CHAPTER 2

Section 2-1

Provide a reasonable description of the sample space for each of the random experiments in Exercises 2-1 to 2-17. There can be more than one acceptable interpretation of each experiment. Describe any assumptions you make.

2-1. Each of three machined parts is classified as either above or below the target specification for the part. Let *a* and *b* denote a part above and below the specification, respectively.

$$S = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$$

2-2. Each of four transmitted bits is classified as either in error or not in error.

Let *e* and *o* denote a bit in error and not in error (*o* denotes okay), respectively.

$$S = \left\{ \begin{matrix} eeee, eoee, oeee, ooee, \\ eeeo, eoeo, oeeo, ooeo, \\ eeoe, eooe, oeoe, oooe, \\ eeoo, eooo, oeoo, oooo \end{matrix} \right\}$$

2-3. In the final inspection of electronic power supplies, either units pass, or three types of nonconformities might occur: functional, minor, or cosmetic. Three units are inspected.

Let *a* denote an acceptable power supply.
Let *f*, *m*, and *c* denote a power supply that has a functional, minor, or cosmetic error, respectively.

$$S = \{a, f, m, c\}$$

2-4. The number of hits (views) is recorded at a high-volume Web site in a day.

$S = \{0,1,2,\ldots\}$ = set of nonnegative integers

2-5. Each of 24 Web sites is classified as containing or not containing banner ads.

Let *y* and *n* denote a web site that contains and does not contain banner ads.
The sample space is the set of all possible sequences of *y* and *n* of length 24. An example outcome in the sample space is $S = \{yynnynyyynnynynnnnyynnyy\}$

2-6. An ammeter that displays three digits is used to measure current in milliamperes.

A vector with three components can describe the three digits of the ammeter. Each digit can be 0,1,2,...,9.
The sample space *S* is 1000 possible three digit integers, $S = \{000, 001, \ldots, 999\}$

2-7. A scale that displays two decimal places is used to measure material feeds in a chemical plant in tons.

S is the sample space of 100 possible two digit integers.

2-8. The following two questions appear on an employee survey questionnaire. Each answer is chosen from the five point
scale 1 (never), 2, 3, 4, 5 (always).

Is the corporation willing to listen to and fairly evaluate new ideas?
How often are my coworkers important in my overall job performance?

Let an ordered pair of numbers, such as 43 denote the response on the first and second question. Then, S consists of the 25 ordered pairs $\{11, 12, \ldots, 55\}$

2-9. The concentration of ozone to the nearest part per billion.

$S = \{0,1,2,...,1E09\}$ in ppb.

2-10. The time until a service transaction is requested of a computer to the nearest millisecond.

$S = \{0,1,2,...,\}$ in milliseconds

2-11. The pH reading of a water sample to the nearest tenth of a unit.

$S = \{1.0,1.1,1.2,\ldots 14.0\}$

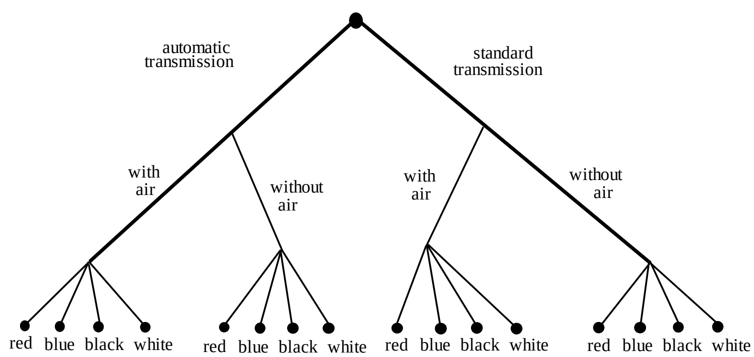
2-12. The voids in a ferrite slab are classified as small, medium, or large. The number of voids in each category is measured by an optical inspection of a sample.

Let *s*, *m*, and *l* denote small, medium, and large, respectively. Then *S* = {*s*, *m*, *l*, *ss*, *sm*, *sl*, ….}

2-13 The time of a chemical reaction is recorded to the nearest millisecond.

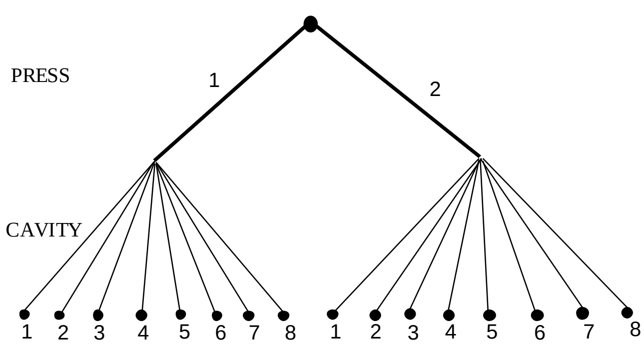
$S = \{0,1,2,...,\}$ in milliseconds.

2-14. An order for an automobile can specify either an automatic or a standard transmission, either with or without air conditioning, and with any one of the four colors red, blue, black, or white. Describe the set of possible orders for this experiment.

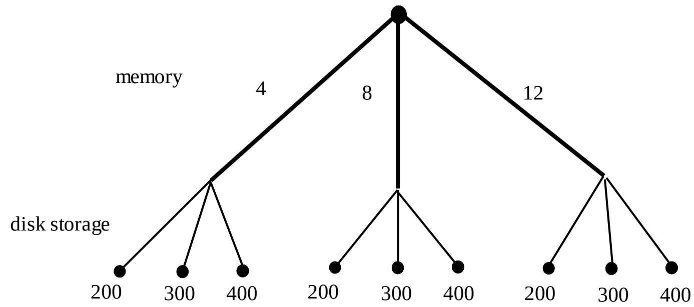


2-15. A sampled injection-molded part could have been produced in either one of two presses and in any one of the eight

cavities in each press.



2-16. An order for a computer system can specify memory of 4, 8, or 12 gigabytes and disk storage of 200, 300, or 400 gigabytes. Describe the set of possible orders.



2-17. Calls are repeatedly placed to a busy phone line until a connection is achieved.

Let c and b denote connect and busy, respectively. Then $S = \{c, bc, bbc, bbbc, bbbbc, \ldots\}$

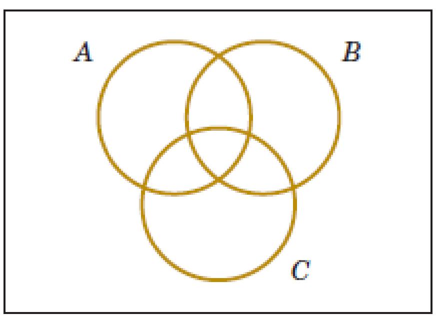
2-18. Three attempts are made to read data in a magnetic storage device before an error recovery procedure that repositions
the magnetic head is used. The error recovery procedure attempts three repositionings before an "abort'' message is sent to the operator. Let

s denote the success of a read operation
f denote the failure of a read operation
S denote the success of an error recovery procedure
F denote the failure of an error recovery procedure
A denote an abort message sent to the operator

Describe the sample space of this experiment with a tree diagram.

$$S = \{s, fs, ffs, fffS, fffFS, fffFFS, fffFFFA\}$$

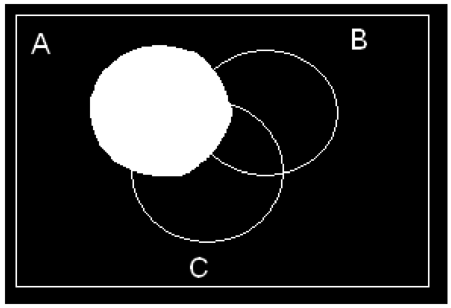
2-19. Three events are shown on the Venn diagram in the following figure:



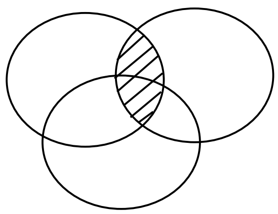
Reproduce the figure and shade the region that corresponds to each of the following events.

(a) A' (b) $A \cap B$ (c) $(A \cap B) \cup C$ (d) $(B \cup C)'$ (e) $(A \cap B)' \cup C$

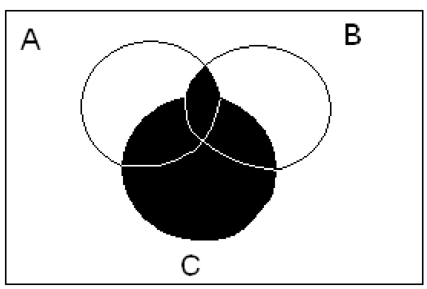
(a)



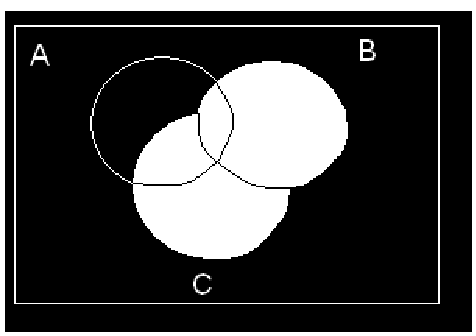
(b)



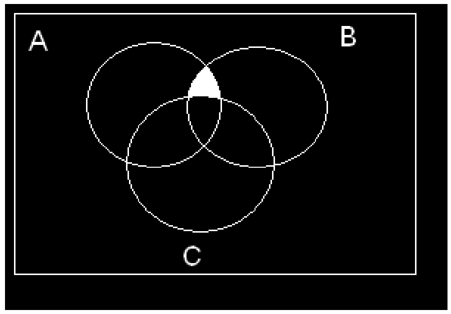
(c)



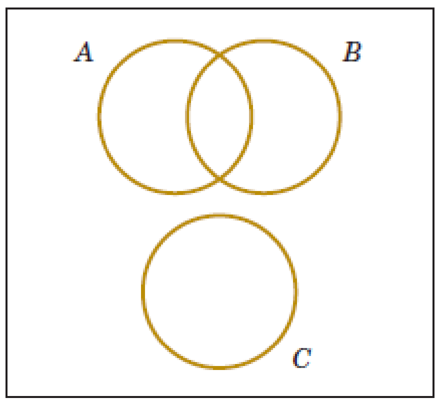
(d)



(e)



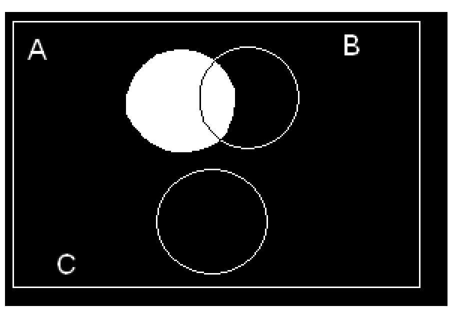
2-20. Three events are shown on the Venn diagram in the following figure:



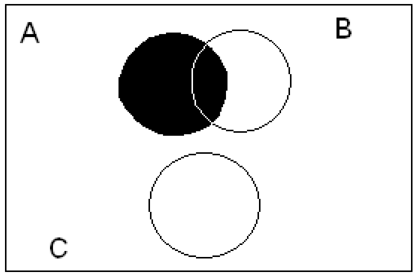
Reproduce the figure and shade the region that corresponds to each of the following events.

(a) A' (b) $(A \cap B) \cup (A \cap B')$ (c) $(A \cap B) \cup C$ (d) $(B \cup C)'$ (e) $(A \cap B)' \cup C$

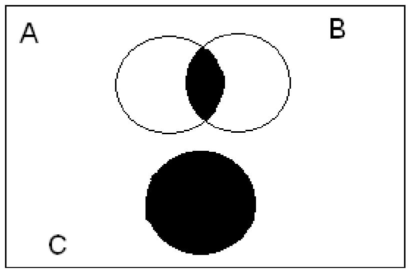
(a)



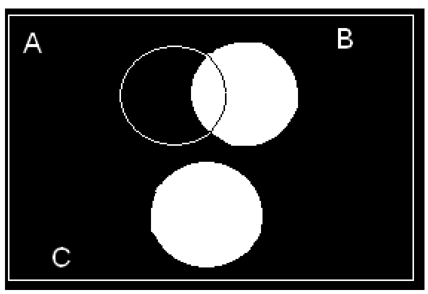
(b)



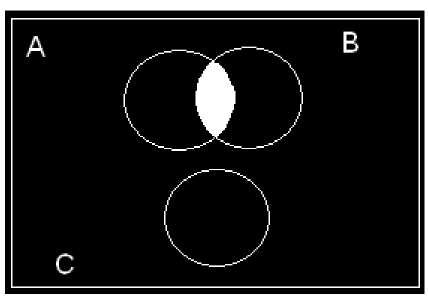
(c)



(d)



(e)



2-21. A digital scale that provides weights to the nearest gram is used.

(a) What is the sample space for this experiment?
Let A denote the event that a weight exceeds 11 grams, let B denote the event that a weight is less than or equal to
15 grams, and let C denote the event that a weight is greater than or equal to 8 grams and less than 12 grams.

Describe the following events.

(b) $A \cup B$ (c) $A \cap B$ (d) A' (e) $A \cup B \cup C$ (f) $(A \cup C)'$ (g) $A \cap B \cap C$ (h) $B' \cap C$ (i) $A \cup (B \cap C)$

(a) Let S = the nonnegative integers from 0 to the largest integer that can be displayed by the scale.
Let X denote the weight.

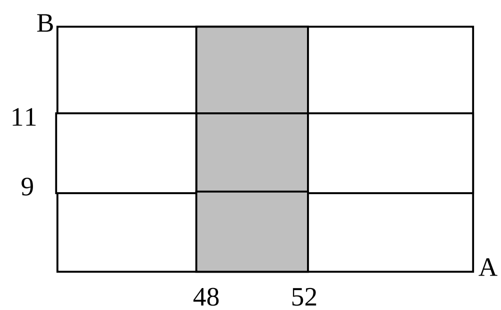
A is the event that $X > 11$ B is the event that $X \le 15$ C is the event that $8 \le X < 12$
$S = \{0, 1, 2, 3, \ldots\}$
(b) S
(c) $11 < X \le 15$ or $\{12, 13, 14, 15\}$
(d) $X \le 11$ or $\{0, 1, 2, \ldots, 11\}$
(e) S
(f) $A \cup C$ contains the values of X such that: $X \ge 8$
Thus $(A \cup C)'$ contains the values of X such that: $X < 8$ or $\{0, 1, 2, \ldots, 7\}$
(g) $\varnothing$
(h) B' contains the values of X such that $X > 15$. Therefore, $B' \cap C$ is the empty set. They have no outcomes in common or $\varnothing$.
(i) $B \cap C$ is the event $8 \le X < 12$. Therefore, $A \cup (B \cap C)$ is the event $X \ge 8$ or $\{8, 9, 10, \ldots\}$

2-22. In an injection-molding operation, the length and width, denoted as X and Y, respectively, of each molded part are evaluated. Let
A denote the event of $48 < X < 52$ centimeters
B denote the event of $9 < Y < 11$ centimeters

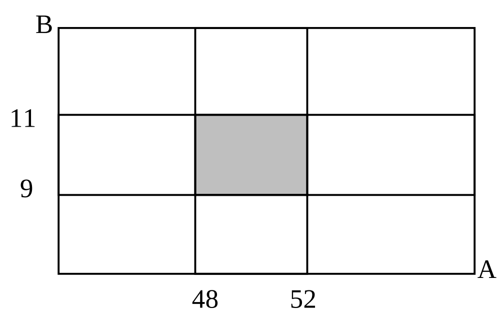
Construct a Venn diagram that includes these events. Shade the areas that represent the following:
(a) A (b) $A \cap B$ (c) $A' \cup B$ (d) $A \cap B$

(e) If these events were mutually exclusive, how successful would this production operation be? Would the process
produce parts with $X = 50$ centimeters and $Y = 10$ centimeters?

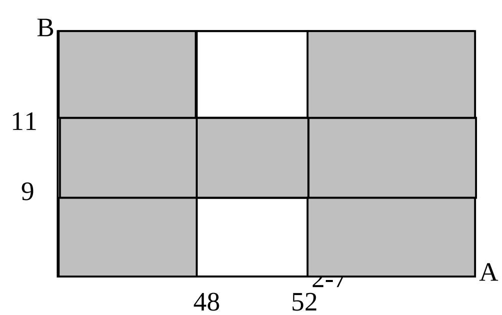
(a)



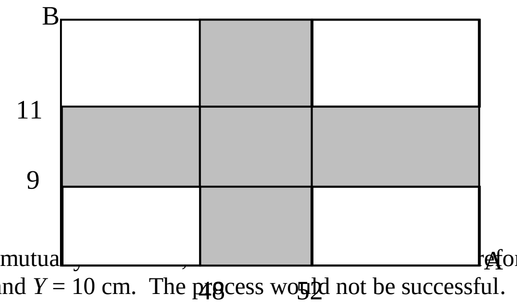
(b)



(c)



(d)



(e) If the events are mutua[illegible]efore, the process does not produce product parts with X = 50 cm and Y = 10 cm. The process would not be successful.

2-23. Four bits are transmitted over a digital communications channel. Each bit is either distorted or received without distortion. Let Ai denote the event that the ith bit is distorted, i = 1,....... ,4.

(a) Describe the sample space for this experiment.
(b) Are the A_i's mutually exclusive?

Describe the outcomes in each of the following events:

(c) A_1 (d) A_1' (e) $A_1 \cap A_2 \cap A_3 \cap A_4$ (f) $(A_1 \cap A_2) \cup (A_3 \cap A_4)$

Let d and o denote a distorted bit and one that is not distorted (o denotes okay), respectively.

(a) $$S = \begin{Bmatrix} dddd, dodd, oddd, oodd, \\ dddo, dodo, oddo, oodo, \\ ddod, dood, odod, oood, \\ ddoo, dooo, odoo, oooo \end{Bmatrix}$$

(b) No, for example $A_1 \cap A_2 = \{dddd, dddo, ddod, ddoo\}$

(c) $$A_1 = \begin{Bmatrix} dddd, dodd, \\ dddo, dodo \\ ddod, dood \\ ddoo, dooo \end{Bmatrix}$$

(d) $$A_1' = \begin{Bmatrix} oddd, oodd, \\ oddo, oodo, \\ odod, oood, \\ odoo, oooo \end{Bmatrix}$$

(e) $A_1 \cap A_2 \cap A_3 \cap A_4 = \{dddd\}$

(f) $(A_1 \cap A_2) \cup (A_3 \cap A_4) = \{dddd, dodd, dddo, oddd, ddod, oodd, ddoo\}$

2-24. In light-dependent photosynthesis, light quality refers to the wavelengths of light that are important. The wavelength of a sample of photosynthetically active radiations (PAR) is measured to the nearest nanometer. The red range is 675–700 nm and the blue range is 450–500 nm. Let *A* denote the event that PAR occurs in the red range, and let *B* denote the event that PAR occurs in the blue range. Describe the sample space and indicate each of the following events:

(a) A (b) B (c) $A \cap B$ (d) $A \cup B$

Let w denote the wavelength. The sample space is $\{w \mid w = 0, 1, 2, \ldots\}$

(a) $A=\{w \mid w = 675, 676, \ldots, 700 \text{ nm}\}$

(b) $B=\{ w \mid w = 450, 451, \ldots, 500 \text{ nm}\}$

(c) $A \cap B = \Phi$

(d) $A \cup B = \{w \mid w = 450, 451, \ldots, 500, 675, 676, \ldots, 700 \text{ nm}\}$

2-25. In control replication, cells are replicated over a period of two days. Not until mitosis is completed can freshly synthesized DNA be replicated again. Two control mechanisms have been identified—one positive and one negative. Suppose that a replication is observed in three cells. Let *A* denote the event that all cells are identified as positive, and let *B* denote the event that all cells are negative. Describe the sample space graphically and display each of the following events:

(a) A (b) B (c) $A \cap B$ (d) $A \cup B$

Let P and N denote positive and negative, respectively.

The sample space is $\{PPP, PPN, PNP, NPP, PNN, NPN, NNP, NNN\}$.

(a) $A=\{ PPP \}$

(b) $B=\{ NNN \}$

(c) $A \cap B = \Phi$

(d) $A \cup B = \{ PPP, NNN \}$

2-26. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized here:

		Shock Resistance	
		High	Low
Scratch	High	70	9
Resistance	Low	16	5

Let *A* denote the event that a disk has high shock resistance, and let *B* denote the event that a disk has high scratch resistance. Determine the number of disks in $A \cap B$, A', and $A \cup B$.

$A \cap B = 70$, $A' = 14$, $A \cup B = 95$

2-27. Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and edge finish. The results of 100 parts are summarized as follows:

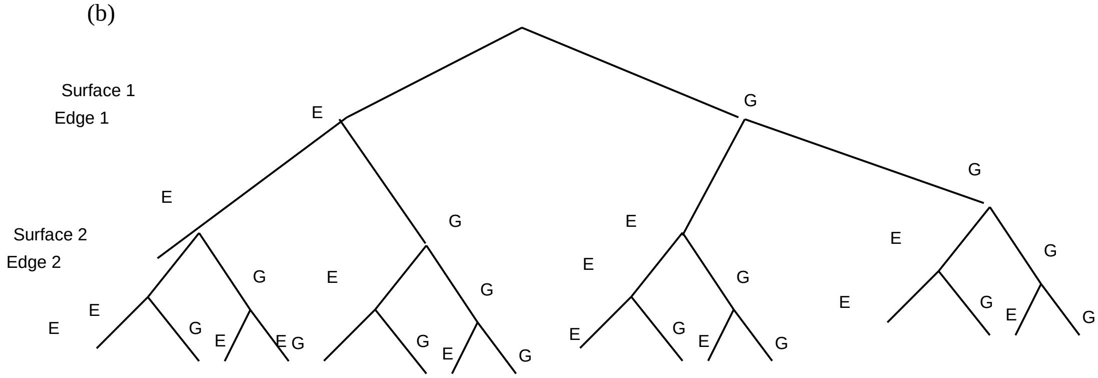
		Edge Finish	
		Excellent	Good
Surface	Excellent	80	2
Finish	Good	10	8

(a) Let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent edge finish. Determine the number of samples in $A' \cap B$, B' and in $A \cup B$.

(b) Assume that each of two samples is to be classified on the basis of surface finish, either excellent or good, and on the basis of edge finish, either excellent or good. Use a tree diagram to represent the possible outcomes of this experiment.

(a) $A' \cap B$ = 10, B' =10, $A \cup B$ = 92

(b)



2-28. Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized as follows:

		Conforms	
		Yes	No
	1	22	8
Supplier	2	25	5
	3	30	10

Let A denote the event that a sample is from supplier 1, and let B denote the event that a sample conforms to specifications. Determine the number of samples in $A' \cap B$, B' and in $A \cup B$.

$A' \cap B$ = 55, B' =23, $A \cup B$ = 85

2-29. The rise time of a reactor is measured in minutes (and fractions of minutes). Let the sample space be positive, real numbers. Define the events A and B as follows:

$A = \{x \mid x < 72.5\}$ and $B = \{x \mid x > 52.5\}$.

Describe each of the following events:

(a) A' (b) B' (c) $A \cap B$ (d) $A \cup B$

(a) $A' = \{x \mid x \geq 72.5\}$
(b) $B' = \{x \mid x \leq 52.5\}$
(c) $A \cap B = \{x \mid 52.5 < x < 72.5\}$
(d) $A \cup B = \{x \mid x > 0\}$

2-30. A sample of two items is selected without replacement from a batch. Describe the (ordered) sample space for each of the following batches:
(a) The batch contains the items $\{a, b, c, d\}$.
(b) The batch contains the items $\{a, b, c, d, e, f, g\}$.
(c) The batch contains 4 defective items and 20 good items.
(d) The batch contains 1 defective item and 20 good items.

(a) {*ab, ac, ad, bc, bd, cd, ba, ca, da, cb, db, dc*}
(b) {*ab, ac, ad, ae, af, ag, ba, bc, bd, be, bf, bg, ca, cb, cd, ce, cf, cg, da, db, dc, de, df, dg, ea, eb, ec, ed, ef, eg, fa, fb, fc, fg, fd, fe, ga, gb, gc, gd, ge, gf*}, contains 42 elements
(c) Let *d* and *g* denote defective and good, respectively. Then *S* = {*gg, gd, dg, dd*}
(d) S = {*gd, dg, gg*}

2-31. A sample of two printed circuit boards is selected without replacement from a batch. Describe the (ordered) sample space for each of the following batches:

(a) The batch contains 90 boards that are not defective, 8 boards with minor defects, and 2 boards with major defects.
(b) The batch contains 90 boards that are not defective, 8 boards with minor defects, and 1 board with major defects.

