $(-2)^3 = (-2)(-2)(-2) = -8$
- $(-3)^2 = (-3)(-3) = 9$
- $-3^2 = -(3 \cdot 3) = -9$

**Even and Odd Powers of a Negative Number:** When a negative number is raised to an even power, the result is positive. When a negative number is raised to an odd power, the result is negative.

**Student Activity:** *The Exponent Trio.* This activity will give students practice with converting between expanded exponential expressions, exponential form, and evaluating exponential expressions. This lays nice groundwork for eventually understanding why, for example, the solution to  $x^2 = 9$  is both 3 and  $-3$ . (RNUM-20)

### The Order of Operations:

The diagram found on the last page of this teaching guide can be copied to an overhead, projected from a portable video projector, or projected from a computer and ceiling projector to a whiteboard.

**Student Activity:** *Order Operation.* In this activity, there are many examples where students “jump” at an incorrect first step. The idea is to carefully decide on a first step; then evaluate each expression. Watch for those “tempting” mistakes that students make in this activity. Each student will need a highlighter for this activity. (RNUM-21)

**Arithmetic mean (or average):**

- To find the mean of a set of values, divide the sum of the values by the number of values.

**Example:** Find the mean temperature for the following readings in degrees Celsius:

$-3^{\circ}$ ,  $6^{\circ}$ ,  $20^{\circ}$ , and  $5^{\circ}$  (answer is  $7^{\circ}\text{C}$ )

**Assess Your Understanding** *Real Numbers*. This activity is designed to help students see the big picture and to go back through all that they have learned in this chapter. Students should try to write out, in words, how to start each problem, as if they were explaining it to a friend. (RNUM-22)

**Metacognitive Skills** *Real Numbers*. This assessment provides a list of learning objectives for the chapter. Students can use this to gain some insight into how well they actually know what they think they know. (RNUM-23)

# Order of Operations

Perform calculations inside **parentheses** (fraction bars, and absolute values) working from innermost to outermost pairs.

Evaluate **exponential expressions**.

Perform **multiplications** and **divisions** as they occur from left to right.

Perform **additions** and **subtractions** as they occur from left to right.



$( )$   $\pi$   $\cdot$   $\%$   $+$



please excuse my dear little relative, Aunt Sally ...



**Solution** *Building Blocks* (RNUM-1)

Goal:			
$\frac{1}{4}$	$\frac{6}{24}$	$\frac{9}{36}$	$\frac{4}{16}$

Goal:			
$\frac{3}{16}$	$\frac{6}{32}$	<del><math>\frac{4}{4}</math></del>	$\frac{15}{80}$

Goal:			
$\frac{3}{2}$	$\frac{75}{50}$	$\frac{12}{8}$	$\frac{1500}{1000}$

Goal:			
$\frac{5}{8}$	$\frac{30}{48}$	$\frac{20}{32}$	<del><math>\frac{60}{60}</math></del>

Goal:			
$\frac{3}{3}$	$\frac{9}{9}$	$\frac{15}{15}$	$\frac{99}{99}$

Goal:			
$\frac{7}{9}$	$\frac{77}{99}$	$\frac{63}{81}$	$\frac{42}{54}$

Goal:			
$\frac{2}{3}$	$\frac{12}{18}$	$\frac{10}{15}$	<del><math>\frac{1}{1}</math></del>

Goal:			
$\frac{7}{5}$	$\frac{35}{25}$	$\frac{49}{35}$	<del><math>\frac{14}{14}</math></del>

**Solution** *Factor Pairings* (RNUM-2)

<b>12</b> 1·12 2·6 3·4	<b>20</b> 1·20 2·10 4·5	<b>24</b> 1·24 2·12 3·8 4·6	<b>28</b> 1·28 2·14 4·7	<b>30</b> 1·30 2·15 3·10 5·6	<b>32</b> 1·32 2·16 4·8
<b>36</b> 1·36 2·18 3·12 4·9 6·6	<b>42</b> 1·42 2·21 3·14 6·7	<b>45</b> 1·45 3·15 5·9	<b>50</b> 1·50 2·25 5·10	<b>60</b> 1·60 2·30 3·20 4·15 5·12 6·10	<b>64</b> 1·64 2·32 4·16 8·8

