CHAPTER 2

Section 2-1

Provide a reasonable description of the sample space for each of the random experiments in Exercises 2.1.1 to 2.1.11. There can be more than one acceptable interpretation of each experiment. Describe any assumptions you make.

2.1.1. Each of four transmitted bits is classified as either in error or not in error.

Let *e* and *o* denote a bit in error and not in error (*o* denotes okay), respectively.

$$S = \left\{ \begin{matrix} eeee, eoee, oeee, ooee, \\ eeeo, eoeo, oeeo, ooeo, \\ eeoe, eooe, oeoe, oooe, \\ eeoo, eooo, oeoo, oooo \end{matrix} \right\}$$

2.1.2. The number of hits (views) is recorded at a high-volume Web site in a day.

$S = \{0,1,2,...\}$ = set of nonnegative integers

2.1.3. In the final inspection of electronic power supplies, either units pass, or three types of nonconformities might occur: functional, minor, or cosmetic. Three units are inspected.

Let *a* denote an acceptable power supply.
Let *f*, *m*, and *c* denote a power supply that has a functional, minor, or cosmetic error, respectively.

$S = \{a, f, m, c\}$

2.1.4. An ammeter that displays three digits is used to measure current in milliamperes.

A vector with three components can describe the three digits of the ammeter. Each digit can be 0,1,2,...,9.
The sample space *S* is 1000 possible three digit integers, $S = \{000, 001, ..., 999\}$

2.1.5. The following two questions appear on an employee survey questionnaire. Each answer is chosen from the five point scale 1 (never), 2, 3, 4, 5 (always).

Is the corporation willing to listen to and fairly evaluate new ideas?
How often are my coworkers important in my overall job performance?

Let an ordered pair of numbers, such as 43 denote the response on the first and second question. Then, S consists of the 25 ordered pairs $\{11, 12, ..., 55\}$

2.1.6. The time until a service transaction is requested of a computer to the nearest millisecond.

$S = \{0,1,2,...,\}$ in milliseconds

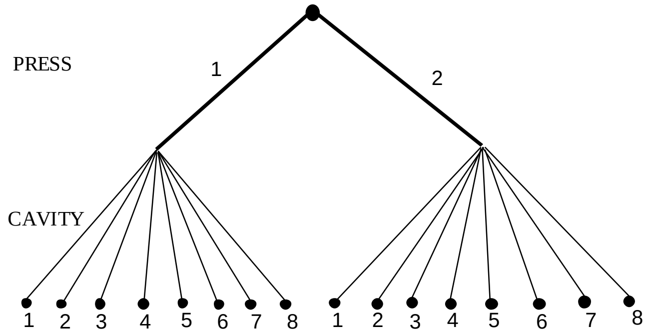
2.1.7. The pH reading of a water sample to the nearest tenth of a unit.

$S = \{1.0, 1.1, 1.2, \ldots 14.0\}$

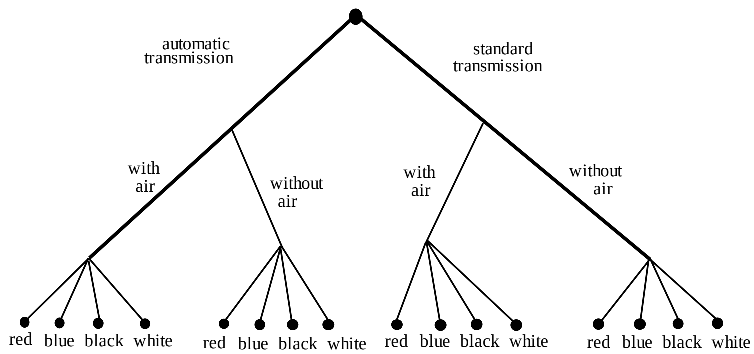
2.1.8. The voids in a ferrite slab are classified as small, medium, or large. The number of voids in each category is measured by an optical inspection of a sample.

Let *s*, *m*, and *l* denote small, medium, and large, respectively. Then *S* = {*s*, *m*, *l*, *ss*, *sm*, *sl*, ….}

2.1.9. A sampled injection-molded part could have been produced in either one of two presses and in any one of the eight cavities in each press.



2.1.10. An order for an automobile can specify either an automatic or a standard transmission, either with or without air conditioning, and with any one of the four colors red, blue, black, or white. Describe the set of possible orders for this experiment.



2.1.11. Calls are repeatedly placed to a busy phone line until a connection is achieved.

Let *c* and *b* denote connect and busy, respectively. Then $S = \{c, bc, bbc, bbbc, bbbbc, \ldots\}$

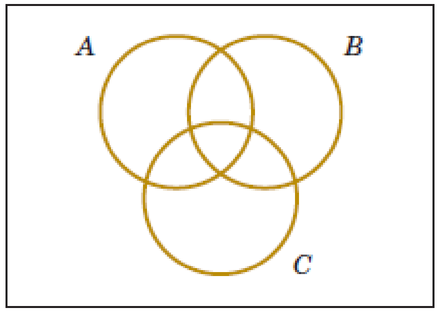
2.1.12. Three attempts are made to read data in a magnetic storage device before an error recovery procedure that repositions the magnetic head is used. The error recovery procedure attempts three repositionings before an "abort'' message is sent to the operator. Let

s denote the success of a read operation
f denote the failure of a read operation
S denote the success of an error recovery procedure
F denote the failure of an error recovery procedure
A denote an abort message sent to the operator

Describe the sample space of this experiment with a tree diagram.

$$S = \{s, fs, ffs, fffS, fffFS, fffFFS, fffFFFA\}$$

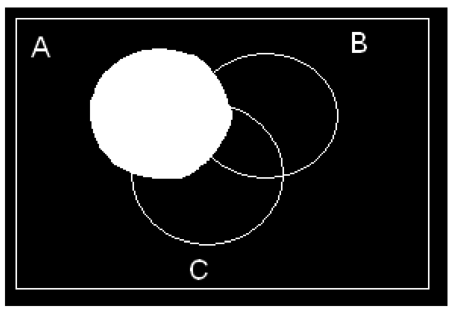
2.1.13. Three events are shown on the Venn diagram in the following figure:



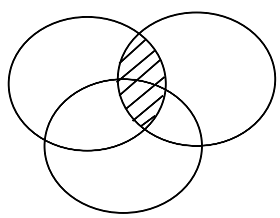
Reproduce the figure and shade the region that corresponds to each of the following events.

(a) A' (b) $A \cap B$ (c) $(A \cap B) \cup C$ (d) $(B \cup C)'$ (e) $(A \cap B)' \cup C$

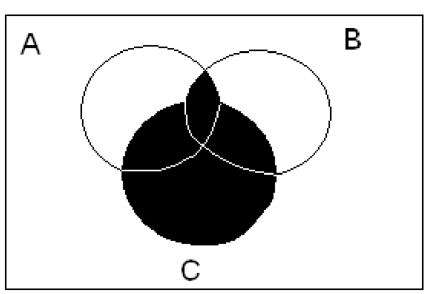
(a)



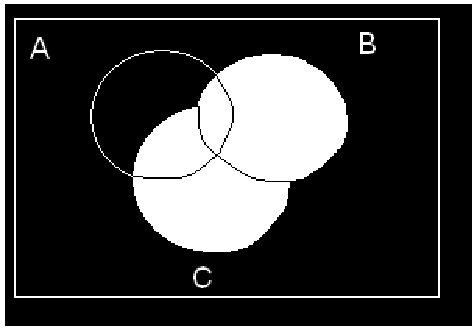
(b)



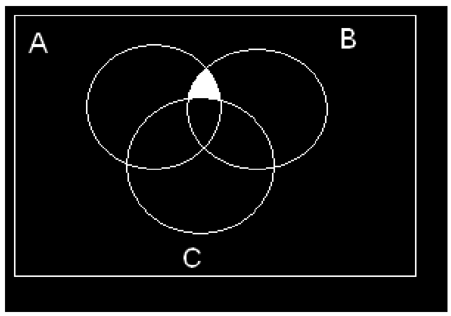
(c)



(d)



(e)



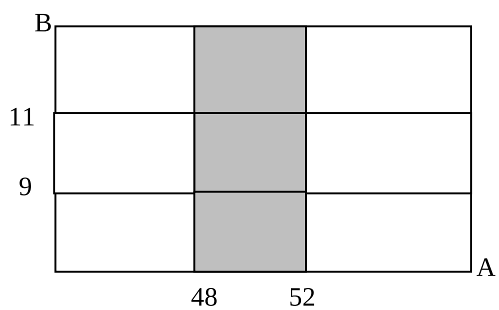
2.1.14 In an injection-molding operation, the length and width, denoted as X and Y, respectively, of each molded part are evaluated. Let

A denote the event of $48 < X < 52$ centimeters
B denote the event of $9 < Y < 11$ centimeters

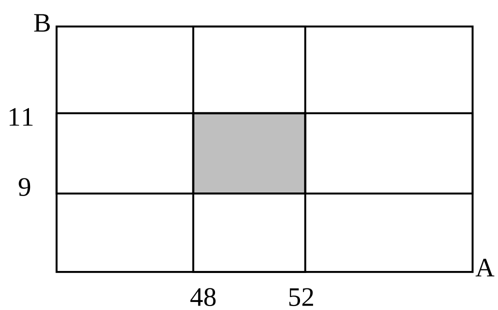
Construct a Venn diagram that includes these events. Shade the areas that represent the following:
(a) A (b) $A \cap B$ (c) $A' \cup B$ (d) $A \cap B$

(e) If these events were mutually exclusive, how successful would this production operation be? Would the process produce parts with $X = 50$ centimeters and $Y = 10$ centimeters?

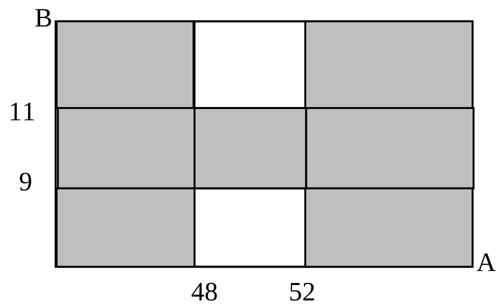
(a)



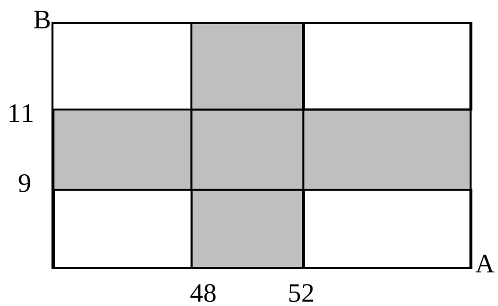
(b)



(c)



(d)



(e) If the events are mutually exclusive, then $A \cap B$ is the null set. Therefore, the process does not produce product parts with $X = 50$ cm and $Y = 10$ cm. The process would not be successful.

2.1.15. A digital scale that provides weights to the nearest gram is used.

(a) What is the sample space for this experiment?
Let A denote the event that a weight exceeds 11 grams, let B denote the event that a weight is less than or equal to 15 grams, and let C denote the event that a weight is greater than or equal to 8 grams and less than 12 grams.

Describe the following events.

(b) $A \cup B$ (c) $A \cap B$ (d) A' (e) $A \cup B \cup C$ (f) $(A \cup C)'$ (g) $A \cap B \cap C$ (h) $B' \cap C$ (i) $A \cup (B \cap C)$

(a) Let S = the nonnegative integers from 0 to the largest integer that can be displayed by the scale.
Let X denote the weight.

A is the event that $X > 11$ B is the event that $X \le 15$ C is the event that $8 \le X < 12$
$S = \{0, 1, 2, 3, \ldots\}$
(b) S
(c) $11 < X \le 15$ or $\{12, 13, 14, 15\}$
(d) $X \le 11$ or $\{0, 1, 2, \ldots, 11\}$
(e) S
(f) $A \cup C$ contains the values of X such that: $X \ge 8$
Thus $(A \cup C)'$ contains the values of X such that: $X < 8$ or $\{0, 1, 2, \ldots, 7\}$
(g) $\varnothing$
(h) B' contains the values of X such that $X > 15$. Therefore, $B' \cap C$ is the empty set. They have no outcomes in common or $\varnothing$.
(i) $B \cap C$ is the event $8 \le X < 12$. Therefore, $A \cup (B \cap C)$ is the event $X \ge 8$ or $\{8, 9, 10, \ldots\}$

2.1.16. In light-dependent photosynthesis, light quality refers to the wavelengths of light that are important. The wavelength of a sample of photosynthetically active radiations (PAR) is measured to the nearest nanometer. The red range is 675–700 nm and the blue range is 450–500 nm. Let A denote the event that PAR occurs in the red range, and let B denote the event that PAR occurs in the blue range. Describe the sample space and indicate each of the following events:

(a) A (b) B (c) $A \cap B$ (d) $A \cup B$

Let w denote the wavelength. The sample space is $\{w \mid w = 0, 1, 2, \ldots\}$
(a) $A=\{w \mid w = 675, 676, \ldots, 700 \text{ nm}\}$
(b) $B=\{ w \mid w = 450, 451, \ldots, 500 \text{ nm}\}$
(c) $A \cap B = \Phi$
(d) $A \cup B = \{w \mid w = 450, 451, \ldots, 500, 675, 676, \ldots, 700 \text{ nm}\}$

2.1.17. Four bits are transmitted over a digital communications channel. Each bit is either distorted or received without distortion. Let Ai denote the event that the ith bit is distorted, $i = 1, \ldots, 4$.

(a) Describe the sample space for this experiment.
(b) Are the A_i's mutually exclusive?

Describe the outcomes in each of the following events:

(c) A_1 (d) A_1' (e) $A_1 \cap A_2 \cap A_3 \cap A_4$ (f) $(A_1 \cap A_2) \cup (A_3 \cap A_4)$

Let d and o denote a distorted bit and one that is not distorted (o denotes okay), respectively.

(a) $S = \begin{Bmatrix} dddd, dodd, oddd, oodd, \\ dddo, dodo, oddo, oodo, \\ ddod, dood, odod, oood, \\ ddoo, dooo, odoo, oooo \end{Bmatrix}$

(b) No, for example $A_1 \cap A_2 = \{dddd, dddo, ddod, ddoo\}$

(c) $A_1 = \begin{Bmatrix} dddd, dodd, \\ dddo, dodo \\ ddod, dood \\ ddoo, dooo \end{Bmatrix}$

(d) $A_1' = \begin{Bmatrix} oddd, oodd, \\ oddo, oodo, \\ odod, oood, \\ odoo, oooo \end{Bmatrix}$

(e) $A_1 \cap A_2 \cap A_3 \cap A_4 = \{dddd\}$

(f) $(A_1 \cap A_2) \cup (A_3 \cap A_4) = \{dddd, dodd, dddo, oddd, ddod, oodd, ddoo\}$

2.1.18. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized here:

		Shock Resistance	
		High	Low
Scratch	High	70	9
Resistance	Low	16	5

Let A denote the event that a disk has high shock resistance, and let B denote the event that a disk has high scratch resistance. Determine the number of disks in $A \cap B$, A', and $A \cup B$.

