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PREFACE

This *Complete Solutions Manual* contains solutions to all of the exercises in my textbook *Applied Calculus for the Managerial, Life, and Social Sciences: A Brief Approach, Tenth Edition*. The corresponding *Student Solutions Manual* contains solutions to the odd-numbered exercises and the even-numbered exercises in the "Before Moving On" quizzes. It also offers problem-solving tips for many sections.

I would like to thank Andy Bulman-Fleming for checking the accuracy of the answers to the new exercises in this edition of the text, rendering the art, and typesetting this manual. I also wish to thank my development editor Laura Wheel and my editor Rita Lombard of Cengage Learning for their help and support in bringing this supplement to market.

Please submit any errors in the solutions manual or suggestions for improvements to me in care of the publisher: Math Editorial, Cengage Learning, 20 Channel Center Street, Boston, MA, 02210.

Soo T. Tan

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1 PRELIMINARIES

1.1 Precalculus Review I

Exercises

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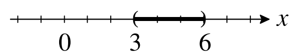
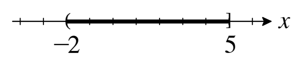
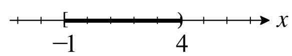
1. The interval $(3, 6)$ is shown on the number line below. Note that this is an open interval indicated by "(" and ")".



2. The interval $(-2, 5]$ is shown on the number line below.

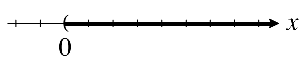


3. The interval $[-1, 4)$ is shown on the number line below. Note that this is a half-open interval indicated by "**[**'' (closed) and "**)**"(open).

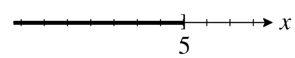


4. The closed interval $\left[-\frac{6}{5}, -\frac{1}{2}\right]$ is shown on the number line below.

5. The infinite interval $(0, \infty)$ is shown on the number line below.



6. The infinite interval $(-\infty, 5]$ is shown on the number line below.



7. $27^{2/3} = \left(3^3\right)^{2/3} = 3^2 = 9.$

8. $8^{-4/3} = \left(\dfrac{1}{8^{4/3}}\right) = \dfrac{1}{2^4} = \dfrac{1}{16}.$

9. $\left(\frac{1}{\sqrt{3}}\right)^0 = 1$. Recall that any number raised to the zeroth power is 1.

10. $\left(7^{1/2}\right)^4 = 7^{4/2} = 7^2 = 49.$

11. $\left[\left(\frac{1}{8}\right)^{1/3}\right]^{-2} = \left(\frac{1}{2}\right)^{-2} = \left(2^2\right) = 4.$

12. $\left[\left(-\frac{1}{3}\right)^2\right]^{-3} = \left(\frac{1}{9}\right)^{-3} = (9)^3 = 729.$

13. $\left(\dfrac{7^{-5} \cdot 7^2}{7^{-2}}\right)^{-1} = \left(7^{-5+2+2}\right)^{-1} = \left(7^{-1}\right)^{-1} = 7^1 = 7.$

14. $\left(\frac{9}{16}\right)^{-1/2} = \left(\frac{16}{9}\right)^{1/2} = \frac{4}{3}.$

15. $\left(125^{2/3}\right)^{-1/2} = 125^{(2/3)(-1/2)} = 125^{-1/3} = \dfrac{1}{125^{1/3}}$
$= \frac{1}{5}.$

16. $\sqrt[3]{2^6} = \left(2^6\right)^{1/3} = 2^{6(1/3)} = 2^2 = 4.$

17. $\dfrac{\sqrt{32}}{\sqrt{8}} = \sqrt{\dfrac{32}{8}} = \sqrt{4} = 2.$

18. $\sqrt[3]{\dfrac{-8}{27}} = \dfrac{\sqrt[3]{-8}}{\sqrt[3]{27}} = -\dfrac{2}{3}.$

19. $\dfrac{16^{5/8}16^{1/2}}{16^{7/8}} = 16^{(5/8)+(1/2)-(7/8)} = 16^{1/4} = 2.$

20. $\left(\dfrac{9^{-3}\cdot 9^{5}}{9^{-2}}\right)^{-1/2} = 9^{(-3+5+2)(-1/2)} = 9^{4(-1/2)} = \dfrac{1}{81}.$

21. $16^{1/4}\cdot 8^{-1/3} = 2\cdot\left(\frac{1}{8}\right)^{1/3} = 2\cdot\frac{1}{2} = 1.$

22. $\dfrac{6^{2.5}\cdot 6^{-1.9}}{6^{-1.4}} = 6^{2.5-1.9-(-1.4)} = 6^{2.5-1.9+1.4} = 6^{2}$

$= 36.$

23. True.

24. True. $3^2\times 2^2 = (3\times 2)^2 = 6^2 = 36.$

25. False. $x^3\times 2x^2 = 2x^{3+2} = 2x^5 \neq 2x^6.$

26. False. $3^3+3 = 27+3 = 30 \neq 3^4.$

27. False. $\dfrac{2^{4x}}{1^{3x}} = \dfrac{2^{4x}}{1} = 2^{4x}.$

28. True. $\left(2^2\times 3^2\right)^2 = (4\times 9)^2 = 36^2 = \left(6^2\right)^2 = 6^4.$

29. False. $\dfrac{1}{4^{-3}} = 4^3 = 64.$

30. True. $\dfrac{4^{3/2}}{2^4} = \dfrac{8}{16} = \dfrac{1}{2}.$

31. False. $\left(1.2^{1/2}\right)^{-1/2} = (1.2)^{-1/4} \neq 1.$

32. True.
$5^{2/3}\times 25^{2/3} = 5^{2/3}\left(5^2\right)^{2/3} = 5^{2/3}\times 5^{4/3} = 5^2 = 25.$

33. $(xy)^{-2} = \dfrac{1}{(xy)^2}.$

34. $3s^{1/3}\cdot s^{-7/3} = 3s^{(1/3)-(7/3)} = 3s^{-6/3} = 3s^{-2} = \dfrac{3}{s^2}.$

35. $\dfrac{x^{-1/3}}{x^{1/2}} = x^{(-1/3)-(1/2)} = x^{-5/6} = \dfrac{1}{x^{5/6}}.$

36. $\sqrt{x^{-1}}\cdot\sqrt{9x^{-3}} = x^{-1/2}\cdot 3x^{-3/2} = 3x^{(-1/2)+(-3/2)}$

$= 3x^{-2} = \dfrac{3}{x^2}.$

37. $12^0(s+t)^{-3} = 1\cdot\dfrac{1}{(s+t)^3} = \dfrac{1}{(s+t)^3}.$

38. $(x-y)\left(x^{-1}+y^{-1}\right) = (x-y)\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$

$= (x-y)\left(\dfrac{y+x}{xy}\right) = \dfrac{(x-y)(x+y)}{xy} = \dfrac{x^2-y^2}{xy}.$

39. $\dfrac{x^{7/3}}{x^{-2}} = x^{(7/3)+2} = x^{(7/3)+(6/3)} = x^{13/3}.$

40. $\left(49x^{-2}\right)^{-1/2} = (49)^{-1/2}x^{(-2)(-1/2)} = \frac{1}{7}x.$

41. $\left(x^2y^{-3}\right)\left(x^{-5}y^3\right) = x^{2-5}y^{-3+3} = x^{-3}y^0 = x^{-3} = \dfrac{1}{x^3}.$

42. $\dfrac{5x^6y^3}{2x^2y^7} = \frac{5}{2}x^{6-2}y^{3-7} = \frac{5}{2}x^4y^{-4} = \dfrac{5x^4}{2y^4}.$

43. $\dfrac{x^{3/4}}{x^{-1/4}} = x^{(3/4)-(-1/4)} = x^{4/4} = x.$

44. $\left(\dfrac{x^3y^2}{z^2}\right)^2 = \dfrac{x^{3\cdot 2}y^{2\cdot 2}}{z^{2(2)}} = \dfrac{x^6y^4}{z^4}.$

45. $\left(\dfrac{x^3}{-27y^{-6}}\right)^{-2/3} = x^{3(-2/3)}\left(-\dfrac{1}{27}\right)^{-2/3}y^{6(-2/3)}$

$= x^{-2}\left(-\dfrac{1}{3}\right)^{-2}y^{-4} = \dfrac{9}{x^2y^4}.$

46. $\left(\dfrac{e^x}{e^{x-2}}\right)^{-1/2} = e^{[x-(x-2)](-1/2)} = e^{-1} = \dfrac{1}{e}.$

47. $\left(\dfrac{x^{-3}}{y^{-2}}\right)^2\left(\dfrac{y}{x}\right)^4 = \dfrac{x^{-3\cdot 2}y^4}{y^{-2\cdot 2}x^4} = \dfrac{y^{4+4}}{x^{4+6}} = \dfrac{y^8}{x^{10}}.$

48. $\dfrac{(r^n)^4}{r^{5-2n}} = r^{4n-(5-2n)} = r^{4n+2n-5} = r^{6n-5}.$

49. $\sqrt[3]{x^{-2}} \cdot \sqrt{4x^5} = x^{-2/3} \cdot 4^{1/2} \cdot x^{5/2} = x^{(-2/3)+(5/2)} \cdot 2 = 2x^{11/6}$.

50. $\sqrt{81x^6y^{-4}} = (81)^{1/2} \cdot x^{6/2} \cdot y^{-4/2} = \dfrac{9x^3}{y^2}$.

51. $-\sqrt[4]{16x^4y^8} = -\left(16^{1/4} \cdot x^{4/4} \cdot y^{8/4}\right) = -2xy^2$.

52. $\sqrt[3]{x^{3a+b}} = x^{(3a+b)(1/3)} = x^{a+(b/3)}$.

53. $\sqrt[6]{64x^8y^3} = 64^{1/6} \cdot x^{8/6}y^{3/6} = 2x^{4/3}y^{1/2}$.

54. $\sqrt[3]{27r^6} \cdot \sqrt{s^2t^4} = 27^{1/3}\left(r^6\right)^{1/3}\left(s^2\right)^{1/2}\left(t^4\right)^{1/2} = 3r^2st^2$.

55. $2^{3/2} = 2\left(2^{1/2}\right) \approx 2\,(1.414) = 2.828$.

56. $8^{1/2} = \left(2^3\right)^{1/2} = 2^{3/2} = 2\left(2^{1/2}\right) \approx 2.828$.

57. $9^{3/4} = \left(3^2\right)^{3/4} = 3^{6/4} = 3^{3/2} = 3 \cdot 3^{1/2} \approx 3\,(1.732) = 5.196$.

58. $6^{1/2} = (2 \cdot 3)^{1/2} = 2^{1/2} \cdot 3^{1/2} \approx (1.414)\,(1.732) \approx 2.449$.

59. $10^{3/2} = 10^{1/2} \cdot 10 \approx (3.162)\,(10) = 31.62$.

60. $1000^{3/2} = \left(10^3\right)^{3/2} = 10^{9/2} = 10^4 \times 10^{1/2} \approx (10000)\,(3.162) = 31{,}620$.

61. $10^{2.5} = 10^2 \cdot 10^{1/2} \approx 100\,(3.162) = 316.2$.

62. $(0.0001)^{-1/3} = \left(10^{-4}\right)^{-1/3} = 10^{4/3} = 10 \cdot 10^{1/3} \approx 10\,(2.154) = 21.54$.