Let *g* denote a good board, *m* a board with minor defects, and *j* a board with major defects.

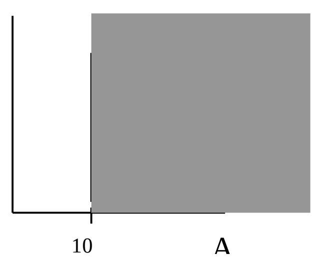
(a) *S* = {*gg, gm, gj, mg, mm, mj, jg, jm, jj*}
(b) *S* ={*gg,gm,gj,mg,mm,mj,jg,jm*}

2-32. Counts of the Web pages provided by each of two computer servers in a selected hour of the day are recorded. Let *A* denote the event that at least 10 pages are provided by server 1, and let *B* denote the event that at least 20 pages are provided by server 2. Describe the sample space for the numbers of pages for the two servers graphically in an *x* □ *y* plot. Show each of the following events on the sample space graph:

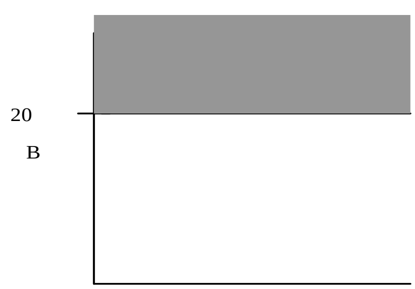
(a) A (b) B (c) $A \cap B$ (d) $A \cup B$

(a) The sample space contains all points in the nonnegative *X*-*Y* plane.

(b)



(c)



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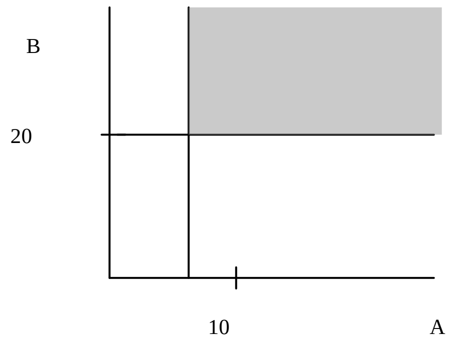

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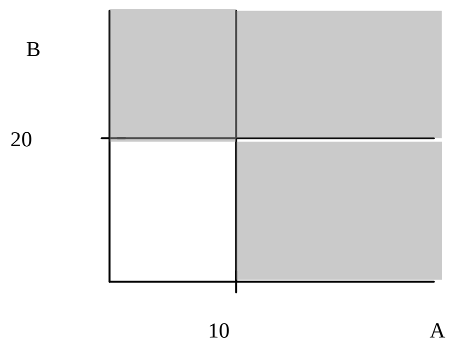


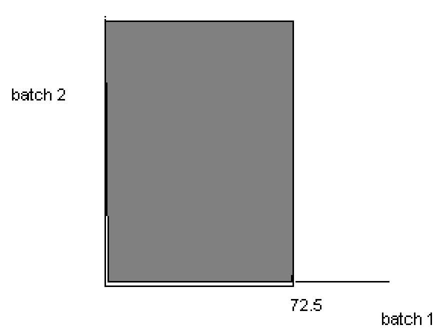
2-33. A reactor's rise time is measured in minutes (and fractions of minutes). Let the sample space for the rise time of each

batch be positive, real numbers. Consider the rise times of *two* batches. Let A denote the event that the rise time of batch 1 is less than 72.5 minutes, and let B denote the event that the rise time of batch 2 is greater than 52.5 minutes.

Describe the sample space for the rise time of two batches graphically and show each of the following events on a two dimensional plot:

(a) A (b) B' (c) $A \cap B$ (d) $A \cup B$

(a)



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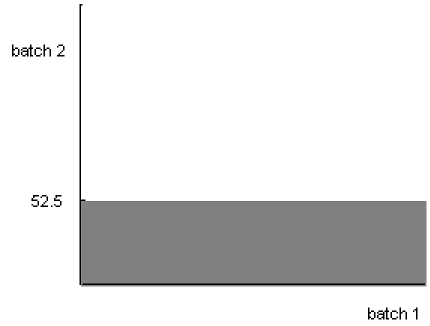

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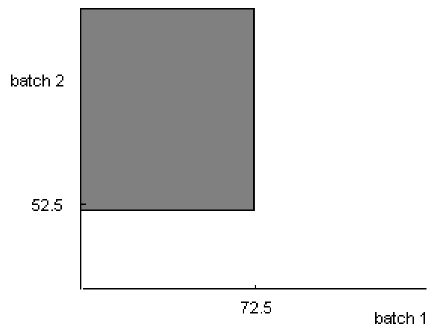

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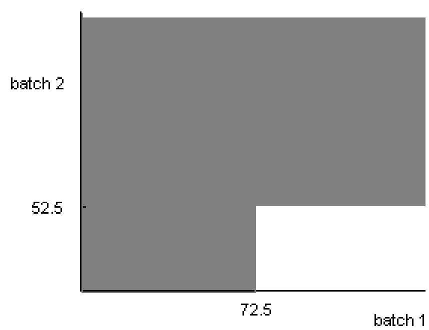


2-34. A wireless garage door opener has a code determined by the up or down setting of 12 switches. How many outcomes are in the sample space of possible codes?

$2^{12} = 4096$

2-35. An order for a computer can specify any one of five memory sizes, any one of three types of displays, and any one
of four sizes of a hard disk, and can either include or not include a pen tablet. How many different systems can be ordered?

From the multiplication rule, the answer is $5 \times 3 \times 4 \times 2 = 120$

2-36. In a manufacturing operation, a part is produced by machining, polishing, and painting. If there are three machine
tools, four polishing tools, and three painting tools, how many different routings (consisting of machining, followed by polishing, and followed by painting) for a part are possible?

From the multiplication rule, $3 \times 4 \times 3 = 36$

2-37. New designs for a wastewater treatment tank have proposed three possible shapes, four possible sizes, three locations
for input valves, and four locations for output valves. How many different product designs are possible?

From the multiplication rule, $3 \times 4 \times 3 \times 4 = 144$

2-38. A manufacturing process consists of 10 operations that can be completed in any order. How many different production
sequences are possible?

From equation 2-1, the answer is 10! = 3,628,800

2-39. A manufacturing operation consists of 10 operations. However, five machining operations must be completed before any of the remaining five assembly operations can begin. Within each set of five, operations can be completed in any order. How many different production sequences are possible?

From the multiplication rule and equation 2-1, the answer is 5!5! = 14,400

2-40. In a sheet metal operation, three notches and four bends are required. If the operations can be done in any order,
how many different ways of completing the manufacturing are possible?

From equation 2-3, $\frac{7!}{3!4!} = 35$ sequences are possible

2-41. A batch of 140 semiconductor chips is inspected by choosing a sample of 5 chips. Assume 10 of the chips do not conform to customer requirements.

(a) How many different samples are possible?
(b) How many samples of five contain exactly one nonconforming chip?
(c) How many samples of five contain at least one nonconforming chip?

(a) From equation 2-4, the number of samples of size five is $\binom{140}{5} = \frac{140!}{5!135!} = 416{,}965{,}528$

(b) There are 10 ways of selecting one nonconforming chip and there are $\binom{130}{4} = \frac{130!}{4!126!} = 11{,}358{,}880$

ways of selecting four conforming chips. Therefore, the number of samples that contain exactly one nonconforming chip is $10 \times \binom{130}{4} = 113{,}588{,}800$

(c) The number of samples that contain at least one nonconforming chip is the total number of samples $\binom{140}{5}$ minus the number of samples that contain no nonconforming chips $\binom{130}{5}$. That is

$$\binom{140}{5} - \binom{130}{5} = \frac{140!}{5!135!} - \frac{130!}{5!125!} = 130{,}721{,}752$$

2-42. In the layout of a printed circuit board for an electronic product, 12 different locations can accommodate chips.

(a) If five different types of chips are to be placed on the board, how many different layouts are possible?
(b) If the five chips that are placed on the board are of the same type, how many different layouts are possible?

(a) If the chips are of different types, then every arrangement of 5 locations selected from the 12 results in a different layout. Therefore, $P_5^{12} = \frac{12!}{7!} = 95{,}040$ layouts are possible.

(b) If the chips are of the same type, then every subset of 5 locations chosen from the 12 results in a different layout. Therefore, $\binom{12}{5} = \frac{12!}{5!7!} = 792$ layouts are possible.

2-43. In the laboratory analysis of samples from a chemical process, five samples from the process are analyzed daily. In addition, a control sample is analyzed twice each day to check the calibration of the laboratory instruments.

(a) How many different sequences of process and control samples are possible each day? Assume that the five process samples are considered identical and that the two control samples are considered identical.
(b) How many different sequences of process and control samples are possible if we consider the five process samples to be different and the two control samples to be identical?
(c) For the same situation as part (b), how many sequences are possible if the first test of each day must be a control sample?

(a) $\frac{7!}{2!5!} = 21$ sequences are possible.

(b) $\frac{7!}{1!1!1!1!1!2!} = 2520$ sequences are possible.

(c) 6! = 720 sequences are possible.

2-44. In the design of an electromechanical product, 12 components are to be stacked into a cylindrical casing in a manner that minimizes the impact of shocks. One end of the casing is designated as the bottom and the other end is the top.

(a) If all components are different, how many different designs are possible?
(b) If seven components are identical to one another, but the others are different, how many different designs are possible?
(c) If three components are of one type and identical to one another, and four components are of another type and identical to one another, but the others are different, how many different designs are possible?

(a) Every arrangement selected from the 12 different components comprises a different design.
Therefore, $12! = 479{,}001{,}600$ designs are possible.

(b) 7 components are the same, others are different, $\frac{12!}{7!1!1!1!1!1!} = 95040$ designs are possible.

(c) $\frac{12!}{3!4!} = 3326400$ designs are possible.

2-45. Consider the design of a communication system.

(a) How many three-digit phone prefixes that are used to represent a particular geographic area (such as an area code) can be created from the digits 0 through 9?
(b) As in part (a), how many three-digit phone prefixes are possible that do not start with 0 or 1, but contain 0 or 1 as the middle digit?
(c) How many three-digit phone prefixes are possible in which no digit appears more than once in each prefix?

(a) From the multiplication rule, $10^3 = 1000$ prefixes are possible
(b) From the multiplication rule, $8 \times 2 \times 10 = 160$ are possible
(c) Every arrangement of three digits selected from the 10 digits results in a possible prefix.
$P_3^{10} = \frac{10!}{7!} = 720$ prefixes are possible.

2-46. A *byte* is a sequence of eight bits and each bit is either 0 or 1.

(a) How many different bytes are possible?
(b) If the first bit of a byte is a parity check, that is, the first byte is determined from the other seven bits, how many
different bytes are possible?

(a) From the multiplication rule, $2^8 = 256$ bytes are possible
(b) From the multiplication rule, $2^7 = 128$ bytes are possible

2-47. In a chemical plant, 24 holding tanks are used for final product storage. Four tanks are selected at random and without
replacement. Suppose that six of the tanks contain material in which the viscosity exceeds the customer requirements.

(a) What is the probability that exactly one tank in the sample contains high-viscosity material?
(b) What is the probability that at least one tank in the sample contains high-viscosity material?
(c) In addition to the six tanks with high viscosity levels, four different tanks contain material with high impurities.
What is the probability that exactly one tank in the sample contains high-viscosity material and exactly one tank in
the sample contains material with high impurities?

(a) The total number of samples possible is $\binom{24}{4} = \frac{24!}{4!20!} = 10{,}626.$ The number of samples in which exactly one tank has high viscosity is $\binom{6}{1}\binom{18}{3} = \frac{6!}{1!5!} \times \frac{18!}{3!15!} = 4896$. Therefore, the probability is

$$\frac{4896}{10626} = 0.461$$

(b) The number of samples that contain no tank with high viscosity is $\binom{18}{4} = \frac{18!}{4!14!} = 3060.$ Therefore, the requested probability is $1 - \frac{3060}{10626} = 0.712$.

(c) The number of samples that meet the requirements is $\binom{6}{1}\binom{4}{1}\binom{14}{2} = \frac{6!}{1!5!} \times \frac{4!}{1!3!} \times \frac{14!}{2!12!} = 2184$.

Therefore, the probability is $\frac{2184}{10626} = 0.206$

2-48. Plastic parts produced by an injection-molding operation are checked for conformance to specifications. Each tool contains 12 cavities in which parts are produced, and these parts fall into a conveyor when the press opens. An inspector chooses 3 parts from among the 12 at random. Two cavities are affected by a temperature malfunction that results in parts that do not conform to specifications.

(a) How many samples contain exactly 1 nonconforming part?
(b) How many samples contain at least 1 nonconforming part?

(a) The total number of samples is $\binom{12}{3} = \frac{12!}{3!9!} = 220$. The number of samples that result in one nonconforming part is $\binom{2}{1}\binom{10}{2} = \frac{2!}{1!1!} \times \frac{10!}{2!8!} = 90.$ Therefore, the requested probability is 90/220 = 0.409.

(b) The number of samples with no nonconforming part is $\binom{10}{3} = \frac{10!}{3!7!} = 120.$ The probability of at least one nonconforming part is $1 - \frac{120}{220} = 0.455$.

2-49. A bin of 50 parts contains 5 that are defective. A sample of 10 parts is selected at random, without replacement. How many samples contain at least four defective parts?

From the 5 defective parts, select 4, and the number of ways to complete this step is 5!/(4!1!) = 5
From the 45 non-defective parts, select 6, and the number of ways to complete this step is 45!/(6!39!) = 8,145,060
Therefore, the number of samples that contain exactly 4 defective parts is 5(8,145,060) = 40,725,300
Similarly, from the 5 defective parts, the number of ways to select 5 is 5!(5!1!) = 1
From the 45 non-defective parts, select 5, and the number of ways to complete this step is 45!/(5!40!) = 1,221,759
Therefore, the number of samples that contain exactly 5 defective parts is
1(1,221,759) = 1,221,759

Therefore, the number of samples that contain at least 4 defective parts is
40,725,300 + 1,221,759 = 41,947,059

2-50. The following table summarizes 204 endothermic reactions involving sodium bicarbonate.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Let A denote the event that a reaction's final temperature is 271 K or less. Let B denote the event that the heat absorbed is below target. Determine the number of reactions in each of the following events.

(a) $A \cap B$ (b) A' (c) $A \cup B$ (d) $A \cup B'$ (e) $A' \cap B'$

(a) $A \cap B$ = 56
(b) A' = 36 + 56 = 92
(c) $A \cup B$ = 40 + 12 + 16 + 44 + 56 = 168
(d) $A \cup B'$ = 40+12+16+44+36=148
(e) $A' \cap B'$ = 36

2-51. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. How many different designs are possible?

Total number of possible designs = $4 \times 3 \times 5 \times 3 \times 5 = 900$

2-52. Consider the hospital emergency department data given below. Let A denote the event that a visit is to hospital 1, and let B denote the event that a visit results in admittance to any hospital.

Hospital					
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the number of persons in each of the following events.

(a) $A \cap B$ (b) A' (c) $A \cup B$ (d) $A \cup B'$ (e) $A' \cap B'$

(a) $A \cap B$ = 1277
(b) A' = 22252 – 5292 = 16960
(c) $A \cup B$ = 1685 + 3733 + 1403 + 2 + 14 + 29 + 46 + 3 = 6915
(d) $A \cup B'$ = 195 + 270 + 246 + 242+ 3820 + 5163 + 4728 + 3103 + 1277 = 19044
(e) $A' \cap B'$ = 270 + 246 + 242 + 5163 + 4728 + 3103 = 13752

2-53. An article in *The Journal of Data Science* ["A Statistical Analysis of Well Failures in Baltimore County" (2009, Vol. 7,

pp. 111–127)] provided the following table of well failures for different geological formation groups in Baltimore County.

Geological Formation Group	Wells Failed	Wells Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed. Determine the number of wells in each of the following events.

(a) $A \cap B$ (b) A' (c) $A \cup B$ (d) $A \cap B'$ (e) $A' \cap B'$

(a) $A \cap B$ = 170 + 443 + 60 = 673
(b) A' = 28 + 363 + 309 + 933 + 39 = 1672
(c) $A \cup B$ = 1685 + 3733 + 1403 + 2 + 14 + 29 + 46 + 3 = 6915
(d) $A \cup B'$ = 1685 + (28 – 2) + 3733 + (363 – 14) + (309 – 29) + 1403 + (933 – 46) + (39 – 3) = 8399
(e) $A' \cap B'$ = 28 – 2 + 363 – 14 + 306 – 29 + 933 – 46 + 39 – 3 = 1578

2-54. A hospital operating room needs to schedule three knee surgeries and two hip surgeries in a day. Suppose that an operating room needs to handle three knee, four hip, and five shoulder surgeries.

(a) How many different sequences are possible?
(b) How many different sequences have all hip, knee, and shoulder surgeries scheduled consecutively?
(c) How many different schedules begin and end with a knee surgery?

(a) From the formula for the number of sequences $\frac{12!}{3!4!5!}$ = 27,720 sequences are possible.

(b) Combining all hip surgeries into one single unit, all knee surgeries into one single unit and all shoulder surgeries into one unit, the possible number of sequences of these units = 3! = 6

(c)With two surgeries specified, 10 remain and there are $\frac{10!}{4!5!1!}$ = 1,260 different sequences.

2-55. Consider the bar code code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements

(five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white)
space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if b and B denote narrow and wide (black) bars, respectively, and w and W denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6). One code is still held back as a delimiter. For each of the following cases, how many characters can be encoded?

(a) The constraint of exactly two wide bars is replaced with one that requires exactly one wide bar.
(b) The constraint of exactly two wide bars is replaced with one that allows either one or two wide bars.
(c) The constraint of exactly two wide bars is dropped.
(d) The constraints of exactly two wide bars and one wide space are dropped.

(a) The constraint of exactly two wide bars is replaced with one that requires exactly one wide bar.
Focus first on the bars. There are $5!/(4!1!) = 5$ permutations of the bars with one wide bar and four narrow bars. As in the example, the number of permutations of the spaces = 4. Therefore, the possible number of codes = 5(4) = 20, and if one is held back as a delimiter, 19 characters can be coded.

(b) The constraint of exactly two wide bars is replaced with one that allows either one or two wide bars.
As in the example, the number of codes with exactly two wide bars = 40. From part (a), the number of codes with exactly one wide bar = 20. Therefore, is the possible codes are 40 + 20 = 60, and if one code is held back as a delimiter, 59 characters can be coded.

(c) The constraint of exactly two wide bars is dropped.
There are 2 choices for each bar (wide or narrow) and 5 bars are used in total. Therefore, the number of possibilities for the bars = $2^5 = 32$. As in the example, there are 4 possibilities for the spaces. Therefore, the number of codes is 32(3) = 128, and if one is held back as a delimiter, 127 characters can be coded.
(d) The constraints of exactly 2 wide bars and 1 wide space is dropped.
As in part (c), there are 32 possibilities for the bars, and there are also $2^4 = 16$ possibilities for the spaces. Therefore, 32(16) = 512 codes are possible, and if one is held back as a delimiter, 511 characters can be coded.

2-56. A computer system uses passwords that contain exactly eight characters, and each character is 1 of the 26 lowercase
letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Let Ω denote the set of all possible passwords, and let *A* and *B* denote the events that consist of passwords with only letters or only integers, respectively. Determine the number of passwords in each of the following events.

(a) Ω (b) *A* (c) $A' \cap B'$
(d) Passwords that contain at least 1 integer
(e) Passwords that contain exactly 1 integer

Let |A| denote the number of elements in the set A.
(a) The number of passwords in Ω is $|\Omega| = 62^8$ (from multiplication rule).
(b) The number of passwords in A is $|A| = 52^8$ (from multiplication rule)
(c) $A' \cap B' = (A \cup B)'$. Also, $|A| = 52^8$ and $|B| = 10^8$ and $A \cap B$ = null. Therefore,
$(A \cup B)' = |\Omega| - |A| - |B| = 62^8 - 52^8 - 10^8 \approx 1.65 \times 10^{14}$
(d) Passwords that contain at least 1 integer = $|\Omega| - |A| = 62^8 - 52^8 \approx 1.65 \times 10^{14}$
(e) Passwords that contain exactly 1 integer. The number of passwords with 7 letters is 52^7. Also, 1 integer is selected in 10 ways, and can be inserted into 8 positions in the password. Therefore, the solution is $8(10)(52^7) \approx 8.22 \times 10^{13}$

2-57. The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let A denote the event that the patient was treated with ribavirin plus interferon alfa, and let B denote the event that the

response was complete. Determine the number of patients in each of the following events.

(a) A (b) $A \cap B$ (c) $A \cup B$ (d) $A' \cap B'$

Let |A| denote the number of elements in the set A.
(a) |A| = 21
(b) |A∩B| = 16
(c) |AUB| = A+B - (A∩B) = 21+22 – 16 = 27
(d) |A'∩B'| = 60 - |AUB| = 60 – 27 = 33

Section 2-2

2-58. Each of the possible five outcomes of a random experiment is equally likely. The sample space is {*a*, *b*, *c*, *d*, *e*}. Let *A*

denote the event {*a*, *b*}, and let *B* denote the event {*c*, *d*, *e*}. Determine the following:

(a) P(A) (b) P(B) (c) P(A') (d) P(A∪B) (e) P(A∩B)

All outcomes are equally likely
(a) P(A) = 2/5
(b) P(B) = 3/5
(c) P(A') = 3/5
(d) P(A∪B) = 1
(e) P(A∩B) = P(∅)= 0

2-59. The sample space of a random experiment is {*a*, *b*,*c*, *d*, *e*} with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let *A* denote the event {*a*, *b*, *c*}, and let *B* denote the event {*c*, *d*, *e*}. Determine the following:

(a) P(A) (b) P(B) (c) P(A') (d) P(A∪B) (e) P(A∩B)

(a) P(A) = 0.4
(b) P(B) = 0.8
(c) P(A') = 0.6
(d) P(A∪B) = 1
(e) P(A∩B) = 0.2

2-60. Orders for a computer are summarized by the optional features that are requested as follows:

	Proportion of Orders
No optional features	0.3
One optional feature	0.5
More than one optional feature	0.2

(a) What is the probability that an order requests at least one optional feature?
(b) What is the probability that an order does not request more than one optional feature?

(a) 0.5 + 0.2 = 0.7
(b) 0.3 + 0.5 = 0.8

2-61. If the last digit of a weight measurement is equally likely to be any of the digits 0 through 9,

(a) What is the probability that the last digit is 0?

(b) What is the probability that the last digit is greater than or equal to 5?

(a) 1/10
(b) 5/10

2-62. A part selected for testing is equally likely to have been produced on any one of six cutting tools.

(a) What is the sample space?
(b) What is the probability that the part is from tool 1?
(c) What is the probability that the part is from tool 3 or tool 5?
(d) What is the probability that the part is not from tool 4?

(a) S = {1, 2, 3, 4, 5, 6}
(b) 1/6
(c) 2/6
(d) 5/6

2-63. An injection-molded part is equally likely to be obtained from any one of the eight cavities on a mold.

(a) What is the sample space?
(b) What is the probability that a part is from cavity 1 or 2?
(c) What is the probability that a part is from neither cavity 3 nor 4?

(a) S = {1,2,3,4,5,6,7,8}
(b) 2/8
(c) 6/8

2-64. In an acid-base titration, a base or acid is gradually added to the other until they have completely neutralized each

other. Because acids and bases are usually colorless (as are the water and salt produced in the neutralization reaction), pH is measured to monitor the reaction. Suppose that the equivalence point is reached after approximately 100 mL of an NaOH solution has been added (enough to react with all the acetic acid present) but that replicates are equally likely to indicate from 95 to 104 mL to the nearest mL. Assume that volumes are measured to the nearest mL and describe the sample space.

(a) What is the probability that equivalence is indicated at 100 mL?
(b) What is the probability that equivalence is indicated at less than 100 mL?
(c) What is the probability that equivalence is indicated between 98 and 102 mL (inclusive)?

The sample space is {95, 96, 97,…, 103, and 104}.

(a) Because the replicates are equally likely to indicate from 95 to 104 mL, the probability that equivalence is indicated at 100 mL is 0.1.
(b) The event that equivalence is indicated at less than 100 mL is {95, 96, 97, 98, 99}. The probability that the event occurs is 0.5.
(c) The event that equivalence is indicated between 98 and 102 mL is {98, 99, 100, 101, 102}. The probability that the event occurs is 0.5.