$A \cap B = 70$, $A' = 14$, $A \cup B = 95$

2.1.19. In control replication, cells are replicated over a period of two days. Not until mitosis is completed can freshly synthesized DNA be replicated again. Two control mechanisms have been identified—one positive and one negative. Suppose that a replication is observed in three cells. Let A denote the event that all cells are identified as positive, and let B denote the event that all cells are negative. Describe the sample space graphically and display each of the following events:

(a) A (b) B (c) $A \cap B$ (d) $A \cup B$

Let P and N denote positive and negative, respectively.
The sample space is $\{PPP, PPN, PNP, NPP, PNN, NPN, NNP, NNN\}$.

(a) $A = \{ PPP \}$
(b) $B = \{ NNN \}$
(c) $A \cap B = \Phi$
(d) $A \cup B = \{ PPP, NNN \}$

2.1.20. Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized as follows:

		Conforms	
		Yes	No
	1	22	8
Supplier	2	25	5
	3	30	10

Let A denote the event that a sample is from supplier 1, and let B denote the event that a sample conforms to specifications. Determine the number of samples in $A' \cap B$, B' and in $A \cup B$.

$A' \cap B = 55$, $B' = 23$, $A \cup B = 85$

2.1.21. The rise time of a reactor is measured in minutes (and fractions of minutes). Let the sample space be positive, real numbers. Define the events A and B as follows:

$A = \{x \mid x < 72.5\}$ and $B = \{x \mid x > 52.5\}$.

Describe each of the following events:

(a) A' (b) B' (c) $A \cap B$ (d) $A \cup B$

(a) $A' = \{x \mid x \geq 72.5\}$
(b) $B' = \{x \mid x \leq 52.5\}$
(c) $A \cap B = \{x \mid 52.5 < x < 72.5\}$

(d) $A \cup B = \{x \mid x > 0\}$

2.1.22. The following table summarizes 204 endothermic reactions involving sodium bicarbonate.

Final Temperature Conditions	**Heat Absorbed (cal)** Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Let A denote the event that a reaction's final temperature is 271 K or less. Let B denote the event that the heat absorbed is below target. Determine the number of reactions in each of the following events.

(a) $A \cap B$ (b) A' (c) $A \cup B$ (d) $A \cup B'$ (e) $A' \cap B'$

(a) $A \cap B = 56$
(b) $A' = 36 + 56 = 92$
(c) $A \cup B = 40 + 12 + 16 + 44 + 56 = 168$
(d) $A \cup B' = 40+12+16+44+36=148$
(e) $A' \cap B' = 36$

2.1.23. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. How many different designs are possible?

Total number of possible designs = $4 \times 3 \times 5 \times 3 \times 5 = 900$

2.1.24. Consider the hospital emergency department data given below. Let A denote the event that a visit is to hospital 1, and let B denote the event that a visit results in admittance to any hospital.

	Hospital				
	1	**2**	**3**	**4**	**Total**
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the number of persons in each of the following events.

(a) $A \cap B$ (b) A' (c) $A \cup B$ (d) $A \cup B'$ (e) $A' \cap B'$

(a) $A \cap B = 1277$
(b) $A' = 22252 - 5292 = 16960$
(c) $A \cup B = 1685 + 3733 + 1403 + 2 + 14 + 29 + 46 + 3 = 6915$
(d) $A \cup B' = 195 + 270 + 246 + 242 + 3820 + 5163 + 4728 + 3103 + 1277 = 19044$
(e) $A' \cap B' = 270 + 246 + 242 + 5163 + 4728 + 3103 = 13752$

2.1.25. The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let A denote the event that the patient was treated with ribavirin plus interferon alfa, and let B denote the event that the response was complete. Determine the number of patients in each of the following events.

(a) A (b) $A \cap B$ (c) $A \cup B$ (d) $A' \cap B'$

Let |A| denote the number of elements in the set A.
(a) $|A| = 21$
(b) $|A \cap B| = 16$
(c) $|A \cup B| = A+B - (A \cap B) = 21+22 - 16 = 27$
(d) $|A' \cap B'| = 60 - |A \cup B| = 60 - 27 = 33$

2.1.26. A computer system uses passwords that contain exactly eight characters, and each character is 1 of the 26 lowercase letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Let Ω denote the set of all possible passwords, and let *A* and *B* denote the events that consist of passwords with only letters or only integers, respectively. Determine the number of passwords in each of the following events.

(a) Ω (b) A (c) $A' \cap B'$
(d) Passwords that contain at least 1 integer
(e) Passwords that contain exactly 1 integer

Let |A| denote the number of elements in the set A.
(a) The number of passwords in Ω is $|\Omega| = 62^8$ (from multiplication rule).
(b) The number of passwords in A is $|A| = 52^8$ (from multiplication rule)
(c) $A' \cap B' = (A \cup B)'$. Also, $|A| = 52^8$ and $|B| = 10^8$ and $A \cap B$ = null. Therefore,
$(A \cup B)' = |\Omega| - |A| - |B| = 62^8 - 52^8 - 10^8 \approx 1.65 \times 10^{14}$
(d) Passwords that contain at least 1 integer = $|\Omega| - |A| = 62^8 - 52^8 \approx 1.65 \times 10^{14}$
(e) Passwords that contain exactly 1 integer. The number of passwords with 7 letters is 52^7. Also, 1 integer is selected in 10 ways, and can be inserted into 8 positions in the password. Therefore, the solution is $8(10)(52^7) \approx 8.22 \times 10^{13}$

Section 2-2

2.2.1. A sample of two printed circuit boards is selected without replacement from a batch. Describe the (ordered) sample space for each of the following batches:

(a) The batch contains 90 boards that are not defective, 8 boards with minor defects, and 2 boards with major defects.
(b) The batch contains 90 boards that are not defective, 8 boards with minor defects, and 1 board with major defects.

Let *g* denote a good board, *m* a board with minor defects, and *j* a board with major defects.

(a) $S = \{gg, gm, gj, mg, mm, mj, jg, jm, jj\}$
(b) $S = \{gg,gm,gj,mg,mm,mj,jg,jm\}$

2.2.2. A sample of two items is selected without replacement from a batch. Describe the (ordered) sample space for each of the following batches:
(a) The batch contains the items $\{a, b, c, d\}$.
(b) The batch contains the items $\{a, b, c, d, e, f, g\}$.
(c) The batch contains 4 defective items and 20 good items.
(d) The batch contains 1 defective item and 20 good items.

(a) $\{ab, ac, ad, bc, bd, cd, ba, ca, da, cb, db, dc\}$

(b) {*ab, ac, ad, ae, af, ag, ba, bc, bd, be, bf, bg, ca, cb, cd, ce, cf, cg, da, db, dc, de, df, dg, ea, eb, ec, ed, ef, eg, fa, fb, fc, fg, fd, fe, ga, gb, gc, gd, ge, gf*}, contains 42 elements
(c) Let *d* and *g* denote defective and good, respectively. Then *S* = {*gg, gd, dg, dd*}
(d) S = {*gd, dg, gg*}

2.2.3. A wireless garage door opener has a code determined by the up or down setting of 12 switches. How many outcomes are in the sample space of possible codes?

$2^{12} = 4096$

2.2.4. In a manufacturing operation, a part is produced by machining, polishing, and painting. If there are three machine tools, four polishing tools, and three painting tools, how many different routings (consisting of machining, followed by polishing, and followed by painting) for a part are possible?

From the multiplication rule, $3 \times 4 \times 3 = 36$

2.2.5. New designs for a wastewater treatment tank have proposed three possible shapes, four possible sizes, three locations for input valves, and four locations for output valves. How many different product designs are possible?

From the multiplication rule, $3 \times 4 \times 3 \times 4 = 144$

2.2.6. A manufacturing process consists of 10 operations that can be completed in any order. How many different production sequences are possible?

From equation 2.1, the answer is 10! = 3,628,800

2.2.7. A batch of 140 semiconductor chips is inspected by choosing a sample of 5 chips. Assume 10 of the chips do not conform to customer requirements.

(a) How many different samples are possible?
(b) How many samples of five contain exactly one nonconforming chip?
(c) How many samples of five contain at least one nonconforming chip?

(a) From equation 2-4, the number of samples of size five is $\binom{140}{5} = \frac{140!}{5!135!} = 416{,}965{,}528$

(b) There are 10 ways of selecting one nonconforming chip and there are $\binom{130}{4} = \frac{130!}{4!126!} = 11{,}358{,}880$ ways of selecting four conforming chips. Therefore, the number of samples that contain exactly one nonconforming chip is $10 \times \binom{130}{4} = 113{,}588{,}800$

(c) The number of samples that contain at least one nonconforming chip is the total number of samples $\binom{140}{5}$ minus the number of samples that contain no nonconforming chips $\binom{130}{5}$. That is

$$\binom{140}{5} - \binom{130}{5} = \frac{140!}{5!135!} - \frac{130!}{5!125!} = 130{,}721{,}752$$

2.2.8. In a sheet metal operation, three notches and four bends are required. If the operations can be done in any order, how many different ways of completing the manufacturing are possible?

From equation 2-3, $\frac{7!}{3!4!} = 35$ sequences are possible

2.2.9. In the laboratory analysis of samples from a chemical process, five samples from the process are analyzed daily. In addition, a control sample is analyzed twice each day to check the calibration of the laboratory instruments.

(a) How many different sequences of process and control samples are possible each day? Assume that the five process samples are considered identical and that the two control samples are considered identical.

(b) How many different sequences of process and control samples are possible if we consider the five process samples to be different and the two control samples to be identical?
(c) For the same situation as part (b), how many sequences are possible if the first test of each day must be a control sample?

(a) $\dfrac{7!}{2!5!} = 21$ sequences are possible.
(b) $\dfrac{7!}{1!1!1!1!1!2!} = 2520$ sequences are possible.
(c) 6! = 720 sequences are possible.

2.2.10. In the layout of a printed circuit board for an electronic product, 12 different locations can accommodate chips.

(a) If five different types of chips are to be placed on the board, how many different layouts are possible?
(b) If the five chips that are placed on the board are of the same type, how many different layouts are possible?

(a) If the chips are of different types, then every arrangement of 5 locations selected from the 12 results in a different layout. Therefore, $P_5^{12} = \dfrac{12!}{7!} = 95{,}040$ layouts are possible.
(b) If the chips are of the same type, then every subset of 5 locations chosen from the 12 results in a different layout. Therefore, $\binom{12}{5} = \dfrac{12!}{5!7!} = 792$ layouts are possible.

2.2.11. Consider the design of a communication system.

(a) How many three-digit phone prefixes that are used to represent a particular geographic area (such as an area code) can be created from the digits 0 through 9?
(b) As in part (a), how many three-digit phone prefixes are possible that do not start with 0 or 1, but contain 0 or 1 as the middle digit?
(c) How many three-digit phone prefixes are possible in which no digit appears more than once in each prefix?

(a) From the multiplication rule, $10^3 = 1000$ prefixes are possible
(b) From the multiplication rule, $8 \times 2 \times 10 = 160$ are possible
(c) Every arrangement of three digits selected from the 10 digits results in a possible prefix.
$P_3^{10} = \dfrac{10!}{7!} = 720$ prefixes are possible.

2.2.12. In the design of an electromechanical product, 12 components are to be stacked into a cylindrical casing in a manner that minimizes the impact of shocks. One end of the casing is designated as the bottom and the other end is the top.

(a) If all components are different, how many different designs are possible?
(b) If seven components are identical to one another, but the others are different, how many different designs are possible?
(c) If three components are of one type and identical to one another, and four components are of another type and identical to one another, but the others are different, how many different designs are possible?

(a) Every arrangement selected from the 12 different components comprises a different design. Therefore, $12! = 479{,}001{,}600$ designs are possible.

(b) 7 components are the same, others are different, $\dfrac{12!}{7!1!1!1!1!1!} = 95040$ designs are possible.

(c) $\dfrac{12!}{3!4!} = 3326400$ designs are possible.

2.2.13. A bin of 50 parts contains 5 that are defective. A sample of 10 parts is selected at random, without replacement. How many samples contain at least four defective parts?

From the 5 defective parts, select 4, and the number of ways to complete this step is 5!/(4!1!) = 5
From the 45 non-defective parts, select 6, and the number of ways to complete this step is 45!/(6!39!) = 8,145,060
Therefore, the number of samples that contain exactly 4 defective parts is 5(8,145,060) = 40,725,300
Similarly, from the 5 defective parts, the number of ways to select 5 is 5!(5!1!) = 1
From the 45 non-defective parts, select 5, and the number of ways to complete this step is 45!/(5!40!) = 1,221,759
Therefore, the number of samples that contain exactly 5 defective parts is
1(1,221,759) = 1,221,759

Therefore, the number of samples that contain at least 4 defective parts is
40,725,300 + 1,221,759 = 41,947,059

2.2.14. Plastic parts produced by an injection-molding operation are checked for conformance to specifications. Each tool contains 12 cavities in which parts are produced, and these parts fall into a conveyor when the press opens. An inspector chooses 3 parts from among the 12 at random. Two cavities are affected by a temperature malfunction that results in parts that do not conform to specifications.

(a) How many samples contain exactly 1 nonconforming part?
(b) How many samples contain at least 1 nonconforming part?

(a) The total number of samples is $\binom{12}{3} = \frac{12!}{3!\,9!} = 220.$ The number of samples that result in one nonconforming part is $\binom{2}{1}\binom{10}{2} = \frac{2!}{1!1!} \times \frac{10!}{2!8!} = 90.$ Therefore, the requested probability is 90/220 = 0.409.

(b) The number of samples with no nonconforming part is $\binom{10}{3} = \frac{10!}{3!7!} = 120.$ The probability of at least one nonconforming part is $1 - \frac{120}{220} = 0.455$.

2.2.15. A hospital operating room needs to schedule three knee surgeries and two hip surgeries in a day. Suppose that an operating room needs to handle three knee, four hip, and five shoulder surgeries.

(a) How many different sequences are possible?
(b) How many different sequences have all hip, knee, and shoulder surgeries scheduled consecutively?
(c) How many different schedules begin and end with a knee surgery?

(a) From the formula for the number of sequences $\frac{12!}{3!4!5!}$ = 27,720 sequences are possible.
(b) Combining all hip surgeries into one single unit, all knee surgeries into one single unit and all shoulder surgeries into one unit, the possible number of sequences of these units = 3! = 6
(c)With two surgeries specified, 10 remain and there are $\frac{10!}{4!5!1!}$ = 1,260 different sequences.