63. $\dfrac{3}{2\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{3\sqrt{x}}{2x}$.

64. $\dfrac{3}{\sqrt{xy}} \cdot \dfrac{\sqrt{xy}}{\sqrt{xy}} = \dfrac{3\sqrt{xy}}{xy}$.

65. $\dfrac{2y}{\sqrt{3y}} \cdot \dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{2y\sqrt{3y}}{3y} = \dfrac{2\sqrt{3y}}{3}$.

66. $\dfrac{5x^2}{\sqrt{3x}} \cdot \dfrac{\sqrt{3x}}{\sqrt{3x}} = \dfrac{5x^2\sqrt{3x}}{3x} = \dfrac{5x\sqrt{3x}}{3}$.

67. $\dfrac{1}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \dfrac{\sqrt[3]{x^2}}{x}$.

68. $\sqrt{\dfrac{2x}{y}} = \dfrac{\sqrt{2x}}{\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{\sqrt{2xy}}{y}$.

69. $\dfrac{2\sqrt{x}}{3} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{2x}{3\sqrt{x}}$.

70. $\dfrac{\sqrt[3]{x}}{24} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{x}{24\sqrt[3]{x^2}}$.

71. $\sqrt{\dfrac{2y}{x}} = \dfrac{\sqrt{2y}}{\sqrt{x}} \cdot \dfrac{\sqrt{2y}}{\sqrt{2y}} = \dfrac{2y}{\sqrt{2xy}}$.

72. $\sqrt[3]{\dfrac{2x}{3y}} = \dfrac{\sqrt[3]{2x}}{\sqrt[3]{3y}} \cdot \dfrac{\sqrt[3]{(2x)^2}}{\sqrt[3]{(2x)^2}} = \dfrac{2x}{\sqrt[3]{12x^2y}}$.

73. $\dfrac{\sqrt[3]{x^2z}}{y} \cdot \dfrac{\sqrt[3]{xz^2}}{\sqrt[3]{xz^2}} = \dfrac{\sqrt[3]{x^3z^3}}{y\sqrt[3]{xz^2}} = \dfrac{xz}{y\sqrt[3]{xz^2}}$.

74. $\dfrac{\sqrt[3]{x^2y}}{2x} \cdot \dfrac{\sqrt[3]{xy^2}}{\sqrt[3]{xy^2}} = \dfrac{xy}{2x\sqrt[3]{xy^2}} = \dfrac{y}{2\sqrt[3]{xy^2}}$.

75. $\left(7x^2 - 2x + 5\right) + \left(2x^2 + 5x - 4\right) = 7x^2 - 2x + 5 + 2x^2 + 5x - 4 = 9x^2 + 3x + 1$.

76. $\left(3x^2 + 5xy + 2y\right) + \left(4 - 3xy - 2x^2\right) = 3x^2 + 5xy + 2y + 4 - 3xy - 2x^2 = x^2 + 2xy + 2y + 4$.

77. $\left(5y^2 - 2y + 1\right) - \left(y^2 - 3y - 7\right) = 5y^2 - 2y + 1 - y^2 + 3y + 7 = 4y^2 + y + 8$.

78. $3\,(2a - b) - 4\,(b - 2a) = 6a - 3b - 4b + 8a = 14a - 7b = 7\,(2a - b)$.

79. $x - \{2x - [-x - (1 - x)]\} = x - \{2x - [-x - 1 + x]\} = x - (2x + 1) = x - 2x - 1 = -x - 1.$

80. $$\begin{aligned} 3x^2 - \left\{x^2 + 1 - x\,[x - (2x - 1)]\right\} + 2 &= 3x^2 - \left[x^2 + 1 - x\,(x - 2x + 1)\right] + 2 \\ &= 3x^2 - \left[x^2 + 1 - x\,(-x + 1)\right] + 2 = 3x^2 - \left(x^2 + 1 + x^2 - x\right) + 2 \\ &= 3x^2 - \left(2x^2 - x + 1\right) + 2 = x^2 - 1 + x + 2 = x^2 + x + 1. \end{aligned}$$

81. $\left(\dfrac{1}{3} - 1 + e\right) - \left(-\dfrac{1}{3} - 1 + e^{-1}\right) = \dfrac{1}{3} - 1 + e + \dfrac{1}{3} + 1 - \dfrac{1}{e} = \dfrac{2}{3} + e - \dfrac{1}{e} = \dfrac{3e^2 + 2e - 3}{3e}.$

82. $-\frac{3}{4}y - \frac{1}{4}x + 100 + \frac{1}{2}x + \frac{1}{4}y - 120 = -\frac{3}{4}y + \frac{1}{4}y - \frac{1}{4}x + \frac{1}{2}x + 100 - 120 = -\frac{1}{2}y + \frac{1}{4}x - 20.$

83. $3\sqrt{8} + 8 - 2\sqrt{y} + \frac{1}{2}\sqrt{x} - \frac{3}{4}\sqrt{y} = 3\sqrt{8} + 8 + \frac{1}{2}\sqrt{x} - \frac{11}{4}\sqrt{y} = 6\sqrt{2} + 8 + \frac{1}{2}\sqrt{x} - \frac{11}{4}\sqrt{y}.$

84. $$\begin{aligned} \tfrac{8}{9}x^2 + \tfrac{2}{3}x + \tfrac{16}{3}x^2 - \tfrac{16}{3}x - 2x + 2 &= \frac{8x^2 + 6x + 48x^2 - 48x - 18x + 18}{9} = \frac{56x^2 - 60x + 18}{9} \\ &= \tfrac{2}{9}\left(28x^2 - 30x + 9\right). \end{aligned}$$

85. $(x + 8)(x - 2) = x(x - 2) + 8(x - 2) = x^2 - 2x + 8x - 16 = x^2 + 6x - 16.$

86. $(5x + 2)(3x - 4) = 5x(3x - 4) + 2(3x - 4) = 15x^2 - 20x + 6x - 8 = 15x^2 - 14x - 8.$

87. $(a + 5)^2 = (a + 5)(a + 5) = a(a + 5) + 5(a + 5) = a^2 + 5a + 5a + 25 = a^2 + 10a + 25.$

88. $$\begin{aligned} (3a - 4b)^2 &= (3a - 4b)(3a - 4b) = 3a(3a - 4b) - 4b(3a - 4b) = 9a^2 - 12ab - 12ab + 16b^2 \\ &= 9a^2 - 24ab + 16b^2. \end{aligned}$$

89. $(x + 2y)^2 = (x + 2y)(x + 2y) = x(x + 2y) + 2y(x + 2y) = x^2 + 2xy + 2yx + 4y^2 = x^2 + 4xy + 4y^2.$

90. $(6 - 3x)^2 = (6 - 3x)(6 - 3x) = 6(6 - 3x) - 3x(6 - 3x) = 36 - 18x - 18x + 9x^2 = 36 - 36x + 9x^2.$

91. $(2x + y)(2x - y) = 2x(2x - y) + y(2x - y) = 4x^2 - 2xy + 2xy - y^2 = 4x^2 - y^2.$

92. $(3x + 2)(2 - 3x) = 3x(2 - 3x) + 2(2 - 3x) = 6x - 9x^2 + 4 - 6x = -9x^2 + 4.$

93. $\left(2x^2 - 1\right)\left(3x^2\right) + \left(x^2 + 3\right)(4x) = 6x^4 - 3x^2 + 4x^3 + 12x = 6x^4 + 4x^3 - 3x^2 + 12x = x\left(6x^3 + 4x^2 - 3x + 12\right).$

94. $\left(x^2 - 1\right)(2x) - x^2(2x) = 2x^3 - 2x - 2x^3 = -2x.$

95. $6x\left(\frac{1}{2}\right)\left(2x^2 + 3\right)^{-1/2}(4x) + 6\left(2x^2 + 3\right)^{1/2} = 3\left(2x^2 + 3\right)^{-1/2}\left[x(4x) + 2\left(2x^2 + 3\right)\right] = \dfrac{6\left(4x^2 + 3\right)}{\left(2x^2 + 3\right)^{1/2}}.$

96. $\left(x^{1/2} + 1\right)\left(\frac{1}{2}x^{-1/2}\right) - \left(x^{1/2} - 1\right)\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}\left[\left(x^{1/2} + 1\right) - \left(x^{1/2} - 1\right)\right] = \frac{1}{2}x^{-1/2}(2) = \dfrac{1}{x^{1/2}} = \dfrac{1}{\sqrt{x}}.$

97. $100\left(-10te^{-0.1t} - 100e^{-0.1t}\right) = -1000(10 + t)e^{-0.1t}.$

98. $$\begin{aligned} 2\left(t + \sqrt{t}\right)^2 - 2t^2 &= 2\left(t + \sqrt{t}\right)\left(t + \sqrt{t}\right) - 2t^2 = 2\left(t^2 + 2t\sqrt{t} + t\right) - 2t^2 = 2t^2 + 4t\sqrt{t} + 2t - 2t^2 \\ &= 4t\sqrt{t} + 2t = 2t\left(2\sqrt{t} + 1\right). \end{aligned}$$

99. $4x^5 - 12x^4 - 6x^3 = 2x^3\left(2x^2 - 6x - 3\right)$.

100. $4x^2y^2z - 2x^5y^2 + 6x^3y^2z^2 = 2x^2y^2\left(2z - x^3 + 3xz^2\right)$.

101. $7a^4 - 42a^2b^2 + 49a^3b = 7a^2\left(a^2 + 7ab - 6b^2\right)$.

102. $3x^{2/3} - 2x^{1/3} = x^{1/3}\left(3x^{1/3} - 2\right)$.

103. $e^{-x} - xe^{-x} = e^{-x}(1 - x)$.

104. $2ye^{xy^2} + 2xy^3e^{xy^2} = 2ye^{xy^2}\left(1 + xy^2\right)$.

105. $2x^{-5/2} - \frac{3}{2}x^{-3/2} = \frac{1}{2}x^{-5/2}(4 - 3x)$.

106. $\frac{1}{2}\left(\frac{2}{3}u^{3/2} - 2u^{1/2}\right) = \frac{1}{2}\cdot\frac{2}{3}u^{1/2}(u - 3) = \frac{1}{3}u^{1/2}(u - 3)$.

107. $6ac + 3bc - 4ad - 2bd = 3c(2a + b) - 2d(2a + b) = (2a + b)(3c - 2d)$.

108. $3x^3 - x^2 + 3x - 1 = x^2(3x - 1) + 1(3x - 1) = \left(x^2 + 1\right)(3x - 1)$.

109. $4a^2 - b^2 = (2a + b)(2a - b)$, a difference of two squares.

110. $12x^2 - 3y^2 = 3\left(4x^2 - y^2\right) = 3(2x + y)(2x - y)$.

111. $10 - 14x - 12x^2 = -2\left(6x^2 + 7x - 5\right) = -2(3x + 5)(2x - 1)$.

112. $x^2 - 2x - 15 = (x - 5)(x + 3)$.

113. $3x^2 - 6x - 24 = 3\left(x^2 - 2x - 8\right) = 3(x - 4)(x + 2)$.

114. $3x^2 - 4x - 4 = (3x + 2)(x - 2)$.

115. $12x^2 - 2x - 30 = 2\left(6x^2 - x - 15\right) = 2(3x - 5)(2x + 3)$.