2-65. In a NiCd battery, a fully charged cell is composed of nickelic hydroxide. Nickel is an element that has multiple oxidation states and that is usually found in the following states:

Nickel Charge	Proportions Found
0	0.17
+2	0.35
+3	0.33
+4	0.15

(a) What is the probability that a cell has at least one of the positive nickel-charged options?
(b) What is the probability that a cell is *not* composed of a positive nickel charge greater than +3?

The sample space is {0, +2, +3, and +4}.

(a) The event that a cell has at least one of the positive nickel charged options is {+2, +3, and +4}. The probability is 0.35 + 0.33 + 0.15 = 0.83.
(b) The event that a cell is not composed of a positive nickel charge greater than +3 is {0, +2, and +3}. The probability is 0.17 + 0.35 + 0.33 = 0.85.

2-66. A credit card contains 16 digits between 0 and 9. However, only 100 million numbers are valid. If a number is entered randomly, what is the probability that it is a valid number?

Total possible: 10^{16}, but only 10^8 are valid. Therefore, P(valid) = $10^8/10^{16} = 1/10^8$

2-67. Suppose your vehicle is licensed in a state that issues license plates that consist of three digits (between 0 and 9) followed by three letters (between *A* and *Z*). If a license number is selected randomly, what is the probability that yours is the one selected?

3 digits between 0 and 9, so the probability of any three numbers is 1/(10*10*10).
3 letters A to Z, so the probability of any three numbers is 1/(26*26*26). The probability your license plate is chosen is then $(1/10^3)*(1/26^3) = 5.7 \times 10^{-8}$

2-68. A message can follow different paths through servers on a network. The sender's message can go to one of five servers for the first step; each of them can send to five servers at the second step; each of those can send to four servers at the third step; and then the message goes to the recipient's server.

(a) How many paths are possible?
(b) If all paths are equally likely, what is the probability that a message passes through the first of four servers at the third step?

(a) 5*5*4 = 100
(b) (5*5)/100 = 25/100=1/4

2-69. Magnesium alkyls are used as homogenous catalysts in the production of linear low-density polyethylene (LLDPE), which requires a finer magnesium powder to sustain a reaction. Redox reaction experiments using four different amounts of magnesium powder are performed. Each result may or may not be further reduced in a second step using three different magnesium powder amounts. Each of these results may or may not be further reduced in a third step using three different amounts of magnesium powder.

(a) How many experiments are possible?
(b) If all outcomes are equally likely, what is the probability that the best result is obtained from an experiment that uses all three steps?
(c) Does the result in part (b) change if five or six or seven different amounts are used in the first step? Explain.

(a) The number of possible experiments is $4 + 4 \times 3 + 4 \times 3 \times 3 = 52$
(b) There are 36 experiments that use all three steps. The probability the best result uses all three steps is 36/52 = 0.6923.
(c) No, it will not change. With k amounts in the first step the number of experiments is $k + 3k + 9k = 13k$. The number of experiments that complete all three steps is $9k$ out of $13k$. The probability is 9/13 = 0.6923.

2-70. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		Shock Resistance	
		High	Low
Scratch	High	70	9
Resistance	Low	16	5

Let *A* denote the event that a disk has high shock resistance, and let *B* denote the event that a disk has high scratch resistance. If a disk is selected at random, determine the following probabilities:

(a) P(A) (b) P(B) (c) P(A') (d) P(A∩B) (e) P(A∪B) (f) P(A'∪B)

(a) P(A) = 86/100 = 0.86
(b) P(B) = 79/100 = 0.79
(c) P(A') = 14/100 = 0.14
(d) P(A∩B) = 70/100 = 0.70
(e) P(A∪B) = (70+9+16)/100 = 0.95
(f) P(A'∪B) = (70+9+5)/100 = 0.84

2-71. Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from
100 samples are summarized as follows:

		Conforms	
		Yes	No
	1	22	8
Supplier	2	25	5
	3	30	10

Let *A* denote the event that a sample is from supplier 1, and let *B* denote the event that a sample conforms to specifications. If a sample is selected at random, determine the following probabilities:

(a) P(A) (b) P(B) (c) P(A') (d) P(A∩B) (e) P(A∪B) (f) P(A'∪B)

(a) P(A) = 30/100 = 0.30
(b) P(B) = 77/100 = 0.77
(c) P(A') = 1 – 0.30 = 0.70
(d) P(A∩B) = 22/100 = 0.22
(e) P(A∪B) = 85/100 = 0.85
(f) P(A'∪B) =92/100 = 0.92

2-72. An article in the *Journal of Database Management* ["Experimental Study of a Self-Tuning Algorithm for DBMS Buffer Pools" (2005, Vol. 16, pp. 1–20)] provided the workload used in the TPC-C OLTP (Transaction Processing Performance Council's Version C On-Line Transaction Processing) benchmark, which simulates a typical order entry application.

TABLE • 2E-1 Average Frequencies and Operations in TPC-C

Transaction	Frequency	Selects	Updates	Inserts	Deletes	Nonunique Selects	Joins
New order	43	23	11	12	0	0	0
Payment	44	4.2	3	1	0	0.6	0
Order status	4	11.4	0	0	0	0.6	0
Delivery	5	130	120	0	10	0	0
Stock level	4	0	0	0	0	0	1

The frequency of each type of transaction (in the second column) can be used as the percentage of each type of transaction. The average number of *selects* operations required for each type of transaction is shown. Let *A* denote the event of transactions with an average number of *selects* operations of 12 or fewer. Let *B* denote the event of transactions with an average number of *updates* operations of 12 or fewer. Calculate the following probabilities.

(a) P(A) (b) P(B) (c) P(A∩B) (d) P(A∩B') (f) P(A∪B)

(a) The total number of transactions is 43+44+4+5+4=100

$$P(A) = \frac{44+4+4}{100} = 0.52$$

(b) $P(B)=\dfrac{100-5}{100}=0.95$

(c) $P(A\cap B)=\dfrac{44+4+4}{100}=0.52$

(d) $P(A\cap B')=0$

(e) $P(A\cup B)=\dfrac{100-5}{100}=0.95$

2-73. Use the axioms of probability to show the following: $A\cup B$ (d) $A\cap B'$

(a) For any event E, $P(E')=1-P(E)$. (b) $P(\varnothing)=0$ (c) If A is contained in B, then $P(A)\le P(B)$.

(a) Because E and E' are mutually exclusive events and $E\cup E'=S$
$1 = P(S) = P(E\cup E') = P(E) + P(E')$. Therefore, $P(E') = 1 - P(E)$

(b) Because S and $\varnothing$ are mutually exclusive events with $S = S\cup\varnothing$
$P(S) = P(S) + P(\varnothing)$. Therefore, $P(\varnothing) = 0$

(c) Now, $B = A\cup(A'\cap B)$ and the events A and $A'\cap B$ are mutually exclusive. Therefore,
$P(B) = P(A) + P(A'\cap B)$. Because $P(A'\cap B)\ge 0$, $P(B)\ge P(A)$.

2.74. Consider the endothermic reaction's table given below. Let A denote the event that a reaction's final temperature is 271 K or less. Let B denote the event that the heat absorbed is above target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Determine the following probabilities.

(a) $P(A\cap B)$ (b) $P(A')$ (c) $P(A\cup B)$ (d) $P(A\cup B')$ (e) $P(A'\cap B')$

(a) $P(A\cap B) = (40 + 16)/204 = 0.2745$
(b) $P(A') = (36 + 56)/204 = 0.4510$
(c) $P(A\cup B) = (40 + 12 + 16 + 44 + 36)/204 = 0.7255$
(d) $P(A\cup B') = (40 + 12 + 16 + 44 + 56)/204 = 0.8235$
(e) $P(A'\cap B') = 56/204 = 0.2745$

2-75. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. If you visit the site five times, what is the probability that you will not see the same design?

A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. If you visit the site five times, what is the probability that you will not see the same design?

Total number of possible designs is 900. The sample space of all possible designs that may be seen on five visits. This space contains 900^5 outcomes.

The number of outcomes in which all five visits are different can be obtained as follows. On the first visit any one of 900 designs may be seen. On the second visit there are 899 remaining designs. On the third visit there are 898 remaining designs. On the fourth and fifth visits there are 897 and 896 remaining designs, respectively. From the multiplication rule, the number of outcomes where all designs are different is 900*899*898*897*896. Therefore, the probability that a design is not seen again is

$(900*899*898*897*896)/ 900^5 = 0.9889$

2-76. Consider the hospital emergency room data is given below. Let A denote the event that a visit is to hospital 4, and let B
denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A')$ (c) $P(A \cup B)$ (d) $P(A \cup B')$ (e) $P(A' \cap B')$

(a) $P(A \cap B) = 242/22252 = 0.0109$
(b) $P(A') = (5292+6991+5640)/22252 = 0.8055$
(c) $P(A \cup B) = (195 + 270 + 246 + 242 + 984 + 3103)/22252 = 0.2265$
(d) $P(A \cup B') = (4329 + (5292 – 195) + (6991 – 270) + 5640 – 246))/22252 = 0.9680$
(e) $P(A' \cap B') = (1277 + 1558 + 666 + 3820 + 5163 + 4728)/22252 = 0.7735$

2-77. Consider the well failure data is given below. Let A denote the event that the geological formation has more than 1000 wells, and let B denote the event that a well failed.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A')$ (c) $P(A \cup B)$ (d) $P(A \cup B')$ (e) $P(A' \cap B')$

(a) $P(A \cap B) = (170 + 443 + 60)/8493 = 0.0792$
(b) $P(A') = (28 + 363 + 309 + 933 + 39)/8493 = 1672/8493 = 0.1969$
(c) $P(A \cup B) = (1685+3733+1403+2+14+29+46+3)/8493 = 6915/8493 = 0.8142$
(d) $P(A \cup B') = (1685 + (28 – 2) + 3733 + (363 – 14) + (309 – 29) + 1403 + (933 – 46) + (39 – 3))/8493 = 8399/8493 = 0.9889$
(e) $P(A' \cap B') = (28 – 2 + 363 – 14 + 306 – 29 + 933 – 46 + 39 – 3)/8493 = 1578/8493 = 0.1858$

2-78. Consider the bar code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if b and B denote narrow and wide (black) bars, respectively, and w and W denote narrow and wide (white) spaces, a valid character is $bwBwBWbwb$ (the number 6). Suppose that all 40 codes are equally likely (none is held back as a delimiter).

Determine the probability for each of the following:
(a) A wide space occurs before a narrow space.
(b) Two wide bars occur consecutively.

(c) Two consecutive wide bars are at the start or end.
(d) The middle bar is wide.

(a) There are 4 spaces and exactly one is wide.
Number of permutations of the spaces where the wide space appears first is 1.
Number of permutations of the bars is 5!/(2!3!) = 10.
Total number of permutations where a wide space occurs before a narrow space 1(10) = 10.
P(wide space occurs before a narrow space) =10/40 = 1/4

(b) There are 5 bars and 2 are wide.
The spaces are handled as in part (a).
Number of permutations of the bars where 2 wide bars are consecutive is 4.
Therefore, the probability is 16/40 = 0.4

(c) The spaces are handled as in part (a).
Number of permutations of the bars where the 2 consecutive wide bars are at the start or end is 2. Therefore, the probability is 8/40 = 0.2
(d) The spaces are handled as in part (a).
Number of permutations of the bars where a wide bar is at the center is 4 because there are 4 remaining positions for the second wide bar. Therefore, the probability is 16/40 = 0.4.

2-79. A hospital operating room needs to schedule three knee surgeries and two hip surgeries in a day. Suppose that an operating room needs to schedule three knee, four hip, and five shoulder surgeries. Assume that all schedules are equally likely.

Determine the probability for each of the following:
(a) All hip surgeries are completed before another type of surgery.
(b) The schedule begins with a hip surgery.
(c) The fi rst and last surgeries are hip surgeries.
(d) The fi rst two surgeries are hip surgeries.

(a) P(all hip surgeries before another type)= $\frac{\frac{8!}{3!5!}}{\frac{12!}{3!4!5!}} = \frac{8!4!}{12!} = \frac{1}{495} = 0.00202$

(b) P(begins with hip surgery)= $\frac{\frac{11!}{3!3!5!}}{\frac{12!}{3!4!5!}} = \frac{11!4!}{12!3!} = \frac{1}{3}$

(c) P(first and last are hip surgeries)= $\frac{\frac{10!}{2!3!5!}}{\frac{12!}{3!4!5!}} = \frac{1}{11}$

(d) P(first two are hip surgeries)= $\frac{\frac{10!}{2!3!5!}}{\frac{12!}{3!4!5!}} = \frac{1}{11}$

2-80. Suppose that a patient is selected randomly from the those described ,The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let A denote the event that the patient is in the group treated with interferon alfa, and let B denote the event that the patient has a complete response.
Determine the following probabilities.

(a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$ (e) $P(A' \cup B)$

(a) P(A)= 19/60 = 0.3167
(b) P(B)= 22/60 = 0.3667
(c) P(A∩B) = 6/60 = 0.1
(d) P(AUB) = P(A)+P(B)-P(A∩B) = (19+22-6)/60 =0.5833
(e) P(A'UB) = P(A')+P(B)-P(A'∩B) = $\frac{21+20}{60}+\frac{22}{60}-\frac{16}{60}=0.7833$

2-81. A computer system uses passwords that contain exactly eight characters, and each character is one of 26 lowercase
letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Let Ω denote the set of all possible passwords, and let *A* and *B* denote the events that consist of passwords with only letters or only integers, respectively. Suppose that
all passwords in Ω are equally likely.
Determine the probability of each of the following:
(a) *A* (b) *B*
(c) A password contains at least 1 integer.
(d) A password contains exactly 2 integers.

(a) P(A) = $\frac{52^8}{62^8}=0.2448$
(b) P(B) = $\frac{10^8}{62^8}$ = $4.58\text{x}10^{-7}$
(c) P(contains at least 1 integer) =1 - P(password contains no integer) = 1 - $\frac{52^8}{62^8}$ = 0.7551
(d) P(contains exactly 2 integers)
Number of positions for the integers is 8!/(2!6!) = 28
Number of permutations of the two integers is 10^2 = 100
Number of permutations of the six letters is 52^6
Total number of permutations is 62^8
Therefore, the probability is
$\frac{28(100)(52^6)}{62^8}=0.254$

<u>Section 2-3</u>

2-82. If $P(A) = 0.3$, $P(B) = 0.2$, and $P(A \cap B) = 0.1$, determine the following probabilities:

(a) $P(A')$ (b) $P(A \cup B)$ (c) $P(A' \cap B)$ (d) $P(A \cap B')$ (e) $P[(A \cup B)']$ (f) $P(A' \cup B)$

(a) P(A') = 1- P(A) = 0.7
(b) P ($A \cup B$) = P(A) + P(B) - P($A \cap B$) = 0.3+0.2 - 0.1 = 0.4
(c) P($A' \cap B$) + P($A \cap B$) = P(B). Therefore, P($A' \cap B$) = 0.2 - 0.1 = 0.1
(d) P(A) = P($A \cap B$) + P($A \cap B'$). Therefore, P($A \cap B'$) = 0.3 - 0.1 = 0.2
(e) P(($A \cup B$)') = 1 - P($A \cup B$) = 1 - 0.4 = 0.6
(f) P($A' \cup B$) = P(A') + P(B) - P($A' \cap B$) = 0.7 + 0.2 - 0.1 = 0.8

2-83. If *A*, *B*, and *C* are mutually exclusive events with $P(A) = 0.2$, $P(B) = 0.3$, and $P(C) = 0.4$, determine the following probabilities:

(a) $P(A \cup B \cup C)$ (b) $P(A \cap B \cap C)$ (c) $P(A \cap B)$ (d) $P[(A \cup B) \cap C]$ (e) $P(A' \cap B' \cap C')$

(a) P($A \cup B \cup C$) = P(A) + P(B) + P(C), because the events are mutually exclusive. Therefore,
P($A \cup B \cup C$) = 0.2+0.3+0.4 = 0.9
(b) P ($A \cap B \cap C$) = 0, because $A \cap B \cap C = \varnothing$
(c) P($A \cap B$) = 0 , because $A \cap B = \varnothing$
(d) P($(A \cup B) \cap C$) = 0, because $(A \cup B) \cap C = (A \cap C) \cup (B \cap C) = \varnothing$

(e) P($A' \cap B' \cap C'$) =1-[P(A) + P(B) + P(C)] = 1-(0.2+0.3+0.4) = 0.1

2-84. In the article "ACL Reconstruction Using Bone-Patellar Tendon-Bone Press-Fit Fixation: 10-Year Clinical Results" in *Knee Surgery*, *Sports Traumatology*, *Arthroscopy* (2005, Vol. 13, pp. 248–255), the following causes for knee injuries were considered:

Activity	Percentage of Knee Injuries
Contact sport	46%
Noncontact sport	44%
Activity of daily living	9%
Riding motorcycle	1%

(a) What is the probability that a knee injury resulted from a sport (contact or noncontact)?
(b) What is the probability that a knee injury resulted from an activity other than a sport?

(a) P(Caused by sports) = P(Caused by contact sports or by noncontact sports)
= P(Caused by contact sports) + P(Caused by noncontact sports)
= 0.46 + 0.44 = 0.9
(b) 1- P(Caused by sports) = 0.1

2.85. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100

disks are summarized as follows:

		Shock Resistance	
		High	Low
Scratch	High	70	9
Resistance	Low	16	5

(a) If a disk is selected at random, what is the probability that its scratch resistance is high and its shock resistance is
high?
(b) If a disk is selected at random, what is the probability that its scratch resistance is high or its shock resistance is
high?
(c) Consider the event that a disk has high scratch resistance and the event that a disk has high shock resistance. Are
these two events mutually exclusive?

(a) 70/100 = 0.70
(b) (79+86-70)/100 = 0.95
(c) No, P($A \cap B$) $\neq 0$

2-86. Strands of copper wire from a manufacturer are analyzed for strength and conductivity. The results from 100 strands are as follows:

	Strength	
	High	Low
High conductivity	74	8
Low conductivity	15	3

(a) If a strand is randomly selected, what is the probability that its conductivity is high and its strength is high?
(b) If a strand is randomly selected, what is the probability that its conductivity is low or its strength is low?
(c) Consider the event that a strand has low conductivity and the event that the strand has low strength. Are these two
events mutually exclusive?

(a) P(High temperature and high conductivity)= 74/100 =0.74
(b) P(Low temperature or low conductivity)
= P(Low temperature) + P(Low conductivity) – P(Low temperature and low conductivity)
= (8+3)/100 + (15+3)/100 – 3/100
= 0.26

(c) No, they are not mutually exclusive. Because P(Low temperature) + P(Low conductivity)
= (8+3)/100 + (15+3)/100
= 0.29, which is not equal to P(Low temperature or low conductivity).

2-87. The analysis of shafts for a compressor is summarized by conformance to specifications.

		Roundness Conforms	
		Yes	No
Surface Finish	Yes	345	5
Conforms	No	12	8

(a) If a shaft is selected at random, what is the probability that it conforms to surface finish requirements?
(b) What is the probability that the selected shaft conforms to surface finish requirements or to roundness requirements?
(c) What is the probability that the selected shaft either conforms to surface finish requirements or does not conform to roundness requirements?
(d) What is the probability that the selected shaft conforms to both surface finish and roundness requirements?

(a) 350/370

(b) $\dfrac{345+5+12}{370}=\dfrac{362}{370}$

(c) $\dfrac{345+5+8}{370}=\dfrac{358}{370}$

(d) 345/370

2-88. Cooking oil is produced in two main varieties: mono and polyunsaturated. Two common sources of cooking oil are corn and canola. The following table shows the number of bottles of these oils at a supermarket:

		Type of oil	
		Canola	Corn
Type of Unsaturation	Mono	7	13
	Poly	93	77

(a) If a bottle of oil is selected at random, what is the probability that it belongs to the polyunsaturated category?
(b) What is the probability that the chosen bottle is monounsaturated canola oil?

(a) 170/190 = 17/19
(b) 7/190

2-89. A manufacturer of front lights for automobiles tests lamps under a high-humidity, high-temperature environment using intensity and useful life as the responses of interest. The following table shows the performance of 130 lamps:

		Useful life	
		Satisfactory	Unsatisfactory
Intensity	Satisfactory	117	3
	Unsatisfactory	8	2

(a) Find the probability that a randomly selected lamp will yield unsatisfactory results under any criteria.
(b) The customers for these lamps demand 95% satisfactory results. Can the lamp manufacturer meet this demand?

(a) P(unsatisfactory) = (5 + 10 – 2)/130 = 13/130
(b) P(both criteria satisfactory) = 117/130 = 0.90, No

2-90. A computer system uses passwords that are six characters, and each character is one of the 26 letters (*a*–*z*) or

10 integers (0–9). Uppercase letters are not used. Let *A* denote the event that a password begins with a vowel (either *a*, *e*, *i*, *o*, or *u*), and let *B* denote the event that a password ends with an even number (either 0, 2, 4, 6, or 8). Suppose a hacker selects a password at random. Determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$

(a) 5/36
(b) 5/36
(c) $P(A \cap B) = P(A)P(B) = 25/1296$
(d) $P(A \cup B) = P(A) + P(B) - P(A)P(B) = 10/36 - 25/1296 = 0.2585$

2-91. Consider the endothermic reactions given below. Let *A* denote the event that a reaction's final temperature is 271 K or less. Let *B* denote the event that the heat absorbed is above target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Use the addition rules to calculate the following probabilities.

(a) $P(A \cup B)$ (b) $P(A \cap B')$ (c) $P(A' \cup B')$

$P(A) = 112/204 = 0.5490$, $P(B) = 92/204 = 0.4510$, $P(A \cap B) = (40+16)/204 = 0.2745$
(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5490 + 0.4510 - 0.2745 = 0.7255$
(b) $P(A \cap B') = (12 + 44)/204 = 0.2745$ and $P(A \cup B') = P(A) + P(B') - P(A \cap B') = 0.5490 + (1 - 0.4510) - 0.2745 = 0.8235$
(c) $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.2745 = 0.7255$

2-92. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let *A* denote the event that the design color is red, and let *B* denote the event that the font size is not the smallest one. Use the addition rules to calculate the following probabilities.

(a) $P(A \cup B)$ (b) $P(A \cup B')$ (c) $P(A' \cup B')$

$P(A) = 1/4 = 0.25$, $P(B) = 4/5 = 0.80$, $P(A \cap B) = P(A)P(B) = (1/4)(4/5) = 1/5 = 0.20$
(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.80 - 0.20 = 0.85$
(b) First $P(A \cap B') = P(A)P(B') = (1/4)(1/5) = 1/20 = 0.05$. Then $P(A \cup B') = P(A) + P(B') - P(A \cap B') = 0.25 + 0.20 - 0.05 = 0.40$
(c) $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.20 = 0.80$

2-93. Consider the hospital emergency room data given below. Let *A* denote the event that a visit is to hospital 4, and let *B*

denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Use the addition rules to calculate the following probabilities.

(a) $P(A \cup B)$ (b) $P(A \cup B')$ (c) $P(A' \cup B')$

$P(A) = 4329/22252 = 0.1945$, $P(B) = 953/22252 = 0.0428$, $P(A \cap B) = 242/22252 = 0.0109$,

$P(A \cap B') = (984+3103)/22252 = 0.1837$
(a) $P(A \cup B) = P(A) + P(B) – P(A \cap B) = 0.1945 + 0.0428 – 0.0109 = 0.2264$
(b) $P(A \cup B') = P(A) + P(B') – P(A \cap B') = 0.1945 + (1 – 0.0428) – 0.1837 = 0.9680$
(c) $P(A' \cup B') = 1 – P(A \cap B) = 1 – 0.0109 = 0.9891$

2-94. Consider the well failure data given below. Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Use the addition rules to calculate the following probabilities.

(a) $P(A \cup B)$ (b) $P(A \cup B')$ (c) $P(A' \cup B')$

$P(A) = (1685 + 3733 + 1403)/8493 = 0.8031$, $P(B) = (170 + 2 + 443 + 14 + 29 + 60 + 46 + 3)/8493 = 0.0903$,
$P(A \cap B) = (170 + 443 + 60)/8493 = 0.0792$, $P(A \cap B') = (1515+3290+1343)/8493 = 0.7239$
a) $P(A \cup B) = P(A) + P(B) – P(A \cap B) = 0.8031 + 0.0903 – 0.0792 = 0.8142$
b) $P(A \cup B') = P(A) + P(B') – P(A \cap B') = 0.8031 + (1 – 0.0903) – 0.7239 = 0.9889$
c) $P(A' \cup B') = 1 – P(A \cap B) = 1 – 0.0792 = 0.9208$

2-95. Consider the bar code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6). Suppose that all 40 codes are equally likely (none is held back as a delimiter).
Determine the probability for each of the following:

(a) The first bar is wide or the second bar is wide.
(b) Neither the first nor the second bar is wide.
(c) The first bar is wide or the second bar is not wide.
(d) The first bar is wide or the first space is wide.