Section 2-3

2.3.1. The sample space of a random experiment is {*a*, *b*,*c*, *d*, *e*} with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let *A* denote the event {*a*, *b*, *c*}, and let *B* denote the event {*c*, *d*, *e*}. Determine the following:

(a) P(A) (b) P(B) (c) P(A') (d) P(A∪B) (e) P(A∩B)

(a) P(A) = 0.4
(b) P(B) = 0.8
(c) P(A') = 0.6
(d) P(A∪B) = 1

(e) $P(A \cap B) = 0.2$

2.3.2. A part selected for testing is equally likely to have been produced on any one of six cutting tools.

(a) What is the sample space?
(b) What is the probability that the part is from tool 1?
(c) What is the probability that the part is from tool 3 or tool 5?
(d) What is the probability that the part is not from tool 4?

(a) S = {1, 2, 3, 4, 5, 6}
(b) 1/6
(c) 2/6
(d) 5/6

2.3.3. An injection-molded part is equally likely to be obtained from any one of the eight cavities on a mold.

(a) What is the sample space?
(b) What is the probability that a part is from cavity 1 or 2?
(c) What is the probability that a part is from neither cavity 3 nor 4?

(a) S = {1,2,3,4,5,6,7,8}
(b) 2/8
(c) 6/8

2.3.4. A credit card contains 16 digits between 0 and 9. However, only 100 million numbers are valid. If a number is entered randomly, what is the probability that it is a valid number?

Total possible: 10^{16}, but only 10^8 are valid. Therefore, P(valid) = $10^8/10^{16} = 1/10^8$

2.3.5. In a NiCd battery, a fully charged cell is composed of nickelic hydroxide. Nickel is an element that has multiple oxidation states and that is usually found in the following states:

Nickel Charge	Proportions Found
0	0.17
+2	0.35
+3	0.33
+4	0.15

(a) What is the probability that a cell has at least one of the positive nickel-charged options?
(b) What is the probability that a cell is *not* composed of a positive nickel charge greater than +3?

The sample space is {0, +2, +3, and +4}.
(a) The event that a cell has at least one of the positive nickel charged options is {+2, +3, and +4}. The probability is 0.35 + 0.33 + 0.15 = 0.83.
(b) The event that a cell is not composed of a positive nickel charge greater than +3 is {0, +2, and +3}. The probability is 0.17 + 0.35 + 0.33 = 0.85.

2.3.6. A message can follow different paths through servers on a network. The sender's message can go to one of five servers for the first step; each of them can send to five servers at the second step; each of those can send to four servers at the third step; and then the message goes to the recipient's server.

(a) How many paths are possible?
(b) If all paths are equally likely, what is the probability that a message passes through the first of four servers at the third step?

(a) 5*5*4 = 100
(b) (5*5)/100 = 25/100=1/4

2.3.7. Suppose your vehicle is licensed in a state that issues license plates that consist of three digits (between 0 and 9) followed by three letters (between *A* and *Z*). If a license number is selected randomly, what is the probability that yours is the one selected?

3 digits between 0 and 9, so the probability of any three numbers is 1/(10*10*10).
3 letters A to Z, so the probability of any three numbers is 1/(26*26*26). The probability your license plate is chosen is then $(1/10^3)*(1/26^3) = 5.7 \times 10^{-8}$

2.3.8. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		Shock Resistance	
		High	Low
Scratch Resistance	High	70	9
	Low	16	5

Let *A* denote the event that a disk has high shock resistance, and let *B* denote the event that a disk has high scratch resistance. If a disk is selected at random, determine the following probabilities:

(a) P(A) (b) P(B) (c) P(A') (d) $P(A\cap B)$ (e) $P(A\cup B)$ (f) $P(A'\cup B)$

(a) P(A) = 86/100 = 0.86
(b) P(B) = 79/100 = 0.79
(c) P(A') = 14/100 = 0.14
(d) $P(A\cap B)$ = 70/100 = 0.70
(e) $P(A\cup B)$ = (70+9+16)/100 = 0.95
(f) $P(A'\cup B)$ = (70+9+5)/100 = 0.84

2.3.9. Magnesium alkyls are used as homogenous catalysts in the production of linear low-density polyethylene (LLDPE), which requires a finer magnesium powder to sustain a reaction. Redox reaction experiments using four different amounts of magnesium powder are performed. Each result may or may not be further reduced in a second step using three different magnesium powder amounts. Each of these results may or may not be further reduced in a third step using three different amounts of magnesium powder.

(a) How many experiments are possible?
(b) If all outcomes are equally likely, what is the probability that the best result is obtained from an experiment that uses all three steps?
(c) Does the result in part (b) change if five or six or seven different amounts are used in the first step? Explain.

(a) The number of possible experiments is $4 + 4 \times 3 + 4 \times 3 \times 3 = 52$
(b) There are 36 experiments that use all three steps. The probability the best result uses all three steps is 36/52 = 0.6923.
(c) No, it will not change. With *k* amounts in the first step the number of experiments is $k + 3k + 9k = 13k$. The number of experiments that complete all three steps is $9k$ out of $13k$. The probability is 9/13 = 0.6923.

2.3.10. An article in the *Journal of Database Management* ["Experimental Study of a Self-Tuning Algorithm for DBMS Buffer Pools" (2005, Vol. 16, pp. 1–20)] provided the workload used in the TPC-C OLTP (Transaction Processing Performance Council's Version C On-Line Transaction Processing) benchmark, which simulates a typical order entry application.

Average Frequencies and Operations in TPC-C

Transaction	Frequency	Selects	Updates
New order	43	23	11
Payment	44	4.2	3
Order status	4	11.4	0
Delivery	5	130	120
Stock level	4	0	0

The frequency of each type of transaction (in the second column) can be used as the percentage of each type of transaction. The average number of *selects* operations required for each type of transaction is shown. Let *A* denote the event of transactions with an average number of *selects* operations of 12 or fewer. Let *B* denote the event of transactions with an average number of *updates* operations of 12 or fewer. Calculate the following probabilities.

(a) P(A) (b) P(B) (c) P(A∩B) (d) P(A∩B') (f) P(A∪B)

(a) The total number of transactions is 43+44+4+5+4=100

$$P(A) = \frac{44+4+4}{100} = 0.52$$

(b) $P(B) = \frac{100-5}{100} = 0.95$

(c) $P(A \cap B) = \frac{44+4+4}{100} = 0.52$

(d) $P(A \cap B') = 0$

(e) $P(A \cup B) = \frac{100-5}{100} = 0.95$

2.3.11. Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized as follows:

		Conforms	
		Yes	No
	1	22	8
Supplier	2	25	5
	3	30	10

Let *A* denote the event that a sample is from supplier 1, and let *B* denote the event that a sample conforms to specifications. If a sample is selected at random, determine the following probabilities:

(a) P(A) (b) P(B) (c) P(A') (d) P(A∩B) (e) P(A∪B) (f) P(A'∪B)

(a) P(A) = 30/100 = 0.30
(b) P(B) = 77/100 = 0.77
(c) P(A') = 1 – 0.30 = 0.70
(d) P(A∩B) = 22/100 = 0.22
(e) P(A∪B) = 85/100 = 0.85
(f) P(A'∪B) =92/100 = 0.92

2.3.12. Consider the hospital emergency room data is given below. Let *A* denote the event that a visit is to hospital 4, and let *B* denote the event that a visit results in LWBS (at any hospital).

Hospital					
	1	**2**	**3**	**4**	**Total**
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A')$ (c) $P(A \cup B)$ (d) $P(A \cup B')$ (e) $P(A' \cap B')$

(a) $P(A \cap B)$ = 242/22252 = 0.0109
(b) $P(A')$ = (5292+6991+5640)/22252 = 0.8055
(c) $P(A \cup B)$ = (195 + 270 + 246 + 242 + 984 + 3103)/22252 = 0.2265
(d) $P(A \cup B')$ = (4329 + (5292 – 195) + (6991 – 270) + 5640 – 246))/22252 = 0.9680
(e) $P(A' \cap B')$ = (1277 + 1558 + 666 + 3820 + 5163 + 4728)/22252 = 0.7735

2.3.13. Use the axioms of probability to show the following: $A \cup B$(d) $A \cap B'$
(a) For any event E, $P(E') = 1 - P(E)$. (b) $P(\varnothing) = 0$ (c) If A is contained in B, then $P(A) \le P(B)$.

(a) Because E and E' are mutually exclusive events and $E \cup E'$ = S
1 = P(S) = P($E \cup E'$) = P(E) + P(E'). Therefore, P(E') = 1 - P(E)
(b) Because S and $\varnothing$ are mutually exclusive events with S = $S \cup \varnothing$
P(S) = P(S) + P($\varnothing$). Therefore, P($\varnothing$) = 0
(c) Now, B = $A \cup (A' \cap B)$ and the events A and $A' \cap B$ are mutually exclusive. Therefore,
P(B) = P(A) + P($A' \cap B$). Because P($A' \cap B$) $\geq$ 0 , P(B) $\geq$ P(A).

2.3.14. Suppose that a patient is selected randomly from the those described ,The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

	Complete Response	**Total**
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let A denote the event that the patient is in the group treated with interferon alfa, and let B denote the event that the patient has a complete response.
Determine the following probabilities.

(a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$ (e) $P(A' \cup B)$

(a) P(A)= 19/60 = 0.3167
(b) P(B)= 22/60 = 0.3667
(c) P(A∩B) = 6/60 = 0.1
(d) P(AUB) = P(A)+P(B)-P(A∩B) = (19+22-6)/60 =0.5833
(e) P(A'UB) = P(A')+P(B)-P(A'∩B) = $\frac{21+20}{60} + \frac{22}{60} - \frac{16}{60} = 0.7833$

2.3.15. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. If you visit the site five times, what is the probability that you will not see the same design?

A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. If you visit the site five times, what is the probability that you will not see the same design?

Total number of possible designs is 900. The sample space of all possible designs that may be seen on five visits. This space contains 900^5 outcomes.

The number of outcomes in which all five visits are different can be obtained as follows. On the first visit any one of 900 designs may be seen. On the second visit there are 899 remaining designs. On the third visit there are 898 remaining designs. On the fourth and fifth visits there are 897 and 896 remaining designs, respectively. From the multiplication rule, the number of outcomes where all designs are different is 900*899*898*897*896. Therefore, the probability that a design is not seen again is

$$(900*899*898*897*896)/ 900^5 = 0.9889$$

2.3.16. A hospital operating room needs to schedule three knee surgeries and two hip surgeries in a day. Suppose that an operating room needs to schedule three knee, four hip, and five shoulder surgeries. Assume that all schedules are equally likely.

Determine the probability for each of the following:
(a) All hip surgeries are completed before another type of surgery.
(b) The schedule begins with a hip surgery.
(c) The fi rst and last surgeries are hip surgeries.
(d) The fi rst two surgeries are hip surgeries.

(a) P(all hip surgeries before another type)= $\frac{\frac{8!}{3!5!}}{\frac{12!}{3!4!5!}} = \frac{8!4!}{12!} = \frac{1}{495} = 0.00202$

(b) P(begins with hip surgery)= $\frac{\frac{11!}{3!3!5!}}{\frac{12!}{3!4!5!}} = \frac{11!4!}{12!3!} = \frac{1}{3}$

(c) P(first and last are hip surgeries)= $\frac{\frac{10!}{2!3!5!}}{\frac{12!}{3!4!5!}} = \frac{1}{11}$

(d) P(first two are hip surgeries)= $\frac{\frac{10!}{2!3!5!}}{\frac{12!}{3!4!5!}} = \frac{1}{11}$

2.3.17. A computer system uses passwords that contain exactly eight characters, and each character is one of 26 lowercase letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Let Ω denote the set of all possible passwords, and let *A* and *B* denote the events that consist of passwords with only letters or only integers, respectively. Suppose that all passwords in Ω are equally likely.
Determine the probability of each of the following:
(a) *A* (b) *B*
(c) A password contains at least 1 integer.
(d) A password contains exactly 2 integers.

(a) P(A) = $\frac{52^8}{62^8} = 0.2448$

(b) P(B) = $\frac{10^8}{62^8} = 4.58\text{x}10^{-7}$

(c) P(contains at least 1 integer) =1 - P(password contains no integer) = 1 - $\frac{52^8}{62^8} = 0.7551$

(d) P(contains exactly 2 integers)
Number of positions for the integers is 8!/(2!6!) = 28
Number of permutations of the two integers is $10^2 = 100$
Number of permutations of the six letters is 52^6
Total number of permutations is 62^8
Therefore, the probability is
$\frac{28(100)(52^6)}{62^8} = 0.254$

<u>Section 2-4</u>

2.4.1. If A, B, and C are mutually exclusive events with $P(A) = 0.2$, $P(B) = 0.3$, and $P(C) = 0.4$, determine the following probabilities:

(a) $P(A \cup B \cup C)$ (b) $P(A \cap B \cap C)$ (c) $P(A \cap B)$ (d) $P[(A \cup B) \cap C]$ (e) $P(A' \cap B' \cap C')$

(a) P($A \cup B \cup C$) = P(A) + P(B) + P(C), because the events are mutually exclusive. Therefore,
P($A \cup B \cup C$) = 0.2+0.3+0.4 = 0.9
(b) P ($A \cap B \cap C$) = 0, because $A \cap B \cap C = \varnothing$
(c) P($A \cap B$) = 0 , because $A \cap B = \varnothing$
(d) P($(A \cup B) \cap C$) = 0, because $(A \cup B) \cap C = (A \cap C) \cup (B \cap C) = \varnothing$
(e) P($A' \cap B' \cap C'$) =1-[P(A) + P(B) + P(C)] = 1-(0.2+0.3+0.4) = 0.1

2.4.2. If $P(A) = 0.3$, $P(B) = 0.2$, and $P(A \cap B) = 0.1$, determine the following probabilities:

(a) $P(A')$ (b) $P(A \cup B)$ (c) $P(A' \cap B)$ (d) $P(A \cap B')$ (e) $P[(A \cup B)']$ (f) $P(A' \cup B)$

(a) P(A') = 1- P(A) = 0.7
(b) P ($A \cup B$) = P(A) + P(B) - P($A \cap B$) = 0.3+0.2 - 0.1 = 0.4
(c) P($A' \cap B$) + P($A \cap B$) = P(B). Therefore, P($A' \cap B$) = 0.2 - 0.1 = 0.1
(d) P(A) = P($A \cap B$) + P($A \cap B'$). Therefore, P($A \cap B'$) = 0.3 - 0.1 = 0.2
(e) P(($A \cup B$)') = 1 - P($A \cup B$) = 1 - 0.4 = 0.6
(f) P($A' \cup B$) = P(A') + P(B) - P($A' \cap B$) = 0.7 + 0.2 - 0.1 = 0.8

2.4.3. A manufacturer of front lights for automobiles tests lamps under a high-humidity, high-temperature environment using intensity and useful life as the responses of interest. The following table shows the performance of 130 lamps:

		Useful life	
		Satisfactory	Unsatisfactory
Intensity	Satisfactory	117	3
	Unsatisfactory	8	2

(a) Find the probability that a randomly selected lamp will yield unsatisfactory results under any criteria.
(b) The customers for these lamps demand 95% satisfactory results. Can the lamp manufacturer meet this demand?