116. $(x + y)^2 - 1 = (x + y - 1)(x + y + 1)$.

117. $9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x - 4y)(3x + 4y)$.

118. $8a^2 - 2ab - 6b^2 = 2\left(4a^2 - ab - 3b^2\right) = 2(a - b)(4a + 3b)$.

119. $x^6 + 125 = \left(x^2\right)^3 + (5)^3 = \left(x^2 + 5\right)\left(x^4 - 5x^2 + 25\right)$.

120. $x^3 - 27 = x^3 - 3^3 = (x - 3)\left(x^2 + 3x + 9\right)$.

121. $\left(x^2 + y^2\right)x - xy(2y) = x^3 + xy^2 - 2xy^2 = x^3 - xy^2$.

122. $2kr(R - r) - kr^2 = 2kRr - 2kr^2 - kr^2 = 2kRr - 3kr^2 = kr(2R - 3r)$.

123. $$\begin{aligned}2(x-1)(2x+2)^3[4(x-1)+(2x+2)] &= 2(x-1)(2x+2)^3(4x-4+2x+2)\\ &= 2(x-1)(2x+2)^3(6x-2) = 4(x-1)(3x-1)(2x+2)^3\\ &= 32(x-1)(3x-1)(x+1)^3.\end{aligned}$$

124. $5x^2\left(3x^2+1\right)^4(6x)+\left(3x^2+1\right)^5(2x) = (2x)\left(3x^2+1\right)^4\left[15x^2+\left(3x^2+1\right)\right] = 2x\left(3x^2+1\right)^4\left(18x^2+1\right).$

125. $$\begin{aligned}4(x-1)^2(2x+2)^3(2)+(2x+2)^4(2)(x-1) &= 2(x-1)(2x+2)^3[4(x-1)+(2x+2)]\\ &= 2(x-1)(2x+2)^3(6x-2) = 4(x-1)(3x-1)(2x+2)^3\\ &= 32(x-1)(3x-1)(x+1)^3.\end{aligned}$$

126. $$\begin{aligned}\left(x^2+1\right)\left(4x^3-3x^2+2x\right)-\left(x^4-x^3+x^2\right)(2x) &= 4x^5-3x^4+2x^3+4x^3-3x^2+2x-2x^5+2x^4-2x^3\\ &= 2x^5-x^4+4x^3-3x^2+2x.\end{aligned}$$

127. $\left(x^2+2\right)^2\left[5\left(x^2+2\right)^2-3\right](2x) = \left(x^2+2\right)^2\left[5\left(x^4+4x^2+4\right)-3\right](2x) = (2x)\left(x^2+2\right)^2\left(5x^4+20x^2+17\right).$

128. $$\begin{aligned}\left(x^2-4\right)\left(x^2+4\right)(2x+8)-\left(x^2+8x-4\right)\left(4x^3\right) &= \left(x^4-16\right)(2x+8)-4x^5-32x^4+16x^3\\ &= 2x^5+8x^4-32x-128-4x^5-32x^4+16x^3 = -2x^5-24x^4+16x^3-32x-128\\ &= -2\left(x^5+12x^4-8x^3+16x+64\right).\end{aligned}$$

129. We factor the left-hand side of $x^2+x-12=0$ to obtain $(x+4)(x-3)=0$, so $x=-4$ or $x=3$. We conclude that the roots are $x=-4$ and $x=3$.

130. We factor the left-hand side of $3x^2-x-4=0$ to obtain $(3x-4)(x+1)=0$. Thus, $3x=4$ or $x=-1$, and we conclude that the roots are $x=\frac{4}{3}$ and $x=-1$.

131. $4t^2+2t-2=(2t-1)(2t+2)=0$. Thus, the roots are $t=\frac{1}{2}$ and $t=-1$.

132. $-6x^2+x+12=(3x+4)(-2x+3)=0$. Thus, $x=-\frac{4}{3}$ and $x=\frac{3}{2}$ are the roots of the equation.

133. $\frac{1}{4}x^2-x+1=\left(\frac{1}{2}x-1\right)\left(\frac{1}{2}x-1\right)=0$. Thus $\frac{1}{2}x=1$, and so $x=2$ is a double root of the equation.

134. $\frac{1}{2}a^2+a-12=a^2+2a-24=(a+6)(a-4)=0$. Thus, $a=-6$ and $a=4$ are the roots of the equation.

135. We use the quadratic formula to solve the equation $4x^2+5x-6=0$. In this case, $a=4$, $b=5$, and $c=-6$. Therefore, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-5\pm\sqrt{5^2-4(4)(-6)}}{2(4)}=\dfrac{-5\pm\sqrt{121}}{8}=\dfrac{-5\pm 11}{8}$. Thus, $x=-\frac{16}{8}=-2$ and $x=\frac{6}{8}=\frac{3}{4}$ are the roots of the equation.

136. We use the quadratic formula to solve the equation $3x^2-4x+1=0$. Here $a=3$, $b=-4$, and $c=1$, so $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-4)\pm\sqrt{(-4)^2-4(3)(1)}}{2(3)}=\dfrac{4\pm\sqrt{4}}{6}$. Thus, $x=\frac{6}{6}=1$ and $x=\frac{2}{6}=\frac{1}{3}$ are the roots of the equation.

137. We use the quadratic formula to solve the equation $8x^2 - 8x - 3 = 0$. Here $a = 8$, $b = -8$, and $c = -3$, so $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(8)(-3)}}{2(8)} = \frac{8 \pm \sqrt{160}}{16} = \frac{8 \pm 4\sqrt{10}}{16} = \frac{2 \pm \sqrt{10}}{4}$. Thus, $x = \frac{1}{2} + \frac{1}{4}\sqrt{10}$ and $x = \frac{1}{2} - \frac{1}{4}\sqrt{10}$ are the roots of the equation.

138. We use the quadratic formula to solve the equation $x^2 - 6x + 6 = 0$. Here $a = 1$, $b = -6$, and $c = 6$. Therefore, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(6)}}{2(1)} = \frac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3}$. Thus, the roots are $3 + \sqrt{3}$ and $3 - \sqrt{3}$.

139. We use the quadratic formula to solve $2x^2 + 4x - 3 = 0$. Here $a = 2$, $b = 4$, and $c = -3$, so $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$. Thus, $x = -1 + \frac{1}{2}\sqrt{10}$ and $x = -1 - \frac{1}{2}\sqrt{10}$ are the roots of the equation.

140. We use the quadratic formula to solve the equation $2x^2 + 7x - 15 = 0$. Then $a = 2$, $b = 7$, and $c = -15$. Therefore, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)} = \frac{-7 \pm \sqrt{169}}{4} = \frac{-7 \pm 13}{4}$. We conclude that $x = \frac{3}{2}$ and $x = -5$ are the roots of the equation.

141. The total revenue is given by $\left(0.2t^2 + 150t\right) + \left(0.5t^2 + 200t\right) = 0.7t^2 + 350t$ thousand dollars t months from now, where $0 \le t \le 12$.

142. In month t, the revenue of the second gas station will exceed that of the first gas station by $\left(0.5t^2 + 200t\right) - \left(0.2t^2 + 150t\right) = 0.3t^2 + 50t$ thousand dollars, where $0 < t \le 12$.

143. **a.** $f(30{,}000) = \left(5.6 \times 10^{11}\right)(30{,}000)^{-1.5} \approx 107{,}772$, or 107,772 families.

b. $f(60{,}000) = \left(5.6 \times 10^{11}\right)(60{,}000)^{-1.5} \approx 38{,}103$, or 38,103 families.

c. $f(150{,}000) = \left(5.6 \times 10^{11}\right)(150{,}000)^{-1.5} \approx 9639$, or 9639 families.

144. $-t^3 + 6t^2 + 15t = -t\left(t^2 - 6t - 15\right)$.

145. $8000x - 100x^2 = 100x(80 - x)$.

146. True. The two real roots are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

147. True. If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is not a real number.

148. True, because $(a + b)(b - a) = b^2 - a^2$.

1.2 Precalculus Review II

Exercises page 23

1. $\dfrac{x^2+x-2}{x^2-4}=\dfrac{(x+2)(x-1)}{(x+2)(x-2)}=\dfrac{x-1}{x-2}.$

2. $\dfrac{2a^2-3ab-9b^2}{2ab^2+3b^3}=\dfrac{(2a+3b)(a-3b)}{b^2(2a+3b)}=\dfrac{a-3b}{b^2}.$

3. $\dfrac{12t^2+12t+3}{4t^2-1}=\dfrac{3\left(4t^2+4t+1\right)}{4t^2-1}=\dfrac{3(2t+1)(2t+1)}{(2t+1)(2t-1)}=\dfrac{3(2t+1)}{2t-1}.$

4. $\dfrac{x^3+2x^2-3x}{-2x^2-x+3}=\dfrac{x\left(x^2+2x-3\right)}{-\left(2x^2+x-3\right)}=\dfrac{x(x+3)(x-1)}{-(2x+3)(x-1)}=-\dfrac{x(x+3)}{2x+3}.$

5. $\dfrac{(4x-1)(3)-(3x+1)(4)}{(4x-1)^2}=\dfrac{12x-3-12x-4}{(4x-1)^2}=-\dfrac{7}{(4x-1)^2}.$

6. $\dfrac{\left(1+x^2\right)^2(2)-2x(2)\left(1+x^2\right)(2x)}{\left(1+x^2\right)^4}=\dfrac{\left(1+x^2\right)(2)\left(1+x^2-4x^2\right)}{\left(1+x^2\right)^4}=\dfrac{\left(1+x^2\right)(2)\left(-3x^2+1\right)}{\left(1+x^2\right)^4}=\dfrac{2\left(1-3x^2\right)}{\left(1+x^2\right)^3}.$

7. $\dfrac{2a^2-2b^2}{b-a}\cdot\dfrac{4a+4b}{a^2+2ab+b^2}=\dfrac{2(a+b)(a-b)4(a+b)}{-(a-b)(a+b)(a+b)}=-8.$

8. $\dfrac{x^2-6x+9}{x^2-x-6}\cdot\dfrac{3x+6}{2x^2-7x+3}=\dfrac{3(x-3)^2(x+2)}{(x-3)(x+2)(2x-1)(x-3)}=\dfrac{3}{2x-1}.$

9. $\dfrac{3x^2+2x-1}{2x+6}\div\dfrac{x^2-1}{x^2+2x-3}=\dfrac{(3x-1)(x+1)}{2(x+3)}\cdot\dfrac{(x+3)(x-1)}{(x+1)(x-1)}=\dfrac{3x-1}{2}.$

10. $\dfrac{3x^2-4xy-4y^2}{x^2y}\div\dfrac{(2y-x)^2}{x^3y}=\dfrac{(3x+2y)(x-2y)}{x^2y}\cdot\dfrac{x^3y}{(2y-x)(2y-x)}=\dfrac{x(3x+2y)}{x-2y}.$

11. $\dfrac{58}{3(3t+2)}+\dfrac{1}{3}=\dfrac{58+3t+2}{3(3t+2)}=\dfrac{3t+60}{3(3t+2)}=\dfrac{t+20}{3t+2}.$

12. $\dfrac{a+1}{3a}+\dfrac{b-2}{5b}=\dfrac{5b(a+1)+3a(b-2)}{15ab}=\dfrac{5ab+5b+3ab-6a}{15ab}=\dfrac{-6a+8ab+5b}{15ab}.$