1.  $10, \frac{2}{3}$       2.  $4, \frac{7}{9}$       3.  $12, \frac{3}{5}$       4.  $6, \frac{4}{7}$

**Solution** Match up on Fractions (RNUM-3)

B	C	E	C
C	A	B	D
C	B		
A	D		
E	A		

**Solution** Fractions Using a Calculator (RNUM-4)

$$1. \frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$$

$$2. \frac{15}{8} = 1.875; \frac{3}{20} = 0.15; \frac{1}{4} = 0.25; \frac{7}{8} = 0.875; \frac{2}{3} = 0.6666... = 0.\bar{6}$$

3. No

4-5. Keystrokes will vary by calculator.

$$6. \left(\frac{1}{2}\right) \cdot \left(\frac{4}{9}\right) = \frac{4}{18} = 0.\bar{2}; \left(\frac{3}{4}\right) \div \left(\frac{2}{15}\right) = \frac{45}{8} = 5.625;$$

$$\left(\frac{3}{8}\right) + \left(\frac{4}{5}\right) = \frac{47}{40} = 1.175; \left(\frac{7}{12}\right) - \left(\frac{1}{5}\right) = \frac{23}{60} = 0.38\bar{3}$$

7. 2.6

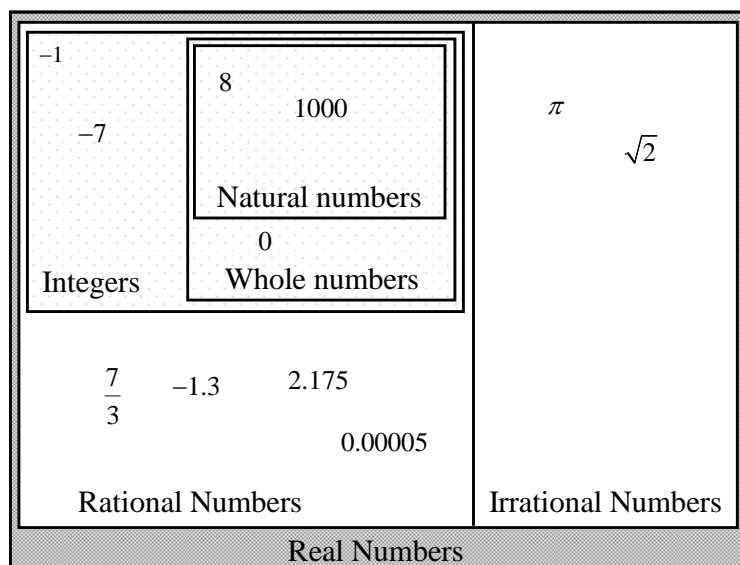
**Solution** Charting the Real Numbers (RNUM-5)**Part I:**

		Natural	Whole	Integer	Rational	Irrational	Positive	Negative
a.	5	X	X	X	X		X	
b.	1	X	X	X	X		X	
c.	0		X	X	X			
d.	-2			X	X			X
e.	$\sqrt{3}$					X	X	
f.	$\frac{2}{3}$				X		X	
g.	$-1.\bar{4}$				X			X
h.	0.75				X		X	
i.	$\frac{\pi}{3}$					X	X	

**Part II:** Answers will vary; an example of each is given below. Ask students to put all their answers on the board for parts a through h so that the class can discuss which are correct.