(a) Number of permutations of the bars with the first bar wide is 4 .
Number of permutations of the bars with the second bar wide is 4 .
Number of permutations of the bars with both the first & second bar wide is 1 .
Number of permutations of the bars with either the first bar wide or the last bar wide = 4 + 4 – 1 = 7.
Number of codes is multiplied this by the number of permutations for the spaces = 4.
P(first bar is wide) = 16/40 = 0.4, P(second bar is wide) = 16/40 = 0.4, P(first & second bar is wide) = 4/40 = 0.1
P(first or second bar is wide) = 4/10 + 4/10 – 1/10 = 7/10

(b) Neither the first or second bar wide implies the two wide bars occur in the last 3 positions.
Number of permutations of the bars with the wide bars in the last 3 positions is 3!/2!1! = 3
P(neither first nor second bar is wide) = 12/40 = 0.3

(c) The spaces are handled as in part (a).
P(first bar is wide) = 16/40 = 0.4
Number of permutations of the bars with the second bar narrow is 4!/(2!2!) = 6
P(second bar is narrow) = 24/40 = 0.6

Number of permutations with the first bar wide and the second bar narrow is 3!/(1!2!) = 3
P(first bar wide and the second bar narrow) = 12/40 = 0.3
P(first bar is wide or the second bar is narrow) = 0.4 + 0.6 – 0.3 = 0.7

(d) The spaces are handled as in part (a).
Number of permutations of the bars with the first bar wide is 4. Therefore, P(first bar is wide) = 16/40 = 0.4
The number of permutations of the bars = 10. Number of permutations of the spaces with the first space wide is 1.
Therefore, P(first space is wide) = 1(10)/40 = 0.25
Number codes with the first bar wide and the first space wide is 4(1) = 4
P(first bar wide & the first space wide) = 4/40 = 0.1
P(first bar is wide or the first space is wide) = 0.4 + 0.25 – 0.1 = 0.55

2-96. Consider the three patient groups. The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let A denote the event that the patient was treated with ribavirin plus interferon alfa, and let B denote the event that the response was complete. Determine the following probabilities:

(a) $P(A \cup B)$ (b) $P(A' \cup B)$ (c) $P(A \cup B')$

(a) P(AUB)= P(A) + P(B) - P(A∩B) = 21/60 + 22/60 – 16/60 = 9/20= 0.45
(b) P(A'UB)= P(A') + P(B) - P(A'∩B) = (19+20)/60 + 22/60 – 6/60 = 11/12 = 0.9166
(c) P(AUB')= P(A) + P(B') - P(A∩B')= 21/60 + (60-22)/60 – 5/60 = 9/10 = 0.9

2-97. A computer system uses passwords that contain exactly eight characters, and each character is one of the 26 lowercase
letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Assume all passwords are equally likely. Let A and B denote the events that consist of passwords with only letters or only integers, respectively. Determine the following probabilities:

(a) $P(A \cup B)$ (b) $P(A' \cup B)$ (c) P (Password contains exactly 1 or 2 integers)

(a) P(AUB) = P(A) + P(B) = $\frac{52^8}{62^8} + \frac{10^8}{62^8}$ = 0.245
(b) P(A'UB) = P(A') = $\frac{10^8}{62^8}$ = 1 – 0.2448 = 0.755
(c) P(contains exactly 1 integer)
Number of positions for the integer is 8!/(1!7!) = 8
Number of values for the integer = 10
Number of permutations of the seven letters is 52^7
Total number of permutations is 62^8
Therefore, the probability is
$\frac{8(10)(52^7)}{62^8} = 0.377$

P(contains exactly 2 integers)
Number of positions for the integers is 8!/(2!6!) = 28
Number of permutations of the two integers is 100
Number of permutations of the 6 letters is 52^6
Total number of permutations is 62^8
Therefore, the probability is

$\frac{28(100)(52^6)}{62^8} = 0.254$

Therefore, P(exactly one integer or exactly two integers) = 0.377 + 0.254 = 0.630

2-98. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose that a patient is selected randomly. Let *A* denote the event that the patient is in group 1, and let *B* denote the event that there is no progression. Determine the following probabilities:

(a) $P(A \cup B)$ (b) $P(A' \cup B')$ (c) $P(A \cup B')$

P(A)= $\frac{114}{114+112+120+121} = \frac{114}{467} = 0.244$

P(B)= $\frac{76+82+104+113}{114+112+120+121} = \frac{375}{467} = 0.8029$

P(A∩B)= $\frac{76}{467} = 0.162$

(a) P(AUB) = P(A) + P(B) - P(A∩B)= 114/467 + 375/467 – 76/467 = 0.884

(b) P(A'UB') = 1 - (A∩B) = 1 – 76/467 =0.838

(c) P(AUB') = P(A) + P(B') - P(A∩B') = 114/467 + (1 – 375/467) – (114 – 76)/467 = 0.359

Section 2-4

2-99. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100

disks are summarized as follows:

		Shock Resistance	
		High	Low
Scratch	High	70	9
Resistance	Low	16	5

Let *A* denote the event that a disk has high shock resistance, and let *B* denote the event that a disk has high scratch resistance. Determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A \mid B)$ (d) $P(B \mid A)$

(a) P(A) = 86/100

(b) P(B) = 79/100

(c) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{70/100}{79/100} = \frac{70}{79}$

(d) $P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{70/100}{86/100} = \frac{70}{86}$

2-100. Samples of skin experiencing desquamation are analyzed for both moisture and melanin content. The results from 100 skin samples are as follows:

		Melanin Content	
		High	Low
Moisture	High	13	7
Content	Low	48	32

Let A denote the event that a sample has low melanin content, and let B denote the event that a sample has high moisture content. Determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A \mid B)$ (d) $P(B \mid A)$

(a) $P(A) = \dfrac{7+32}{100} = 0.39$

(b) $P(B) = \dfrac{13+7}{100} = 0.2$

(c) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{7/100}{20/100} = 0.35$

(d) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{7/100}{39/100} = 0.1795$

2-101. The analysis of results from a leaf transmutation experiment (turning a leaf into a petal) is summarized by type of transformation completed:

		Total Textural Transformation	
		Yes	No
Total Color	Yes	243	26
Transformation	No	13	18

(a) If a leaf completes the color transformation, what is the probability that it will complete the textural transformation?

(b) If a leaf does not complete the textural transformation, what is the probability it will complete the color transformation?

Let A denote the event that a leaf completes the color transformation and let B denote the event that a leaf completes the textural transformation. The total number of experiments is 300.

(a) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{243/300}{(243+26)/300} = 0.903$

(b) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{26/300}{(18+26)/300} = 0.591$

2-102. Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and length measurements. The results of 100 parts are summarized as follows:

		Length	
		Excellent	Good
Surface	Excellent	80	2
Finish	Good	10	8

Let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent length. Determine:

(a) $P(A)$ (b) $P(B)$ (c) $P(A \mid B)$ (d) $P(B \mid A)$

(e) If the selected part has excellent surface finish, what is the probability that the length is excellent?

(f) If the selected part has good length, what is the probability that the surface finish is excellent?

(a) 0.82

(b) 0.90
(c) 8/9 = 0.889
(d) 80/82 = 0.9756
(e) 80/82 = 0.9756
(f) 2/10 = 0.20

2-103. The following table summarizes the analysis of samples of galvanized steel for coating weight and surface roughness:

		Coating Weight	
		High	Low
Surface	High	12	16
Roughness	Low	88	34

(a) If the coating weight of a sample is high, what is the probability that the surface roughness is high?
(b) If the surface roughness of a sample is high, what is the probability that the coating weight is high?
(c) If the surface roughness of a sample is low, what is the probability that the coating weight is low?

(a) 12/100 (b) 12/28 (c) 34/122

2-104. Consider the data on wafer contamination and location in the sputtering tool shown in Table 2-2. Assume that one wafer is selected at random from this set. Let *A* denote the event that a wafer contains four or more particles, and let *B* denote the event that a wafer is from the center of the sputtering tool. Determine:

(a) $P(A)$ (b) $P(A \mid B)$ (c) $P(B)$ (d) $P(B \mid A)$ (e) $P(A \cap B)$ (f) $P(A \cup B)$

(a) P(A) = 0.05 + 0.10 = 0.15
(b) P(A|B) = $\dfrac{P(A \cap B)}{P(B)} = \dfrac{0.04 + 0.07}{0.72} = 0.153$
(c) P(B) = 0.72
(d) P(B|A) = $\dfrac{P(A \cap B)}{P(A)} = \dfrac{0.04 + 0.07}{0.15} = 0.733$
(e) P(A $\cap$ B) = 0.04 +0.07 = 0.11
(f) P(A $\cup$ B) = 0.15 + 0.72 – 0.11 = 0.76

2-105. The following table summarizes the number of deceased beetles under autolysis (the destruction of a cell after its death by the action of its own enzymes) and putrefaction (decomposition of organic matter, especially protein, by microorganisms, resulting in production of foul-smelling matter):

		Autolysis	
		High	Low
Putrefaction	High	14	59
	Low	18	9

(a) If the autolysis of a sample is high, what is the probability that the putrefaction is low?
(b) If the putrefaction of a sample is high, what is the probability that the autolysis is high?
(c) If the putrefaction of a sample is low, what is the probability that the autolysis is low?

Let *A* denote the event that autolysis is high and let *B* denote the event that putrefaction is high. The total number of experiments is 100.

(a) $P(B' \mid A) = \dfrac{P(A \cap B')}{P(A)} = \dfrac{18/100}{(14+18)/100} = 0.5625$

(b) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{14/100}{(14+59)/100} = 0.1918$

(c) $P(A'|B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{9/100}{(18+9)/100} = 0.333$

2-106. A maintenance firm has gathered the following information regarding the failure mechanisms for air conditioning systems:

		Evidence of Gas Leaks	
		Yes	No
Evidence of electrical failure	Yes	55	17
	No	32	3

The units without evidence of gas leaks or electrical failure showed other types of failure. If this is a representative sample of AC failure, find the probability
(a) That failure involves a gas leak
(b) That there is evidence of electrical failure given that there was a gas leak
(c) That there is evidence of a gas leak given that there is evidence of electrical failure

(a) P(gas leak) = (55 + 32)/107 = 0.813
(b) P(electric failure | gas leak) = (55/107)/(87/102) = 0.632
(c) P(gas leak | electric failure) = (55/107)/(72/107) = 0.764

2-107. A lot of 100 semiconductor chips contains 20 that are defective. Two are selected randomly, without replacement, from the lot.
(a) What is the probability that the first one selected is defective?
(b) What is the probability that the second one selected is defective given that the first one was defective?
(c) What is the probability that both are defective?
(d) How does the answer to part (b) change if chips selected were replaced prior to the next selection?

(a) 20/100
(b) 19/99
(c) (20/100)(19/99) = 0.038
(d) If the chips were replaced, the probability would be (20/100) = 0.2

2-108. A batch of 500 containers for frozen orange juice contains 5 that are defective. Two are selected, at random, without replacement from the batch.
(a) What is the probability that the second one selected is defective given that the first one was defective?
(b) What is the probability that both are defective?
(c) What is the probability that both are acceptable?
Three containers are selected, at random, without replacement, from the batch.
(d) What is the probability that the third one selected is defective given that the first and second ones selected were defective?
(e) What is the probability that the third one selected is defective given that the first one selected was defective and the second one selected was okay?
(f) What is the probability that all three are defective?

(a) 4/499 = 0.0080
(b) (5/500)(4/499) = 0.000080
(c) (495/500)(494/499) = 0.98
(d) 3/498 = 0.0060
(e) 4/498 = 0.0080
(f) $\left(\dfrac{5}{500}\right)\left(\dfrac{4}{499}\right)\left(\dfrac{3}{498}\right) = 4.82\text{x}10^{-7}$

2-109 A batch of 350 samples of rejuvenated mitochondria contains 8 that are mutated (or defective). Two are selected from the batch, at random, without replacement.

(a) What is the probability that the second one selected is defective given that the first one was defective?
(b) What is the probability that both are defective?
(c) What is the probability that both are acceptable?

(a) P = (8-1)/(350-1)=0.020
(b) P = (8/350) $\times$ [(8-1)/(350-1)]=0.000458
(c) P = (342/350) $\times$ [(342-1)/(350-1)]=0.9547

2-110. A computer system uses passwords that are exactly seven characters and each character is one of the 26 letters (*a*–*z*)
or 10 integers (0–9). You maintain a password for this computer system. Let *A* denote the subset of passwords that begin with a vowel (either *a*, *e*, *i*, *o*, or *u*) and let *B* denote the subset of passwords that end with an even number (either 0, 2, 4, 6, or 8).
(a) Suppose a hacker selects a password at random. What is the probability that your password is selected?
(b) Suppose a hacker knows that your password is in event *A* and selects a password at random from this subset. What
is the probability that your password is selected?
(c) Suppose a hacker knows that your password is in *A* and *B*and selects a password at random from this subset. What
is the probability that your password is selected?

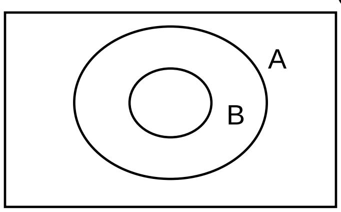
(a) $\dfrac{1}{36^7}$

(b) $\dfrac{1}{5(36^6)}$

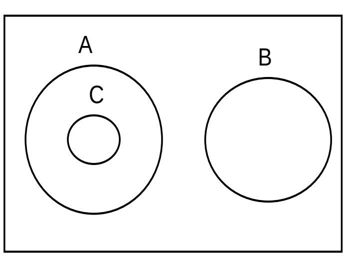
(c) $\dfrac{1}{5(36^5)5}$

2-111. If $P(A \mid B) = 1$, must $A = B$? Draw a Venn diagram to explain your answer.

No, if $B \subset A$, then P(A/B) = $\dfrac{P(A \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$



2-112. Suppose *A* and *B* are mutually exclusive events. Construct a Venn diagram that contains the three events *A*, *B*, and *C* such that $P(A \mid C) = 1$ and $P(B \mid C) = 0$.



2-113. Consider the endothermic reactions given below. Let *A* denote the event that a reaction's final temperature is 271
K or less. Let *B* denote the event that the heat absorbed is above target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Determine the following probabilities.

(a) $P(A \mid B)$ (b) $P(A' \mid B)$ (c) $P(A \mid B')$ (d) $P(B \mid A)$

(a) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{(40+16)/204}{(40+16+36)/204} = \dfrac{56}{92} = 0.6087$

(b) $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{36/204}{(40+16+36)/204} = \dfrac{36}{92} = 0.3913$

(c) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{56/204}{(12+44+56)/204} = \dfrac{56}{112} = 0.5$

(d) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{(40+16)/204}{(40+12+16+44)/204} = \dfrac{40+16}{112} = 0.5$

2-114. Consider the hospital emergency room data given below. Let *A* denote the event that a visit is to hospital 4, and let *B* denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(A \mid B)$ (b) $P(A' \mid B)$ (c) $P(A \mid B')$ (d) $P(B \mid A)$

(a) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{242/22252}{953/22252} = \dfrac{242}{953} = 0.2539$

(b) $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{(195+270+246)/22252}{953/22252} = \dfrac{711}{953} = 0.7461$

(c) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{(984+3103)/22252}{(22252-953)/22252} = \dfrac{4087}{21299} = 0.1919$

(d) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{242/22252}{4329/22252} = \dfrac{242}{4329} = 0.0559$

2-115. Consider the well failure data given below.

Geological Formation Group	Wells Failed	Wells Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

(a) What is the probability of a failure given there are more than 1,000 wells in a geological formation?
(b) What is the probability of a failure given there are fewer than 500 wells in a geological formation?

(a) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{(170+443+60)/8493}{(1685+3733+1403)/8493} = \dfrac{673}{6821} = 0.0987$

Also the probability of failure for fewer than 1000 wells is

$$P(B \mid A') = \frac{P(B \cap A')}{P(B')} = \frac{(2+14+29+46+3)/8493}{(28+363+309+933+39)/8493} = \frac{92}{1672} = 0.0562$$

(b) Let C denote the event that fewer than 500 wells are present.

$$P(B \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{(2+14+29+46+3)/8493}{(28+363+309+39)/8493} = \frac{48}{739} = 0.0650$$

2-116. An article in the *The Canadian Entomologist* (Harcourt et al., 1977, Vol. 109, pp. 1521–1534) reported on the life of the alfalfa weevil from eggs to adulthood. The following table shows the number of larvae that survived at each stage of development from eggs to adults.

Eggs	Early Larvae	Late Larvae	Pre-pupae	Late Pupae	Adults
421	412	306	45	35	31

(a) What is the probability an egg survives to adulthood?
(b) What is the probability of survival to adulthood given survival to the late larvae stage?
(c) What stage has the lowest probability of survival to the next stage?

Let A denote the event that an egg survives to an adult
Let EL denote the event that an egg survives at early larvae stage
Let LL denote the event that an egg survives at late larvae stage
Let PP denote the event that an egg survives at pre-pupae larvae stage
Let LP denote the event that an egg survives at late pupae stage

(a) $P(A) = 31/421 = 0.0736$

(b) $P(A \mid LL) = \dfrac{P(A \cap LL)}{P(LL)} = \dfrac{31/421}{306/421} = 0.1013$

(c) $P(EL) = 412/421 = 0.9786$

$$P(LL \mid EL) = \frac{P(LL \cap EL)}{P(EL)} = \frac{306/421}{412/421} = 0.7427$$

$$P(PP \mid LL) = \frac{P(PP \cap LL)}{P(LL)} = \frac{45/421}{306/421} = 0.1471$$

$$P(LP \mid PP) = \frac{P(LP \cap PP)}{P(PP)} = \frac{35/421}{45/421} = 0.7778$$

$$P(A \mid LP) = \frac{P(A \cap LP)}{P(LP)} = \frac{31/421}{35/421} = 0.8857$$

The late larvae stage has the lowest probability of survival to the pre-pupae stage.

2-117. Consider the bar code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white)
space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6).. Suppose that all 40 codes are equally likely (none is held back as a delimiter).
Determine the probability for each of the following:

(a) The second bar is wide given that the first bar is wide.
(b) The third bar is wide given that the first two bars are not wide.
(c) The first bar is wide given that the last bar is wide.

(a) A = permutations with first bar wide, B = permutations with second bar wide

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

There are 5 bars and 2 are wide. Number of permutations of the bars with 2 wide and 3 narrow bars is 5!/(2!3!) = 10
Number of permutations of the 4 spaces is 4!/(1!3!) = 4

Number of permutations of the bars with the first bar wide is 4!/(3!1!) = 4. Spaces are handled as previously. Therefore, P(A) = 16/40 = 0.4
Number of permutations of the bars with the first and second bar wide is 1. Spaces are handled as previously.
Therefore, P(A ∩ B) = 4/40 = 0.1
Therefore, P(B|A) = 0.1/0.4 = 0.25
Or can use the fact that if the first bar is wide there are 4 other equally likely positions for the wide bar. Therefore, P(B|A) = 0.25

(b) A = first two bars not wide, B = third bar wide

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Number of permutations of the bars with the first two bars not wide is 3!/2!1! = 3. Spaces are handled as in part (a).
Therefore, P(A) = 12/40 =0.3
Number of permutations of the bars with the first two bars not wide and the third bar wide is 2. Spaces are handled as in part (a). Therefore, P(A ∩ B) = 8/40
Therefore, P(B|A) = 0.2/0.3 = 2/3
Or can use the fact that if the first two bars are not wide, there are only 3 equally likely positions for the 2 wide bars and 2 of these positions result in a wide bar in the third position. Therefore, P(B|A) = 2/3

(c) A = first bar wide, B = last bar wide

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Number of permutations of the bars with last bar wide is 4!/(3!1!) = 4. Spaces are handled as in part (a). Therefore, P(B) = 16/40 = 0.4
Number of permutations of the bar with the first and last bar wide is 1. Spaces are handled as in part (a).
Therefore, P(A ∩ B) = 4/40 = 0.1 and P(A|B) = 0.1/0.4 = 0.25
Or can use the fact that if the last bar is wide there are 4 other equally likely positions for the wide bar. Therefore, P(B|A) = 0.25

2-118. The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let *A* denote the event that the patient is treated with ribavirin plus interferon alfa, and let *B* denote the event that the response is complete. Determine the following probabilities:

(a) $P(B \mid A)$ (b) $P(A \mid B)$ (c) $P(A \mid B')$ (d) $P(A' \mid B)$

P(A) = 21/60= 0.35, P(B) = 22/60 = 0.366, P(A∩B) = 16/60 = 0.266

(a) $P(B|A)= \frac{P(A\cap B)}{P(A)} = \frac{16/60}{21/60} = 0.762$

(b) $P(A|B)= \frac{P(A\cap B)}{P(B)} = \frac{16/60}{22/60} = 0.727$

(c) $P(A|B')= \frac{P(A\cap B')}{P(B')} = \frac{5/60}{38/60} = 0.131$

(d) $P(A'|B)= \frac{P(A'\cap B)}{P(B)} = \frac{6/60}{22/60} = 0.272$

2-119. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose that a patient is selected randomly. Let *A* denote the event that the patient is in group 1, and let *B* denote the event that there is no progression. Determine the following probabilities:

(a) $P(B \mid A)$ (b) $P(A \mid B)$ (c) $P(A \mid B')$ (d) $P(A' \mid B)$

P(A) = 114/467 P(B) = 375/467 P(A ∩ B) = 76/467

(a) $P(B|A)= \frac{P(A\cap B)}{P(A)} = \frac{76/467}{114/467} = 0.667$

(b) $P(A|B)= \frac{P(A\cap B)}{P(B)} = \frac{76/467}{375/467} = 0.203$

(c) $P(A|B')= \frac{P(A\cap B')}{P(B')} = \frac{^{38}/_{467}}{^{92}/_{467}} = 0.413$

(d) $P(A'|B)= \frac{P(A'\cap B)}{P(B)} = \frac{^{299}/_{467}}{375/467} = 0.797$

2-120. A computer system uses passwords that contain exactly eight characters, and each character is one of the 26 lowercase letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Let Ω denote the set of all possible passwords. Suppose that all passwords in Ω are equally likely. Determine the probability for each of the following:
(a) Password contains all lowercase letters given that it contains only letters
(b) Password contains at least 1 uppercase letter given that it contains only letters
(c) Password contains only even numbers given that is contains all numbers

Let A= passwords with all letters, B = passwords with all lowercase letters

(a) $P(B|A)= \frac{P(A\cap B)}{P(A)} = \frac{\frac{26^8}{62^8}}{\frac{52^8}{62^8}} = \frac{26^8}{52^8} = 0.0039$

(b) C = passwords with at least 1 uppercase letter

P(C|A)= $\frac{P(A\cap C)}{P(A)}$

P(A ∩ C) = P(A) – P(A ∩ C') = $\frac{52^8}{62^8} - \frac{26^8}{62^8}$

P(A) = $\frac{52^8}{62^8}$

Therefore, P(C | A) = 1 - $\frac{26^8}{52^8}$=0.996

(c) P(containing all even numbers | contains all numbers) = $\frac{5^8}{10^8} = 0.0039$

Section 2-5

2-121. Suppose that $P(A \mid B) = 0.4$ and $P(B) = 0.5$. Determine the following:

(a) $P(A \cap B)$ (b) $P(A' \cap B)$

(a) $P(A \cap B) = P(A|B)P(B) = (0.4)(0.5) = 0.20$

(b) $P(A' \cap B) = P(A'|B)P(B) = (0.6)(0.5) = 0.30$

2-122. Suppose that $P(A \mid B) = 0.2$, $P(A \mid B') = 0.3$, and $P(B) = 0.8$. What is $P(A)$?

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ &= P(A|B)P(B) + P(A|B')P(B') \\ &= (0.2)(0.8) + (0.3)(0.2) \\ &= 0.16 + 0.06 = 0.22 \end{aligned}$$

2-123. The probability is 1% that an electrical connector that is kept dry fails during the warranty period of a portable computer. If the connector is ever wet, the probability of a failure during the warranty period is 5%. If 90% of the connectors are kept dry and 10% are wet, what proportion of connectors fail during the warranty period?
Let F denote the event that a connector fails and let W denote the event that a connector is wet.

$$\begin{aligned} P(F) &= P(F|W)P(W) + P(F|W')P(W') \\ &= (0.05)(0.10) + (0.01)(0.90) = 0.014 \end{aligned}$$

2-124. Suppose 2% of cotton fabric rolls and 3% of nylon fabric rolls contain flaws. Of the rolls used by a manufacturer, 70% are cotton and 30% are nylon. What is the probability that a randomly selected roll used by the manufacturer contains flaws?