13. $\dfrac{2x}{2x-1}-\dfrac{3x}{2x+5}=\dfrac{2x(2x+5)-3x(2x-1)}{(2x-1)(2x+5)}=\dfrac{4x^2+10x-6x^2+3x}{(2x-1)(2x+5)}=\dfrac{-2x^2+13x}{(2x-1)(2x+5)}$
$=-\dfrac{x(2x-13)}{(2x-1)(2x+5)}.$

14. $\dfrac{-xe^x}{x+1}+e^x=\dfrac{-xe^x+(x+1)e^x}{x+1}=\dfrac{-xe^x+xe^x+e^x}{x+1}=\dfrac{e^x}{x+1}.$

15. $\dfrac{4}{x^2-9}-\dfrac{5}{x^2-6x+9}=\dfrac{4}{(x+3)(x-3)}-\dfrac{5}{(x-3)^2}=\dfrac{4(x-3)-5(x+3)}{(x-3)^2(x+3)}=-\dfrac{x+27}{(x-3)^2(x+3)}.$

16. $\dfrac{x}{1-x}+\dfrac{2x+3}{x^2-1}=\dfrac{-x\,(x+1)+2x+3}{(x+1)\,(x-1)}=\dfrac{-x^2-x+2x+3}{x^2-1}=-\dfrac{x^2-x-3}{x^2-1}.$

17. $\dfrac{1+\dfrac{1}{x}}{1-\dfrac{1}{x}}=\dfrac{\dfrac{x+1}{x}}{\dfrac{x-1}{x}}=\dfrac{x+1}{x}\cdot\dfrac{x}{x-1}=\dfrac{x+1}{x-1}.$

18. $\dfrac{\dfrac{1}{x}+\dfrac{1}{y}}{1-\dfrac{1}{xy}}=\dfrac{\dfrac{x+y}{xy}}{\dfrac{xy-1}{xy}}=\dfrac{x+y}{xy}\cdot\dfrac{xy}{xy-1}=\dfrac{x+y}{xy-1}.$

19. $\dfrac{4x^2}{2\sqrt{2x^2+7}}+\sqrt{2x^2+7}=\dfrac{4x^2+2\sqrt{2x^2+7}\sqrt{2x^2+7}}{2\sqrt{2x^2+7}}=\dfrac{4x^2+4x^2+14}{2\sqrt{2x^2+7}}=\dfrac{4x^2+7}{\sqrt{2x^2+7}}.$

20. $6\,(2x+1)^2\sqrt{x^2+x}+\dfrac{(2x+1)^4}{2\sqrt{x^2+x}}=\dfrac{6\,(2x+1)^2\sqrt{x^2+x}\,(2)\sqrt{x^2+x}+(2x+1)^4}{2\sqrt{x^2+x}}$

$=\dfrac{(2x+1)^2\left[12\left(x^2+x\right)+4x^2+4x+1\right]}{2\sqrt{x^2+x}}=\dfrac{(2x+1)^2\left(16x^2+16x+1\right)}{2\sqrt{x^2+x}}.$

21. $5\left[\dfrac{\left(t^2+1\right)(1)-t\,(2t)}{\left(t^2+1\right)^2}\right]=\dfrac{5\left(t^2+1-2t^2\right)}{\left(t^2+1\right)^2}=\dfrac{5\left(1-t^2\right)}{\left(t^2+1\right)^2}=-\dfrac{5\left(t^2-1\right)}{\left(t^2+1\right)^2}.$

22. $\dfrac{2x\,(x+1)^{-1/2}-(x+1)^{1/2}}{x^2}=\dfrac{(x+1)^{-1/2}\,(2x-x-1)}{x^2}=\dfrac{(x+1)^{-1/2}\,(x-1)}{x^2}=\dfrac{x-1}{x^2\sqrt{x+1}}.$

23. $\dfrac{\left(x^2+1\right)^2(-2)+(2x)\,2\left(x^2+1\right)(2x)}{\left(x^2+1\right)^4}=\dfrac{2\left(x^2+1\right)\left[-\left(x^2+1\right)+4x^2\right]}{\left(x^2+1\right)^4}=\dfrac{2\left(3x^2-1\right)}{\left(x^2+1\right)^3}.$

24. $\dfrac{\left(x^2+1\right)^{1/2}-2x^2\left(x^2+1\right)^{-1/2}}{1-x^2}=\dfrac{\left(x^2+1\right)^{-1/2}\left(x^2+1-2x^2\right)}{1-x^2}=\dfrac{\left(x^2+1\right)^{-1/2}\left(-x^2+1\right)}{1-x^2}=\dfrac{1}{\sqrt{x^2+1}}.$

25. $3\left(\dfrac{2x+1}{3x+2}\right)^2\left[\dfrac{(3x+2)\,(2)-(2x+1)\,(3)}{(3x+2)^2}\right]=\dfrac{3\,(2x+1)^2\,(6x+4-6x-3)}{(3x+2)^4}=\dfrac{3\,(2x+1)^2}{(3x+2)^4}.$

26. $\dfrac{(2x+1)^{1/2}-(x+2)\,(2x+1)^{-1/2}}{2x+1}=\dfrac{(2x+1)^{-1/2}\,(2x+1-x-2)}{2x+1}=\dfrac{(2x+1)^{-1/2}\,(x-1)}{2x+1}=\dfrac{x-1}{(2x+1)^{3/2}}.$

27. $100\left[\dfrac{\left(t^2+20t+100\right)(2t+10)-\left(t^2+10t+100\right)(2t+20)}{\left(t^2+20t+100\right)^2}\right]$

$=100\left[\dfrac{2t^3+40t^2+200t+10t^2+200t+1000-2t^3-20t^2-200t-20t^2-200t-2000}{\left(t^2+20t+100\right)^2}\right]$

$=\dfrac{100\left(10t^2-1000\right)}{\left(t^2+20t+100\right)^2}=\dfrac{1000\,(t-10)}{(t+10)^3}.$

28. $$\frac{2(2x-3)^{1/3}-(x-1)(2x-3)^{-2/3}}{(2x-3)^{2/3}} = \frac{(2x-3)^{-2/3}[2(2x-3)-(x-1)]}{(2x-3)^{2/3}} = \frac{(2x-3)^{-2/3}(4x-6-x+1)}{(2x-3)^{2/3}} = \frac{3x-5}{(2x-3)^{4/3}}.$$

29. $\dfrac{1}{\sqrt{3}-1}\cdot\dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{\sqrt{3}+1}{3-1} = \dfrac{\sqrt{3}+1}{2}.$

30. $\dfrac{1}{\sqrt{x}+5}\cdot\dfrac{\sqrt{x}-5}{\sqrt{x}-5} = \dfrac{\sqrt{x}-5}{x-25}.$

31. $\dfrac{1}{\sqrt{x}-\sqrt{y}}\cdot\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \dfrac{\sqrt{x}+\sqrt{y}}{x-y}.$

32. $\dfrac{a}{1-\sqrt{a}}\cdot\dfrac{1+\sqrt{a}}{1+\sqrt{a}} = \dfrac{a\left(1+\sqrt{a}\right)}{1-a}.$

33. $\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\cdot\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \dfrac{\left(\sqrt{a}+\sqrt{b}\right)^2}{a-b}.$

34. $\dfrac{2\sqrt{a}+\sqrt{b}}{2\sqrt{a}-\sqrt{b}}\cdot\dfrac{2\sqrt{a}+\sqrt{b}}{2\sqrt{a}+\sqrt{b}} = \dfrac{\left(2\sqrt{a}+\sqrt{b}\right)^2}{4a-b}.$

35. $\dfrac{\sqrt{x}}{3}\cdot\dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{x}{3\sqrt{x}}.$

36. $\dfrac{\sqrt[3]{y}}{x}\cdot\dfrac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \dfrac{y}{x\sqrt[3]{y^2}}.$

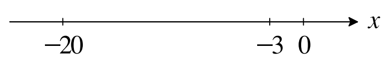
37. $\dfrac{1-\sqrt{3}}{3}\cdot\dfrac{1+\sqrt{3}}{1+\sqrt{3}} = \dfrac{1^2-\left(\sqrt{3}\right)^2}{3\left(1+\sqrt{3}\right)} = -\dfrac{2}{3\left(1+\sqrt{3}\right)}.$

38. $\dfrac{\sqrt{x}-1}{x}\cdot\dfrac{\sqrt{x}+1}{\sqrt{x}+1} = \dfrac{x-1}{x\left(\sqrt{x}+1\right)}.$

39. $\dfrac{1+\sqrt{x+2}}{\sqrt{x+2}}\cdot\dfrac{1-\sqrt{x+2}}{1-\sqrt{x+2}} = \dfrac{1-(x+2)}{\sqrt{x+2}\left(1-\sqrt{x+2}\right)} = -\dfrac{x+1}{\sqrt{x+2}\left(1-\sqrt{x+2}\right)}.$

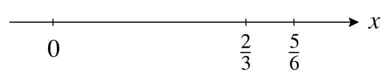
40. $\dfrac{\sqrt{x+3}-\sqrt{x}}{3}\cdot\dfrac{\sqrt{x+3}+\sqrt{x}}{\sqrt{x+3}+\sqrt{x}} = \dfrac{x+3-x}{3\left(\sqrt{x+3}+\sqrt{x}\right)} = \dfrac{1}{\sqrt{x+3}+\sqrt{x}}.$

41. The statement is false because -3 is greater than -20. See the number line below.



42. The statement is true because -5 is equal to -5.

43. The statement is false because $\frac{2}{3}=\frac{4}{6}$ is less than $\frac{5}{6}$.



44. The statement is false because $-\frac{5}{6}=-\frac{10}{12}$ is greater than $-\frac{11}{12}$.

45. We are given $2x+4<8$. Add -4 to each side of the inequality to obtain $2x<4$, then multiply each side of the inequality by $\frac{1}{2}$ to obtain $x<2$. We write this in interval notation as $(-\infty, 2)$.

46. We are given $-6>4+5x$. Add -4 to each side of the inequality to obtain $-6-4>5x$, so $-10>5x$. Dividing by 2, we obtain $-2>x$, so $x<-2$. We write this in interval notation as $(-\infty,-2)$.

47. We are given the inequality $-4x\geq 20$. Multiply both sides of the inequality by $-\frac{1}{4}$ and reverse the sign of the inequality to obtain $x\leq -5$. We write this in interval notation as $(-\infty,-5]$.

48. $-12 \le -3x \Rightarrow 4 \ge x$, or $x \le 4$. We write this in interval notation as $(-\infty, 4]$.

49. We are given the inequality $-6 < x - 2 < 4$. First add 2 to each member of the inequality to obtain $-6 + 2 < x < 4 + 2$ and $-4 < x < 6$, so the solution set is the open interval $(-4, 6)$.

50. We add -1 to each member of the given double inequality $0 \le x + 1 \le 4$ to obtain $-1 \le x \le 3$, and the solution set is $[-1, 3]$.