**Example answers:** a. -6   b. 3.2   c. 0   d.  $-\sqrt{3}$    e.  $-\frac{1}{4}$    f. impossible   g. 4   h.  $\pi$



**Solution** Venn Diagram of the Real Numbers (RNUM-6)

1. The only number that is a whole number and not a natural number is 0.
2. No. 3. Yes. All integers are rational numbers, examples:  $5 = \frac{5}{1}$ ,  $2 = \frac{2}{1}$ ,  $-3 = \frac{-3}{1}$
4. All of the negative integers are whole numbers, examples:  $\dots, -4, -3, -2, -1$

**Solution** Linking Rational Numbers with Decimals (RNUM-7)

1-4. See grid answers below.

$\frac{2}{3} = 0.\overline{6}$	$\frac{1}{4} = 0.25$	$\frac{7}{8} = 0.875$	$\frac{5}{9} = 0.\overline{5}$
$\frac{1}{2} = 0.5$	$\frac{4}{9} = 0.\overline{4}$	$\frac{8}{5} = 1.6$	$\frac{17}{25} = 0.68$
$\frac{1}{1000} = 0.001$	$\frac{5}{12} = 0.4\overline{16}$	$\frac{3}{4} = 0.75$	$\frac{7}{27} = 0.\overline{259}$
$\frac{1}{5} = 0.2$	$\frac{12}{5} = 2.4$	$\frac{4}{3} = 1.\overline{3}$	$\frac{1}{6} = 0.1\overline{6}$
Answers will vary.	Answers will vary.	Answers will vary.	Answers will vary.

5. No. All the boxes in the grid should be shaded.
6. Fractions of integers will always be either a repeating or terminating decimal.

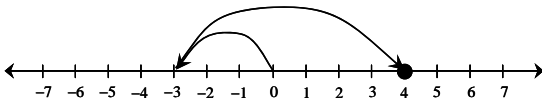
**Solution** *Using Addition Models (RNUM-8)*

**Part I:**

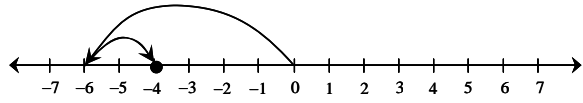
- a.  $4 + (-7) = -3$  or  $-7 + 4 = -3$       b.  $5 + (-3) = 2$  or  $-3 + 5 = 2$   
 c.  $7 + (-2) = 5$  or  $-2 + 7 = 5$       d.  $-5 + (-3) = -8$  or  $0 + (-8) = -8$   
 e.  $4 + (-4) = 0$  or  $-4 + 4 = 0$       f.  $1 + (-6) = -5$  or  $-6 + 1 = -5$

**Part II:**

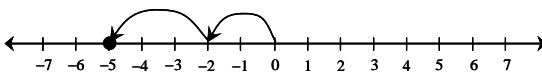
a.  $-3 + 7 = 4$



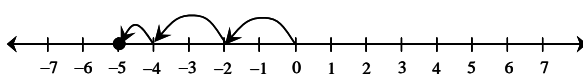
d.  $-6 + 2 = -4$



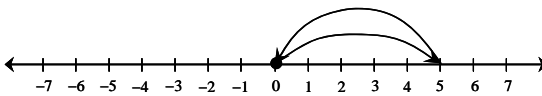
b.  $-2 + (-3) = -5$



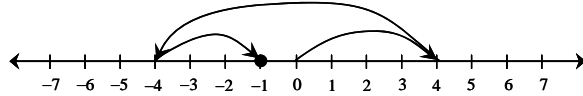
e.  $-2 + (-2) + (-1) = -5$



c.  $5 + (-5) = 0$



f.  $4 + (-8) + 3 = -1$



**Solution** *Match Up on Addition of Real Numbers (RNUM-10)*

B	C	A	B
A	E	B	C
C	C	D	D
E	B	A	E
B	E	A	D

**Solution** *Scrambled Addition Tables* (RNUM-11)

+	-3	-2	-1	0	1	2	3
-3	-6	-5	-4	-3	-2	-1	0
-2	-5	-4	-3	-2	-1	0	1
-1	-4	-3	-2	-1	0	1	2
0	-3	-2	-1	0	1	2	3
1	-2	-1	0	1	2	3	4
2	-1	0	1	2	3	4	5
3	0	1	2	3	4	5	6