(a) P(unsatisfactory) = (5 + 10 – 2)/130 = 13/130
(b) P(both criteria satisfactory) = 117/130 = 0.90, No

2.4.4. In the article "ACL Reconstruction Using Bone-Patellar Tendon-Bone Press-Fit Fixation: 10-Year Clinical Results" in *Knee Surgery, Sports Traumatology, Arthroscopy* (2005, Vol. 13, pp. 248–255), the following causes for knee injuries were considered:

Activity	Percentage of Knee Injuries
Contact sport	46%
Noncontact sport	44%
Activity of daily living	9%
Riding motorcycle	1%

(a) What is the probability that a knee injury resulted from a sport (contact or noncontact)?
(b) What is the probability that a knee injury resulted from an activity other than a sport?

(a) P(Caused by sports) = P(Caused by contact sports or by noncontact sports)
= P(Caused by contact sports) + P(Caused by noncontact sports)
= 0.46 + 0.44 = 0.9
(b) 1- P(Caused by sports) = 0.1

2.4.5. Consider the hospital emergency room data given below. Let A denote the event that a visit is to hospital 4, and let B

denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Use the addition rules to calculate the following probabilities.

(a) $P(A \cup B)$ (b) $P(A \cup B')$ (c) $P(A' \cup B')$

P(A) = 4329/22252 = 0.1945, P(B) = 953/22252 = 0.0428, $P(A \cap B)$ = 242/22252 = 0.0109,
$P(A \cap B')$ = (984+3103)/22252 = 0.1837

(a) $P(A \cup B) = P(A) + P(B) – P(A \cap B)$ = 0.1945 + 0.0428 – 0.0109 = 0.2264

(b) $P(A \cup B') = P(A) + P(B') – P(A \cap B')$ = 0.1945 + (1 – 0.0428) – 0.1837 = 0.9680

(c) $P(A' \cup B') = 1 – P(A \cap B)$ = 1 – 0.0109 = 0.9891

2.4.6. Strands of copper wire from a manufacturer are analyzed for strength and conductivity. The results from 100 strands are as follows:

	Strength	
	High	Low
High conductivity	74	8
Low conductivity	15	3

(a) If a strand is randomly selected, what is the probability that its conductivity is high and its strength is high?
(b) If a strand is randomly selected, what is the probability that its conductivity is low or its strength is low?
(c) Consider the event that a strand has low conductivity and the event that the strand has low strength. Are these two events mutually exclusive?

(a) P(High temperature and high conductivity)= 74/100 =0.74
(b) P(Low temperature or low conductivity)
= P(Low temperature) + P(Low conductivity) – P(Low temperature and low conductivity)
= (8+3)/100 + (15+3)/100 – 3/100
= 0.26
(c) No, they are not mutually exclusive. Because P(Low temperature) + P(Low conductivity)
= (8+3)/100 + (15+3)/100
= 0.29, which is not equal to P(Low temperature or low conductivity).

2.4.7. A computer system uses passwords that are six characters, and each character is one of the 26 letters (*a*–*z*) or 10 integers (0–9). Uppercase letters are not used. Let *A* denote the event that a password begins with a vowel (either *a*, *e*, *i*, *o*, or *u*), and let *B* denote the event that a password ends with an even number (either 0, 2, 4, 6, or 8). Suppose a hacker selects a password at random. Determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$

(a) 5/36
(b) 5/36
(c) $P(A \cap B) = P(A)P(B) = 25/1296$
(d) $P(A \cup B) = P(A) + P(B) - P(A)P(B) = 10/36 - 25/1296 = 0.2585$

2.4.8. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let *A* denote the event that the design color is red, and let *B* denote the event that the font size is not the smallest one. Use the addition rules to calculate the following probabilities.

(a) $P(A \cup B)$ (b) $P(A \cup B')$ (c) $P(A' \cup B')$

P(A) = 1/4 = 0.25, P(B) = 4/5 = 0.80, P(A $\cap$ B) = P(A)P(B) = (1/4)(4/5) = 1/5 = 0.20
(a) P(A $\cup$ B) = P(A) + P(B) – P(A $\cap$ B) = 0.25 + 0.80 – 0.20 = 0.85
(b) First P(A $\cap$ B') = P(A)P(B′) = (1/4)(1/5) = 1/20 = 0.05. Then P(A $\cup$ B′) = P(A) + P(B′) – P(A $\cap$ B') = 0.25 + 0.20 – 0.05= 0.40
(c) P(A′ $\cup$ B′) = 1 – P(A $\cap$ B) = 1 – 0.20 = 0.80

2.4.9. A computer system uses passwords that contain exactly eight characters, and each character is one of the 26 lowercase letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Assume all passwords are equally likely. Let *A* and *B* denote the events that consist of passwords with only letters or only integers, respectively. Determine the following probabilities:

(a) $P(A \cup B)$ (b) $P(A' \cup B)$ (c) *P* (Password contains exactly 1 or 2 integers)

(a) P(AUB) = P(A) + P(B) = $\frac{52^8}{62^8} + \frac{10^8}{62^8}$ = 0.245
(b) P(A'UB) = P(A') = $\frac{10^8}{62^8}$ = 1 – 0.2448 = 0.755
(c) P(contains exactly 1 integer)
Number of positions for the integer is 8!/(1!7!) = 8
Number of values for the integer = 10
Number of permutations of the seven letters is 52^7
Total number of permutations is 62^8
Therefore, the probability is
$\frac{8(10)(52^7)}{62^8} = 0.377$

P(contains exactly 2 integers)
Number of positions for the integers is 8!/(2!6!) = 28
Number of permutations of the two integers is 100
Number of permutations of the 6 letters is 52^6
Total number of permutations is 62^8
Therefore, the probability is
$\frac{28(100)(52^6)}{62^8} = 0.254$
Therefore, P(exactly one integer or exactly two integers) = 0.377 + 0.254 = 0.630

2.4.10. Consider the three patient groups. The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let *A* denote the event that the patient was treated with ribavirin plus interferon alfa, and let *B* denote the event that the response was complete. Determine the following probabilities:

(a) $P(A \cup B)$ (b) $P(A' \cup B)$ (c) $P(A \cup B')$

(a) P(AUB)= P(A) + P(B) - P(A∩B) = 21/60 + 22/60 – 16/60 = 9/20= 0.45
(b) P(A'UB)= P(A') + P(B) - P(A'∩B) = (19+20)/60 + 22/60 – 6/60 = 11/12 = 0.9166
(c) P(AUB')= P(A) + P(B') - P(A∩B')= 21/60 + (60-22)/60 – 5/60 = 9/10 = 0.9

2.4.11. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose that a patient is selected randomly. Let *A* denote the event that the patient is in group 1, and let *B* denote the event that there is no progression. Determine the following probabilities:

(a) $P(A \cup B)$ (b) $P(A' \cup B')$ (c) $P(A \cup B')$

P(A)= $\frac{114}{114+112+120+121} = \frac{114}{467} = 0.244$
P(B)= $\frac{76+82+104+113}{114+112+120+121} = \frac{375}{467} = 0.8029$
P(A∩B)= $\frac{76}{467} = 0.162$
(a) P(AUB) = P(A) + P(B) - P(A∩B)= 114/467 + 375/467 – 76/467 = 0.884
(b) P(A'UB') = 1 - (A∩B) = 1 – 76/467 =0.838
(c) P(AUB') = P(A) + P(B') - P(A∩B') = 114/467 + (1 – 375/467) – (114 – 76)/467 = 0.359

Section 2-5

2.5.1. The analysis of results from a leaf transmutation experiment (turning a leaf into a petal) is summarized by type of transformation completed:

		Total Textural Transformation	
		Yes	No
Total color transformation	Yes	243	26
	No	13	18

(a) If a leaf completes the color transformation, what is the probability that it will complete the textural transformation?
(b) If a leaf does not complete the textural transformation, what is the probability it will complete the color transformation?

Let *A* denote the event that a leaf completes the color transformation and let *B* denote the event that a leaf completes the textural transformation. The total number of experiments is 300.

(a) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{243/300}{(243+26)/300} = 0.903$

(b) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{26/300}{(18+26)/300} = 0.591$

2.5.2. Samples of skin experiencing desquamation are analyzed for both moisture and melanin content. The results from 100 skin samples are as follows:

		Melanin Content	
		High	Low
Moisture content	High	13	7
	Low	48	32

Let *A* denote the event that a sample has low melanin content, and let *B* denote the event that a sample has high moisture content. Determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A \mid B)$ (d) $P(B \mid A)$

(a) $P(A)=\dfrac{7+32}{100}=0.39$

(b) $P(B)=\dfrac{13+7}{100}=0.2$

(c) $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{7/100}{20/100}=0.35$

(d) $P(B\mid A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{7/100}{39/100}=0.1795$

2.5.3. The following table summarizes the number of deceased beetles under autolysis (the destruction of a cell after its death by the action of its own enzymes) and putrefaction (decomposition of organic matter, especially protein, by microorganisms, resulting in production of foul-smelling matter):

		Autolysis	
		High	Low
Putrefaction	High	14	59
	Low	18	9

(a) If the autolysis of a sample is high, what is the probability that the putrefaction is low?
(b) If the putrefaction of a sample is high, what is the probability that the autolysis is high?
(c) If the putrefaction of a sample is low, what is the probability that the autolysis is low?

Let *A* denote the event that autolysis is high and let *B* denote the event that putrefaction is high. The total number of experiments is 100.

(a) $P(B'\mid A)=\dfrac{P(A\cap B')}{P(A)}=\dfrac{18/100}{(14+18)/100}=0.5625$

(b) $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{14/100}{(14+59)/100}=0.1918$

(c) $P(A'\mid B')=\dfrac{P(A'\cap B')}{P(B')}=\dfrac{9/100}{(18+9)/100}=0.333$

2.5.4. A maintenance firm has gathered the following information regarding the failure mechanisms for air conditioning systems:

		Evidence of Gas Leaks	
		Yes	No
Evidence of electrical failure	Yes	55	17
	No	32	3

The units without evidence of gas leaks or electrical failure showed other types of failure. If this is a representative sample of AC failure, find the probability
(a) That failure involves a gas leak
(b) That there is evidence of electrical failure given that there was a gas leak
(c) That there is evidence of a gas leak given that there is evidence of electrical failure

(a) P(gas leak) = (55 + 32)/107 = 0.813
(b) P(electric failure | gas leak) = (55/107)/(87/102) = 0.632

(c) P(gas leak | electric failure) = (55/107)/(72/107) = 0.764

2.5.5. Consider the endothermic reactions given below. Let *A* denote the event that a reaction's final temperature is 271 K or less. Let *B* denote the event that the heat absorbed is above target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Determine the following probabilities.

(a) $P(A \mid B)$ (b) $P(A' \mid B)$ (c) $P(A \mid B')$ (d) $P(B \mid A)$

(a) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{(40+16)/204}{(40+16+36)/204} = \dfrac{56}{92} = 0.6087$

(b) $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{36/204}{(40+16+36)/204} = \dfrac{36}{92} = 0.3913$

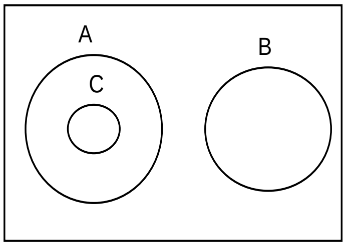
(c) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{56/204}{(12+44+56)/204} = \dfrac{56}{112} = 0.5$

(d) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{(40+16)/204}{(40+12+16+44)/204} = \dfrac{40+16}{112} = 0.5$

2.5.6. A batch of 500 containers for frozen orange juice contains 5 that are defective. Two are selected, at random, without replacement from the batch.
(a) What is the probability that the second one selected is defective given that the first one was defective?
(b) What is the probability that both are defective?
(c) What is the probability that both are acceptable?
Three containers are selected, at random, without replacement, from the batch.
(d) What is the probability that the third one selected is defective given that the first and second ones selected were defective?
(e) What is the probability that the third one selected is defective given that the first one selected was defective and the second one selected was okay?
(f) What is the probability that all three are defective?

(a) 4/499 = 0.0080
(b) (5/500)(4/499) = 0.000080
(c) (495/500)(494/499) = 0.98
(d) 3/498 = 0.0060
(e) 4/498 = 0.0080
(f) $\left(\dfrac{5}{500}\right)\left(\dfrac{4}{499}\right)\left(\dfrac{3}{498}\right) = 4.82\text{x}10^{-7}$

2.5.7. Suppose *A* and *B* are mutually exclusive events. Construct a Venn diagram that contains the three events *A*, *B*, and *C* such that $P(A \mid C) = 1$ and $P(B \mid C) = 0$.



2.5.8. An article in *The Canadian Entomologist* (Harcourt et al., 1977, Vol. 109, pp. 1521–1534) reported on the life of the alfalfa weevil from eggs to adulthood. The following table shows the number of larvae that survived at each stage of development from eggs to adults.

Eggs	Early Larvae	Late Larvae	Pre-pupae	Late Pupae	Adults
421	412	306	45	35	31

(a) What is the probability an egg survives to adulthood?
(b) What is the probability of survival to adulthood given survival to the late larvae stage?
(c) What stage has the lowest probability of survival to the next stage?

Let A denote the event that an egg survives to an adult
Let EL denote the event that an egg survives at early larvae stage
Let LL denote the event that an egg survives at late larvae stage
Let PP denote the event that an egg survives at pre-pupae larvae stage
Let LP denote the event that an egg survives at late pupae stage

(a) $P(A) = 31/421 = 0.0736$

(b) $P(A \mid LL) = \dfrac{P(A \cap LL)}{P(LL)} = \dfrac{31/421}{306/421} = 0.1013$

(c) $P(EL) = 412/421 = 0.9786$

$$P(LL \mid EL) = \frac{P(LL \cap EL)}{P(EL)} = \frac{306/421}{412/421} = 0.7427$$

$$P(PP \mid LL) = \frac{P(PP \cap LL)}{P(LL)} = \frac{45/421}{306/421} = 0.1471$$

$$P(LP \mid PP) = \frac{P(LP \cap PP)}{P(PP)} = \frac{35/421}{45/421} = 0.7778$$

$$P(A \mid LP) = \frac{P(A \cap LP)}{P(LP)} = \frac{31/421}{35/421} = 0.8857$$

The late larvae stage has the lowest probability of survival to the pre-pupae stage.