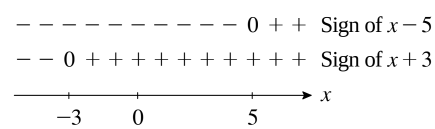
51. We want to find the values of x that satisfy at least one of the inequalities $x + 1 > 4$ and $x + 2 < -1$. Adding -1 to both sides of the first inequality, we obtain $x + 1 - 1 > 4 - 1$, so $x > 3$. Similarly, adding -2 to both sides of the second inequality, we obtain $x + 2 - 2 < -1 - 2$, so $x < -3$. Therefore, the solution set is $(-\infty, -3) \cup (3, \infty)$.

52. We want to find the values of x that satisfy at least one of the inequalities $x + 1 > 2$ and $x - 1 < -2$. Solving these inequalities, we find that $x > 1$ or $x < -1$, and the solution set is $(-\infty, -1) \cup (1, \infty)$.

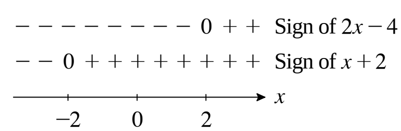
53. We want to find the values of x that satisfy the inequalities $x + 3 > 1$ and $x - 2 < 1$. Adding -3 to both sides of the first inequality, we obtain $x + 3 - 3 > 1 - 3$, or $x > -2$. Similarly, adding 2 to each side of the second inequality, we obtain $x - 2 + 2 < 1 + 2$, so $x < 3$. Because both inequalities must be satisfied, the solution set is $(-2, 3)$.

54. We want to find the values of x that satisfy the inequalities $x - 4 \le 1$ and $x + 3 > 2$. Solving these inequalities, we find that $x \le 5$ and $x > -1$, and the solution set is $(-1, 5]$.

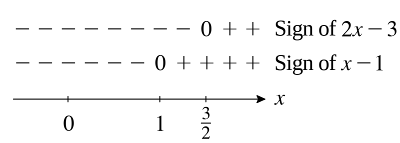
55. We want to find the values of x that satisfy the inequality $(x + 3)(x - 5) \le 0$. From the sign diagram, we see that the given inequality is satisfied when $-3 \le x \le 5$, that is, when the signs of the two factors are different or when one of the factors is equal to zero.



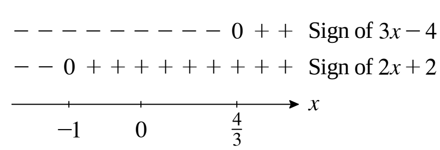
56. We want to find the values of x that satisfy the inequality $(2x - 4)(x + 2) \ge 0$. From the sign diagram, we see that the given inequality is satisfied when $x \le -2$ or $x \ge 2$; that is, when the signs of both factors are the same or one of the factors is equal to zero.



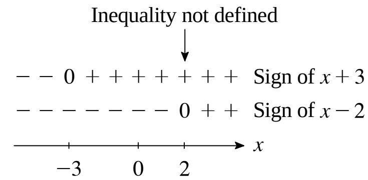
57. We want to find the values of x that satisfy the inequality $(2x - 3)(x - 1) \ge 0$. From the sign diagram, we see that the given inequality is satisfied when $x \le 1$ or $x \ge \frac{3}{2}$; that is,when the signs of both factors are the same, or one of the factors is equal to zero.



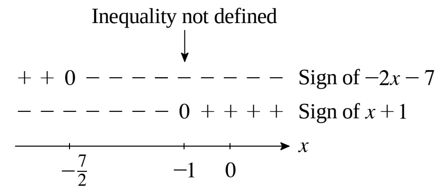
58. We want to find the values of x that satisfy the inequalities $(3x - 4)(2x + 2) \le 0$.
From the sign diagram, we see that the given inequality is satisfied when $-1 \le x \le \frac{4}{3}$, that is, when the signs of the two factors are different or when one of the factors is equal to zero.



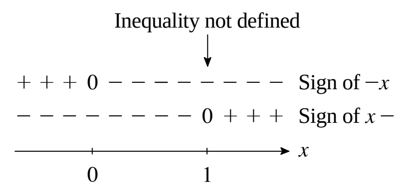
59. We want to find the values of x that satisfy the inequalities $\dfrac{x+3}{x-2} \geq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq -3$ or $x > 2$, that is, when the signs of the two factors are the same. Notice that $x = 2$ is not included because the inequality is not defined at that value of x.



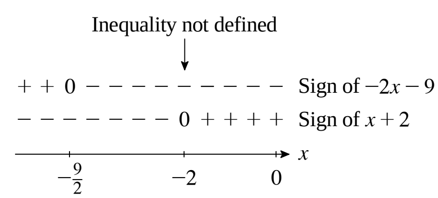
60. We want to find the values of x that satisfy the inequality $\dfrac{2x-3}{x+1} \geq 4$. We rewrite the inequality as $\dfrac{2x-3}{x+1} - 4 \geq 0$, $\dfrac{2x-3-4x-4}{x+1} \geq 0$, and $\dfrac{-2x-7}{x+1} \geq 0$. From the sign diagram, we see that the given inequality is satisfied when $-\frac{7}{2} \leq x < -1$; that is, when the signs of the two factors are the same. Notice that $x = -1$ is not included because the inequality is not defined at that value of x.



61. We want to find the values of x that satisfy the inequality $\dfrac{x-2}{x-1} \leq 2$. Subtracting 2 from each side of the given inequality and simplifying gives $\dfrac{x-2}{x-1} - 2 \leq 0$, $\dfrac{x-2-2(x-1)}{x-1} \leq 0$, and $-\dfrac{x}{x-1} \leq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq 0$ or $x > 1$; that is, when the signs of the two factors differ. Notice that $x = 1$ is not included because the inequality is undefined at that value of x.



62. We want to find the values of x that satisfy the inequality $\dfrac{2x-1}{x+2} \leq 4$. Subtracting 4 from each side of the given inequality and simplifying gives $\dfrac{2x-1}{x+2} - 4 \leq 0$, $\dfrac{2x-1-4(x+2)}{x+2} \leq 0$, $\dfrac{2x-1-4x-8}{x+2} \leq 0$, and finally $\dfrac{-2x-9}{x+2} \leq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq -\frac{9}{2}$ or $x > -2$.



63. $|-6+2| = 4$.

64. $4 + |-4| = 4 + 4 = 8$.

65. $\dfrac{|-12+4|}{|16-12|} = \dfrac{|-8|}{|4|} = 2$.

66. $\left|\dfrac{0.2-1.4}{1.6-2.4}\right| = \left|\dfrac{-1.2}{-0.8}\right| = 1.5$.

67. $\sqrt{3}\,|-2| + 3\left|-\sqrt{3}\right| = \sqrt{3}\,(2) + 3\sqrt{3} = 5\sqrt{3}$.

68. $|-1| + \sqrt{2}\,|-2| = 1 + 2\sqrt{2}$.

69. $|\pi - 1| + 2 = \pi - 1 + 2 = \pi + 1$.

70. $|\pi - 6| - 3 = 6 - \pi - 3 = 3 - \pi$.

71. $\left|\sqrt{2}-1\right| + \left|3-\sqrt{2}\right| = \sqrt{2} - 1 + 3 - \sqrt{2} = 2$.

72. $\left|2\sqrt{3}-3\right|-\left|\sqrt{3}-4\right|=2\sqrt{3}-3-\left(4-\sqrt{3}\right)=3\sqrt{3}-7.$

73. False. If $a > b$, then $-a < -b$, $-a+b < -b+b$, and $b-a < 0$.

74. False. Let $a=-2$ and $b=-3$. Then $a/b=\frac{-2}{-3}=\frac{2}{3}<1$.

75. False. Let $a=-2$ and $b=-3$. Then $a^2=4$ and $b^2=9$, and $4<9$. Note that we need only to provide a counterexample to show that the statement is not always true.

76. False. Let $a=-2$ and $b=-3$. Then $\dfrac{1}{a}=-\dfrac{1}{2}$ and $\dfrac{1}{b}=-\dfrac{1}{3}$, and $-\dfrac{1}{2}<-\dfrac{1}{3}$.

77. True. There are three possible cases.
Case 1: If $a>0$ and $b>0$, then $a^3>b^3$, since $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)>0$.
Case 2: If $a>0$ and $b<0$, then $a^3>0$ and $b^3<0$, and it follows that $a^3>b^3$.
Case 3: If $a<0$ and $b<0$, then $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)>0$, and we see that $a^3>b^3$. (Note that $a-b>0$ and $ab>0$.)

78. True. If $a>b$, then it follows that $-a<-b$ because an inequality symbol is reversed when both sides of the inequality are multiplied by a negative number.

79. False. If we take $a=-2$, then $|-a|=|-(-2)|=|2|=2\neq a$.

80. True. If $b<0$, then $b^2>0$, and $\left|b^2\right|=b^2$.

81. True. If $a-4<0$, then $|a-4|=4-a=|4-a|$. If $a-4>0$, then $|4-a|=a-4=|a-4|$.

82. False. If we let $a=-2$, then $|a+1|=|-2+1|=|-1|=1\neq|-2|+1=3$.

83. False. If we take $a=3$ and $b=-1$, then $|a+b|=|3-1|=2\neq|a|+|b|=3+1=4$.

84. False. If we take $a=3$ and $b=-1$, then $|a-b|=4\neq|a|-|b|=3-1=2$.

85. If the car is driven in the city, then it can be expected to cover $(18.1)(20)=362\ \frac{\text{miles}}{\text{gal}}\cdot\text{gal}$, or 362 miles, on a full tank. If the car is driven on the highway, then it can be expected to cover $(18.1)(27)=488.7\ \frac{\text{miles}}{\text{gal}}\cdot\text{gal}$, or 488.7 miles, on a full tank. Thus, the driving range of the car may be described by the interval $[362, 488.7]$.

86. Simplifying $5(C-25)\geq 1.75+2.5C$, we obtain $5C-125\geq 1.75+2.5C$, $5C-2.5C\geq 1.75+125$, $2.5C\geq 126.75$, and finally $C\geq 50.7$. Therefore, the minimum cost is \$50.70.

87. $6(P-2500)\leq 4(P+2400)$ can be rewritten as $6P-15{,}000\leq 4P+9600$, $2P\leq 24{,}600$, or $P\leq 12{,}300$. Therefore, the maximum profit is \$12,300.

88. **a.** We want to find a formula for converting Centigrade temperatures to Fahrenheit temperatures. Thus, $C=\frac{5}{9}(F-32)=\frac{5}{9}F-\frac{160}{9}$. Therefore, $\frac{5}{9}F=C+\frac{160}{9}$, $5F=9C+160$, and $F=\frac{9}{5}C+32$. Calculating the lower temperature range, we have $F=\frac{9}{5}(-15)+32=5$, or 5 degrees. Calculating the upper temperature range, $F=\frac{9}{5}(-5)+32=23$, or 23 degrees. Therefore, the temperature range is $5^\circ<F<23^\circ$.

b. For the lower temperature range, $C = \frac{5}{9}(63 - 32) = \frac{155}{9} \approx 17.2$, or 17.2 degrees. For the upper temperature range, $C = \frac{5}{9}(80 - 32) = \frac{5}{9}(48) \approx 26.7$, or 26.7 degrees. Therefore, the temperature range is $17.2^\circ < C < 26.7^\circ$.

89. Let x represent the salesman's monthly sales in dollars. Then $0.15(x - 12{,}000) \geq 6000$, $15(x - 12{,}000) \geq 600{,}000$, $15x - 180{,}000 \geq 600{,}000$, $15x \geq 780{,}000$, and $x \geq 52{,}000$. We conclude that the salesman must have sales of at least \$52,000 to reach his goal.