Let F denote the event that a roll contains a flaw and let C denote the event that a roll is cotton.

$$\begin{aligned} P(F) &= P(F|C)P(C) + P(F|C')P(C') \\ &= (0.02)(0.70) + (0.03)(0.30) = 0.023 \end{aligned}$$

2-125. The edge roughness of slit paper products increases as knife blades wear. Only 1% of products slit with new blades

have rough edges, 3% of products slit with blades of average sharpness exhibit roughness, and 5% of products slit with worn blades exhibit roughness. If 25% of the blades in manufacturing are new, 60% are of average sharpness, and 15% are worn, what is the proportion of products that exhibit edge roughness?

Let R denote the event that a product exhibits surface roughness. Let N, A, and W denote the events that the blades are new, average, and worn, respectively. Then,

P(R)= P(R|N)P(N) + P(R|A)P(A) + P(R|W)P(W)
= (0.01)(0.25) + (0.03) (0.60) + (0.05)(0.15)
= 0.028

2-126. In the 2012 presidential election, exit polls from the critical state of Ohio provided the following results:

Total	Obama	Romney
No college degree (60%)	52%	45%
College graduate (40%)	47%	51%

What is the probability a randomly selected respondent voted for Obama?

Let A denote the event that a respondent is a college graduate and let B denote the event that an individual votes for Obama.
P(B) = P(A)P(B|A) + P(A')P(B|A') = $0.40\times0.47+0.60\times0.52 = 0.50$

2-127. Computer keyboard failures are due to faulty electrical connects (12%) or mechanical defects (88%). Mechanical defects are related to loose keys (27%) or improper assembly (73%). Electrical connect defects are caused by defective wires (35%), improper connections (13%), or poorly welded wires (52%).
(a) Find the probability that a failure is due to loose keys.
(b) Find the probability that a failure is due to improperly connected or poorly welded wires.

(a) (0.88)(0.27) = 0.2376
(b) (0.12)(0.13+0.52) = 0.0.078

2-128. Heart failures are due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances (73%) or foreign objects (27%). Natural occurrences are caused by arterial blockage (56%), disease (27%), and infection (e.g., staph infection) (17%).
(a) Determine the probability that a failure is due to an induced substance.
(b) Determine the probability that a failure is due to disease or infection

(a)P = $0.13\times0.73=0.0949$
(b)P = $0.87\times(0.27+0.17)=0.3828$

2-129. A batch of 25 injection-molded parts contains 5 parts that have suffered excessive shrinkage.
(a) If two parts are selected at random, and without replacement, what is the probability that the second part selected is one with excessive shrinkage?
(b) If three parts are selected at random, and without replacement, what is the probability that the third part selected is one with excessive shrinkage?

Let A and B denote the event that the first and second part selected has excessive shrinkage, respectively.
(a) $P(B)= P(B|A)P(A) + P(B|A')P(A')$
$= (4/24)(5/25) + (5/24)(20/25) = 0.20$
(b) Let C denote the event that the third part selected has excessive shrinkage.

$$P(C) = P(C|A\cap B)P(A\cap B) + P(C|A\cap B')P(A\cap B')$$
$$+P(C|A'\cap B)P(A'\cap B) + P(C|A'\cap B')P(A'\cap B')$$
$$= \frac{3}{23}\left(\frac{4}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{20}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{5}{24}\right)\left(\frac{20}{25}\right) + \frac{5}{23}\left(\frac{19}{24}\right)\left(\frac{20}{25}\right)$$
$$= 0.20$$

2-130. A lot of 100 semiconductor chips contains 20 that are defective.
(a) Two are selected, at random, without replacement, from the lot. Determine the probability that the second chip selected is defective.
(b) Three are selected, at random, without replacement, from the lot. Determine the probability that all are defective.

Let A and B denote the events that the first and second chips selected are defective, respectively.
(a) P(B) = P(B|A)P(A) + P(B|A')P(A') = (19/99)(20/100) + (20/99)(80/100) = 0.2
(b) Let C denote the event that the third chip selected is defective.

$$P(A \cap B \cap C) = P(C|A \cap B)P(A \cap B) = P(C|A \cap B)P(B|A)P(A)$$
$$= \frac{18}{98}\left(\frac{19}{99}\right)\left(\frac{20}{100}\right)$$
$$= 0.00705$$

2-131. An article in the *British Medical Journal* ["Comparison of treatment of renal calculi by operative surgery, percutaneous nephrolithotomy, and extracorporeal shock wave lithotripsy" (1986, Vol. 82, pp. 879–892)] provided the following discussion of success rates in kidney stone removals. Open surgery had a success rate of 78% (273/350) and a newer method, percutaneous nephrolithotomy (PN), had a success rate of 83% (289/350). This newer method looked better, but the results changed when stone diameter was considered. For stones with diameters less than 2 centimeters, 93% (81/87) of cases of open surgery were successful compared with only 83% (234/270) of cases of PN. For stones greater than or equal to 2 centimeters, the success rates were 73% (192/263) and 69% (55/80) for open surgery and PN, respectively. Open surgery is better for both stone sizes, but less successful in total. In 1951, E. H. Simpson pointed out this apparent contradiction (known as **Simpson's paradox**), and the hazard still persists today. Explain how open surgery can be better for both stone sizes but worse in total.

Open surgery

	success	failure	sample size	sample percentage	conditional success rate
large stone	192	71	263	75%	73%
small stone	81	6	87	25%	93%
overall summary	273	77	350	100%	78%

PN

	success	failure	sample size	sample percentage	conditional success rate
large stone	55	25	80	23%	69%
small stone	234	36	270	77%	83%
overall summary	289	61	350	100%	83%

The overall success rate depends on the success rates for each stone size group, but also the probability of the groups. It is the weighted average of the group success rate weighted by the group size as follows

P(overall success) = P(success| large stone)P(large stone)) + P(success| small stone)P(small stone).

For open surgery, the dominant group (large stone) has a smaller success rate while for PN, the dominant group (small stone) has a larger success rate.

2-132. Consider the endothermic reactions given below. Let A denote the event that a reaction's final temperature is 271 K or less. Let B denote the event that the heat absorbed is above target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A \cup B)$ (c) $P(A' \cup B')$ (d) Use the total probability rule to determine $P(A)$

P(A) = 112/204 = 0.5490, P(B) = 92/204 = 0.4510

(a) $P(A \cap B) = P(A | B)P(B) = (56/92)(92/204) = 0.2745$

(b) $P(A \cup B) = P(A) + P(B) – P(A \cap B) = 0.5490 + 0.4510 – 0.2745 = 0.7255$

(c) $P(A' \cup B') = 1 – P(A \cap B) = 1 – 0.2745 = 0.7255$

(d) $P(A) = P(A | B)P(B) + P(A | B')P(B') = (56/92) (92/204) + (56/112)(112/204) = 112/204 = 0.5490$

2-133. Consider the hospital emergency room data given below. Let A denote the event that a visit is to hospital 4 and let B denote the event that a visit results in LWBS (at any hospital).

	Hospital				
	1	**2**	**3**	**4**	**Total**
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Determine the following probabilities.

(a) $P(A \cap B)$ (b) $P(A \cup B)$ (c) $P(A' \cup B')$ (d)Use the total probability rule to determine $P(A)$

P(A) = 4329/22252 = 0.1945, P(B) = 953/22252 = 0.0428
(a) $P(A \cap B) = P(A \mid B)P(B) = (242/953)(953/22252) = 0.0109$
(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.1945 + 0.0428 - 0.0109 = 0.2264$
(c) $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.0109 = 0.9891$
(d) $P(A) = P(A \mid B)P(B) + P(A \mid B')P(B') = (242/953)(953/22252) + (4087/21299)(21299/22252) = 0.1945$

2-134. Consider the hospital emergency room data given below. Suppose that three visits that resulted in LWBS are selected randomly (without replacement) for a follow-up interview.

	Hospital				
	1	**2**	**3**	**4**	**Total**
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

(a) What is the probability that all three are selected from hospital 2?
(b) What is the probability that all three are from the same hospital?

(a) $$P = \frac{\binom{270}{3}}{\binom{953}{3}} = 0.0226$$

(b) $$P = \frac{\binom{195}{3} + \binom{270}{3} + \binom{246}{3} + \binom{242}{3}}{\binom{953}{3}} = 0.0643$$

2-135. Consider the well failure data given below. Let A denote the event that the geological formation has more than 1000 wells, and let B denote the event that a well failed.

	Wells	
Geological Formation Group	**Failed**	**Total**
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Determine the following probabilities.

(a) $P(A\_ B)$ (b) $P(A\_ B)$ (c) $P(A\_\_ B\_)$ (d) Use the total probability rule to determine $P(A)$

P(A) = (1685 + 3733 + 1403)/8493 = 0.8031, P(B) = (170 + 2 + 443 + 14 + 29 + 60 + 46 + 3)/8493 = 0.0903
(a) $P(A \cap B) = P(B \mid A)P(A) = (673/6821)(6821/8493) = 0.0792$
(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8031 + 0.0903 - 0.0792 = 0.8142$
(c) $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.0792 = 0.9208$
(d) $P(A) = P(A \mid B)P(B) + P(A \mid B')P(B') = (673/767)(767/8493) + (6148/7726)(7726/8493) = 0.8031$

2-136. Consider the well failure data given below. Suppose that two failed wells are selected randomly (without replacement) for a follow-up review.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

(a) What is the probability that both are from the gneiss geological formation group?
(b) What is the probability that both are from the same geological formation group?

(a) $$P = \frac{\binom{170}{2}}{\binom{767}{2}} = 0.0489$$

(b) $$P = \frac{\binom{170}{2} + \binom{2}{2} + \binom{443}{2} + \binom{14}{2} + \binom{29}{2} + \binom{60}{2} + \binom{46}{2} + \binom{3}{2}}{\binom{767}{2}} = 0.3934$$

2-137. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Determine the probability that the ad color is red and the font size is not the smallest one.

Let R denote red color and F denote that the font size is not the smallest. Then P(R) = 1/4, P(F) = 4/5. Because the Web sites are generated randomly these events are independent. Therefore, $P(R \cap F) = P(R)P(F) = (1/4)(4/5) = 0.2$

2-138. Consider the code 39 is a common bar code system that consists of narrow and wide bars (black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white) space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6). Suppose that all 40 codes are equally likely (none is held back as a delimiter). Determine the probability for each of the following:

(a) The code starts and ends with a wide bar.
(b) Two wide bars occur consecutively.
(c) Two consecutive wide bars occur at the start or end.
(d) The middle bar is wide.

(a) Number of permutations of the bars that start and end with a wide bar is 1. Number of permutations of the spaces is 4!/(1!3!) = 4.
Number of codes that start and end with a wide bar = 4. P(code starts and ends with a wide bar) = 4/40 = 0.1
(b) Number of permutations of the bars where two wide bars are consecutive = 4. Spaces are handled as in part (a). P(two wide bars are consecutive) =16/40 = 0.4
(c) Number of permutations of the bars with two consecutive wide bars at the start or end = 2. Spaces are handled as in part (a). P(two consecutive wide bars at the start or end) = 8/40 = 0.2
(d) Number of permutations of the bars with the middle bar wide = 4. Spaces are handled as in part (a). P(middle bar is wide) = 16/40 = 0.4

2-139. A hospital operating room needs to schedule three knee surgeries and two hip surgeries in a day. Suppose that an operating room needs to schedule three knee, four hip, and five shoulder surgeries. Assume that all schedules are equally likely. Determine the following probabilities:
(a) All hip surgeries are completed first given that all knee surgeries are last.
(b) The schedule begins with a hip surgery given that all knee surgeries are last.
(c) The first and last surgeries are hip surgeries given that knee surgeries are scheduled in time periods 2 through 4.
(d) The first two surgeries are hip surgeries given that all knee surgeries are last.

(a) P(all hip surgeries are completed first given that all knee surgeries are last)
A = schedules with all hip surgeries completed first
B = schedules with all knee surgeries last
Total number of schedules $=\frac{12!}{3!4!5!}$
Number of schedules with all knee surgeries last = $\frac{9!}{4!5!}$
Number of schedules with all hip surgeries first and all knee surgeries last = 1
P(B) = $\frac{9!}{4!5!}/\frac{12!}{3!4!5!}$, P(A ∩ B) = $1/\frac{12!}{3!4!5!}$
Therefore, P(A | B) = P(A ∩ B)/P(B) = $1/\frac{9!}{4!5!}$=1/126
Alternatively, one can reason that when all knee surgeries are last, there are $\frac{9!}{4!5!}$ remaining schedules and one of these has all knee surgeries first. Therefore, the solution is $1/\frac{9!}{4!5!} = 1/126$

(b) P(schedule begins with a hip surgery given that all knee surgeries are last)
C = schedules that begin with a hip surgery
B = schedules with all knee surgeries last
P(B) = $\frac{9!}{4!5!}/\frac{12!}{3!4!5!}$, P(C ∩ B) = $\frac{8!}{3!5!}/\frac{12!}{3!4!5!}$
Therefore, P(C | B) = P(C ∩ B)/P(B) = $\frac{8!}{3!5!}/\frac{9!}{4!5!}$ = 4/9
Alternatively, one can reason that when all knee surgeries are last, there are 4 hip and 5 shoulder surgeries that remain to schedule. The probability the first one is a hip surgery is then 4/9

(c) P(first and last surgeries are hip surgeries given that knee surgeries are scheduled in time periods 2 through 4)
D = schedules with first and last hip surgeries
E = schedules with knee surgeries in periods 2 through 4
P(E) $=\frac{9!}{4!5!}/\frac{12!}{3!4!5!}$
P(D∩E) = $\frac{7!}{2!5!}/\frac{12!}{3!4!5!}$
P(D | E) = P(D∩E)/P(E) = $\frac{7!}{2!5!}/\frac{9!}{4!5!}$ = 1/6
Alternatively, one can conclude that with knee surgeries in periods 2 through 4, there are $\frac{9!}{4!5!}$ remaining schedules and $\frac{7!}{2!5!}$ of these have hip surgeries first and last. Therefore, P(D | E) = P(D∩E)/P(E) = $\frac{7!}{2!5!}/\frac{9!}{4!5!}$ = 1/6

(d) P(first two surgeries are hip surgeries given that all knee surgeries are last)
F = schedules with the first two surgeries as hip surgeries
B = schedules with all knee surgeries last
P(B) = $\frac{9!}{4!5!}/\frac{12!}{3!4!5!}$,
P(F∩B) = $\frac{7!}{2!5!}/\frac{12!}{3!4!5!}$

P(F | B) = P(F∩B)/P(B) = $\frac{7!}{2!5!} / \frac{9!}{4!5!}$ = 1/6

Alternatively, one can conclude that with knee surgeries last, there are $\frac{9!}{4!5!}$ remaining schedules and $\frac{7!}{2!5!}$ have hip surgeries in the first two periods. Therefore, P(F | B) = P(F∩B)/P(B) = $\frac{7!}{2!5!} / \frac{9!}{4!5!}$ = 1/6

2-140. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose that a patient is selected randomly. Let *A* denote the event that the patient is in group 1, and let *B* denote the event for which there is no progression. Determine the following probabilities:

(a) *P*(*A*) *B*) (b) *P*(*B*) (c) *P*(*A*__ *B*) (d) *P*(*A*∗ *B*) (e) *P*(*A*__ *B*)

A = group 1, B = no progression

(a) P(A∩B) = P(B|A)P(A)= (76/114)(114/467) = 0.162

(b) P(B) = P(B|G1)P(G1) + P(B|G2)P(G2) + P(B|G3)P(G3) + P(P|G4)P(G4) = 0.802

(c) P(A'∩B) = P(B|A') P(A') = (299/353)(353/467) = 0.6403

(d) P(A U B) = P(A) + P(B) - P(A∩B) = 114/467 + 375/467 – 76/467 = 0.884

(e) P(A' U B) = P(A') + P(B) - P(A'∩B) = 353/467+375/467 – 299/467 = 0.919

2-141. A computer system uses passwords that contain exactly eight characters, and each character is one of the 26 lowercase letters (*a*–*z*) or 26 uppercase letters (*A*–*Z*) or 10 integers (0–9). Let Ω denote the set of all possible password, and let *A* and *B* denote the events that consist of passwords with only letters or only integers, respectively. Suppose that all passwords in Ω are equally likely. Determine the following probabilities:

(a) *P*(*A*|*B*2)
(b) *P*(*A*__ *B*)
(c) *P* (password contains exactly 2 integers given that it contains at least 1 integer)

A = all letters, B = all integers

(a) P(A|B') = P(A∩B')/P(B') = P(A)/P(B') = P(A)/[1 - P(B)]

P(A)= $\frac{52^8}{62^8}$, P(B)= $\frac{10^8}{62^8}$

P(A|B') = $\frac{\frac{52^8}{62^8}}{1-\frac{10^8}{62^8}}$ = 0.245

(b) P(A' ∩ B') = P(A'|B')P(B')

From part (a), P(B') = 1 – P(B) = 1 - $\frac{10^8}{62^8}$ and P(A'|B') = 1 - P(A|B') = $1-\frac{\frac{52^8}{62^8}}{1-\frac{10^8}{62^8}}$

Therefore P(A' ∩ B') = $1-\frac{52^8}{62^8}-\frac{10^8}{62^8}$ = 0.755

This can also be solved as P(A' ∩ B') = 1 - P(A U B) = $1-\frac{52^8}{62^8}-\frac{10^8}{62^8}$ because A and B are mutually exclusive.

(c) Let C = passwords with exactly 2 integers

Let D = passwords with at least one integer

P(C|D) = P(C ∩ D) / P(D) = P(C) / P(D)

P(C):

Number of positions for the integers is 8!/(2!6!) = 28

Number of ordering of the two integers is 10^2 = 100

Number of orderings of the six letters is 52^6

Total number of orderings is 62^8

Therefore, the probability is
$\frac{28(100)(52^6)}{62^8} = 0.254$
P(D):
$1 - P(D) = (52/62)^8$
$P(D) = 1 - (52/62)^8 = 0.755$

P(C | D) = 0.254/0.755 = 0.336

Section 2-6

2-142. If $P(A \mid B) = 0.4$, $P(B) = 0.8$, and $P(A) = 0.5$, are the events A and B independent?

Because P(A | B) $\neq$ P(A), the events are not independent.

2-143. If $P(A \mid B) = 0.3$, $P(B) = 0.8$, and $P(A) = 0.3$, are the events B and the complement of A independent?
P(A') = 1 – P(A) = 0.7 and $P(A'|B)$ = 1 – P(A | B) = 0.7
Therefore, A' and B are independent events.

2-144. If $P(A) = 0.2$, $P(B) = 0.2$, and A and B are mutually exclusive, are they independent?

If A and B are mutually exclusive, then $P(A \cap B) = 0$ and P(A)P(B) = 0.04.
Therefore, A and B are not independent.

2-145. A batch of 500 containers of frozen orange juice contains 5 that are defective. Two are selected, at random, without
replacement, from the batch. Let A and B denote the events that the first and second containers selected are defective, respectively.
(a) Are A and B independent events?
(b) If the sampling were done with replacement, would A and B be independent?

(a) $P(B \mid A) = 4/499$ and
$P(B) = P(B|A)P(A) + P(B|A')P(A') = (4/499)(5/500) + (5/499)(495/500) = 5/500$
Therefore, A and B are not independent.
(b) A and B are independent.

2-146. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100
disks are summarized as follows:.

		Shock Resistance	
		High	Low
Scratch	High	70	9
Resistance	Low	16	5

Let A denote the event that a disk has high shock resistance, and let B denote the event that a disk has high scratch resistance. Are events A and B independent?

$P(A \cap B)$ = 70/100, P(A) = 86/100, P(B) = 77/100.
Then, $P(A \cap B) \neq P(A)P(B)$, so A and B are not independent.

2-147. Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results
from 100 samples are summarized as follows:

		Conforms	
		Yes	No
Supplier	1	22	8
	2	25	5
	3	30	10

Let *A* denote the event that a sample is from supplier 1, and let *B* denote the event that a sample conforms to specifications.
(a) Are events *A* and *B* independent? (b) Determine $P(B \mid A)$.

(a) $P(A \cap B)$= 22/100, P(A) = 30/100, P(B) = 77/100, Then $P(A \cap B) \neq P(A)P(B)$
Therefore, A and B are not independent.
(b) $P(B|A) = P(A \cap B)/P(A) = (22/100)/(30/100) = 0.733$

2-148. Redundant array of inexpensive disks (RAID) is a technology that uses multiple hard drives to increase the speed of data transfer and provide instant data backup. Suppose that the probability of any hard drive failing in a day is 0.001 and the drive failures are independent.
(a) A RAID 0 scheme uses two hard drives, each containing a mirror image of the other. What is the probability of data loss? Assume that data loss occurs if both drives fail within the same day.
(b) A RAID 1 scheme splits the data over two hard drives. What is the probability of data loss? Assume that data loss occurs if at least one drive fails within the same day.

(a) $P = (0.001)^2 = 10^{-6}$

(b) $P = 1 - (0.999)^2 = 0.002$

2-149. The probability that a lab specimen contains high levels of contamination is 0.10. Five samples are checked, and the samples are independent.
(a) What is the probability that none contain high levels of contamination?
(b) What is the probability that exactly one contains high levels of contamination?
(c) What is the probability that at least one contains high levels of contamination?

It is useful to work one of these exercises with care to illustrate the laws of probability. Let H_i denote the event that the *i*th sample contains high levels of contamination.

(a) $P(H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5') = P(H_1')P(H_2')P(H_3')P(H_4')P(H_5')$

by independence. Also, $P(H_i') = 0.9$. Therefore, the answer is $0.9^5 = 0.59$

(b) $A_1 = (H_1 \cap H_2' \cap H_3' \cap H_4' \cap H_5')$

$A_2 = (H_1' \cap H_2 \cap H_3' \cap H_4' \cap H_5')$

$A_3 = (H_1' \cap H_2' \cap H_3 \cap H_4' \cap H_5')$

$A_4 = (H_1' \cap H_2' \cap H_3' \cap H_4 \cap H_5')$

$A_5 = (H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5)$

The requested probability is the probability of the union $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$ and these events are mutually exclusive. Also, by independence $P(A_i) = 0.9^4(0.1) = 0.0656$. Therefore, the answer is 5(0.0656) = 0.328.

(c) Let B denote the event that no sample contains high levels of contamination. The requested probability is P(B') = 1 - P(B). From part (a), P(B') = 1 - 0.59 = 0.41.

2-150. In a test of a printed circuit board using a random test pattern, an array of 10 bits is equally likely to be 0 or 1. Assume the bits are independent.
(a) What is the probability that all bits are 1s?
(b) What is the probability that all bits are 0s?
(c) What is the probability that exactly 5 bits are 1s and 5 bits are 0s?

Let A_i denote the event that the ith bit is a one.

(a) By independence $P(A_1 \cap A_2 \cap ... \cap A_{10}) = P(A_1)P(A_2)...P(A_{10}) = (\frac{1}{2})^{10} = 0.000976$

(b) By independence, $P(A_1^{'} \cap A_2^{'} \cap ... \cap A_{10}^{'}) = P(A_1^{'})P(A_2^{'})...P(A_{10}^{c}) = (\frac{1}{2})^{10} = 0.000976$

(c) The probability of the following sequence is

$P(A_1^{'} \cap A_2^{'} \cap A_3^{'} \cap A_4^{'} \cap A_5^{'} \cap A_6 \cap A_7 \cap A_8 \cap A_9 \cap A_{10}) = (\frac{1}{2})^{10}$, by independence. The number of sequences consisting of five "1"'s, and five "0"'s is $\binom{10}{5} = \frac{10!}{5!5!} = 252$. The answer is $252\left(\frac{1}{2}\right)^{10} = 0.246$

2-151. Six tissues are extracted from an ivy plant infested by spider mites. The plant in infested in 20% of its area. Each tissue is chosen from a randomly selected area on the ivy plant.
(a) What is the probability that four successive samples show the signs of infestation?
(b) What is the probability that three out of four successive samples show the signs of infestation?