2.5.9. Consider the hospital emergency room data given below. Let *A* denote the event that a visit is to hospital 4, and let *B* denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(A \mid B)$ (b) $P(A' \mid B)$ (c) $P(A \mid B')$ (d) $P(B \mid A)$

(a) $P(A|B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{242/22252}{953/22252}=\dfrac{242}{953}=0.2539$

(b) $P(A'|B)=\dfrac{P(A'\cap B)}{P(B)}=\dfrac{(195+270+246)/22252}{953/22252}=\dfrac{711}{953}=0.7461$

(c) $P(A|B')=\dfrac{P(A\cap B')}{P(B')}=\dfrac{(984+3103)/22252}{(22252-953)/22252}=\dfrac{4087}{21299}=0.1919$

(d) $P(B|A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{242/22252}{4329/22252}=\dfrac{242}{4329}=0.0559$

2.5.10. A computer system uses passwords that contain exactly eight characters, and each character is one of the 26 lowercase letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Let Ω denote the set of all possible passwords. Suppose that all passwords in Ω are equally likely. Determine the probability for each of the following:
(a) Password contains all lowercase letters given that it contains only letters
(b) Password contains at least 1 uppercase letter given that it contains only letters
(c) Password contains only even numbers given that is contains all numbers

Let A= passwords with all letters, B = passwords with all lowercase letters

(a) P(B|A)= $\frac{P(A\cap B)}{P(A)}=\frac{\frac{26^8}{62^8}}{\frac{52^8}{62^8}}=\frac{26^8}{52^8}=0.0039$

(b) C = passwords with at least 1 uppercase letter

P(C|A)= $\frac{P(A\cap C)}{P(A)}$

P(A ∩ C) = P(A) – P(A ∩ C') = $\frac{52^8}{62^8}-\frac{26^8}{62^8}$

P(A) = $\frac{52^8}{62^8}$

Therefore, P(C | A) = 1 - $\frac{26^8}{52^8}$=0.996

(c) P(containing all even numbers | contains all numbers) = $\frac{5^8}{10^8}=0.0039$

2.5.11. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose that a patient is selected randomly. Let *A* denote the event that the patient is in group 1, and let *B* denote the event that there is no progression. Determine the following probabilities:

(a) $P(B \mid A)$ (b) $P(A \mid B)$ (c) $P(A \mid B')$ (d) $P(A' \mid B)$

P(A) = 114/467 P(B) = 375/467 P(A ∩ B) = 76/467

(a) P(B|A)= $\frac{P(A\cap B)}{P(A)} = \frac{76/467}{114/467} = 0.667$

(b) P(A|B)= $\frac{P(A\cap B)}{P(B)} = \frac{76/467}{375/467} = 0.203$

(c) P(A|B')= $\frac{P(A\cap B')}{P(B')} = \frac{38/467}{92/467} = 0.413$

(d) P(A'|B)= $\frac{P(A'\cap B)}{P(B)} = \frac{299/467}{375/467} = 0.797$

Section 2-6

2.6.1. Suppose that $P(A \mid B) = 0.4$ and $P(B) = 0.5$. Determine the following:

(a) $P(A \cap B)$ (b) $P(A' \cap B)$

(a) $P(A \cap B) = P(A|B)P(B) = (0.4)(0.5) = 0.20$

(b) $P(A' \cap B) = P(A'|B)P(B) = (0.6)(0.5) = 0.30$

2.6.2. Suppose that $P(A \mid B) = 0.2$, $P(A \mid B') = 0.3$, and $P(B) = 0.8$. What is $P(A)$?

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ &= P(A|B)P(B) + P(A|B')P(B') \\ &= (0.2)(0.8) + (0.3)(0.2) \\ &= 0.16 + 0.06 = 0.22 \end{aligned}$$

2.6.3. The probability is 1% that an electrical connector that is kept dry fails during the warranty period. If the connector is ever wet, the probability of a failure during the warranty period is 5%. If 90% of the connectors are kept dry and 10% are wet, what proportion of connectors fail during the warranty period? Let F denote the event that a connector fails and let W denote the event that a connector is wet.

$$\begin{aligned} P(F) &= P(F|W)P(W) + P(F|W')P(W') \\ &= (0.05)(0.10) + (0.01)(0.90) = 0.014 \end{aligned}$$

2.6.4. Heart failures are due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances (73%) or foreign objects (27%). Natural occurrences are caused by arterial blockage (56%), disease (27%), and infection (e.g., staph infection) (17%).

(a) Determine the probability that a failure is due to an induced substance.

(b) Determine the probability that a failure is due to disease or infection

(a)P = 0.13 × 0.73=0.0949

(b)P = 0.87 × (0.27+0.17)=0.3828

2.6.5. The edge roughness of slit paper products increases as knife blades wear. Only 1% of products slit with new blades have rough edges, 3% of products slit with blades of average sharpness exhibit roughness, and 5% of products slit with worn blades exhibit roughness. If 25% of the blades in manufacturing are new, 60% are of average sharpness, and 15% are worn, what is the proportion of products that exhibit edge roughness?

Let R denote the event that a product exhibits surface roughness. Let N, A, and W denote the events that the blades are new, average, and worn, respectively. Then,

P(R)= P(R|N)P(N) + P(R|A)P(A) + P(R|W)P(W)

= (0.01)(0.25) + (0.03) (0.60) + (0.05)(0.15)

= 0.028

2.6.6. A lot of 100 semiconductor chips contains 20 that are defective.

(a) Two are selected, at random, without replacement, from the lot. Determine the probability that the second chip selected is defective.

(b) Three are selected, at random, without replacement, from the lot. Determine the probability that all are defective.

Let A and B denote the events that the first and second chips selected are defective, respectively.
(a) P(B) = P(B|A)P(A) + P(B|A')P(A') = (19/99)(20/100) + (20/99)(80/100) = 0.2
(b) Let C denote the event that the third chip selected is defective.

$$P(A \cap B \cap C) = P(C|A \cap B)P(A \cap B) = P(C|A \cap B)P(B|A)P(A)$$
$$= \frac{18}{98}\left(\frac{19}{99}\right)\left(\frac{20}{100}\right)$$
$$= 0.00705$$

2.6.7. Computer keyboard failures are due to faulty electrical connects (12%) or mechanical defects (88%). Mechanical defects are related to loose keys (27%) or improper assembly (73%). Electrical connect defects are caused by defective wires (35%), improper connections (13%), or poorly welded wires (52%).
(a) Find the probability that a failure is due to loose keys.
(b) Find the probability that a failure is due to improperly connected or poorly welded wires.

(a) (0.88)(0.27) = 0.2376
(b) (0.12)(0.13+0.52) = 0.0.078

2.6.8. An article in the *British Medical Journal* ["Comparison of treatment of renal calculi by operative surgery, percutaneous nephrolithotomy, and extracorporeal shock wave lithotripsy" (1986, Vol. 82, pp. 879–892)] provided the following discussion of success rates in kidney stone removals. Open surgery had a success rate of 78% (273/350) and a newer method, percutaneous nephrolithotomy (PN), had a success rate of 83% (289/350). This newer method looked better, but the results changed when stone diameter was considered. For stones with diameters less than 2 centimeters, 93% (81/87) of cases of open surgery were successful compared with only 83% (234/270) of cases of PN. For stones greater than or equal to 2 centimeters, the success rates were 73% (192/263) and 69% (55/80) for open surgery and PN, respectively. Open surgery is better for both stone sizes, but less successful in total. In 1951, E. H. Simpson pointed out this apparent contradiction (known as **Simpson's paradox**), and the hazard still persists today. Explain how open surgery can be better for both stone sizes but worse in total.

Open surgery

	success	failure	sample size	sample percentage	conditional success rate
large stone	192	71	263	75%	73%
small stone	81	6	87	25%	93%
overall summary	273	77	350	100%	78%

PN

	success	failure	sample size	sample percentage	conditional success rate
large stone	55	25	80	23%	69%
small stone	234	36	270	77%	83%
overall summary	289	61	350	100%	83%

The overall success rate depends on the success rates for each stone size group, but also the probability of the groups. It is the weighted average of the group success rate weighted by the group size as follows

P(overall success) = P(success| large stone)P(large stone)) + P(success| small stone)P(small stone).

For open surgery, the dominant group (large stone) has a smaller success rate while for PN, the dominant group (small stone) has a larger success rate.

2.6.9. Consider the hospital emergency room data given below. Let *A* denote the event that a visit is to hospital 4 and let *B* denote the event that a visit results in LWBS (at any hospital).

Hospital					
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A \cup B)$ (c) $P(A' \cup B')$ (d)Use the total probability rule to determine $P(A)$

P(A) = 4329/22252 = 0.1945, P(B) = 953/22252 = 0.0428
(a) $P(A \cap B) = P(A \mid B)P(B)$ = (242/953)(953/22252) = 0.0109
(b) $P(A \cup B) = P(A) + P(B) – P(A \cap B)$ = 0.1945 + 0.0428 – 0.0109 = 0.2264
(c) $P(A' \cup B') = 1 – P(A \cap B)$ = 1 – 0.0109 = 0.9891
(d) P(A) = P(A | B)P(B) + P(A | B')P(B') = (242/953)(953/22252) + (4087/21299)(21299/22252) = 0.1945

2.6.10. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose that a patient is selected randomly. Let *A* denote the event that the patient is in group 1, and let *B* denote the event for which there is no progression. Determine the following probabilities:

(a) *P*(*A*) *B*) (b) *P*(*B*) (c) *P*(*A*__ *B*) (d) *P*(*A*∗ *B*) (e) *P*(*A*__ *B*)

A = group 1, B = no progression
(a) P(A∩B) = P(B|A)P(A)= (76/114)(114/467) = 0.162
(b) P(B) = P(B|G1)P(G1) + P(B|G2)P(G2) + P(B|G3)P(G3) + P(P|G4)P(G4) = 0.802
(c) P(A'∩B) = P(B|A') P(A') = (299/353)(353/467) = 0.6403
(d) P(A U B) = P(A) + P(B) - P(A∩B) = 114/467 + 375/467 – 76/467 = 0.884
(e) P(A' U B) = P(A') + P(B) - P(A'∩B) = 353/467+375/467 – 299/467 = 0.919

2.6.11. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Determine the probability that the ad color is red and the font size is not the smallest one.

Let R denote red color and F denote that the font size is not the smallest. Then P(R) = 1/4, P(F) = 4/5.
Because the Web sites are generated randomly these events are independent. Therefore, P(R ∩ F) = P(R)P(F) = (1/4)(4/5) = 0.2

2.6.12. A hospital operating room needs to schedule three knee surgeries and two hip surgeries in a day. Suppose that an operating room needs to schedule three knee, four hip, and five shoulder surgeries. Assume that all schedules are equally likely. Determine the following probabilities:
(a) All hip surgeries are completed first given that all knee surgeries are last.
(b) The schedule begins with a hip surgery given that all knee surgeries are last.
(c) The first and last surgeries are hip surgeries given that knee surgeries are scheduled in time periods 2 through 4.
(d) The first two surgeries are hip surgeries given that all knee surgeries are last.

(a) P(all hip surgeries are completed first given that all knee surgeries are last)
A = schedules with all hip surgeries completed first
B = schedules with all knee surgeries last
Total number of schedules $=\frac{12!}{3!4!5!}$
Number of schedules with all knee surgeries last = $\frac{9!}{4!5!}$

Number of schedules with all hip surgeries first and all knee surgeries last = 1
P(B) = $\frac{9!}{4!5!}/\frac{12!}{3!4!5!}$, P(A ∩ B) = $1/\frac{12!}{3!4!5!}$
Therefore, P(A | B) = P(A ∩ B)/P(B) = 1/$\frac{9!}{4!5!}$=1/126
Alternatively, one can reason that when all knee surgeries are last, there are $\frac{9!}{4!5!}$ remaining schedules and one of these has all knee surgeries first. Therefore, the solution is 1/$\frac{9!}{4!5!} = 1/126$

(b) P(schedule begins with a hip surgery given that all knee surgeries are last)
C = schedules that begin with a hip surgery
B = schedules with all knee surgeries last
P(B) = $\frac{9!}{4!5!}/\frac{12!}{3!4!5!}$, P(C ∩ B) = $\frac{8!}{3!5!}/\frac{12!}{3!4!5!}$
Therefore, P(C | B) = P(C ∩ B)/P(B) = $\frac{8!}{3!5!}/\frac{9!}{4!5!}$ = 4/9
Alternatively, one can reason that when all knee surgeries are last, there are 4 hip and 5 shoulder surgeries that remain to schedule. The probability the first one is a hip surgery is then 4/9

(c) P(first and last surgeries are hip surgeries given that knee surgeries are scheduled in time periods 2 through 4)
D = schedules with first and last hip surgeries
E = schedules with knee surgeries in periods 2 through 4
P(E) =$\frac{9!}{4!5!}/\frac{12!}{3!4!5!}$
P(D∩E) = $\frac{7!}{2!5!}/\frac{12!}{3!4!5!}$
P(D | E) = P(D∩E)/P(E) = $\frac{7!}{2!5!}/\frac{9!}{4!5!}$ = 1/6
Alternatively, one can conclude that with knee surgeries in periods 2 through 4, there are $\frac{9!}{4!5!}$ remaining schedules and $\frac{7!}{2!5!}$ of these have hip surgeries first and last. Therefore, P(D | E) = P(D∩E)/P(E) = $\frac{7!}{2!5!}/\frac{9!}{4!5!}$ = 1/6

(d) P(first two surgeries are hip surgeries given that all knee surgeries are last)
F = schedules with the first two surgeries as hip surgeries
B = schedules with all knee surgeries last
P(B) = $\frac{9!}{4!5!}/\frac{12!}{3!4!5!}$,
P(F∩B) = $\frac{7!}{2!5!}/\frac{12!}{3!4!5!}$
P(F | B) = P(F∩B)/P(B) = $\frac{7!}{2!5!}/\frac{9!}{4!5!}$ = 1/6
Alternatively, one can conclude that with knee surgeries last, there are $\frac{9!}{4!5!}$ remaining schedules and $\frac{7!}{2!5!}$ have hip surgeries in the first two periods. Therefore, P(F | B) = P(F∩B)/P(B) = $\frac{7!}{2!5!}/\frac{9!}{4!5!}$ = 1/6

Section 2-7

2.7.1. If $P(A \mid B) = 0.3$, $P(B) = 0.8$, and $P(A) = 0.3$, are the events *B* and the complement of *A* independent?

P(A') = 1 – P(A) = 0.7 and P($A' \mid B$) = 1 – P(A | B) = 0.7

Therefore, A' and B are independent events.

2.7.2. If $P(A) = 0.2$, $P(B) = 0.2$, and *A* and *B* are mutually exclusive, are they independent?

If A and B are mutually exclusive, then P($A \cap B$) = 0 and P(A)P(B) = 0.04.
Therefore, A and B are <u>not</u> independent.