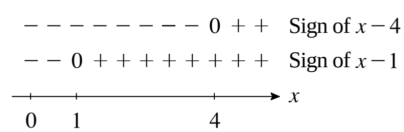
90. Let x represent the wholesale price of the car. Then $\dfrac{\text{Selling price}}{\text{Wholesale price}} - 1 \geq \text{Markup}$; that is, $\dfrac{11{,}200}{x} - 1 \geq 0.30$, whence $\dfrac{11{,}200}{x} \geq 1.30$, $1.3x \leq 11{,}200$, and $x \leq 8615.38$. We conclude that the maximum wholesale price is \$8615.38.

91. The rod is acceptable if $0.49 \leq x \leq 0.51$ or $-0.01 \leq x - 0.5 \leq 0.01$. This gives the required inequality, $|x - 0.5| \leq 0.01$.

92. $|x - 0.1| \leq 0.01$ is equivalent to $-0.01 \leq x - 0.1 \leq 0.01$ or $0.09 \leq x \leq 0.11$. Therefore, the smallest diameter a ball bearing in the batch can have is 0.09 inch, and the largest diameter is 0.11 inch.

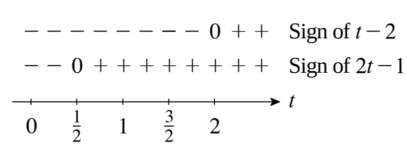
93. We want to solve the inequality $-6x^2 + 30x - 10 \geq 14$. (Remember that x is expressed in thousands.) Adding -14 to both sides of this inequality, we have $-6x^2 + 30x - 10 - 14 \geq 14 - 14$, or $-6x^2 + 30x - 24 \geq 0$. Dividing both sides of the inequality by -6 (which reverses the sign of the inequality), we have $x^2 - 5x + 4 \leq 0$. Factoring this last expression, we have $(x - 4)(x - 1) \leq 0$.

From the sign diagram, we see that x must lie between 1 and 4. (The inequality is satisfied only when the two factors have opposite signs.) Because x is expressed in thousands of units, we see that the manufacturer must produce between 1000 and 4000 units of the commodity.



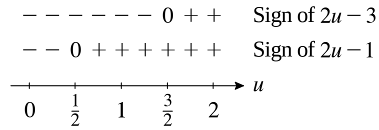
94. We solve the inequality $\dfrac{0.2t}{t^2 + 1} \geq 0.08$, obtaining $0.08t^2 + 0.08 \leq 0.2t$, $0.08t^2 - 0.2t + 0.08 \leq 0$, $2t^2 - 5t + 2 \leq 0$, and $(2t - 1)(t - 2) \leq 0$.

From the sign diagram, we see that the required solution is $\left[\frac{1}{2}, 2\right]$, so the concentration of the drug is greater than or equal to 0.08 mg/cc between $\frac{1}{2}$ hr and 2 hr after injection.



95. We solve the inequalities $25 \leq \dfrac{0.5x}{100 - x} \leq 30$, obtaining $2500 - 25x \leq 0.5x \leq 3000 - 30x$, which is equivalent to $2500 - 25x \leq 0.5x$ and $0.5x \leq 3000 - 30x$. Simplifying further, $25.5x \geq 2500$ and $30.5x \leq 3000$, so $x \geq \dfrac{2500}{25.5} \approx 98.04$ and $x \leq \dfrac{3000}{30.5} \approx 98.36$. Thus, the city could expect to remove between 98.04% and 98.36% of the toxic pollutant.

96. We simplify the inequality $20t - 40\sqrt{t} + 50 \le 35$ to $20t - 40\sqrt{t} + 15 \le 0$ (1). Let $u = \sqrt{t}$. Then $u^2 = t$, so we have $20u^2 - 40u + 15 \le 0$, $4u^2 - 8u + 3 \le 0$, and $(2u - 3)(2u - 1) \le 0$.
From the sign diagram, we see that we must have u in $\left[\frac{1}{2}, \frac{3}{2}\right]$.



Because $t = u^2$, we see that the solution to Equation (1) is $\left[\frac{1}{4}, \frac{9}{4}\right]$.
Thus, the average speed of a vehicle is less than or equal to 35 miles per hour between 6:15 a.m. and 8:15 a.m.

97. We solve the inequality $\dfrac{136}{1 + 0.25(t - 4.5)^2} + 28 \ge 128$ or $\dfrac{136}{1 + 0.25(t - 4.5)^2} \ge 100$. Next, $136 \ge 100\left[1 + 0.25(t - 4.5)^2\right]$, so $136 \ge 100 + 25(t - 4.5)^2$, $36 \ge 25(t - 4.5)^2$, $(t - 4.5)^2 \le \frac{36}{25}$, and $t - 4.5 \le \pm\frac{6}{5}$. Solving this last inequality, we have $t \le 5.7$ and $t \ge 3.3$. Thus, the amount of nitrogen dioxide is greater than or equal to 128 PSI between 10:18 a.m. and 12:42 p.m.

98. False. Take $a = 2$, $b = 3$, and $c = 4$. Then $\dfrac{a}{b+c} = \dfrac{2}{3+4} = \dfrac{2}{7}$, but $\dfrac{a}{b} + \dfrac{a}{c} = \dfrac{2}{3} + \dfrac{2}{4} = \dfrac{8+6}{12} = \dfrac{14}{12} = \dfrac{7}{6}$.

99. False. Take $a = 1$, $b = 2$, and $c = 3$. Then $a < b$, but $a - c = 1 - 3 = -2 \not> 2 - 3 = -1 = b - c$.

100. True. $|b - a| = |(-1)(a - b)| = |-1|\,|a - b| = |a - b|$.

101. True. $|a - b| = |a + (-b)| \le |a| + |-b| = |a| + |b|$.

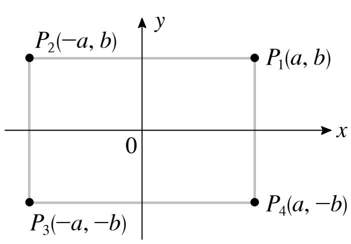
102. False. Take $a = 3$ and $b = 1$. Then $\sqrt{a^2 - b^2} = \sqrt{9 - 1} = \sqrt{8} = 2\sqrt{2}$, but $|a| - |b| = 3 - 1 = 2$.

1.3 The Cartesian Coordinate System

Concept Questions page 29

1. a. $a < 0$ and $b > 0$ **b.** $a < 0$ and $b < 0$ **c.** $a > 0$ and $b < 0$

2. a.



b. $d(P_1(a, b), (0, 0)) = \sqrt{(0 - a)^2 + (0 - b)^2} = \sqrt{a^2 + b^2}$,
$d(P_2(-a, b), (0, 0)) = \sqrt{[0 - (-a)]^2 + (0 - b)^2} = \sqrt{a^2 + b^2}$,
$d(P_3(-a, -b), (0, 0)) = \sqrt{[0 - (-a)]^2 + [0 - (-b)]^2} = \sqrt{a^2 + b^2}$,
and $d(P_4(a, -b), (0, 0)) = \sqrt{(0 - a)^2 + [0 - (-b)]^2} = \sqrt{a^2 + b^2}$,
so the points $P_1(a, b)$, $P_2(-a, b)$, $P_3(-a, -b)$, and $P_4(a, -b)$ are all the same distance from the origin.

Exercises page 30

1. The coordinates of A are $(3, 3)$ and it is located in Quadrant I.

2. The coordinates of B are $(-5, 2)$ and it is located in Quadrant II.

3. The coordinates of C are $(2, -2)$ and it is located in Quadrant IV.

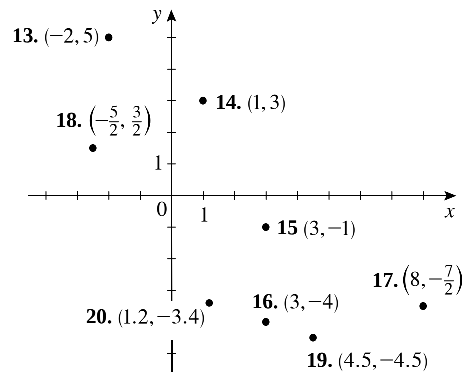
4. The coordinates of D are $(-2, 5)$ and it is located in Quadrant II.

5. The coordinates of E are $(-4, -6)$ and it is located in Quadrant III.

6. The coordinates of F are $(8, -2)$ and it is located in Quadrant IV.

7. A **8.** $(-5, 4)$ **9.** E, F, and G **10.** E **11.** F **12.** D

For Exercises 13—20, refer to the following figure.



21. Using the distance formula, we find that $\sqrt{(4-1)^2+(7-3)^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5$.

22. Using the distance formula, we find that $\sqrt{(4-1)^2+(4-0)^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5$.

23. Using the distance formula, we find that $\sqrt{[4-(-1)]^2+(9-3)^2} = \sqrt{5^2+6^2} = \sqrt{25+36} = \sqrt{61}$.

24. Using the distance formula, we find that $\sqrt{[10-(-2)]^2+(6-1)^2} = \sqrt{12^2+5^2} = \sqrt{144+25} = \sqrt{169} = 13$.

25. The coordinates of the points have the form $(x, -6)$. Because the points are 10 units away from the origin, we have $(x-0)^2+(-6-0)^2 = 10^2$, $x^2 = 64$, or $x = \pm 8$. Therefore, the required points are $(-8, -6)$ and $(8, -6)$.

26. The coordinates of the points have the form $(3, y)$. Because the points are 5 units away from the origin, we have $(3-0)^2+(y-0)^2 = 5^2$, $y^2 = 16$, or $y = \pm 4$. Therefore, the required points are $(3, 4)$ and $(3, -4)$.

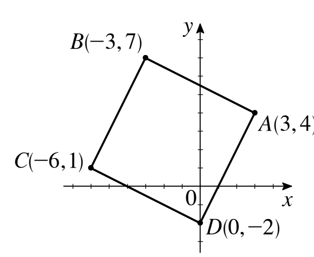
27. The points are shown in the diagram. To show that the four sides are equal, we compute

$d(A, B) = \sqrt{(-3-3)^2+(7-4)^2} = \sqrt{(-6)^2+3^2} = \sqrt{45}$,

$d(B, C) = \sqrt{[-6-(-3)]^2+(1-7)^2} = \sqrt{(-3)^2+(-6)^2} = \sqrt{45}$,

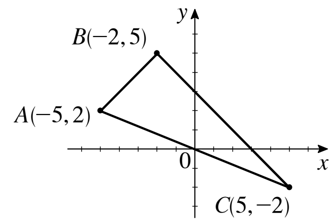
$d(C, D) = \sqrt{[0-(-6)]^2+[(-2)-1]^2} = \sqrt{(6)^2+(-3)^2} = \sqrt{45}$,

and $d(A, D) = \sqrt{(0-3)^2+(-2-4)^2} = \sqrt{(3)^2+(-6)^2} = \sqrt{45}$.



Next, to show that ΔABC is a right triangle, we show that it satisfies the Pythagorean Theorem. Thus, $d(A, C) = \sqrt{(-6-3)^2+(1-4)^2} = \sqrt{(-9)^2+(-3)^2} = \sqrt{90} = 3\sqrt{10}$ and $[d(A, B)]^2 + [d(B, C)]^2 = 90 = [d(A, C)]^2$. Similarly, $d(B, D) = \sqrt{90} = 3\sqrt{10}$, so ΔBAD is a right triangle as well. It follows that $\angle B$ and $\angle D$ are right angles, and we conclude that $ADCB$ is a square.