+	15	-5	20	-10	0	5	10
10	25	5	30	0	10	15	20
5	20	0	25	-5	5	10	15
-5	10	-10	15	-15	-5	0	5
-15	0	-20	5	-25	-15	-10	-5
20	35	15	40	10	20	25	30
0	15	-5	20	-10	0	5	10
15	30	10	35	5	15	20	25

+	2	-4	-1	3	0
-5	-3	-9	-6	-2	-5
-2	0	-6	-3	1	-2
4	6	0	3	7	4
6	8	2	5	9	6
0	2	-4	-1	3	0

+	5	-10	15	0	-5
-20	-15	-30	-5	-20	-25
-10	-5	-20	5	-10	-15
0	5	-10	15	0	-5
10	15	0	25	10	5
5	10	-5	20	5	0

**Solution** *Signed Number Magic Puzzles* (RNUM-12)

Magic Puzzle #1

-2	8	-1
10	-9	4
-3	6	2

Magic Number = 5

Magic Puzzle #2

-3	-1	4
-5	3	2
8	-2	-6

Magic Number = 0

Magic Puzzle #3

$-\frac{1}{4}$	$-1\frac{1}{2}$	$2\frac{1}{4}$
$\frac{1}{2}$	$2\frac{1}{2}$	$-2\frac{1}{2}$
$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{3}{4}$

Magic Number =  $\frac{1}{2}$ 

Magic Puzzle #4

-2	1	9	-7
4	-9	-4	10
8	-4	2	-5
-9	13	-6	3

Magic Number = 1

Magic Puzzle #5

-8	-3	2	7
-3	14	-5	-8
1	-11	2	6
8	-2	-1	-7

Magic Number = -2

**Solution** *Language of Subtraction* (RNUM-13)

a.	$0 < 8$	Inequality	Zero is less than 8.
b.	$10 - 2 = 8$	Equation	The difference of 10 and 2 is 8. Subtract 2 from 10 to get 8. 10 minus 2 is 8. 2 less than 10 is 8.
c.	$2 - (-5)$	Expression	the difference of 2 and negative 5 subtract negative 5 from 2 2 minus negative 5
d.	$-(-10)$	Expression	the opposite of the opposite of 10 the opposite of negative 10
e.	$5 - (-10) = 15$	Equation	The difference of 5 and negative 10 is 15. Subtract negative 10 from 5 to get 15. 5 minus negative 10 is 15. Negative 10 less than 5 is 15.
f.	$-6 < -3$	Inequality	Negative 6 is less than negative 3.
g.	$-(-5) = 5$	Equation	The opposite of negative 5 is 5.
h.	$3 < -(-6)$	Inequality	3 is less than the opposite of the opposite of 6. 3 is less than the opposite of negative 6.
i.	$\frac{1}{2} - \frac{3}{4}$	Expression	$\frac{1}{2}$ minus $\frac{3}{4}$ $\frac{3}{4}$ less than $\frac{1}{2}$ the difference of $\frac{1}{2}$ and $\frac{3}{4}$ subtract $\frac{3}{4}$ from $\frac{1}{2}$
j.	$-19 < -18$	Inequality	Negative 19 is less than negative 18.

**Solution** *Match Up on Subtraction of Real Numbers* (RNUM-15)

A	D	B	C
A	B	E	E
A	E	D	E
C	D	C	A
D			

**Solution** *Signed Numbers Using a Calculator* (RNUM-16)

1. Keystrokes will vary by calculator.
2. 3; keystrokes will vary by calculator
3. Negative 8 minus 3;  $-11$ ; keystrokes will vary by calculator.
4. See table below.