2.7.3. A batch of 500 containers of frozen orange juice contains 5 that are defective. Two are selected, at random, without replacement, from the batch. Let *A* and *B* denote the events that the first and second containers selected are defective, respectively.
(a) Are *A* and *B* independent events?
(b) If the sampling were done with replacement, would *A* and *B* be independent?

(a) $P(B \mid A)$ = 4/499 and
$P(B) = P(B|A)P(A) + P(B|A')P(A') = (4/499)(5/500) + (5/499)(495/500) = 5/500$
Therefore, *A* and *B* are <u>not</u> independent.

(b) *A* and *B* are independent.

2.7.4. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		Shock Resistance	
		High	Low
Scratch Resistance	High	70	9
	Low	16	5

Let *A* denote the event that a disk has high shock resistance, and let *B* denote the event that a disk has high scratch resistance. Are events *A* and *B* independent?

$P(A\cap B) = 70/100$, $P(A) = 86/100$, $P(B) = 77/100$.
Then, $P(A\cap B) \neq P(A)P(B)$, so A and B are not independent.

2.7.5. Redundant array of inexpensive disks (RAID) is a technology that uses multiple hard drives to increase the speed of data transfer and provide instant data backup. Suppose that the probability of any hard drive failing in a day is 0.001 and the drive failures are independent.
(a) A RAID 0 scheme uses two hard drives, each containing a mirror image of the other. What is the probability of data loss? Assume that data loss occurs if both drives fail within the same day.
(b) A RAID 1 scheme splits the data over two hard drives. What is the probability of data loss? Assume that data loss occurs if at least one drive fails within the same day.

(a) $P = (0.001)^2 = 10^{-6}$

(b) $P = 1-(0.999)^2 = 0.002$

2.7.6. A test of a printed circuit board uses a random test pattern with an array of 10 bits is equally likely to be 0 or 1. Assume the bits are independent.
(a) What is the probability that all bits are 1s?
(b) What is the probability that all bits are 0s?
(c) What is the probability that exactly 5 bits are 1s and 5 bits are 0s?

Let A_i denote the event that the ith bit is a one.

(a) By independence $P(A_1 \cap A_2 \cap ... \cap A_{10}) = P(A_1)P(A_2)...P(A_{10}) = (\frac{1}{2})^{10} = 0.000976$

(b) By independence, $P(A_1^{'} \cap A_2^{'} \cap ... \cap A_{10}^{'}) = P(A_1^{'})P(A_2^{'})...P(A_{10}^{c}) = (\frac{1}{2})^{10} = 0.000976$

(c) The probability of the following sequence is

$P(A_1^{'} \cap A_2^{'} \cap A_3^{'} \cap A_4^{'} \cap A_5^{'} \cap A_6 \cap A_7 \cap A_8 \cap A_9 \cap A_{10}) = (\frac{1}{2})^{10}$, by independence. The number of sequences consisting of five "1"'s, and five "0"'s is $\binom{10}{5} = \frac{10!}{5!5!} = 252$. The answer is $252\left(\frac{1}{2}\right)^{10} = 0.246$

2.7.7. The probability that a lab specimen contains high levels of contamination is 0.10. Five samples are checked, and the samples are independent.
(a) What is the probability that none contain high levels of contamination?
(b) What is the probability that exactly one contains high levels of contamination?
(c) What is the probability that at least one contains high levels of contamination?

It is useful to work one of these exercises with care to illustrate the laws of probability. Let H_i denote the event that the *i*th sample contains high levels of contamination.

(a) $P(H_1^{'} \cap H_2^{'} \cap H_3^{'} \cap H_4^{'} \cap H_5^{'}) = P(H_1^{'})P(H_2^{'})P(H_3^{'})P(H_4^{'})P(H_5^{'})$

by independence. Also, $P(H_i') = 0.9$. Therefore, the answer is $0.9^5 = 0.59$

(b) $A_1 = (H_1 \cap H_2' \cap H_3' \cap H_4' \cap H_5')$

$A_2 = (H_1' \cap H_2 \cap H_3' \cap H_4' \cap H_5')$

$A_3 = (H_1' \cap H_2' \cap H_3 \cap H_4' \cap H_5')$

$A_4 = (H_1' \cap H_2' \cap H_3' \cap H_4 \cap H_5')$

$A_5 = (H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5)$

The requested probability is the probability of the union $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$ and these events are mutually exclusive. Also, by independence $P(A_i) = 0.9^4(0.1) = 0.0656$. Therefore, the answer is 5(0.0656) = 0.328.

(c) Let B denote the event that no sample contains high levels of contamination. The requested probability is P(B') = 1 - P(B). From part (a), P(B') = 1 - 0.59 = 0.41.

2.7.8. A player of a video game is confronted with a series of four opponents and an 80% probability of defeating each opponent. Assume that the results from opponents are independent (and that when the player is defeated by an opponent the game ends).
(a) What is the probability that a player defeats all four opponents in a game?
(b) What is the probability that a player defeats at least two opponents in a game?
(c) If the game is played three times, what is the probability that the player defeats all four opponents at least once?

(a) $P = (0.8)^4 = 0.4096$

(b) $P = 1 - 0.2 - 0.8 \times 0.2 = 0.64$

(c) Probability defeats all four in a game = 0.8^4 = 0.4096. Probability defeats all four at least once = $1 - (1 - 0.4096)^3$ = 0.7942

2.7.9. Eight cavities in an injection-molding tool produce plastic connectors that fall into a common stream. A sample is chosen every several minutes. Assume that the samples are independent.
(a) What is the probability that five successive samples were all produced in cavity 1 of the mold?
(b) What is the probability that five successive samples were all produced in the same cavity of the mold?
(c) What is the probability that four out of five successive samples were produced in cavity 1 of the mold?

Let A denote the event that a sample is produced in cavity one of the mold.

(a) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) = (\frac{1}{8})^5 = 0.00003$

(b) Let B_i be the event that all five samples are produced in cavity i. Because the B's are mutually exclusive, $P(B_1 \cup B_2 \cup ... \cup B_8) = P(B_1) + P(B_2) + ... + P(B_8)$

From part (a), $P(B_i) = (\frac{1}{8})^5$. Therefore, the answer is $8(\frac{1}{8})^5 = 0.00024$

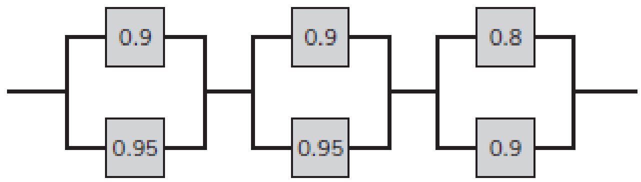
(c) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5') = (\frac{1}{8})^4(\frac{7}{8})$. The number of sequences in which four out of five samples are from cavity one is 5. Therefore, the answer is $5(\frac{1}{8})^4(\frac{7}{8}) = 0.00107$.

2.7.10. A credit card contains 16 digits. It also contains the month and year of expiration. Suppose there are 1 million credit card holders with unique card numbers. A hacker randomly selects a 16-digit credit card number.
(a) What is the probability that it belongs to a user?
(b) Suppose a hacker has a 25% chance of correctly guessing the year your card expires and randomly selects 1 of the 12 months. What is the probability that the hacker correctly selects the month and year of expiration?

(a) $P = \frac{10^6}{10^{16}} = 10^{-10}$

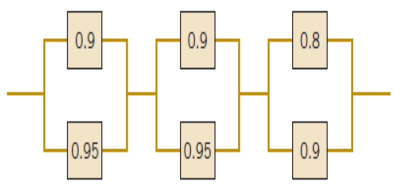
(b) $P = 0.25 \times (\frac{1}{12}) = 0.020833$

2.7.11. The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



Let A denote the upper devices function. Let B denote the lower devices function.
P(A) = (0.9)(0.8)(0.7) = 0.504
P(B) = (0.95)(0.95)(0.95) = 0.8574
P(A∩B) = (0.504)(0.8574) = 0.4321
Therefore, the probability that the circuit operates = P(A∪B) = P(A) +P(B) – P(A∩B) = 0.9293

2.7.12. The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



P = [1 – (0.1)(0.05)][1 – (0.1)(0.05)][1 – (0.2)(0.1)] = 0.9702

2.7.13. Consider the hospital emergency room data given below. Let *A* denote the event that a visit is to hospital 4, and let *B* denote the event that a visit results in LWBS (at any hospital). Are these events independent?

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

P(A) = 4329/22252 = 0.1945, P(B) = 953/22252 = 0.0428, P(A ∩ B) = 242/22252 = 0.0109
Because P(A)*P(B) = (0.1945)(0.0428) = 0.0083 ≠ 0.0109 = P(A ∩ B), A and B are not independent.

2.7.14. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let *A* denote the event that the design color is red, and let *B* denote the event that the font size is not the smallest one. Are *A* and *B* independent events? Explain why or why not.

P(A) = (3*5*3*5)/(4*3*5*3*5) = 0.25, P(B) = (4*3*4*3*5)/(4*3*5*3*5) = 0.8,
P(A ∩ B) = (3*4*3*5) /(4*3*5*3*5) = 0.2
Because P(A)*P(B) = (0.25)(0.8) = 0.2 = P(A ∩ B), A and B are independent.

2.7.15. An integrated circuit contains 10 million logic gates (each can be a logical AND or OR circuit). Assume the probability of a gate failure is *p* and that the failures are independent. The integrated circuit fails to function if any gate fails. Determine the value for *p* so that the probability that the integrated circuit functions is 0.95.

p = probability of gate failure
A = event that the integrated circuit functions
P(A) = 0.95 => P(A') = 0.05
(1 – *p*) = probability of gate functioning
Hence from the independence, $P(A) = (1 - p)^{10,000,000} = 0.95$.
Take logarithms to obtain $10^7 \ln(1 - p) = \ln(0.95)$ and
$p = 1 - \exp[10^{-7}\ln(0.95)] = 5.13\text{x}10^{-9}$

2.7.16. The following table provides data on wafers categorized by location and contamination levels. Let *A* denote the event that contamination is *low*, and let *B* denote the event that the location is *center*. Are *A* and *B* independent? Why or why not?

	Location in Sputtering Tool		
Contamination	Center	Edge	Total
Low	514	68	582
High	112	246	358
Total	626	314	

A: contamination is low, B: location is center
For A and B to be independent, $P(A\cap B) = P(A)\,P(B)$
$P(A\cap B) = 514/940 = 0.546$
P(A) = 582/940 = 0.619; P(B) = 626/940 = 0.665; P(A)P(B) = 0.412. Because the probabilities are not equal, they are not independent.

Section 2-8

2.8.1. Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.
(a) What is the probability that a product attains a good review?
(b) If a new design attains a good review, what is the probability that it will be a highly successful product?
(c) If a product does not attain a good review, what is the probability that it will be a highly successful product?

Let G denote a product that received a good review. Let H, M, and P denote products that were high, moderate, and poor performers, respectively.

(a)
$$\begin{aligned} P(G) &= P(G|H)P(H) + P(G|M)P(M) + P(G|P)P(P) \\ &= 0.95(0.40) + 0.60(0.35) + 0.10(0.25) \\ &= 0.615 \end{aligned}$$

(b) Using the result from part (a)
$$P(H|G) = \frac{P(G|H)P(H)}{P(G)} = \frac{0.95(0.40)}{0.615} = 0.618$$

(c) $$P(H|G') = \frac{P(G'|H)P(H)}{P(G')} = \frac{0.05(0.40)}{1 - 0.615} = 0.052$$

2.8.2. Suppose that *P*(*A* | *B*) = 0.7, *P*(*A*) = 0.5, and *P*(*B*) = 0.2. Determine *P*(*B* | *A*).

Because, $P(A|B)\,P(B) = P(A\cap B) = P(B|A)\,P(A)$,

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.7(0.2)}{0.5} = 0.28$$

2.8.3. A new analytical method to detect pollutants in water is being tested. This new method of chemical analysis is important because, if adopted, it could be used to detect three different contaminants—organic pollutants, volatile solvents, and chlorinated compounds—instead of having to use a single test for each pollutant. The makers of the test claim that it can detect high levels of organic pollutants with 99.7% accuracy, volatile solvents with 99.95% accuracy, and chlorinated compounds with 89.7% accuracy. If a pollutant is not present, the test does not signal. Samples are prepared for the calibration of the test and 60% of them are contaminated with organic pollutants, 27% with volatile solvents, and 13% with traces of chlorinated compounds. A test sample is selected randomly.
(a) What is the probability that the test will signal?
(b) If the test signals, what is the probability that chlorinated compounds are present?

Denote as follows: S = signal, O = organic pollutants, V = volatile solvents, C = chlorinated compounds
(a) P(S) = P(S|O)P(O)+P(S|V)P(V)+P(S|C)P(C) = 0.997(0.60) + 0.9995(0.27) + 0.897(0.13) = 0.9847
(b) P(C|S) = P(S|C)P(C)/P(S) = (0.897)(0.13)/0.9847 = 0.1184

2.8.4. Software to detect fraud in consumer phone cards tracks the number of metropolitan areas where calls originate each day. It is found that 1% of the legitimate users originate calls from two or more metropolitan areas in a single day. However, 30% of fraudulent users originate calls from two or more metropolitan areas in a single day. The proportion of fraudulent users is 0.01%. If the same user originates calls from two or more metropolitan areas in a single day, what is the probability that the user is fraudulent?

Let F denote a fraudulent user and let T denote a user that originates calls from two or more metropolitan areas in a day. Then,

$$P(F|T) = \frac{P(T|F)P(F)}{P(T|F)P(F) + P(T|F')P(F')} = \frac{0.30(0.0001)}{0.30(0.0001) + 0.01(.9999)} = 0.003$$

2.8.5. Consider the hospital emergency room data given below. Use Bayes' theorem to calculate the probability that a person visits hospital 4 given they are LWBS.