28. The triangle is shown in the figure. To prove that ΔABC is a right triangle, we show that $[d(A, C)]^2 = [d(A, B)]^2 + [d(B, C)]^2$ and the result will then follow from the Pythagorean Theorem. Now $[d(A, C)]^2 = (-5 - 5)^2 + [2 - (-2)]^2 = 100 + 16 = 116$. Next, we find



$[d(A, B)]^2 + [d(B, C)]^2 = [-2 - (-5)]^2 + (5 - 2)^2 + [5 - (-2)]^2 + (-2 - 5)^2 = 9 + 9 + 49 + 49 = 116$, and the result follows.

29. The equation of the circle with radius 5 and center $(2, -3)$ is given by $(x - 2)^2 + \left[y - (-3)\right]^2 = 5^2$, or $(x - 2)^2 + (y + 3)^2 = 25$.

30. The equation of the circle with radius 3 and center $(-2, -4)$ is given by $[x - (-2)]^2 + \left[y - (-4)\right]^2 = 9$, or $(x + 2)^2 + (y + 4)^2 = 9$.

31. The equation of the circle with radius 5 and center $(0, 0)$ is given by $(x - 0)^2 + (y - 0)^2 = 5^2$, or $x^2 + y^2 = 25$.

32. The distance between the center of the circle and the point $(2, 3)$ on the circumference of the circle is given by $d = \sqrt{(3 - 0)^2 + (2 - 0)^2} = \sqrt{13}$. Therefore $r = \sqrt{13}$ and the equation of the circle centered at the origin that passes through $(2, 3)$ is $x^2 + y^2 = 13$.

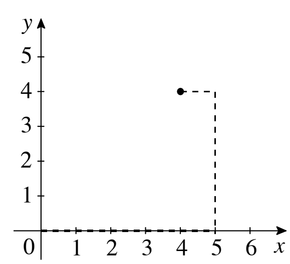
33. The distance between the points $(5, 2)$ and $(2, -3)$ is given by $d = \sqrt{(5 - 2)^2 + [2 - (-3)]^2} = \sqrt{3^2 + 5^2} = \sqrt{34}$. Therefore $r = \sqrt{34}$ and the equation of the circle passing through $(5, 2)$ and $(2, -3)$ is $(x - 2)^2 + \left[y - (-3)\right]^2 = 34$, or $(x - 2)^2 + (y + 3)^2 = 34$.

34. The equation of the circle with center $(-a, a)$ and radius $2a$ is given by $[x - (-a)]^2 + (y - a)^2 = (2a)^2$, or $(x + a)^2 + (y - a)^2 = 4a^2$.

35. **a.** The coordinates of the suspect's car at its final destination are $x = 4$ and $y = 4$.

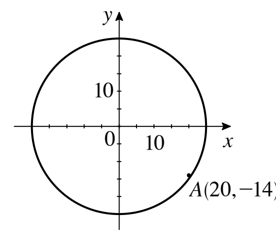
b. The distance traveled by the suspect was $5 + 4 + 1$, or 10 miles.

c. The distance between the original and final positions of the suspect's car was $d = \sqrt{(4 - 0)^2 + (4 - 0)^2} = \sqrt{32} = 4\sqrt{2}$, or approximately 5.66 miles.

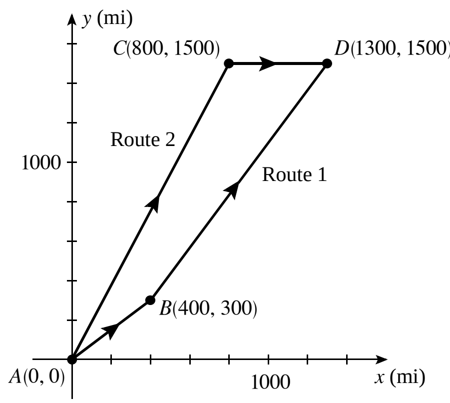


36. Referring to the diagram on page 31 of the text, we see that the distance from A to B is given by $d(A, B) = \sqrt{400^2 + 300^2} = \sqrt{250{,}000} = 500$. The distance from B to C is given by $d(B, C) = \sqrt{(-800 - 400)^2 + (800 - 300)^2} = \sqrt{(-1200)^2 + (500)^2} = \sqrt{1{,}690{,}000} = 1300$. The distance from C to D is given by $d(C, D) = \sqrt{[-800 - (-800)]^2 + (800 - 0)^2} = \sqrt{0 + 800^2} = 800$. The distance from D to A is given by $d(D, A) = \sqrt{[(-800) - 0]^2 + (0 - 0)} = \sqrt{640{,}000} = 800$. Therefore, the total distance covered on the tour is $d(A, B) + d(B, C) + d(C, D) + d(D, A) = 500 + 1300 + 800 + 800 = 3400$, or 3400 miles.

37. Suppose that the furniture store is located at the origin O so that your house is located at $A(20, -14)$. Because $d(O, A) = \sqrt{20^2 + (-14)^2} = \sqrt{596} \approx 24.4$, your house is located within a 25-mile radius of the store and you will not incur a delivery charge.



38.



Referring to the diagram, we see that the distance the salesman would cover if he took Route 1 is given by

$$d(A, B) + d(B, D) = \sqrt{400^2 + 300^2} + \sqrt{(1300 - 400)^2 + (1500 - 300)^2}$$
$$= \sqrt{250{,}000} + \sqrt{2{,}250{,}000} = 500 + 1500 = 2000$$

or 2000 miles. On the other hand, the distance he would cover if he took Route 2 is given by

$$d(A, C) + d(C, D) = \sqrt{800^2 + 1500^2} + \sqrt{(1300 - 800)^2} = \sqrt{2{,}890{,}000} + \sqrt{250{,}000}$$
$$= 1700 + 500 = 2200$$

or 2200 miles. Comparing these results, we see that he should take Route 1.

39. The cost of shipping by freight train is $(0.66)(2000)(100) = 132{,}000$, or \$132,000.
The cost of shipping by truck is $(0.62)(2200)(100) = 136{,}400$, or \$136,400.
Comparing these results, we see that the automobiles should be shipped by freight train. The net savings are $136{,}400 - 132{,}000 = 4400$, or \$4400.

40. The length of cable required on land is $d(S, Q) = 10{,}000 - x$ and the length of cable required under water is $d(Q, M) = \sqrt{(x^2 - 0) + (0 - 3000)^2} = \sqrt{x^2 + 3000^2}$. The cost of laying cable is thus $3(10{,}000 - x) + 5\sqrt{x^2 + 3000^2}$.
If $x = 2500$, then the total cost is given by $3(10{,}000 - 2500) + 5\sqrt{2500^2 + 3000^2} \approx 42{,}025.62$, or \$42,025.62.
If $x = 3000$, then the total cost is given by $3(10{,}000 - 3000) + 5\sqrt{3000^2 + 3000^2} \approx 42{,}213.20$, or \$42,213.20.

41. To determine the VHF requirements, we calculate $d = \sqrt{25^2 + 35^2} = \sqrt{625 + 1225} = \sqrt{1850} \approx 43.01$. Models B, C, and D satisfy this requirement.
To determine the UHF requirements, we calculate $d = \sqrt{20^2 + 32^2} = \sqrt{400 + 1024} = \sqrt{1424} \approx 37.74$. Models C and D satisfy this requirement.
Therefore, Model C allows him to receive both channels at the least cost.

42. a. Let the positions of ships A and B after t hours be $A(0, y)$ and $B(x, 0)$, respectively. Then $x = 30t$ and $y = 20t$. Therefore, the distance in miles between the two ships is $D = \sqrt{(30t)^2 + (20t)^2} = \sqrt{900t^2 + 400t^2} = 10\sqrt{13}t$.

b. The required distance is obtained by letting $t = 2$, giving $D = 10\sqrt{13}(2)$, or approximately 72.11 miles.

43. a. Let the positions of ships A and B be $(0, y)$ and $(x, 0)$, respectively. Then $y = 25\left(t + \frac{1}{2}\right)$ and $x = 20t$. The distance D in miles between the two ships is

$$D = \sqrt{(x-0)^2 + (0-y)^2} = \sqrt{x^2 + y^2} = \sqrt{400t^2 + 625\left(t + \tfrac{1}{2}\right)^2} \quad (1).$$

b. The distance between the ships 2 hours after ship A has left port is obtained by letting $t = \frac{3}{2}$ in Equation (1), yielding $D = \sqrt{400\left(\frac{3}{2}\right)^2 + 625\left(\frac{3}{2} + \frac{1}{2}\right)^2} = \sqrt{3400}$, or approximately 58.31 miles.

44. a. The distance in feet is given by $\sqrt{(4000)^2 + x^2} = \sqrt{16{,}000{,}000 + x^2}$.

b. Substituting the value $x = 20{,}000$ into the above expression gives $\sqrt{16{,}000{,}000 + (20{,}000)^2} \approx 20{,}396$, or 20,396 ft.

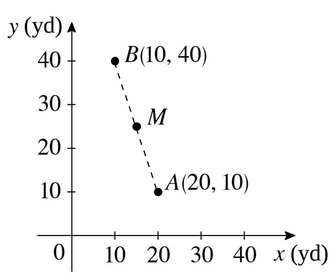
45. a. Suppose that $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are endpoints of the line segment and that the point $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is the midpoint of the line segment PQ. The distance between P and Q is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. The distance between P and M is

$$\sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2} = \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} = \tfrac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

which is one-half the distance from P to Q. Similarly, we obtain the same expression for the distance from M to P.

b. The midpoint is given by $\left(\dfrac{4-3}{2}, \dfrac{-5+2}{2}\right)$, or $\left(\dfrac{1}{2}, -\dfrac{3}{2}\right)$.

46. a.



b. The coordinates of the position of the prize are $x = \dfrac{20 + 10}{2}$ and $y = \dfrac{10 + 40}{2}$, or $x = 15$ yards and $y = 25$ yards.

c. The distance from the prize to the house is

$$d(M(15, 25), (0, 0)) = \sqrt{(15-0)^2 + (25-0)^2} = \sqrt{850} \approx 29.15 \text{ (yards)}.$$

47. True. Plot the points.

48. True. Plot the points.

49. False. The distance between $P_1(a, b)$ and $P_3(kc, kd)$ is

$$d = \sqrt{(kc-a)^2 + (kd-b)^2}$$
$$\neq |k|\, D = |k|\sqrt{(c-a)^2 + (d-b)^2} = \sqrt{k^2(c-a)^2 + k^2(d-b)^2} = \sqrt{[k(c-a)]^2 + [k(d-b)]^2}.$$

50. True. $kx^2 + ky^2 = a^2$ gives $x^2 + y^2 = \dfrac{a^2}{k} < a^2$ if $k > 1$. So the radius of the circle with equation $kx^2 + ky^2 = a^2$ is a circle of radius smaller than a centered at the origin if $k > 1$. Therefore, it lies inside the circle of radius a with equation $x^2 + y^2 = a^2$.