(a) Let I and G denote an infested and good sample. There are 3 ways to obtain four consecutive samples showing the signs of the infestation: IIIIGG, GIIIIG, GGIIII. Therefore, the probability is $3 \times (0.2^4 0.8^2) = 0.003072$
(b) There are 10 ways to obtain three out of four consecutive samples showing the signs of infestation. The probability is $10 \times (0.2^3 * 0.8^3) = 0.04096$

2-152. A player of a video game is confronted with a series of four opponents and an 80% probability of defeating each opponent. Assume that the results from opponents are independent (and that when the player is defeated by an opponent the game ends).
(a) What is the probability that a player defeats all four opponents in a game?
(b) What is the probability that a player defeats at least two opponents in a game?
(c) If the game is played three times, what is the probability that the player defeats all four opponents at least once?

(a) $P = (0.8)^4 = 0.4096$
(b) $P = 1 - 0.2 - 0.8 \times 0.2 = 0.64$
(c) Probability defeats all four in a game = 0.8^4 = 0.4096. Probability defeats all four at least once = $1 – (1 – 0.4096)^3$ = 0.7942

2-153. In an acid-base titration, a base or acid is gradually added to the other until they have completely neutralized each other. Because acids and bases are usually colorless (as are the water and salt produced in the neutralization reaction), pH is measured to monitor the reaction. Suppose that the equivalence point is reached after approximately 100 mL of an NaOH solution has been added (enough to react with all the acetic acid present) but that replicates are equally likely to indicate from 95 to 104 mL, measured to the nearest mL. Assume that two technicians each conduct titrations independently.
(a) What is the probability that both technicians obtain equivalence at 100 mL?
(b) What is the probability that both technicians obtain equivalence between 98 and 104 mL (inclusive)?
(c) What is the probability that the average volume at equivalence from the technicians is 100 mL?

(a) The probability that one technician obtains equivalence at 100 mL is 0.1.
So the probability that both technicians obtain equivalence at 100 mL is $0.1^2 = 0.01$.
(b) The probability that one technician obtains equivalence between 98 and 104 mL is 0.7.
So the probability that both technicians obtain equivalence between 98 and 104 mL is $0.7^2 = 0.49$.
(c) The probability that the average volume at equivalence from the technician is 100 mL is $9(0.1^2) = 0.09$.

2-154. A credit card contains 16 digits. It also contains the month and year of expiration. Suppose there are 1 million credit card holders with unique card numbers. A hacker randomly selects a 16-digit credit card number.
(a) What is the probability that it belongs to a user?

(b) Suppose a hacker has a 25% chance of correctly guessing the year your card expires and randomly selects 1 of the
12 months. What is the probability that the hacker correctly selects the month and year of expiration?

(a) $P = \dfrac{10^6}{10^{16}} = 10^{-10}$

(b) $P = 0.25 \times (\dfrac{1}{12}) = 0.020833$

2-155. Eight cavities in an injection-molding tool produce plastic connectors that fall into a common stream. A sample is chosen every several minutes. Assume that the samples are independent.
(a) What is the probability that five successive samples were all produced in cavity 1 of the mold?
(b) What is the probability that five successive samples were all produced in the same cavity of the mold?
(c) What is the probability that four out of five successive samples were produced in cavity 1 of the mold?

Let A denote the event that a sample is produced in cavity one of the mold.

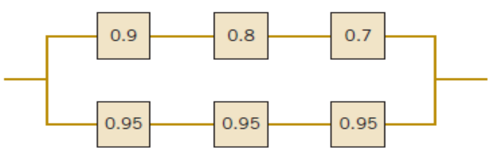
(a) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) = (\frac{1}{8})^5 = 0.00003$

(b) Let B_i be the event that all five samples are produced in cavity i. Because the B's are mutually exclusive, $P(B_1 \cup B_2 \cup \ldots \cup B_8) = P(B_1) + P(B_2) + \ldots + P(B_8)$

From part (a), $P(B_i) = (\frac{1}{8})^5$. Therefore, the answer is $8(\frac{1}{8})^5 = 0.00024$

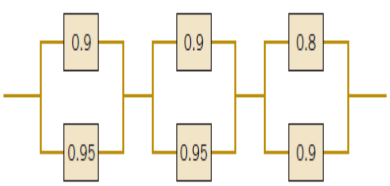
(c) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5') = (\frac{1}{8})^4(\frac{7}{8})$. The number of sequences in which four out of five samples are from cavity one is 5. Therefore, the answer is $5(\frac{1}{8})^4(\frac{7}{8}) = 0.00107$.

2-156. The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



Let A denote the upper devices function. Let B denote the lower devices function.
P(A) = (0.9)(0.8)(0.7) = 0.504
P(B) = (0.95)(0.95)(0.95) = 0.8574
P(A∩B) = (0.504)(0.8574) = 0.4321
Therefore, the probability that the circuit operates = P(A∪B) = P(A) +P(B) – P(A∩B) = 0.9293

2-157. The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



$P = [1 – (0.1)(0.05)][1 – (0.1)(0.05)][1 – (0.2)(0.1)] = 0.9702$

2-158. Consider the endothermic reactions given below. Let *A* denote the event that a reaction's final temperature is 271 K or less. Let *B* denote the event that the heat absorbed is above target. Are these events independent?

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

P(A) = 112/204 = 0.5490, P(B) = 92/204 = 0.4510, P(A $\cap$ B) = 56/204 = 0.2745
Because P(A) P(B) = (0.5490)(0.4510) = 0.2476 $\neq$ 0.2745 = P(A $\cap$ B), A and B are not independent.

2-159. Consider the hospital emergency room data given below. Let *A* denote the event that a visit is to hospital 4, and let *B*

denote the event that a visit results in LWBS (at any hospital). Are these events independent?

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

P(A) = 4329/22252 = 0.1945, P(B) = 953/22252 = 0.0428, P(A $\cap$ B) = 242/22252 = 0.0109
Because P(A)*P(B) = (0.1945)(0.0428) = 0.0083 $\neq$ 0.0109 = P(A $\cap$ B), A and B are not independent.

2-160. Consider the well failure data given below. Let *A* denote the event that the geological formation has more than 1000 wells, and let *B* denote the event that a well failed. Are these events independent?

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

P(A) = (1685+3733+1403)/8493 = 0.8031, P(B) = (170+2+443+14+29+60+46+3)/8493 = 0.0903,
P(A $\cap$ B) = (170+443+60)/8493 = 0.0792
Because P(A)*P(B) = (0.8031)(0.0903) = 0.0725 $\neq$ 0.0792 = P(A $\cap$ B), A and B are not independent.

2-161. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let *A* denote the event that the design color is red, and let *B* denote the event that the font size is not the smallest one. Are *A* and *B* independent events? Explain why or why not.

P(A) = (3*5*3*5)/(4*3*5*3*5) = 0.25, P(B) = (4*3*4*3*5)/(4*3*5*3*5) = 0.8,
P(A $\cap$ B) = (3*4*3*5) /(4*3*5*3*5) = 0.2
Because P(A)*P(B) = (0.25)(0.8) = 0.2 = P(A $\cap$ B), A and B are independent.

2-162. Consider the code 39 is a common bar code system that consists of narrow and wide bars(black) separated by either wide or narrow spaces (white). Each character contains nine elements (five bars and four spaces). The code for a character starts and ends with a bar (either narrow or wide) and a (white)
space appears between each bar. The original specification (since revised) used *exactly* two wide bars and one wide space in each character. For example, if *b* and *B* denote narrow and wide (black) bars, respectively, and *w* and *W* denote narrow and wide (white) spaces, a valid character is *bwBwBWbwb* (the number 6). Suppose that all 40 codes are equally likely (none is held back as a delimiter).
Let *A* and *B* denote the event that the first bar is wide and *B* denote the event that the second bar is wide. Determine the following:

(a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) Are A and B independent events?

(a) The total number of permutations of 2 wide and 3 narrow bars is $\frac{5!}{2!3!} = 10$
The number of permutations that begin with a wide bar is $\frac{4!}{1!3!} = 4$
Therefore, P(A) = 4/10 = 0.4
(b) A similar approach to that used in part (a) implies P(B) = 0.4
(c) Because a code contains exactly 2 wide bars, there is only 1 permutation with wide bars in the first and second positions. Therefore, P(A∩B) = 1/10 = 0.1
(d) Because P(A)P(B) = 0.4(0.4) = 0.16 ≠ 0.1, the events are not independent.

2-163. An integrated circuit contains 10 million logic gates (each can be a logical AND or OR circuit). Assume the probability of a gate failure is p and that the failures are independent. The integrated circuit fails to function if any gate fails. Determine the value for p so that the probability that the integrated circuit functions is 0.95.

p = probability of gate failure
A = event that the integrated circuit functions
P(A) = 0.95 => P(A') = 0.05
$(1 - p)$ = probability of gate functioning
Hence from the independence, $P(A) = (1 - p)^{10,000,000} = 0.95$.
Take logarithms to obtain $10^7 \ln(1 - p) = \ln(0.95)$ and
$p = 1 - \exp[10^{-7}\ln(0.95)] = 5.13\text{x}10^{-9}$

2-164. The following table provides data on wafers categorized by location and contamination levels. Let A denote the event that contamination is *low*, and let B denote the event that the location is *center*. Are A and B independent? Why or why not?

Location in Sputtering Tool			
Contamination	Center	Edge	Total
Low	514	68	582
High	112	246	358
Total	626	314	

A: contamination is low, B: location is center
For A and B to be independent, P(A∩B) = P(A) P(B)
P(A∩B) = 514/940 = 0.546
P(A) = 582/940 = 0.619; P(B) = 626/940 = 0.665; P(A)P(B) = 0.412. Because the probabilities are not equal, they are not independent.

2-165. The following table provides data on wafers categorized by location and contamination levels. More generally, let the number of wafers with *low* contamination from the *center* and *edge* locations be denoted as n_{lc} and n_{le}, respectively. Similarly, let n_{hc} and n_{he} denote the number of wafers with *high* contamination from the *center* and *edge* locations, respectively. Suppose that $n_{lc} = 10n_{hc}$ and $n_{le} = 10n_{he}$. That is, there are 10 times as many *low* contamination wafers as *high* ones from each location. Let A denote the event that contamination is low, and let B denote the event that the location is *center*. Are A and B independent? Does your conclusion change if the multiplier of 10 (between *low* and *high* contamination wafers) is changed from 10 to another positive integer?

Location in Sputtering Tool			
Contamination	Center	Edge	Total
Low	514	68	582
High	112	246	358
Total	626	314	

$n_{lc}=514$; $n_{le}=68$; $n_{hc}=112$; $n_{he}=246$

$n_{lc}=10n_{hc}$; $n_{le}=10n_{he}$

	center	edge
low	$10n_{hc}$	$10n_{he}$
high	n_{hc}	n_{he}

$P(A) = (10n_{hc}+ 10n_{he})/(11n_{hc}+ 11n_{he}) = 10/11$

$P(B) = (11n_{hc})/(11n_{hc}+ 11n_{he})$

$P(A \cap B) = (10n_{hc})/(11n_{hc}+ 11n_{he})$

Because $P(A)P(B) = (10n_{hc})/(11n_{hc}+ 11n_{he})$ the events are independent.

The conclusion does not change. Even though the multiplier is changed, this relation does not change.

Section 2-7

2-166. Suppose that $P(A \mid B) = 0.7$, $P(A) = 0.5$, and $P(B) = 0.2$. Determine $P(B \mid A)$.

Because, $P(A|B)\,P(B) = P(A \cap B) = P(B|A)\,P(A)$,

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.7(0.2)}{0.5} = 0.28$$

2-167. Suppose that $P(A \mid B) = 0.4$, $PA \mid B\_ = 0.2$, and $P(B) = 0.8$. Determine $P(B \mid A)$.

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|B')P(B')}$$

$$= \frac{0.4\times 0.8}{0.4\times 0.8+0.2\times 0.2} = 0.89$$

2-168. Software to detect fraud in consumer phone cards tracks the number of metropolitan areas where calls originate each day. It is found that 1% of the legitimate users originate calls from two or more metropolitan areas in a single day. However, 30% of fraudulent users originate calls from two or more metropolitan areas in a single day. The proportion of fraudulent users is 0.01%. If the same user originates calls from two or more metropolitan areas in a single day, what is the probability that the user is fraudulent?

Let F denote a fraudulent user and let T denote a user that originates calls from two or more metropolitan areas in a day. Then,

$$P(F|T) = \frac{P(T|F)P(F)}{P(T|F)P(F)+P(T|F')P(F')} = \frac{0.30(0.0001)}{0.30(0.0001)+0.01(.9999)} = 0.003$$

2-169. A new process of more accurately detecting anaerobic respiration in cells is being tested. The new process is important due to its high accuracy, its lack of extensive experimentation, and the fact that it could be used to identify five different categories of organisms: obligate anaerobes, facultative anaerobes, aerotolerant, microaerophiles, and nanaerobes instead of using a single test for each category. The process claims that it can identify obligate anaerobes with 97.8% accuracy, facultative anaerobes with 98.1% accuracy, aerotolerants with 95.9% accuracy, microaerophiles with 96.5% accuracy, and nanaerobes with 99.2% accuracy. If any category is not present, the process does not signal. Samples are prepared for the calibration of the process and 31% of them contain obligate anaerobes, 27% contain facultative anaerobes, 21% contain microaerophiles, 13% contain nanaerobes, and 8% contain aerotolerants. A test sample is selected randomly.

(a) What is the probability that the process will signal?
(b) If the test signals, what is the probability that microaerophiles are present?

(a) $P=(0.31)(0.978)+(0.27)(0.981)+(0.21)(0.965)+(0.13)(0.992)+(0.08)(0.959)$
$= 0.97638$

(b) $P = \dfrac{(0.21)(0.965)}{0.97638} = 0.207552$

2-170. In the 2012 presidential election, exit polls from the critical state of Ohio provided the following results:

	Obama	Romney
No college degree (60%)	52%	45%
College graduate (40%)	47%	51%

If a randomly selected respondent voted for Obama, what is the probability that the person has a college degree?

Let O and C denote the respondents for Obama and the respondents with college degrees, respectively. Then
P(C | O) = P(O | C)P(C) /[P(O | C)P(C) + P(O | C')P(C')]
= 0.47(0.40)/[0.47(0.40) + 0.52(0.60)] = 0.376

2-171. Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received
good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.
(a) What is the probability that a product attains a good review?
(b) If a new design attains a good review, what is the probability that it will be a highly successful product?
(c) If a product does not attain a good review, what is the probability that it will be a highly successful product?

Let G denote a product that received a good review. Let H, M, and P denote products that were high, moderate, and poor performers, respectively.

(a)

$$\begin{aligned} P(G) &= P(G|H)P(H) + P(G|M)P(M) + P(G|P)P(P) \\ &= 0.95(0.40) + 0.60(0.35) + 0.10(0.25) \\ &= 0.615 \end{aligned}$$

(b) Using the result from part (a)

$$P(H|G) = \frac{P(G|H)P(H)}{P(G)} = \frac{0.95(0.40)}{0.615} = 0.618$$

(c) $P(H|G') = \dfrac{P(G'|H)P(H)}{P(G')} = \dfrac{0.05(0.40)}{1-0.615} = 0.052$

2-172. An inspector working for a manufacturing company has a 99% chance of correctly identifying defective items and a 0.5% chance of incorrectly classifying a good item as defective. The company has evidence that 0.9% of the items its line produces are nonconforming.
(a) What is the probability that an item selected for inspection is classified as defective?
(b) If an item selected at random is classified as non defective, what is the probability that it is indeed good?

(a) P(D)=P(D|G)P(G)+P(D|G')P(G')=(.005)(.991)+(.99)(.009)=0.013865
(b) $P(G|D')=P(G\cap D')/P(D')=P(D'|G)P(G)/P(D')=(.995)(.991)/(1-.013865)=0.9999$

2-173. A new analytical method to detect pollutants in water is being tested. This new method of chemical analysis is important because, if adopted, it could be used to detect three different contaminants—organic pollutants, volatile solvents, and chlorinated compounds—instead of having to use a single test for each pollutant. The makers of the test claim that it can detect high levels of organic pollutants with 99.7% accuracy, volatile solvents with 99.95% accuracy, and chlorinated compounds with 89.7% accuracy. If a pollutant is not present, the test does not signal. Samples are prepared for the calibration of the test and 60% of them are contaminated with organic pollutants, 27% with volatile solvents, and 13% with traces of chlorinated compounds. A test sample is selected randomly.
(a) What is the probability that the test will signal?
(b) If the test signals, what is the probability that chlorinated compounds are present?

Denote as follows: S = signal, O = organic pollutants, V = volatile solvents, C = chlorinated compounds
(a) P(S) = P(S|O)P(O)+P(S|V)P(V)+P(S|C)P(C) = 0.997(0.60) + 0.9995(0.27) + 0.897(0.13) = 0.9847
(b) P(C|S) = P(S|C)P(C)/P(S) = (0.897)(0.13)/0.9847 = 0.1184

2-174. Consider the endothermic reactions given below. Use Bayes' theorem to calculate the probability that a reaction's
final temperature is 271 K or less given that the heat absorbed is above target.

Final Temperature Conditions	Heat Absorbed (cal)	
	Below Target	Above Target
266 K	12	40
271 K	44	16
274 K	56	36

Let A denote the event that a reaction final temperature is 271 K or less
Let B denote the event that the heat absorbed is above target

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B \mid A)}{P(A)P(B \mid A) + P(A')P(B \mid A')}$$
$$= \frac{(0.5490)(0.5)}{(0.5490)(0.5) + (0.4510)(0.3913)} = 0.6087$$

2-175. Consider the hospital emergency room data given below. Use Bayes' theorem to calculate the probability that a person visits hospital 4 given they are LWBS.

	Hospital				
	1	2	3	4	Total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

Let L denote the event that a person is LWBS
Let A denote the event that a person visits Hospital 1
Let B denote the event that a person visits Hospital 2
Let C denote the event that a person visits Hospital 3
Let D denote the event that a person visits Hospital 4

$$P(D \mid L) = \frac{P(L \mid D)P(D)}{P(L \mid A)P(A) + P(L \mid B)P(B) + P(L \mid C)P(C) + P(L \mid D)P(D)}$$
$$= \frac{(0.0559)(0.1945)}{(0.0368)(0.2378) + (0.0386)(0.3142) + (0.0436)(0.2535) + (0.0559)(0.1945)}$$
$$= 0.2540$$

2-176. Consider the well failure data given below. Use Bayes' theorem to calculate the probability that a randomly selected well is in the gneiss group given that the well has failed.

	Wells	
Geological Formation Group	Failed	Total
Gneiss	170	1685
Granite	2	28
Loch raven schist	443	3733
Mafic	14	363
Marble	29	309
Prettyboy schist	60	1403
Other schists	46	933
Serpentine	3	39

Let A denote the event that a well is failed
Let B denote the event that a well is in Gneiss
Let C denote the event that a well is in Granite
Let D denote the event that a well is in Loch raven schist
Let E denote the event that a well is in Mafic
Let F denote the event that a well is in Marble
Let G denote the event that a well is in Prettyboy schist
Let H denote the event that a well is in Other schist
Let I denote the event that a well is in Serpentine

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A \mid B)P(B) + P(A \mid C)P(C) + P(A \mid D)P(D) + P(A \mid E)P(E) + P(A \mid F)P(F) + P(A \mid G)P(E) + P(A \mid G)P(G) + P(A \mid H)P(H)}$$

$$= \frac{\left(\frac{170}{1685}\right)\left(\frac{1685}{8493}\right)}{\left(\frac{170}{1685}\right)\left(\frac{1685}{8493}\right) + \left(\frac{2}{28}\right)\left(\frac{28}{8493}\right) + \left(\frac{443}{3733}\right)\left(\frac{3733}{8493}\right) + \left(\frac{14}{363}\right)\left(\frac{363}{8493}\right) + \left(\frac{29}{309}\right)\left(\frac{309}{8493}\right) + \left(\frac{60}{1403}\right)\left(\frac{1403}{8493}\right) + \left(\frac{46}{933}\right)\left(\frac{933}{8493}\right) + \left(\frac{3}{39}\right)\left(\frac{39}{8493}\right)}$$

$$= 0.2216$$

2-177. Two Web colors are used for a site advertisement. If a site visitor arrives from an affiliate, the probabilities of the blue or green colors being used in the advertisement are 0.8 and 0.2, respectively. If the site visitor arrives from a search site, the probabilities of blue and green colors in the advertisement are 0.4 and 0.6, respectively. The proportions of visitors from affiliates and search sites are 0.3 and 0.7, respectively. What is the probability that a visitor is from a search site given that the blue ad was viewed?

Denote as follows: A = affiliate site, S = search site, B =blue, G =green

$$P(S \mid B) = \frac{P(B \mid S)P(S)}{P(B \mid S)P(S) + P(B \mid A)P(A)} = \frac{(0.4)(0.7)}{(0.4)(0.7) + (0.8)(0.3)} = 0.5$$

2-178. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose that a patient is selected randomly. Let *A* denote the event that the patient is in group 1, and let *B* denote the event that there is no progression.
Determine the following probabilities:

(a) $P(B)$ (b) $P(B \mid A)$ (c) $P(A \mid B)$

(a) P(B) = P(B|G1) P(G1)+P(B|G2) P(G2)+P(B|G3) P(G3)+P(P|G4) P(G4) = 0.802

(b) P(B|A) = $\frac{P(A \cap B)}{P(A)}$ = 76/114 = 0.667

(c) P(A|B) =

$$\frac{P(B|A)P(A)}{\mathrm{P(B|G1)P(G1)+P(B|G2)P(G2)+P(B|G3)P(G3)+P(P|G4)P(G4)}} = \frac{0.667(0.244)}{0.802} = 0.203$$

2-179. An e-mail filter is planned to separate valid e-mails from spam. The word *free* occurs in 60% of the spam messages and only 4% of the valid messages. Also, 20% of the messages are spam. Determine the following probabilities:
(a) The message contains *free*.
(b) The message is spam given that it contains *free*.
(c) The message is valid given that it does not contain *free*.

F: Free; S: Spam; V: Valid
P(F|S) = 0.6, P(F|V) = 0.04
(a) P(F) = P(F|S) P(S)+P(F|V) P(V) = 0.6(0.2) + 0.04(0.8) = 0.152

(b) P(S|F) = $\frac{P(F|S)P(S)}{P(F)} = \frac{0.6(0.2)}{0.152}$ = 0.789

(c) P(V|F') = $\frac{P(F'|V)P(V)}{P(F')} = \frac{(0.96)0.8}{1-0.152} = 0.906$

2-180. A recreational equipment supplier finds that among orders that include tents, 40% also include sleeping mats. Only 5% of orders that do not include tents do include sleeping mats. Also, 20% of orders include tents. Determine the following probabilities:
(a) The order includes sleeping mats.
(b) The order includes a tent given it includes sleeping mats.

SM: Sleeping Mats; T:Tents;
P(SM|T) = 0.4; P(SM|T') = 0.05; P(T) = 0.2
(a) P(SM) = P(SM|T)P(T) + P(SM|T')P(T') = 0.4(0.2) + 0.05(0.8) = 0.12
(b) P(T|SM) = $\frac{\mathrm{P(SM|T)P(T)}}{P(SM)} = \frac{0.4X0.2}{0.12} = 0.667$

2-181. The probabilities of poor print quality given no printer problem, misaligned paper, high ink viscosity, or printer-head
debris are 0, 0.3, 0.4, and 0.6, respectively. The probabilities of no printer problem, misaligned paper, high ink viscosity, or printer-head debris are 0.8, 0.02, 0.08, and 0.1, respectively.
(a) Determine the probability of high ink viscosity given poor print quality.
(b) Given poor print quality, what problem is most likely?

NP = no problem; PP = poor print; MP = misaligned paper
HV = high ink viscosity; HD = print head debris
P(MP) = 0.02; P(HV) = 0.08; P(HD) = 0.1; P(NP) = 0.8
P(PP|NP) = 0; P(PP|MP) = 0.3; P(PP|HV) = 0.4; P(PP|HD) = 0.6; P(NP) = 0.8
(a) P(HV|PP)= $\frac{P(PP|HV)P(HV)}{P(PP)}$

$$P(PP) = P(PP|HV)P(HV) + P(PP|NP)P(NP) + P(PP|MP)P(MP) + P(PP|HD)P(HD)$$
$= 0.4(0.08) + 0(0.8) + 0.3(0.02) + 0.6(0.1) = 0.98$

Therefore, P(HV|PP) = $\frac{0.4(0.08)}{0.098} = \frac{0.032}{0.098} = 0.327$

(b) P(HV|PP) = P(PP|HV)P(HV)/P(PP) = 0.032/0.098 = 0.327
P(NP|PP) = P(PP|NP)P(NP)/P(PP) = 0
P(MP|PP)= P(PP|MP)P(MP)/P(PP) = 0.006/0.098 = 0.061
P(HD|PP) = P(PP | HD)P(HD)/P(PP) = 0.06/0.098 = 0.612
The problem most likely given poor print quality is head debris.