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Let L denote the event that a person is LWBS
Let A denote the event that a person visits Hospital 1
Let B denote the event that a person visits Hospital 2
Let C denote the event that a person visits Hospital 3
Let D denote the event that a person visits Hospital 4

$$\begin{aligned} P(D \mid L) &= \frac{P(L \mid D)P(D)}{P(L \mid A)P(A) + P(L \mid B)P(B) + P(L \mid C)P(C) + P(L \mid D)P(D)} \\ &= \frac{(0.0559)(0.1945)}{(0.0368)(0.2378) + (0.0386)(0.3142) + (0.0436)(0.2535) + (0.0559)(0.1945)} \\ &= 0.2540 \end{aligned}$$

2.8.6. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose that a

patient is selected randomly. Let *A* denote the event that the patient is in group 1, and let *B* denote the event that there is no progression.
Determine the following probabilities:

(a) $P(B)$ (b) $P(B \mid A)$ (c) $P(A \mid B)$

(a) P(B) = P(B|G1) P(G1)+P(B|G2) P(G2)+P(B|G3) P(G3)+P(P|G4) P(G4) = 0.802

(b) P(B|A) = $\frac{P(A\cap B)}{P(A)}$ = 76/114 = 0.667

(c) P(A|B) =

$$\frac{P(B|A)P(A)}{\mathrm{P(B|G1)P(G1)+P(B|G2)P(G2)+P(B|G3)P(G3)+P(P|G4)P(G4)}} = \frac{0.667(0.244)}{0.802} = 0.203$$

2.8.7. Two Web colors are used for a site advertisement. If a site visitor arrives from an affiliate, the probabilities of the blue or green colors being used in the advertisement are 0.8 and 0.2, respectively. If the site visitor arrives from a search site, the probabilities of blue and green colors in the advertisement are 0.4 and 0.6, respectively. The proportions of visitors from affiliates and search sites are 0.3 and 0.7, respectively. What is the probability that a visitor is from a search site given that the blue ad was viewed?

Denote as follows: A = affiliate site, S = search site, B =blue, G =green

$$P(S \mid B) = \frac{P(B \mid S)P(S)}{P(B \mid S)P(S) + P(B \mid A)P(A)} = \frac{(0.4)(0.7)}{(0.4)(0.7) + (0.8)(0.3)} = 0.5$$

2.8.8. A recreational equipment supplier finds that among orders that include tents, 40% also include sleeping mats. Only 5% of orders that do not include tents do include sleeping mats. Also, 20% of orders include tents. Determine the following probabilities:
(a) The order includes sleeping mats.
(b) The order includes a tent given it includes sleeping mats.

SM: Sleeping Mats; T:Tents;

P(SM|T) = 0.4; P(SM|T') = 0.05; P(T) = 0.2

(a) P(SM) = P(SM|T)P(T) + P(SM|T')P(T') = 0.4(0.2) + 0.05(0.8) = 0.12

(b) P(T|SM) = $\frac{\mathrm{P(SM|T)P(T)}}{P(SM)} = \frac{0.4X0.2}{0.12} = 0.667$

2.8.9. An e-mail filter is planned to separate valid e-mails from spam. The word *free* occurs in 60% of the spam messages and only 4% of the valid messages. Also, 20% of the messages are spam. Determine the following probabilities:
(a) The message contains *free.*
(b) The message is spam given that it contains *free.*
(c) The message is valid given that it does not contain *free.*

F: Free; S: Spam; V: Valid

P(F|S) = 0.6, P(F|V) = 0.04

(a) P(F) = P(F|S) P(S)+P(F|V) P(V) = 0.6(0.2) + 0.04(0.8) = 0.152

(b) P(S|F) = $\frac{P(F|S)P(S)}{P(F)} = \frac{0.6(0.2)}{0.152}$ = 0.789

(c) P(V|F') = $\frac{P(F'|V)P(V)}{P(F')} = \frac{(0.96)0.8}{1-0.152} = 0.906$

Section 2-9

2.9.1. Decide whether a discrete or continuous random variable is the best model for each of the following variables:
(a) The number of cracks exceeding one-half inch in 10 miles of an interstate highway.
(b) The weight of an injection-molded plastic part.
(c) The number of molecules in a sample of gas.
(d) The concentration of output from a reactor.

(e) The current in an electronic circuit.

(a) discrete (b) continuous (c) discrete, but large values might be modeled as continuous
(d) continuous (e) continuous

2.9.2. Decide whether a discrete or continuous random variable is the best model for each of the following variables:
(a) The time until a projectile returns to earth.
(b) The number of times a transistor in a computer memory changes state in one operation.
(c) The volume of gasoline that is lost to evaporation during the filling of a gas tank.
(d) The outside diameter of a machined shaft.

(a) continuous (b) discrete (c) continuous (d) continuous

2.9.3. Decide whether a discrete or continuous random variable is the best model for each of the following variables:
(a) The time for a computer algorithm to assign an image to a category.
(b) The number of bytes used to store a file in a computer.
(c) The ozone concentration in micrograms per cubic meter.
(d) The ejection fraction (volumetric fraction of blood pumped from a heart ventricle with each beat).
(e) The fluid flow rate in liters per minute.

(a) continuous (b) discrete, but large values might be modeled as continuous
(c) continuous (d) continuous (e) continuous

Supplemental Exercises

2.S4. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2% and 1%, respectively, of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

Let B denote the event that a glass breaks.
Let L denote the event that large packaging is used.
P(B)= P(B|L)P(L) + P(B|L')P(L')
= 0.01(0.60) + 0.02(0.40) = 0.014

2.S5. Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and edge finish. The results of 100 parts are summarized as follows:

		Edge Finish	
		Excellent	Good
Surface finish	Excellent	80	2
	Good	10	8

Let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent edge finish. If a part is selected at random, determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A')$ (d) $P(A \cap B)$ (e) $P(A \cup B)$ (f) $P(A' \cup B)$

Let A = excellent surface finish; B = excellent length
(a) P(A) = 82/100 = 0.82
(b) P(B) = 90/100 = 0.90
(c) P(A') = 1 – 0.82 = 0.18
(d) P(A∩B) = 80/100 = 0.80
(e) P(A∪B) = 0.92
(f) P(A’∪B) = 0.98

2.S6. A lot contains 15 castings from a local supplier and 25 castings from a supplier in the next state. Two castings are selected randomly, without replacement, from the lot of 40. Let *A* be the event that the first casting selected is from the local supplier, and let *B* denote the event that the second casting is selected from the local supplier. Determine:

(a) $P(A)$ (b) $P(B \mid A)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$

Suppose that 3 castings are selected at random, without replacement, from the lot of 40. In addition to the definitions of events *A* and *B*, let *C* denote the event that the third casting selected is from the local supplier. Determine:

(e) $P(A \cap B \cap C)$ (f) $P(A \cap B \cap C')$

(a) P(A) = 15/40

(b) P($B|A$) = 14/39

(c) P($A \cap B$) = P(A) P(B/A) = (15/40) (14/39) = 0.135

(d) P($A \cup B$) = 1 – P(A' and B') = $1-\left(\frac{25}{40}\right)\left(\frac{24}{39}\right)=0.615$

A = first is local, B = second is local, C = third is local
(e) P(A ∩ B ∩ C) = (15/40)(14/39)(13/38) = 0.046
(f) P(A ∩ B ∩ C') = (15/40)(14/39)(25/39) = 0.089

2.S7. If *A*, *B*, and *C* are mutually exclusive events, is it possible for $P(A) = 0.3$, $P(B) = 0.4$, and $P(C) = 0.5$? Why or why not?

If A,B,C are mutually exclusive, then P($A \cup B \cup C$) = P(A) + P(B) + P(C) = 0.3 + 0.4 + 0.5 = 1.2, which greater than 1. Therefore, P(A), P(B),and P(C) cannot equal the given values.

2.S8. Incoming calls to a customer service center are classified as complaints (75% of calls) or requests for information (25% of calls). Of the complaints, 40% deal with computer equipment that does not respond and 57% deal with incomplete software installation; in the remaining 3% of complaints, the user has improperly followed the installation instructions. The requests for information are evenly divided on technical questions (50%) and requests to purchase more products (50%).

(a) What is the probability that an incoming call to the customer service center will be from a customer who has not followed installation instructions properly?
(b) Find the probability that an incoming call is a request for purchasing more products.

Let U denote the event that the user has improperly followed installation instructions.
Let C denote the event that the incoming call is a complaint.
Let P denote the event that the incoming call is a request to purchase more products.
Let R denote the event that the incoming call is a request for information.
a) P(U|C)P(C) = (0.75)(0.03) = 0.0225
b) P(P|R)P(R) = (0.50)(0.25) = 0.125

2.S9. In the manufacturing of a chemical adhesive, 3% of all batches have raw materials from two different lots. This occurs when holding tanks are replenished and the remaining portion of a lot is insufficient to fill the tanks. Only 5% of batches with material from a single lot require reprocessing. However, the viscosity of batches consisting of two or more lots of material is more difficult to control, and 40% of such batches require additional processing to achieve the required viscosity.

Let *A* denote the event that a batch is formed from two different lots, and let *B* denote the event that a lot requires additional processing. Determine the following probabilities:

(a) $P(A)$ (b) $P(A')$ (c) $P(B \mid A)$ (d) $P(B \mid A')$
(e) $P(A \cap B)$ (f) $P(A \cap B')$ (g) $P(B)$

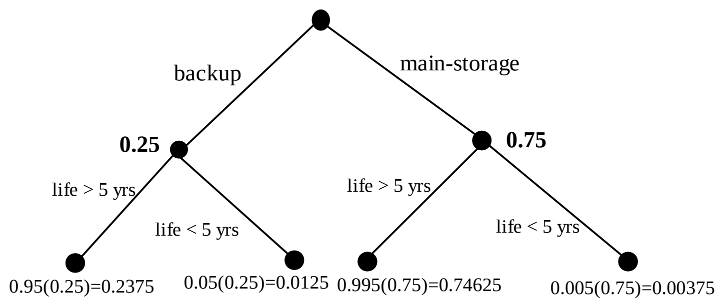
(a) P(A) = 0.03
(b) P(A') = 0.97
(c) P(B|A) = 0.40
(d) P(B|A') = 0.05
(e) P($A \cap B$) = P($B|A$)P(A) = (0.40)(0.03) = 0.012

(f) P($A \cap B'$) = P($B'|A$)P(A) = (0.60)(0.03) = 0.018

(g) P(B) = P($B|A$)P(A) + P($B|A'$)P(A') = (0.40)(0.03) + (0.05)(0.97) = 0.0605

2.S10. Semiconductor lasers used in optical storage products require higher power levels for write operations than for read operations. High-power-level operations lower the useful life of the laser. Lasers in products used for backup of higher-speed magnetic disks primarily write, and the probability that the useful life exceeds five years is 0.95. Lasers that are in products that are used for main storage spend approximately an equal amount of time reading and writing, and the probability that the useful life exceeds five years is 0.995. Now, 25% of the products from a manufacturer are used for backup and 75% of the products are used for main storage.
Let *A* denote the event that a laser's useful life exceeds five years, and let *B* denote the event that a laser is in a product that is used for backup.
Use a tree diagram to determine the following:

(a) $P(B)$ (b) $P(A \mid B)$ (c) $P(A \mid B')$
(d) $P(A \cap B)$ (e) $P(A \cap B')$ (f) $P(A)$
(g) What is the probability that the useful life of a laser exceeds five years?
(h) What is the probability that a laser that failed before five years came from a product used for backup?



(a) P(B) = 0.25

(b) P($A|B$) = 0.95

(c) P($A|B'$) = 0.995

(d) P($A \cap B$) = P($A|B$)P(B) = 0.95(0.25) = 0.2375

(e) P($A \cap B'$) = P($A|B'$)P(B') = 0.995(0.75) = 0.74625

(f) P(A) = P($A \cap B$) + P($A \cap B'$) = 0.95(0.25) + 0.995(0.75) = 0.98375
(g) 0.95(0.25) + 0.995(0.75) = 0.98375.
(h)

$$P(B|A') = \frac{P(A'|B)P(B)}{P(A'|B)P(B) + P(A'|B')P(B')} = \frac{0.05(0.25)}{0.05(0.25) + 0.005(0.75)} = 0.769$$

2.S11. A congested computer network has a 0.002 probability of losing a data packet, and packet losses are independent events. A lost packet must be resent.
(a) What is the probability that an e-mail message with 100 packets will need to be resent?
(b) What is the probability that an e-mail message with 3 packets will need exactly 1 to be resent?
(c) If 10 e-mail messages are sent, each with 100 packets, what is the probability that at least 1 message will need some packets to be resent?

(a) $P = 1 - (1 - 0.002)^{100} = 0.18143$

(b) $P = C_3^1(0.998^2)0.002 = 0.005976$

(c) $P = 1 - [(1 - 0.002)^{100}]^{10} = 0.86494$

2.S12. An electronic storage device uses an error recovery procedure that requires an immediate satisfactory readback of any written data. If the readback is not successful after three writing operations, that sector of the device is eliminated as unacceptable for data storage. On an acceptable portion of the device, the probability of a satisfactory readback is 0.98. Assume the readbacks are independent. What is the probability that an acceptable portion of the device is eliminated as unacceptable for data storage?

Let A_i denote the event that the ith readback is successful. By independence,

$$P(A_1' \cap A_2' \cap A_3') = P(A_1')P(A_2')P(A_3') = (0.02)^3 = 0.000008.$$

2.S13. Energy released from cells breaks the molecular bond and converts ATP (adenosine triphosphate) into ADP (adenosine diphosphate). Storage of ATP in muscle cells (even for an athlete) can sustain maximal muscle power only for less than five seconds (a short dash). Three systems are used to replenish ATP—phosphagen system, glycogen-lactic acid system (anaerobic), and aerobic respiration—but the first is useful only for less than 10 seconds, and even the second system provides less than two minutes of ATP. An endurance athlete needs to perform below the anaerobic threshold to sustain energy for extended periods. A sample of 100 individuals is described by the energy system used in exercise at different intensity levels.

	Primarily Aerobic	
Period	Yes	No
1	50	7
2	13	30

Let A denote the event that an individual is in period 2, and let B denote the event that the energy is primarily aerobic. Determine the number of individuals in
(a) $A' \cap B$ (b) B' (c) $A \cup B$

(a) $A' \cap B = 50$
(b) B'=37
(c) $A \cup B = 93$

2.S14. The probability that a customer's order is not shipped on time is 0.05. A particular customer places three orders, and the orders are placed far enough apart in time that they can be considered to be independent events.
(a) What is the probability that all are shipped on time?
(b) What is the probability that exactly one is not shipped on time?
(c) What is the probability that two or more orders are not shipped on time?

Let A_i denote the event that the *i*th order is shipped on time.

(a) By independence, $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3) = (0.95)^3 = 0.857$

(b) Let

$$B_1 = A_1' \cap A_2 \cap A_3$$
$$B_2 = A_1 \cap A_2' \cap A_3$$
$$B_3 = A_1 \cap A_2 \cap A_3'$$

Then, because the B's are mutually exclusive,

$$P(B_1 \cup B_2 \cup B_3) = P(B_1) + P(B_2) + P(B_3)$$
$$= 3(0.95)^2(0.05)$$
$$= 0.135$$

(c) Let

$B_1 = A_1' \cap A_2' \cap A_3$

$B_2 = A_1' \cap A_2 \cap A_3'$

$B_3 = A_1 \cap A_2' \cap A_3'$

$B_4 = A_1' \cap A_2' \cap A_3'$

Because the B's are mutually exclusive,

$$P(B_1 \cup B_2 \cup B_3 \cup B_4) = P(B_1) + P(B_2) + P(B_3) + P(B_4)$$
$$= 3(0.05)^2(0.95) + (0.05)^3$$
$$= 0.00725$$

2.S15. In circuit testing of printed circuit boards, each board either fails or does not fail the test. A board that fails the test is then checked further to determine which one of five defect types is the primary failure mode. Represent the sample space for this experiment.