Expression	Answer	In words (using negative and/or minus)	Keystrokes
$-6 + 3$	-3	negative 6 plus 3	Will vary by calculator
$5 - (-4)$	9	5 minus negative 4	Will vary by calculator
$-19 - 6$	-25	negative 19 minus 6	Will vary by calculator
$-1.25 - 0.25$	-1.5	negative 1.25 minus 0.25	Will vary by calculator
$10 - (-8)$	18	10 minus negative 8	Will vary by calculator
$-3 - (-3)$	0	negative 3 minus negative 3	Will vary by calculator

**Solution** *Pick Your Property* (RNUM-17)

G	D	A
F	E	C
D	A	B
H	C	E
F	I	H

**Solution** *What's Wrong with Division by Zero?* (RNUM-18)

1.  $0 \div 5 = 0$ ,  $5 \div 0 = \text{error}$ ,  $\frac{12}{0} = \text{error}$ ,  $\frac{0}{12} = 0$       2. Answers will vary.
3.  $5 \div 0.1 = 50$ ,  $5 \div 0.01 = 500$ ,  $5 \div 0.001 = 5000$ ,  $5 \div 0.0001 = 50,000$
4. The positive quotients get larger (approaching infinity) as the denominator approaches 0.
5.  $5 \div (-0.1) = -50$ ,  $5 \div (-0.01) = -500$ ,  $5 \div (-0.001) = -5000$ ,  $5 \div (-0.0001) = -50,000$
6. The negative quotients get larger (approaching negative infinity) as the denominator approaches 0.
7. Not only do the quotients get larger (in magnitude), but on opposite sides of zero, the quotients approach opposite infinities.

**Solution** *Match up on Multiplying and Dividing Real Numbers* (RNUM-19)

A	A	C	E
D	A	B	C
E	E	E	D
D	C	C	A
D	A	D	B

**Solution** *The Exponent Trio* (RNUM-20)

$(-3)(-3)$
$(-3)^2 \mid 9$

$(3)(3)$
$3^2 \mid 9$

$2 \cdot 2 \cdot 2 \cdot 2$
$(2)^4 \mid 16$

$-(3 \cdot 3)$
$-3^2 \mid -9$

$(-2)(-2)(-2)(-2)$
$(-2)^4 \mid 16$

$-(2 \cdot 2 \cdot 2 \cdot 2)$
$-2^4 \mid -16$

$2 \cdot 2 \cdot 2$
$2^3 \mid 8$

$(-2)(-2)(-2)$
$(-2)^3 \mid -8$

$-(2 \cdot 2 \cdot 2)$
$-2^3 \mid -8$

$-(4 \cdot 4)$
$-4^2 \mid -16$

$(-4)(-4)$
$(-4)^2 \mid 16$

$-(10 \cdot 10 \cdot 10)$
$-10^3 \mid -1000$

$(-\frac{1}{2}) \cdot (-\frac{1}{2})$
$(-\frac{1}{2})^2 \mid \frac{1}{4}$

$(\frac{1}{2})(\frac{1}{2})$
$(\frac{1}{2})^2 \mid \frac{1}{4}$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$2^8 \mid 256$

$(-9)(-9)$
$(-9)^2 \mid 81$

$5 \cdot 5 \cdot 5$
$5^3 \mid 125$

**Solution** *Order Operation* (RNUM-21)1. First:  $3 \cdot 5$ ; answer: 182. First:  $12 \div 2$ ; answer: 373. First:  $3^2$ ; answer: 14. First:  $6 \cdot 2$  or  $8 \div 2$ ; answer: 125. First:  $4 + 3$ ; answer: -196. First:  $4 \cdot 3$ ; answer: -247. First:  $2^2$ ; answer: -178. First:  $(-3)^2$ ; answer:  $-\frac{1}{5}$ 9. First:  $2 - 5$ ; answer: 1910. First:  $(\frac{1}{2})^2$ ; answer:  $\frac{7}{12}$ **Solution** *Assess Your Understanding on Real Numbers* (RNUM-22)

The assessment is designed to foster points of discussion in class; answers will vary.