Section 2-8

2-182. Decide whether a discrete or continuous random variable is the best model for each of the following variables:
(a) The time until a projectile returns to earth.
(b) The number of times a transistor in a computer memory changes state in one operation.
(c) The volume of gasoline that is lost to evaporation during the filling of a gas tank.
(d) The outside diameter of a machined shaft.

(a) continuous (b) discrete (c) continuous (d) continuous

2-183. Decide whether a discrete or continuous random variable is the best model for each of the following variables:
(a) The number of cracks exceeding one-half inch in 10 miles of an interstate highway.
(b) The weight of an injection-molded plastic part.
(c) The number of molecules in a sample of gas.
(d) The concentration of output from a reactor.
(e) The current in an electronic circuit.

(a) discrete (b) continuous (c) discrete, but large values might be modeled as continuous

(d) continuous (e) continuous

2-184. Decide whether a discrete or continuous random variable is the best model for each of the following variables:
(a) The time for a computer algorithm to assign an image to a category.
(b) The number of bytes used to store a file in a computer.
(c) The ozone concentration in micrograms per cubic meter.
(d) The ejection fraction (volumetric fraction of blood pumped from a heart ventricle with each beat).
(e) The fluid flow rate in liters per minute.

(a) continuous (b) discrete, but large values might be modeled as continuous
(c) continuous (d) continuous (e) continuous

Supplemental Exercises

2-185. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2% and 1%, respectively, of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

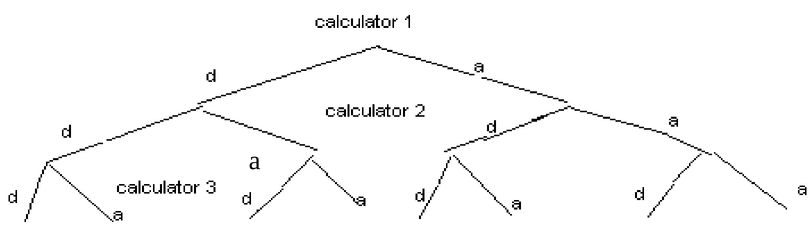
Let B denote the event that a glass breaks.
Let L denote the event that large packaging is used.
P(B)= P(B|L)P(L) + P(B|L')P(L')
= 0.01(0.60) + 0.02(0.40) = 0.014

2-186. A sample of three calculators is selected from a manufacturing line, and each calculator is classified as either defective or acceptable. Let A, B, and C denote the events that the first, second, and third calculators, respectively, are defective.
(a) Describe the sample space for this experiment with a tree diagram.

Use the tree diagram to describe each of the following events:

(b) A (c) B (d) $A \cap B$ (e) $B \cup C$

Let "d" denote a defective calculator and let "a" denote an acceptable calculator



(a) $S = \{ddd, add, dda, ada, dad, aad, daa, aaa\}$
(b) $A = \{ddd, dda, dad, daa\}$
(c) $B = \{ddd, dda, add, ada\}$
(d) $A \cap B = \{ddd, dda\}$
(e) $B \cup C = \{ddd, dda, add, ada, dad, aad\}$

2-187. Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and edge finish. The results of 100 parts are summarized as follows:

		Edge Finish	
		Excellent	Good
Surface	Excellent	80	2
Finish	Good	10	8

Let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent edge finish. If a part is selected at random, determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A')$ (d) $P(A \cap B)$ (e) $P(A \cup B)$ (f) $P(A' \cup B)$

Let A = excellent surface finish; B = excellent length
(a) P(A) = 82/100 = 0.82
(b) P(B) = 90/100 = 0.90
(c) P(A') = 1 – 0.82 = 0.18
(d) P(A∩B) = 80/100 = 0.80
(e) P(A∪B) = 0.92
(f) P(A'∪B) = 0.98

2-188. Shafts are classified in terms of the machine tool that was used for manufacturing the shaft and conformance to surface finish and roundness.

Tool 1		Roundness Conforms	
		Yes	No
Surface Finish	Yes	200	1
Conforms	No	4	2

Tool 2		Roundness Conforms	
		Yes	No
Surface Finish	Yes	145	4
Conforms	No	8	6

(a) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or to roundness requirements or is from tool 1?
(b) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or does not conform to roundness requirements or is from tool 2?
(c) If a shaft is selected at random, what is the probability that the shaft conforms to both surface finish and roundness requirements or the shaft is from tool 2?
(d) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or the shaft is from tool 2?

(a) (207+350+357-201-204-345+200)/370 = 0.9838
(b) 366/370 = 0.989
(c) (200+163)/370 = 363/370 = 0.981
(d) (201+163)/370 = 364/370 = 0.984

2-189. If A, B, and C are mutually exclusive events, is it possible for $P(A) = 0.3$, $P(B) = 0.4$, and $P(C) = 0.5$? Why or why not?

If A,B,C are mutually exclusive, then $P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.3 + 0.4 + 0.5 = 1.2$, which greater than 1. Therefore, P(A), P(B),and P(C) cannot equal the given values.

2-190. The analysis of shafts for a compressor is summarized by conformance to specifications:

		Roundness Conforms	
		Yes	No
Surface finish	Yes	345	5
Conforms	No	12	8

(a) If we know that a shaft conforms to roundness requirements, what is the probability that it conforms to surface finish requirements?

(b) If we know that a shaft does not conform to roundness requirements, what is the probability that it conforms to
surface finish requirements?

(a) 345/357 (b) 5/13

2-191. A researcher receives 100 containers of oxygen. Of those containers, 20 have oxygen that is not ionized, and the rest are ionized. Two samples are randomly selected, without replacement, from the lot.
(a) What is the probability that the first one selected is not ionized?
(b) What is the probability that the second one selected is not ionized given that the first one was ionized?
(c) What is the probability that both are ionized?
(d) How does the answer in part (b) change if samples selected were replaced prior to the next selection?

(a) P(the first one selected is not ionized)=20/100=0.2
(b) P(the second is not ionized given the first one was ionized) =20/99=0.202
(c) P(both are ionized)
= P(the first one selected is ionized) $\times$ P(the second is ionized given the first one was ionized)
= (80/100)$\times$(79/99)=0.638
(d) If samples selected were replaced prior to the next selection,
P(the second is not ionized given the first one was ionized) =20/100=0.2.
The event of the first selection and the event of the second selection are independent.

2-192. A lot contains 15 castings from a local supplier and 25 castings from a supplier in the next state. Two castings are
selected randomly, without replacement, from the lot of 40. Let *A* be the event that the first casting selected is from the local supplier, and let *B* denote the event that the second casting is selected from the local supplier. Determine:

(a) $P(A)$ (b) $P(B \mid A)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$

Suppose that 3 castings are selected at random, without replacement, from the lot of 40. In addition to the definitions of events *A* and *B*, let *C* denote the event that the third casting selected is from the local supplier. Determine:

(e) $P(A \cap B \cap C)$ (f) $P(A \cap B \cap C')$

(a) P(A) = 15/40
(b) P($B|A$) = 14/39
(c) P($A \cap B$) = P(A) P(B/A) = (15/40) (14/39) = 0.135
(d) P($A \cup B$) = 1 – P(A' and B') = $1-\left(\frac{25}{40}\right)\left(\frac{24}{39}\right)=0.615$
A = first is local, B = second is local, C = third is local
(e) P(A ∩ B ∩ C) = (15/40)(14/39)(13/38) = 0.046
(f) P(A ∩ B ∩ C') = (15/40)(14/39)(25/39) = 0.089

2-193. In the manufacturing of a chemical adhesive, 3% of all batches have raw materials from two different lots. This occurs when holding tanks are replenished and the remaining portion of a lot is insufficient to fill the tanks. Only 5% of batches with material from a single lot require reprocessing. However, the viscosity of batches consisting of two or more lots of material is more difficult to control, and 40% of such batches require additional processing to achieve the required viscosity.

Let *A* denote the event that a batch is formed from two different lots, and let *B* denote the event that a lot requires additional processing. Determine the following probabilities:

(a) $P(A)$ (b) $P(A')$ (c) $P(B \mid A)$ (d) $P(B \mid A')$
(e) $P(A \cap B)$ (f) $P(A \cap B')$ (g) $P(B)$

(a) P(A) = 0.03
(b) P(A') = 0.97

(c) P(B|A) = 0.40
(d) P(B|A') = 0.05
(e) P($A \cap B$) = P($B|A$)P(A) = (0.40)(0.03) = 0.012
(f) P($A \cap B'$) = P($B'|A$)P(A) = (0.60)(0.03) = 0.018
(g) P(B) = P($B|A$)P(A) + P($B|A'$)P(A') = (0.40)(0.03) + (0.05)(0.97) = 0.0605

2-194. Incoming calls to a customer service center are classified as complaints (75% of calls) or requests for information
(25% of calls). Of the complaints, 40% deal with computer equipment that does not respond and 57% deal with incomplete software installation; in the remaining 3% of complaints, the user has improperly followed the installation instructions. The requests for information are evenly divided on technical questions (50%) and requests to purchase more products (50%).
(a) What is the probability that an incoming call to the customer service center will be from a customer who has not
followed installation instructions properly?
(b) Find the probability that an incoming call is a request for purchasing more products.

Let U denote the event that the user has improperly followed installation instructions.
Let C denote the event that the incoming call is a complaint.
Let P denote the event that the incoming call is a request to purchase more products.
Let R denote the event that the incoming call is a request for information.
a) P(U|C)P(C) = (0.75)(0.03) = 0.0225
b) P(P|R)P(R) = (0.50)(0.25) = 0.125

2-195. A congested computer network has a 0.002 probability of losing a data packet, and packet losses are independent events. A lost packet must be resent.
(a) What is the probability that an e-mail message with 100 packets will need to be resent?
(b) What is the probability that an e-mail message with 3 packets will need exactly 1 to be resent?
(c) If 10 e-mail messages are sent, each with 100 packets, what is the probability that at least 1 message will need some
packets to be resent?

(a) $P = 1 - (1 - 0.002)^{100} = 0.18143$
(b) $P = C_3^1 (0.998^2) 0.002 = 0.005976$
(c) $P = 1 - [(1 - 0.002)^{100}]^{10} = 0.86494$

2-196. Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and length measurements.
The results of 100 parts are summarized as follows:

		Length	
		Excellent	Good
Surface	Excellent	80	2
Finish	Good	10	8

Let *A* denote the event that a sample has excellent surface finish, and let *B* denote the event that a sample has excellent
length. Are events *A* and *B* independent?
P($A \cap B$) = 80/100, P(A) = 82/100, P(B) = 90/100.
Then, P($A \cap B$) $\neq$ P(A)P(B), so A and B are not independent.

2-197. An optical storage device uses an error recovery procedure that requires an immediate satisfactory readback of any written data. If the readback is not successful after three writing operations, that sector of the disk is eliminated as unacceptable for data storage. On an acceptable portion of the disk, the probability of a satisfactory readback is 0.98. Assume the readbacks are independent. What is the probability that an acceptable portion of the disk is eliminated as unacceptable fordata storage?

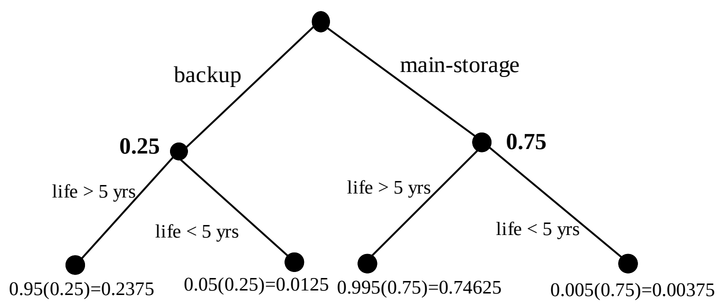
Let A_i denote the event that the ith readback is successful. By independence,

$$P(A_1' \cap A_2' \cap A_3') = P(A_1')P(A_2')P(A_3') = (0.02)^3 = 0.000008.$$

2-198. Semiconductor lasers used in optical storage products require higher power levels for write operations than for read operations. High-power-level operations lower the useful life of the laser. Lasers in products used for backup of higher-speed magnetic disks primarily write, and the probability that the useful life exceeds five years is 0.95. Lasers that are in products that are used for main storage spend approximately an equal amount of time reading and writing, and the probability that the useful life exceeds five years is 0.995. Now, 25% of the products from a manufacturer are used for backup and 75% of the products are used for main storage.
Let A denote the event that a laser's useful life exceeds five years, and let B denote the event that a laser is in a product that is used for backup.

Use a tree diagram to determine the following:

(a) $P(B)$ (b) $P(A \mid B)$ (c) $P(A \mid B')$
(d) $P(A \cap B)$ (e) $P(A \cap B')$ (f) $P(A)$
(g) What is the probability that the useful life of a laser exceeds five years?
(h) What is the probability that a laser that failed before five years came from a product used for backup?



(a) P(B) = 0.25
(b) $P(A|B) = 0.95$
(c) $P(A|B') = 0.995$
(d) $P(A \cap B) = P(A|B)P(B) = 0.95(0.25) = 0.2375$
(e) $P(A \cap B') = P(A|B')P(B') = 0.995(0.75) = 0.74625$
(f) $P(A) = P(A \cap B) + P(A \cap B') = 0.95(0.25) + 0.995(0.75) = 0.98375$
(g) 0.95(0.25) + 0.995(0.75) = 0.98375.
(h)

$$P(B|A') = \frac{P(A'|B)P(B)}{P(A'|B)P(B) + P(A'|B')P(B')} = \frac{0.05(0.25)}{0.05(0.25) + 0.005(0.75)} = 0.769$$

2-199. Energy released from cells breaks the molecular bond and converts ATP (adenosine triphosphate) into ADP (adenosine diphosphate). Storage of ATP in muscle cells (even for an athlete) can sustain maximal muscle power only for less than five seconds (a short dash). Three systems are used to replenish ATP—phosphagen system, glycogen-lactic acid system (anaerobic), and aerobic respiration—but the first is useful only for less than 10 seconds, and even the second system provides less than two minutes of ATP. An endurance athlete needs to perform below the anaerobic threshold to sustain energy for extended periods. A sample of 100 individuals is described by the energy system used in exercise at different intensity levels.

	Primarily Aerobic	
Period	Yes	No
1	50	7
2	13	30

Let A denote the event that an individual is in period 2, and let B denote the event that the energy is primarily aerobic. Determine the number of individuals in
(a) $A' \cap B$ (b) B' (c) $A \cup B$

(a) $A' \cap B = 50$
(b) B'=37
(c) $A \cup B = 93$

2-200. A sample preparation for a chemical measurement is completed correctly by 25% of the lab technicians, completed with a minor error by 70%, and completed with a major error by 5%.
(a) If a technician is selected randomly to complete the preparation, what is the probability that it is completed without
error?
(b) What is the probability that it is completed with either a minor or a major error?

(a) 0.25
(b) 0.75

2-201. In circuit testing of printed circuit boards, each board either fails or does not fail the test. A board that fails the test is then checked further to determine which one of five defect types is the primary failure mode. Represent the sample space for this experiment.

Let D_i denote the event that the primary failure mode is type i and let A denote the event that a board passes the test. The sample space is S = $\{A, A'D_1, A'D_2, A'D_3, A'D_4, A'D_5\}$.

2-202. The data from 200 machined parts are summarized as follows:

	Depth of Bore	
Edge Condition	Above Target	Below Target
Coarse	15	10
Moderate	25	20
Smooth	50	80

(a) What is the probability that a part selected has a moderate edge condition and a below-target bore depth?
(b) What is the probability that a part selected has a moderate edge condition or a below-target bore depth?
(c) What is the probability that a part selected does not have a moderate edge condition or does not have a below-target
bore depth?

(a) 20/200 (b) 135/200 (c) 65/200

2-203. Computers in a shipment of 100 units contain a portable hard drive, solid-state memory, or both, according to the following table:

	Portable Hard Drive	
Solid-state memory	Yes	No
Yes	15	80
No	4	1

Let A denote the event that a computer has a portable hard drive, and let B denote the event that a computer has a solidstate memory. If one computer is selected randomly, compute
(a) $P(A)$ (b) $P(A \cap B)$ (c) $P(A \cup B)$ (d) $P(A' \cap B)$ (e) $P(A \mid B)$

(a) P(A) = 19/100 = 0.19
(b) P(A $\cap$ B) = 15/100 = 0.15
(c) P(A $\cup$ B) = (19 + 95 – 15)/100 = 0.99

(d) $P(A' \cap B) = 80/100 = 0.80$
(e) $P(A|B) = P(A \cap B)/P(B) = 0.158$

2-204. The probability that a customer's order is not shipped on time is 0.05. A particular customer places three orders, and the orders are placed far enough apart in time that they can be considered to be independent events.
(a) What is the probability that all are shipped on time?
(b) What is the probability that exactly one is not shipped on time?
(c) What is the probability that two or more orders are not shipped on time?

Let A_i denote the event that the *i*th order is shipped on time.

(a) By independence, $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3) = (0.95)^3 = 0.857$

(b) Let

$$B_1 = A_1' \cap A_2 \cap A_3$$
$$B_2 = A_1 \cap A_2' \cap A_3$$
$$B_3 = A_1 \cap A_2 \cap A_3'$$

Then, because the B's are mutually exclusive,

$$\begin{aligned} P(B_1 \cup B_2 \cup B_3) &= P(B_1) + P(B_2) + P(B_3) \\ &= 3(0.95)^2(0.05) \\ &= 0.135 \end{aligned}$$

(c) Let

$$B_1 = A_1' \cap A_2' \cap A_3$$
$$B_2 = A_1' \cap A_2 \cap A_3'$$
$$B_3 = A_1 \cap A_2' \cap A_3'$$
$$B_4 = A_1' \cap A_2' \cap A_3'$$

Because the B's are mutually exclusive,

$$\begin{aligned} P(B_1 \cup B_2 \cup B_3 \cup B_4) &= P(B_1) + P(B_2) + P(B_3) + P(B_4) \\ &= 3(0.05)^2(0.95) + (0.05)^3 \\ &= 0.00725 \end{aligned}$$

2-205. Let E_1, E_2, and E_3 denote the samples that conform to a percentage of solids specification, a molecular weight specification, and a color specification, respectively. A total of 240 samples are classified by the E_1, E_2, and E_3 specifications, where *yes* indicates that the sample conforms.

E_3 yes

		E_2		
		Yes	No	Total
E_1	Yes	200	1	201
	No	5	4	9
Total		205	5	210

E_3 no

		E_2		
		Yes	No	Total
E_1	Yes	20	4	24
	No	6	0	6
Total		26	4	30

(a) Are E_1, E_2, and E_3 mutually exclusive events?
(b) Are E_1', E_2', and E_3' mutually exclusive events?
(c) What is $P(E_1'$ or E_2' or $E_3')$?
(d) What is the probability that a sample conforms to all three specifications?
(e) What is the probability that a sample conforms to the E_1 or E_3 specification?
(f) What is the probability that a sample conforms to the E_1 or E_2 or E_3 specification?

(a) No, $P(E_1 \cap E_2 \cap E_3) \neq 0$
(b) No, $E_1' \cap E_2'$ is not $\varnothing$
(c) $P(E_1' \cup E_2' \cup E_3') = P(E_1') + P(E_2') + P(E_3') - P(E_1' \cap E_2') - P(E_1' \cap E_3') - P(E_2' \cap E_3')$
$+ P(E_1' \cap E_2' \cap E_3')$

= 40/240

(d) $P(E_1 \cap E_2 \cap E_3) = 200/240$

(e) $P(E_1 \cup E_3) = P(E_1) + P(E_3) – P(E_1 \cap E_3) = 234/240$

(f) $P(E_1 \cup E_2 \cup E_3) = 1 – P(E_1' \cap E_2' \cap E_3') = 1 - 0 = 1$

2-206. Transactions to a computer database are either new items or changes to previous items. The addition of an item can be completed in less than 100 milliseconds 90% of the time, but only 20% of changes to a previous item can be completed in less than this time. If 30% of transactions are changes, what is the probability that a transaction can be completed in less than 100 milliseconds?

(a) (0.20)(0.30) +(0.7)(0.9) = 0.69

2-207. A steel plate contains 20 bolts. Assume that 5 bolts are not torqued to the proper limit. 4 bolts are selected at random,

without replacement, to be checked for torque.

(a) What is the probability that all 4 of the selected bolts are torqued to the proper limit?

(b) What is the probability that at least 1 of the selected bolts is *not* torqued to the proper limit?

Let A_i denote the event that the *i*th bolt selected is not torqued to the proper limit.

(a) Then,

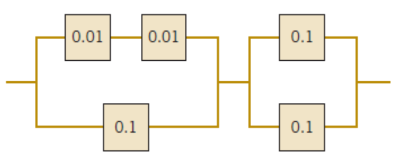
$$P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_4 | A_1 \cap A_2 \cap A_3)P(A_1 \cap A_2 \cap A_3)$$
$$= P(A_4 | A_1 \cap A_2 \cap A_3)P(A_3 | A_1 \cap A_2)P(A_2 | A_1)P(A_1)$$
$$= \left(\frac{12}{17}\right)\left(\frac{13}{18}\right)\left(\frac{14}{19}\right)\left(\frac{15}{20}\right) = 0.282$$

(b) Let B denote the event that at least one of the selected bolts are not properly torqued. Thus, B' is the event that all bolts are properly torqued. Then,

$$P(B) = 1 - P(B') = 1 - \left(\frac{15}{20}\right)\left(\frac{14}{19}\right)\left(\frac{13}{18}\right)\left(\frac{12}{17}\right) = 0.718$$

2-208. The following circuit operates if and only if there is a path of functional devices from left to right. Assume devices fail

independently and that the probability of *failure* of each device is as shown. What is the probability that the circuit operates?



Let A,B denote the event that the first, second portion of the circuit operates.

Then, P(A) = (0.99)(0.99)+0.9-(0.99)(0.99)(0.9) = 0.998

P(B) = 0.9+0.9-(0.9)(0.9) = 0.99 and

$P(A \cap B) = P(A) P(B) = (0.998) (0.99) = 0.988$

2-209. The probability that concert tickets are available by telephone is 0.92. For the same event, the probability that tickets are available through a Web site is 0.95. Assume that these two ways to buy tickets are independent. What is the

probability that someone who tries to buy tickets through the Web and by phone will obtain tickets?

A_1 = by telephone, A_2 = website; $P(A_1) = 0.92$, $P(A_2) = 0.95$;

By independence $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.92 + 0.95 - 0.92(0.95) = 0.996$

2-210. The British government has stepped up its information campaign regarding foot-and-mouth disease by mailing brochures to farmers around the country. It is estimated that 99% of Scottish farmers who receive the brochure possess enough information to deal with an outbreak of the disease, but only 90% of those without the brochure can deal with an outbreak. After the first three months of mailing, 95% of the farmers in Scotland had received the informative brochure. Compute the probability that a randomly selected farmer will have enough information to deal effectively with an outbreak of the disease.

P(Possess) = 0.95(0.99) +(0.05)(0.90) = 0.9855

2-211. In an automated filling operation, the probability of an incorrect fill when the process is operated at a low speed is

0.001. When the process is operated at a high speed, the probability of an incorrect fill is 0.01. Assume that 30% of the containers are filled when the process is operated at a high speed and the remainder are filled when the process is operated at a low speed.
(a) What is the probability of an incorrectly filled container?
(b) If an incorrectly filled container is found, what is the probability that it was filled during the high-speed operation?

Let D denote the event that a container is incorrectly filled and let H denote the event that a container is filled under high-speed operation. Then,

(a) $P(D) = P(D|H)P(H) + P(D|H')P(H') = 0.01(0.30) + 0.001(0.70) = 0.0037$

(b) $P(H|D) = \dfrac{P(D|H)P(H)}{P(D)} = \dfrac{0.01(0.30)}{0.0037} = 0.8108$

2-212. An encryption-decryption system consists of three elements: encode, transmit, and decode. A faulty encode occurs

in 0.5% of the messages processed, transmission errors occur in 1% of the messages, and a decode error occurs in 0.1% of the messages. Assume the errors are independent.
(a) What is the probability of a completely defect-free message?
(b) What is the probability of a message that has either an encode or a decode error?