Let D_i denote the event that the primary failure mode is type i and let A denote the event that a board passes the test. The sample space is S = $\{A, A'D_1, A'D_2, A'D_3, A'D_4, A'D_5\}$.

2.S16. Transactions to a computer database are either new items or changes to previous items. The addition of an item can be completed in less than 100 milliseconds 90% of the time, but only 20% of changes to a previous item can be completed in less than this time. If 30% of transactions are changes, what is the probability that a transaction can be completed in less than 100 milliseconds?

(a) (0.20)(0.30) +(0.7)(0.9) = 0.69

2.S17. Let E_1, E_2, and E_3 denote the samples that conform to a percentage of solids specification, a molecular weight specification, and a color specification, respectively. A total of 240 samples are classified by the E_1, E_2, and E_3 specifications, where *yes* indicates that the sample conforms.

E_3 yes

		E_2		
		Yes	No	Total
E_1	Yes	200	1	201
	No	5	4	9
Total		205	5	210

E_3 no

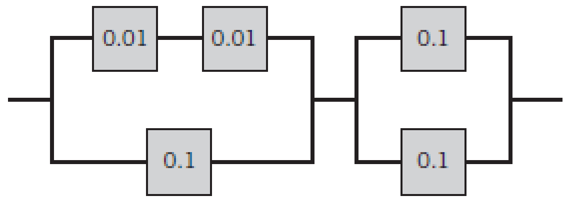
		E_2		
		Yes	No	Total
E_1	Yes	20	4	24
	No	6	0	6
Total		26	4	30

(a) Are E_1, E_2, and E_3 mutually exclusive events?
(b) Are E_1', E_2', and E_3' mutually exclusive events?
(c) What is $P(E_1'$ or E_2' or $E_3')$?
(d) What is the probability that a sample conforms to all three specifications?
(e) What is the probability that a sample conforms to the E_1 or E_3 specification?
(f) What is the probability that a sample conforms to the E_1 or E_2 or E_3 specification?

(a) No, $P(E_1 \cap E_2 \cap E_3) \neq 0$
(b) No, $E_1' \cap E_2'$ is not $\varnothing$
(c) $P(E_1' \cup E_2' \cup E_3') = P(E_1') + P(E_2') + P(E_3') - P(E_1' \cap E_2') - P(E_1' \cap E_3') - P(E_2' \cap E_3')$
$+ P(E_1' \cap E_2' \cap E_3')$

= 40/240
(d) $P(E_1 \cap E_2 \cap E_3) = 200/240$
(e) $P(E_1 \cup E_3) = P(E_1) + P(E_3) - P(E_1 \cap E_3) = 234/240$
(f) $P(E_1 \cup E_2 \cup E_3) = 1 - P(E_1' \cap E_2' \cap E_3') = 1 - 0 = 1$

2.S18. The following circuit operates if and only if there is a path of functional devices from left to right. Assume devices fail independently and that the probability of *failure* of each device is as shown. What is the probability that the circuit operates?



Let A,B denote the event that the first, second portion of the circuit operates.
Then, P(A) = (0.99)(0.99)+0.9-(0.99)(0.99)(0.9) = 0.998
P(B) = 0.9+0.9-(0.9)(0.9) = 0.99 and
$P(A \cap B) = P(A)\, P(B) = (0.998)\,(0.99) = 0.988$

2.S19. A steel plate contains 20 bolts. Assume that 5 bolts are not torqued to the proper limit. 4 bolts are selected at random, without replacement, to be checked for torque.
(a) What is the probability that all 4 of the selected bolts are torqued to the proper limit?
(b) What is the probability that at least 1 of the selected bolts is *not* torqued to the proper limit?

Let A_i denote the event that the *i*th bolt selected is not torqued to the proper limit.
(a) Then,

$$\begin{aligned} P(A_1 \cap A_2 \cap A_3 \cap A_4) &= P(A_4 | A_1 \cap A_2 \cap A_3) P(A_1 \cap A_2 \cap A_3) \\ &= P(A_4 | A_1 \cap A_2 \cap A_3) P(A_3 | A_1 \cap A_2) P(A_2 | A_1) P(A_1) \\ &= \left(\frac{12}{17}\right)\left(\frac{13}{18}\right)\left(\frac{14}{19}\right)\left(\frac{15}{20}\right) = 0.282 \end{aligned}$$

(b) Let B denote the event that at least one of the selected bolts are not properly torqued. Thus, B' is the event that all bolts are properly torqued. Then,

$$P(B) = 1 - P(B') = 1 - \left(\frac{15}{20}\right)\left(\frac{14}{19}\right)\left(\frac{13}{18}\right)\left(\frac{12}{17}\right) = 0.718$$

2.S20. The British government has stepped up its information campaign regarding foot-and-mouth disease by mailing brochures to farmers around the country. It is estimated that 99% of Scottish farmers who receive the brochure possess enough information to deal with an outbreak of the disease, but only 90% of those without the brochure can deal with an outbreak. After the first three months of mailing, 95% of the farmers in Scotland had received the informative brochure. Compute the probability that a randomly selected farmer will have enough information to deal effectively with an outbreak of the disease.

P(Possess) = 0.95(0.99) +(0.05)(0.90) = 0.9855

2.S21. It is known that two defective cellular phones were erroneously sent to a shipping lot that now has a total of 75 phones. A sample of phones will be selected from the lot without replacement.
(a) If three phones are inspected, determine the probability that exactly one of the defective phones will be found.
(b) If three phones of the software are inspected, determine the probability that both defective phones will be found.
(c) If 73 phones are inspected, determine the probability that both defective phones will be found. (*Hint*: Work with the phones that remain in the lot.)

D = defective copy

(a) $P(D = 1) = \left(\frac{2}{75}\right)\left(\frac{73}{74}\right)\left(\frac{72}{73}\right)+\left(\frac{73}{75}\right)\left(\frac{2}{74}\right)\left(\frac{72}{73}\right)+\left(\frac{73}{75}\right)\left(\frac{72}{74}\right)\left(\frac{2}{73}\right) = 0.0778$

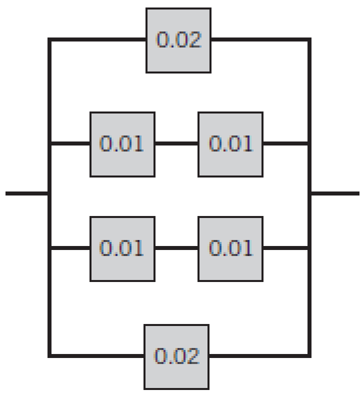
(b) $P(D = 2) = \left(\frac{2}{75}\right)\left(\frac{1}{74}\right)\left(\frac{73}{73}\right)+\left(\frac{2}{75}\right)\left(\frac{73}{74}\right)\left(\frac{1}{73}\right)+\left(\frac{73}{75}\right)\left(\frac{2}{74}\right)\left(\frac{1}{73}\right) = 0.00108$

(c) Let A represent the event that the two items NOT inspected are not defective. Then, P(A)=(73/75)(72/74)=0.947.

2.S22. An encryption-decryption system consists of three elements: encode, transmit, and decode. A faulty encode occurs in 0.5% of the messages processed, transmission errors occur in 1% of the messages, and a decode error occurs in 0.1% of the messages. Assume the errors are independent.
(a) What is the probability of a completely defect-free message?
(b) What is the probability of a message that has either an encode or a decode error?

(a) $P(E' \cap T' \cap D') = (0.995)(0.99)(0.999) = 0.984$
(b) $P(E \cup D) = P(E) + P(D) - P(E \cap D) = 0.005995$

2.S23. The following circuit operates if and only if there is a path of functional devices from left to right. Assume that devices fail independently and that the probability of *failure* of each device is as shown. What is the probability that the circuit does not operate?



Let A_i denote the event that the *i*th row operates. Then,

$P(A_1) = 0.98, P(A_2) = (0.99)(0.99) = 0.9801, P(A_3) = 0.9801, \ P(A_4) = 0.98.$

The probability the circuit does not operate is

$P(A_1')P(A_2')P(A_3')P(A_4') = (0.02)(0.0199)(0.0199)(0.02) = 1.58 \times 10^{-7}$

2.S24. A robotic insertion tool contains 10 primary components. The probability that any component fails during the warranty period is 0.01. Assume that the components fail independently and that the tool fails if any component fails. What is the probability that the tool fails during the warranty period?

The tool fails if any component fails. Let F denote the event that the tool fails. Then, $P(F') = 0.99^{10}$ by independence and $P(F) = 1 - 0.99^{10} = 0.0956$

2.S25. A machine tool is idle 15% of the time. You request immediate use of the tool on five different occasions during the year. Assume that your requests represent independent events.
(a) What is the probability that the tool is idle at the time of all of your requests?
(b) What is the probability that the machine is idle at the time of exactly four of your requests?
(c) What is the probability that the tool is idle at the time of at least three of your requests?

(a) By independence, $0.15^5 = 7.59 \times 10^{-5}$

(b) Let A_i denote the events that the machine is idle at the time of your ith request. Using independence, the requested probability is

$$P(A_1 A_2 A_3 A_4 A_5' \text{ or } A_1 A_2 A_3 A_4' A_5 \text{ or } A_1 A_2 A_3' A_4 A_5 \text{ or } A_1 A_2' A_3 A_4 A_5 \text{ or } A_1' A_2 A_3 A_4 A_5)$$
$$= 5(0.15^4)(0.85^1)$$
$$= 0.00215$$

(c) As in part b, the probability of 3 of the events is

$$P(A_1 A_2 A_3 A_4' A_5' \text{ or } A_1 A_2 A_3' A_4 A_5' \text{ or } A_1 A_2 A_3' A_4' A_5 \text{ or } A_1 A_2' A_3 A_4 A_5' \text{ or } A_1 A_2' A_3 A_4' A_5 \text{ or}$$
$$A_1 A_2' A_3' A_4 A_5 \text{ or } A_1' A_2 A_3 A_4 A_5' \text{ or } A_1' A_2 A_3 A_4' A_5 \text{ or } A_1' A_2 A_3' A_4 A_5 \text{ or } A_1' A_2' A_3 A_4 A_5)$$
$$= 10(0.15^3)(0.85^2)$$
$$= 0.0244$$

For the probability of at least 3, add answer parts a) and b) to the above to obtain the requested probability. Therefore, the answer is 0.0000759 + 0.0022 + 0.0244 = 0.0267

2.S26. A company that tracks the use of its Web site determined that the more pages a visitor views, the more likely the visitor is to request more information (RMI). Use the following table to answer the questions:

Number of pages viewed:	1	2	3	4 or more
Percentage of visitors:	40	30	20	10
Percentage who RMI	10	10	20	40

(a) What is the probability that a visitor to the Web site provides contact information?
(b) If a visitor provides contact information, what is the probability that the visitor viewed four or more pages?

(a) (0.4)(0.1) + (0.3)(0.1) +(0.2)(0.2) + (0.4)(0.1) = 0.15
(b) P(4 or more | provided info) = (0.4)(0.1)/0.15 = 0.267

2.S27. An article in *Genome Research* ["An Assessment of Gene Prediction Accuracy in Large DNA Sequences" (2000, Vol. 10, pp. 1631–1642)], considered the accuracy of commercial software to predict nucleotides in gene sequences. The following table shows the number of sequences for which the programs produced predictions and the number of nucleotides correctly predicted (computed globally from the total number of prediction successes and failures on all sequences).

	Number of Sequences	**Proportion**
GenScan	177	0.93
Blastx default	175	0.91
Blastx topcomboN	174	0.97
Blastx 2 stages	175	0.90
GeneWise	177	0.98
Procrustes	177	0.93

Assume the prediction successes and failures are independent among the programs.
(a) What is the probability that all programs predict a nucleotide correctly?
(b) What is the probability that all programs predict a nucleotide incorrectly?
(c) What is the probability that at least one Blastx program predicts a nucleotide correctly?

(a) P=(0.93)(0.91)(0.97)(0.90)(0.98)(0.93)=0.67336
(b) P=(1-0.93)(1-0.91)(1-0.97)(1-0.90)(1-0.98)(1-0.93)=2.646×10^{-8}
(c) P=1-(1-0.91)(1-0.97)(1-0.90)=0.99973

2.S28. A batch contains 36 bacteria cells. Assume that 12 of the cells are not capable of cellular replication. Of the cells, 6 are selected at random, without replacement, to be checked for replication.
(a) What is the probability that all 6 of the selected cells are able to replicate?

(b) What is the probability that at least 1 of the selected cells is not capable of replication?

(a) P=(24/36)(23/35)(22/34)(21/33)(20/32)(19/31)=0.069
(b) P=1-0.069=0.931

2.S29. A computer system uses passwords that are exactly seven characters, and each character is one of the 26 letters (a–z) or 10 integers (0–9). Uppercase letters are not used.
(a) How many passwords are possible?
(b) If a password consists of exactly 6 letters and 1 number, how many passwords are possible?
(c) If a password consists of 5 letters followed by 2 numbers, how many passwords are possible?

(a) 36^7
(b) Number of permutations of six letters is 26^6. Number of ways to select one number = 10. Number of positions among the six letters to place the one number = 7. Number of passwords = $26^6 \times 10 \times 7$
(c) $26^5 10^2$

2.S30. The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment. Suppose a patient is selected randomly.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let *A* denote the event that the patient is treated with ribavirin plus interferon alfa or interferon alfa, and let *B* denote the event that the response is complete. Determine the following probabilities.

(a) $P(A \mid B)$ (b) $P(B \mid A)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$

P(A|B)= $\frac{P(B|A)P(A)}{P(B)} = \frac{\frac{22}{40}(\frac{40}{60})}{\frac{22}{60}} = 1$
P(B|A) = 22/40 = 0.55
P(A∩B) = P(B|A)P(A)=(22/40)(40/60) = 22/60 = 0.366
P(AUB) = P(A)+P(B) - P(A∩B) = (40/60) + (22/60) – (22/60) = 0.667

2.S31. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose a patient is selected randomly. Let *A* denote the event that the patient is in group 1 or 2, and let *B* denote the event that there is no progression. Determine the following probabilities:

(a) $P(A \mid B)$ (b) $P(B \mid A)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$

(a) P(A|B)= $\frac{158}{375} = 0.421$
(b) P(B|A)= $\frac{158}{226} = 0.699$
(c) P(A∩B)= $\frac{76+82}{467} = 0.338$
(d) P(AUB)= $\frac{226}{467} + \frac{375}{467} - \frac{158}{467} = 0.948$