(a) $P(E' \cap T' \cap D') = (0.995)(0.99)(0.999) = 0.984$
(b) $P(E \cup D) = P(E) + P(D) - P(E \cap D) = 0.005995$

2-213. It is known that two defective copies of a commercial software program were erroneously sent to a shipping lot that now has a total of 75 copies of the program. A sample of copies will be selected from the lot without replacement.
(a) If three copies of the software are inspected, determine the probability that exactly one of the defective copies will be found.
(b) If three copies of the software are inspected, determine the probability that both defective copies will be found.
(c) If 73 copies are inspected, determine the probability that both copies will be found. (*Hint*: Work with the copies that remain in the lot.)

D = defective copy

(a) $P(D = 1) = \left(\frac{2}{75}\right)\left(\frac{73}{74}\right)\left(\frac{72}{73}\right)+\left(\frac{73}{75}\right)\left(\frac{2}{74}\right)\left(\frac{72}{73}\right)+\left(\frac{73}{75}\right)\left(\frac{72}{74}\right)\left(\frac{2}{73}\right) = 0.0778$

(b) $P(D = 2) = \left(\frac{2}{75}\right)\left(\frac{1}{74}\right)\left(\frac{73}{73}\right)+\left(\frac{2}{75}\right)\left(\frac{73}{74}\right)\left(\frac{1}{73}\right)+\left(\frac{73}{75}\right)\left(\frac{2}{74}\right)\left(\frac{1}{73}\right) = 0.00108$

(c) Let A represent the event that the two items NOT inspected are not defective. Then, P(A)=(73/75)(72/74)=0.947.

2-214. A robotic insertion tool contains 10 primary components. The probability that any component fails during the warranty period is 0.01. Assume that the components fail independently and that the tool fails if any component fails. What is the probability that the tool fails during the warranty period?

The tool fails if any component fails. Let F denote the event that the tool fails. Then, $P(F') = 0.99^{10}$ by independence and $P(F) = 1 - 0.99^{10} = 0.0956$

2-215. An e-mail message can travel through one of two server routes. The probability of transmission error in each of the servers and the proportion of messages that travel each route are shown in the following table. Assume that the servers are independent.

	Percentage of Messages	Probability of Error			
		Server 1	Server 2	Server 3	Server 4
Route 1	30	0.01	0.015	—	—
Route 2	70	—	—	0.02	0.003

(a) What is the probability that a message will arrive without error?
(b) If a message arrives in error, what is the probability it was sent through route 1?

(a) (0.3)(0.99)(0.985) + (0.7)(0.98)(0.997) = 0.9764

(b) $P(route1|E) = \dfrac{P(E|route1)P(route1)}{P(E)} = \dfrac{0.02485(0.30)}{1-0.9764} = 0.3159$

2-216. A machine tool is idle 15% of the time. You request immediate use of the tool on five different occasions during the year. Assume that your requests represent independent events.
(a) What is the probability that the tool is idle at the time of all of your requests?
(b) What is the probability that the machine is idle at the time of exactly four of your requests?
(c) What is the probability that the tool is idle at the time of at least three of your requests?

(a) By independence, $0.15^5 = 7.59 \times 10^{-5}$

(b) Let A_i denote the events that the machine is idle at the time of your ith request. Using independence, the requested probability is

$$P(A_1 A_2 A_3 A_4 A_5' \text{ or } A_1 A_2 A_3 A_4' A_5 \text{ or } A_1 A_2 A_3' A_4 A_5 \text{ or } A_1 A_2' A_3 A_4 A_5 \text{ or } A_1' A_2 A_3 A_4 A_5)$$
$$= 5(0.15^4)(0.85^1)$$
$$= 0.00215$$

(c) As in part b, the probability of 3 of the events is

$$P(A_1 A_2 A_3 A_4' A_5' \text{ or } A_1 A_2 A_3' A_4 A_5' \text{ or } A_1 A_2 A_3' A_4' A_5 \text{ or } A_1 A_2' A_3 A_4 A_5' \text{ or } A_1 A_2' A_3 A_4' A_5 \text{ or }$$
$$A_1 A_2' A_3' A_4 A_5 \text{ or } A_1' A_2 A_3 A_4 A_5' \text{ or } A_1' A_2 A_3 A_4' A_5 \text{ or } A_1' A_2 A_3' A_4 A_5 \text{ or } A_1' A_2' A_3 A_4 A_5)$$
$$= 10(0.15^3)(0.85^2)$$
$$= 0.0244$$

For the probability of at least 3, add answer parts a) and b) to the above to obtain the requested probability. Therefore, the answer is 0.0000759 + 0.0022 + 0.0244 = 0.0267

2-217. A lot of 50 spacing washers contains 30 washers that are thicker than the target dimension. Suppose that 3 washers are selected at random, without replacement, from the lot.
(a) What is the probability that all 3 washers are thicker than the target?
(b) What is the probability that the third washer selected is thicker than the target if the first 2 washers selected are thinner than the target?
(c) What is the probability that the third washer selected is thicker than the target?

Let A_i denote the event that the ith washer selected is thicker than target.

(a) $\left(\dfrac{30}{50}\right)\left(\dfrac{29}{49}\right)\left(\dfrac{28}{48}\right) = 0.207$

(b) 30/48 = 0.625

(c) The requested probability can be written in terms of whether or not the first and second washer selected are thicker than the target. That is,

$$\left(\frac{30}{50}\right)\left(\frac{29}{49}\right)\left(\frac{28}{48}\right)+\left(\frac{30}{50}\right)\left(\frac{20}{49}\right)\left(\frac{29}{48}\right)+\left(\frac{20}{50}\right)\left(\frac{30}{49}\right)\left(\frac{29}{48}\right)+\left(\frac{20}{50}\right)\left(\frac{19}{49}\right)\left(\frac{30}{48}\right)=0.60$$

2-218. Washers are selected from the lot at random without replacement.
(a) What is the minimum number of washers that need to be selected so that the probability that all the washers are
thinner than the target is less than 0.10?
(b) What is the minimum number of washers that need to be selected so that the probability that 1 or more washers are
thicker than the target is at least 0.90?

(a) If n washers are selected, then the probability they are all less than the target is $\frac{20}{50}\cdot\frac{19}{49}\cdot\ldots\frac{20-n+1}{50-n+1}$.

n	probability all selected washers are less than target
1	20/50 = 0.4
2	(20/50)(19/49) = 0.155
3	(20/50)(19/49)(18/48) = 0.058

Therefore, the answer is n = 3.

(b) Then event E that one or more washers is thicker than target is the complement of the event that all are less than target. Therefore, P(E) equals one minus the probability in part a. Therefore, n = 3.

2-219. The following table lists the history of 940 orders for features in an entry-level computer product.

		Extra Memory	
		No	Yes
Optional high-speed processor	No	514	68
	Yes	112	246

Let A be the event that an order requests the optional highspeed processor, and let B be the event that an order requests
extra memory. Determine the following probabilities:
(a) $P(A \cup B)$ (b) $P(A \cap B)$ (c) $P(A' \cup B)$ (d) $P(A' \cap B')$
(e) What is the probability that an order requests an optional high-speed processor given that the order requests extra
memory?
(f) What is the probability that an order requests extra memory given that the order requests an optional highspeed
processor?

$$a)\quad P(A \cup B) = \frac{112+68+246}{940} = 0.453$$

$$b)\quad P(A \cap B) = \frac{246}{940} = 0.262$$

$$c)\quad P(A' \cup B) = \frac{514+68+246}{940} = 0.881$$

$$d)\quad P(A' \cap B') = \frac{514}{940} = 0.547$$

. e) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{246/940}{314/940} = 0.783$

f) $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{246/940}{358/940} = 0.687$

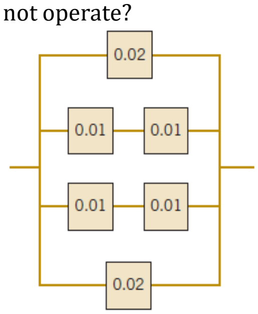
2-220. The alignment between the magnetic media and head in a magnetic storage system affects the system's performance.
Suppose that 10% of the read operations are degraded by skewed alignments, 5% of the read operations are degraded

by off-center alignments, and the remaining read operations are properly aligned. The probability of a read error is 0.01 from a skewed alignment, 0.02 from an off-center alignment, and 0.001 from a proper alignment.
(a) What is the probability of a read error?
(b) If a read error occurs, what is the probability that it is due to a skewed alignment?

Let E denote a read error and let S,O,P denote skewed, off-center, and proper alignments, respectively.
Then,
(a) P(E) = P(E|S) P(S) + P(E|O) P (O) + P(E|P) P(P)
= 0.01(0.10) + 0.02(0.05) + 0.001(0.85)
= 0.00285

(b) $P(S|E) = \dfrac{P(E|S)P(S)}{P(E)} = \dfrac{0.01(0.10)}{0.00285} = 0.351$

2-221. The following circuit operates if and only if there is a path of functional devices from left to right. Assume that devices fail independently and that the probability of *failure* of each device is as shown. What is the probability that the circuit does not operate?



Let A_i denote the event that the *i*th row operates. Then,

$$P(A_1) = 0.98, P(A_2) = (0.99)(0.99) = 0.9801, P(A_3) = 0.9801, \ P(A_4) = 0.98.$$

The probability the circuit does not operate is

$$P(A_1')P(A_2')P(A_3')P(A_4') = (0.02)(0.0199)(0.0199)(0.02) = 1.58 \times 10^{-7}$$

2-222. A company that tracks the use of its Web site determined that the more pages a visitor views, the more likely the visitor

is to provide contact information. Use the following tables to answer the questions:

Number of pages viewed:	1	2	3	4 or more
Percentage of visitors:	40	30	20	10
Percentage of visitors in each page-view category that provides contact information:	10	10	20	40

(a) What is the probability that a visitor to the Web site provides contact information?
(b) If a visitor provides contact information, what is the probability that the visitor viewed four or more pages?

(a) (0.4)(0.1) + (0.3)(0.1) +(0.2)(0.2) + (0.4)(0.1) = 0.15
(b) P(4 or more | provided info) = (0.4)(0.1)/0.15 = 0.267

2-223. An article in *Genome Research* ["An Assessment of Gene Prediction Accuracy in Large DNA Sequences" (2000, Vol. 10, pp. 1631–1642)], considered the accuracy of commercial software to predict nucleotides in gene sequences. The following table shows the number of sequences for which the programs produced predictions and the number of nucleotides correctly predicted (computed globally from the total number of prediction successes and failures on all sequences).

	Number of Sequences	Proportion
GenScan	177	0.93
Blastx default	175	0.91
Blastx topcomboN	174	0.97
Blastx 2 stages	175	0.90
GeneWise	177	0.98
Procrustes	177	0.93

Assume the prediction successes and failures are independent among the programs.
(a) What is the probability that all programs predict a nucleotide correctly?
(b) What is the probability that all programs predict a nucleotide incorrectly?
(c) What is the probability that at least one Blastx program predicts a nucleotide correctly?

(a) P=(0.93)(0.91)(0.97)(0.90)(0.98)(0.93)=0.67336
(b) P=(1-0.93)(1-0.91)(1-0.97)(1-0.90)(1-0.98)(1-0.93)=2.646×10^{-8}
(c) P=1-(1-0.91)(1-0.97)(1-0.90)=0.99973

2-224. A batch contains 36 bacteria cells. Assume that 12 of the cells are not capable of cellular replication. Of the cells, 6 are selected at random, without replacement, to be checked for replication.
(a) What is the probability that all 6 of the selected cells are able to replicate?
(b) What is the probability that at least 1 of the selected cells is not capable of replication?

(a) P=(24/36)(23/35)(22/34)(21/33)(20/32)(19/31)=0.069
(b) P=1-0.069=0.931

2-225. A computer system uses passwords that are exactly seven characters, and each character is one of the 26 letters (a–z) or 10 integers (0–9). Uppercase letters are not used.
(a) How many passwords are possible?
(b) If a password consists of exactly 6 letters and 1 number, how many passwords are possible?
(c) If a password consists of 5 letters followed by 2 numbers, how many passwords are possible?

(a) 36^7
(b) Number of permutations of six letters is 26^6. Number of ways to select one number = 10. Number of positions among the six letters to place the one number = 7. Number of passwords = $26^6 \times 10 \times 7$
(c) $26^5 10^2$

2-226. Natural red hair consists of two genes. People with red hair have two dominant genes, two regressive genes, or one

dominant and one regressive gene. A group of 1000 people was categorized as follows:

	Gene 2		
Gene 1	Dominant	Regressive	Other
Dominant	5	25	30
Regressive	7	63	35
Other	20	15	800

Let *A* denote the event that a person has a dominant red hair gene, and let *B* denote the event that a person has a regressive red hair gene. If a person is selected at random from this group, compute the following:
(a) $P(A)$ (b) $P(A\cap B)$ (c) $P(A\cup B)$ (d) $P(A'\cap B)$ (e) $P(A\mid B)$
(f) Probability that the selected person has red hair

(a) $P(A)=\frac{5+25+30+7+20}{1000}=0.087$

(b) $P(A\cap B)=\frac{25+7}{1000}=0.032$

(c) $P(A\cup B)=1-\frac{800}{1000}=0.20$

(d) $P(A' \cap B) = \dfrac{63+35+15}{1000} = 0.113$

(e) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.032}{(25+63+15+7+35)/1000} = 0.2207$

(f) $P = (5+25+7+63)/1000 = 0.1$

2-227. Two suppliers each supplied 2000 parts that were evaluated for conformance to specifications. One part type was more complex than the other. The proportion of nonconforming parts of each type are shown in the table.

Supplier		Simple Component	Complex Assembly	Total
1	Nonconforming	2	10	12
	Total	1000	1000	2000
2	Nonconforming	4	6	10
	Total	1600	400	2000

One part is selected at random from each supplier. For each supplier, separately calculate the following probabilities:
(a) What is the probability a part conforms to specifications?
(b) What is the probability a part conforms to specifications given it is a complex assembly?
(c) What is the probability a part conforms to specifications given it is a simple component?
(d) Compare your answers for each supplier in part (a) to those in parts (b) and (c) and explain any unusual results.

(a) Let A denote that a part conforms to specifications and let B denote a simple component.
For supplier 1: P(A) = 1988/2000 = 0.994
For supplier 2: P(A)= 1990/2000 = 0.995
(b)
For supplier 1: P(A|B') = 990/1000 = 0.99
For supplier 2: P(A|B') = 394/400 = 0.985
(c)
For supplier 1: P(A|B) = 998/1000 = 0.998
For supplier 2: P(A|B) = 1596/1600 = 0.9975

(d) The unusual result is that for both a simple component and for a complex assembly, supplier 1 has a greater probability that a part conforms to specifications. However, supplier 1 has a lower probability of conformance overall. The overall conforming probability depends on both the conforming probability of each part type and also the probability of each part type. Supplier 1 produces more of the complex parts so that overall conformance from supplier 1 is lower.

2-228. The article "Term Efficacy of Ribavirin Plus Interferon Alfa in the Treatment of Chronic Hepatitis C," [*Gastroenterology* (1996, Vol. 111, no. 5, pp. 1307–1312)], considered the effect of two treatments and a control for treatment of hepatitis C. The following table provides the total patients in each group and the number that showed a complete (positive) response after 24 weeks of treatment. Suppose a patient is selected randomly.

	Complete Response	Total
Ribavirin plus interferon alfa	16	21
Interferon alfa	6	19
Untreated controls	0	20

Let *A* denote the event that the patient is treated with ribavirin plus interferon alfa or interferon alfa, and let *B* denote the event that the response is complete. Determine the following probabilities.

(a) $P(A \mid B)$ (b) $P(B \mid A)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$

P(A|B)= $\dfrac{P(B|A)P(A)}{P(B)} = \dfrac{\frac{22}{40}(\frac{40}{60})}{\frac{22}{60}} = 1$

P(B|A) = 22/40 = 0.55

P(A∩B) = P(B|A)P(A)=(22/40)(40/60) = 22/60 = 0.366
P(AUB) = P(A)+P(B) - P(A∩B) = (40/60) + (22/60) – (22/60) = 0.667

2-229. The article ["Clinical and Radiographic Outcomes of Four Different Treatment Strategies in Patients with Early Rheumatoid Arthritis," *Arthritis & Rheumatism* (2005, Vol. 52, pp. 3381– 3390)] considered four treatment groups. The groups consisted of patients with different drug therapies (such as prednisone and infliximab): sequential monotherapy (group 1), step-up combination therapy (group 2), initial combination therapy (group 3), or initial combination therapy with infliximab (group 4). Radiographs of hands and feet were used to evaluate disease progression. The number of patients without progression of joint damage was 76 of 114 patients (67%), 82 of 112 patients (73%), 104 of 120 patients (87%), and 113 of 121 patients (93%) in groups 1–4, respectively. Suppose a patient is selected randomly. Let *A* denote the event that the patient is in group 1 or 2, and let *B* denote the event that there is no progression. Determine the following probabilities:

(a) $P(A \mid B)$ (b) $P(B \mid A)$ (c) $P(A \cap B)$ (d) $P(A \cup B)$

(a) P(A|B)= $\frac{158}{375} = 0.421$
(b) P(B|A)= $\frac{158}{226} = 0.699$
(c) P(A∩B)= $\frac{76+82}{467} = 0.338$
(d) P(AUB)= $\frac{226}{467} + \frac{375}{467} - \frac{158}{467}$ = 0.948

Mind-Expanding Exercises

2-230. Suppose documents in a lending organization are selected randomly (without replacement) for review. In a set of 50 documents, suppose that 2 actually contain errors.
(a) What is the minimum sample size such that the probability exceeds 0.90 that at least 1 document in error is selected?
(b) Comment on the effectiveness of sampling inspection to detect errors.

(a) Let *X* denote the number of documents in error in the sample and let *n* denote the sample size.

$P(X \geq 1) = 1 - P(X = 0)$ and $P(X = 0) = \frac{\binom{2}{0}\binom{48}{n}}{\binom{50}{n}}$

Trials for *n* result in the following results

n	$P(X = 0)$	$1 - P(X = 0)$
5	0.808163265	0.191836735
10	0.636734694	0.363265306
15	0.485714286	0.514285714
20	0.355102041	0.644897959
25	0.244897959	0.755102041
30	0.155102041	0.844897959
33	0.111020408	0.888979592
34	0.097959184	0.902040816

Therefore $n = 34$.

(b) A large proportion of the set of documents needs to be inspected in order for the probability of a document in error to be detected to exceed 0.9.

2-231. Suppose that a lot of washers is large enough that it can be assumed that the sampling is done with replacement. Assume that 60% of the washers exceed the target thickness.
(a) What is the minimum number of washers that need to be selected so that the probability that none is thicker than the
target is less than 0.10?
(b) What is the minimum number of washers that need to be selected so that the probability that 1 or more washers

are thicker than the target is at least 0.90?

Let n denote the number of washers selected.
(a) The probability that none are thicker, that is, all are less than the target is 0.4^n by independence.
The following results are obtained:

n	0.4^n
1	0.4
2	0.16
3	0.064

Therefore, n = 3.

(b) The requested probability is the complement of the probability requested in part a). Therefore, n = 3

2-232. A biotechnology manufacturing firm can produce diagnostic test kits at a cost of $20. Each kit for which there is a demand in the week of production can be sold for $100. However, the half-life of components in the kit requires the
kit to be scrapped if it is not sold in the week of production. The cost of scrapping the kit is $5. The weekly demand is
summarized as follows:

Weekly Demand				
Number of units	0	50	100	200
Probability of demand	0.05	0.4	0.3	0.25

How many kits should be produced each week to maximize the firm's mean earnings?

Let x denote the number of kits produced.

Revenue at each demand

	0	50	100	200
$0 \le x \le 50$	-5x	100x	100x	100x
Mean profit = 100x(0.95)-5x(0.05)-20x				
$50 \le x \le 100$	-5x	100(50)-5(x-50)	100x	100x
Mean profit = [100(50)-5(x-50)](0.4) + 100x(0.55)-5x(0.05)-20x				
$100 \le x \le 200$	-5x	100(50)-5(x-50)	100(100)-5(x-100)	100x
Mean profit = [100(50)-5(x-50)](0.4) + [100(100)-5(x-100)](0.3) + 100x(0.25) - 5x(0.05) - 20x				

	Mean Profit	Maximum Profit
$0 \le x \le 50$	74.75 x	$ 3737.50 at x=50
$50 \le x \le 100$	32.75 x + 2100	$ 5375 at x=100
$100 \le x \le 200$	1.25 x + 5250	$ 5500 at x=200

Therefore, profit is maximized at 200 kits. However, the difference in profit over 100 kits is small.

2-233. A steel plate contains 20 bolts. Assume that 5 bolts are not torqued to the proper limit. 4 bolts are selected at random,
without replacement, to be checked for torque. If an operator checks a bolt,the probability that an incorrectly torqued bolt is identified is 0.95. If a checked bolt is correctly torqued, the operator's conclusion is always correct. What is the probability that at least one bolt in the sample of four is identified as being incorrectly torqued?

Let E denote the event that none of the bolts are identified as incorrectly torqued.
Let X denote the number of bolts in the sample that are incorrect. The requested probability is P(E').
Then,
P(E) = P(E|X=0)P(X=0) + P(E|X=1) P(X=1) + P(E|X=2) P(X=2) + P(E|X=3) P(X=3) + P(E|X=4)P(X=4)
and P(X=0) = (15/20)(14/19)(13/18)(12/17) = 0.2817.
The remaining probability for X can be determined from the counting methods. Then

$$P(X=1)=\frac{\binom{5}{1}\binom{15}{3}}{\binom{20}{4}}=\frac{\left(\frac{5!}{4!1!}\right)\left(\frac{15!}{3!12!}\right)}{\left(\frac{20!}{4!16!}\right)}=\frac{5!15!4!16!}{4!3!12!20!}=0.4696$$

$$P(X=2)=\frac{\binom{5}{2}\binom{15}{2}}{\binom{20}{4}}=\frac{\left(\frac{5!}{3!2!}\right)\left(\frac{15!}{2!13!}\right)}{\left(\frac{20!}{4!16!}\right)}=0.2167$$

$$P(X=3)=\frac{\binom{5}{3}\binom{15}{1}}{\binom{20}{4}}=\frac{\left(\frac{5!}{3!2!}\right)\left(\frac{15!}{1!14!}\right)}{\left(\frac{20!}{4!16!}\right)}=0.0309$$

P(X = 4) = (5/20)(4/19)(3/18)(2/17) = 0.0010, P(E | X = 0) = 1, P(E | X = 1) = 0.05,
P(E | X = 2) = 0.05^2 = 0.0025, P(E|X=3) = 0.05^3 = $1.25\text{x}10^{-4}$, P(E | X=4) = 0.05^4 = $6.25\text{x}10^{-6}$.
Then, $P(E)=1(0.2817)+0.05(0.4696)+0.0025(0.2167)+1.25\times10^{-4}(0.0309)+6.25\times10^{-6}(0.0010)=0.306$
and P(E') = 0.694

2-234. If the events *A* and *B* are independent, show that *A*′ and *B*′ are independent.

$$\begin{aligned}P(A'\cap B')&=1-P([A'\cap B']')=1-P(A\cup B)\\&=1-[P(A)+P(B)-P(A\cap B)]\\&=1-P(A)-P(B)+P(A)P(B)\\&=[1-P(A)][1-P(B)]\\&=P(A')P(B')\end{aligned}$$

2-235. Suppose that a table of part counts is generalized as follows:

		Conforms	
		Yes	No
Supplier	1	*ka*	*kb*
	2	*a*	*b*

where a, b, and k are positive integers. Let A denote the event that a part is from supplier 1, and let B denote the event that a part conforms to specifications. Show that A and B are independent events. This exercise illustrates the result that whenever the rows of a table (with r rows and c columns) are proportional, an event defined by a row category and an event defined by a column category are independent.

The total sample size is $ka + a + kb + b = (k + 1)a + (k + 1)b$. Therefore

$$P(A) = \frac{k(a+b)}{(k+1)a+(k+1)b}, P(B) = \frac{ka+a}{(k+1)a+(k+1)b}$$

and

$$P(A \cap B) = \frac{ka}{(k+1)a+(k+1)b} = \frac{ka}{(k+1)(a+b)}$$

Then,

$$P(A)P(B) = \frac{k(a+b)(ka+a)}{[(k+1)a+(k+1)b]^2} = \frac{k(a+b)(k+1)a}{(k+1)^2(a+b)^2} = \frac{ka}{(k+1)(a+b)} = P(A \cap B)$$