51. Referring to the figure in the text, we see that the distance between the two points is given by the length of the hypotenuse of the right triangle. That is, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

52. $(x-h)^2 + (y-k)^2 = r^2$; $x^2 - 2xh + h^2 + y^2 - 2ky + k^2 = r^2$. This has the form $x^2 + y^2 + Cx + Dy + E = 0$, where $C = -2h$, $D = -2k$, and $E = h^2 + k^2 - r^2$.

1.4 Straight Lines

Concept Questions page 42

1. The slope is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $P(x_1, y_1)$ and $P(x_2, y_2)$ are any two distinct points on the nonvertical line. The slope of a vertical line is undefined.

2. **a.** $y - y_1 = m(x - x_1)$ **b.** $y = mx + b$ **c.** $ax + by + c = 0$, where a and b are not both zero.

3. **a.** $m_1 = m_2$ **b.** $m_2 = -\dfrac{1}{m_1}$

4. **a.** Solving the equation for y gives $By = -Ax - C$, so $y = -\dfrac{A}{B}x - \dfrac{C}{B}$. The slope of L is the coefficient of x, $-\dfrac{A}{B}$.

b. If $B = 0$, then the equation reduces to $Ax + C = 0$. Solving this equation for x, we obtain $x = -\dfrac{C}{A}$. This is an equation of a vertical line, and we conclude that the slope of L is undefined.

Exercises page 42

1. (e) **2.** (c) **3.** (a) **4.** (d) **5.** (f) **6.** (b)

7. Referring to the figure shown in the text, we see that $m = \dfrac{2-0}{0-(-4)} = \dfrac{1}{2}$.

8. Referring to the figure shown in the text, we see that $m = \dfrac{4-0}{0-2} = -2$.

9. This is a vertical line, and hence its slope is undefined.

10. This is a horizontal line, and hence its slope is 0.

11. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8-3}{5-4} = 5$.

12. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8-5}{3-4} = \dfrac{3}{-1} = -3$.

13. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8-3}{4-(-2)} = \dfrac{5}{6}$.

14. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4-(-2)}{4-(-2)} = \dfrac{-2}{6} = -\dfrac{1}{3}$.

15. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{d-b}{c-a}$, provided $a \neq c$.

16. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-b - (b-1)}{a + 1 - (-a+1)} = -\dfrac{-b - b + 1}{a + 1 + a - 1} = \dfrac{1 - 2b}{2a}$.

17. Because the equation is already in slope-intercept form, we read off the slope $m = 4$.

a. If x increases by 1 unit, then y increases by 4 units.

b. If x decreases by 2 units, then y decreases by $4(-2) = -8$ units.

18. Rewrite the given equation in slope-intercept form: $2x + 3y = 4$, $3y = 4 - 2x$, and so $y = \frac{4}{3} - \frac{2}{3}x$.

a. Because $m = -\frac{2}{3}$, we conclude that the slope is negative.

b. Because the slope is negative, y decreases as x increases.

c. If x decreases by 2 units, then y increases by $\left(-\frac{2}{3}\right)(-2) = \frac{4}{3}$ units.

19. The slope of the line through A and B is $\dfrac{-10 - (-2)}{-3 - 1} = \dfrac{-8}{-4} = 2$. The slope of the line through C and D is $\dfrac{1 - 5}{-1 - 1} = \dfrac{-4}{-2} = 2$. Because the slopes of these two lines are equal, the lines are parallel.

20. The slope of the line through A and B is $\dfrac{-2 - 3}{2 - 2}$. Because this slope is undefined, we see that the line is vertical. The slope of the line through C and D is $\dfrac{5 - 4}{-2 - (-2)}$. Because this slope is undefined, we see that this line is also vertical. Therefore, the lines are parallel.

21. The slope of the line through A and B is $\dfrac{2 - 5}{4 - (-2)} = -\dfrac{3}{6} = -\dfrac{1}{2}$. The slope of the line through C and D is $\dfrac{6 - (-2)}{3 - (-1)} = \dfrac{8}{4} = 2$. Because the slopes of these two lines are the negative reciprocals of each other, the lines are perpendicular.

22. The slope of the line through A and B is $\dfrac{-2 - 0}{1 - 2} = \dfrac{-2}{-1} = 2$. The slope of the line through C and D is $\dfrac{4 - 2}{-8 - 4} = \dfrac{2}{-12} = -\dfrac{1}{6}$. Because the slopes of these two lines are not the negative reciprocals of each other, the lines are not perpendicular.

23. The slope of the line through the point $(1, a)$ and $(4, -2)$ is $m_1 = \dfrac{-2 - a}{4 - 1}$ and the slope of the line through $(2, 8)$ and $(-7, a + 4)$ is $m_2 = \dfrac{a + 4 - 8}{-7 - 2}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{-2 - a}{3} = \dfrac{a - 4}{-9}$, $-9(-2 - a) = 3(a - 4)$, $18 + 9a = 3a - 12$, and $6a = -30$, so $a = -5$.

24. The slope of the line through the point $(a, 1)$ and $(5, 8)$ is $m_1 = \dfrac{8 - 1}{5 - a}$ and the slope of the line through $(4, 9)$ and $(a + 2, 1)$ is $m_2 = \dfrac{1 - 9}{a + 2 - 4}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{7}{5 - a} = \dfrac{-8}{a - 2}$, $7(a - 2) = -8(5 - a)$, $7a - 14 = -40 + 8a$, and $a = 26$.

25. An equation of a horizontal line is of the form $y = b$. In this case $b = -3$, so $y = -3$ is an equation of the line.

26. An equation of a vertical line is of the form $x = a$. In this case $a = 0$, so $x = 0$ is an equation of the line.

27. We use the point-slope form of an equation of a line with the point $(3, -4)$ and slope $m = 2$. Thus $y - y_1 = m(x - x_1)$ becomes $y - (-4) = 2(x - 3)$. Simplifying, we have $y + 4 = 2x - 6$, or $y = 2x - 10$.

28. We use the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = -1$. Thus $y - y_1 = m(x - x_1)$, giving $y - 4 = -1(x - 2)$, $y - 4 = -x + 2$, and finally $y = -x + 6$.

29. Because the slope $m = 0$, we know that the line is a horizontal line of the form $y = b$. Because the line passes through $(-3, 2)$, we see that $b = 2$, and an equation of the line is $y = 2$.

30. We use the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = -\frac{1}{2}$. Thus $y - y_1 = m(x - x_1)$ gives $y - 2 = -\frac{1}{2}(x - 1)$, $2y - 4 = -x + 1$, $2y = -x + 5$, and $y = -\frac{1}{2}x + \frac{5}{2}$.

31. We first compute the slope of the line joining the points $(2, 4)$ and $(3, 7)$ to be $m = \dfrac{7-4}{3-2} = 3$. Using the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = 3$, we find $y - 4 = 3(x - 2)$, or $y = 3x - 2$.

32. We first compute the slope of the line joining the points $(2, 1)$ and $(2, 5)$ to be $m = \dfrac{5-1}{2-2}$. Because this slope is undefined, we see that the line must be a vertical line of the form $x = a$. Because it passes through $(2, 5)$, we see that $x = 2$ is the equation of the line.

33. We first compute the slope of the line joining the points $(1, 2)$ and $(-3, -2)$ to be $m = \dfrac{-2-2}{-3-1} = \dfrac{-4}{-4} = 1$. Using the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = 1$, we find $y - 2 = x - 1$, or $y = x + 1$.

34. We first compute the slope of the line joining the points $(-1, -2)$ and $(3, -4)$ to be $m = \dfrac{-4-(-2)}{3-(-1)} = \dfrac{-2}{4} = -\dfrac{1}{2}$. Using the point-slope form of an equation of a line with the point $(-1, -2)$ and slope $m = -\frac{1}{2}$, we find $y - (-2) = -\frac{1}{2}[x - (-1)]$, $y + 2 = -\frac{1}{2}(x + 1)$, and finally $y = -\frac{1}{2}x - \frac{5}{2}$.

35. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 3$ and $b = 4$, the equation is $y = 3x + 4$.

36. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = -2$ and $b = -1$, the equation is $y = -2x - 1$.

37. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 0$ and $b = 5$, the equation is $y = 5$.

38. We use the slope-intercept form of an equation of a line:$y = mx + b$. Because $m = -\frac{1}{2}$, and $b = \frac{3}{4}$, the equation is $y = -\frac{1}{2}x + \frac{3}{4}$.

39. We first write the given equation in the slope-intercept form: $x - 2y = 0$, so $-2y = -x$, or $y = \frac{1}{2}x$. From this equation, we see that $m = \frac{1}{2}$ and $b = 0$.

40. We write the equation in slope-intercept form: $y - 2 = 0$, so $y = 2$. From this equation, we see that $m = 0$ and $b = 2$.

41. We write the equation in slope-intercept form: $2x - 3y - 9 = 0$, $-3y = -2x + 9$, and $y = \frac{2}{3}x - 3$. From this equation, we see that $m = \frac{2}{3}$ and $b = -3$.

42. We write the equation in slope-intercept form: $3x - 4y + 8 = 0$, $-4y = -3x - 8$, and $y = \frac{3}{4}x + 2$. From this equation, we see that $m = \frac{3}{4}$ and $b = 2$.

43. We write the equation in slope-intercept form: $2x + 4y = 14$, $4y = -2x + 14$, and $y = -\frac{2}{4}x + \frac{14}{4} = -\frac{1}{2}x + \frac{7}{2}$. From this equation, we see that $m = -\frac{1}{2}$ and $b = \frac{7}{2}$.

44. We write the equation in the slope-intercept form: $5x + 8y - 24 = 0$, $8y = -5x + 24$, and $y = -\frac{5}{8}x + 3$. From this equation, we conclude that $m = -\frac{5}{8}$ and $b = 3$.

45. We first write the equation $2x - 4y - 8 = 0$ in slope-intercept form: $2x - 4y - 8 = 0$, $4y = 2x - 8$, $y = \frac{1}{2}x - 2$. Now the required line is parallel to this line, and hence has the same slope. Using the point-slope form of an equation of a line with $m = \frac{1}{2}$ and the point $(-2, 2)$, we have $y - 2 = \frac{1}{2}[x - (-2)]$ or $y = \frac{1}{2}x + 3$.

46. We first write the equation $3x + 4y - 22 = 0$ in slope-intercept form: $3x + 4y - 22 = 0$, so $4y = -3x + 22$ and $y = -\frac{3}{4}x + \frac{11}{2}$ Now the required line is perpendicular to this line, and hence has slope $\frac{4}{3}$ (the negative reciprocal of $-\frac{3}{4}$). Using the point-slope form of an equation of a line with $m = \frac{4}{3}$ and the point $(2, 4)$, we have $y - 4 = \frac{4}{3}(x - 2)$, or $y = \frac{4}{3}x + \frac{4}{3}$.

47. The midpoint of the line segment joining $P_1(-2, -4)$ and $P_2(3, 6)$ is $M\left(\dfrac{-2+3}{2}, \dfrac{-4+6}{2}\right)$ or $M\left(\frac{1}{2}, 1\right)$.

Using the point-slope form of the equation of a line with $m = -2$, we have $y - 1 = -2\left(x - \frac{1}{2}\right)$ or $y = -2x + 2$.

48. The midpoint of the line segment joining $P_1(-1, -3)$ and $P_2(3, 3)$ is $M_1\left(\dfrac{-1+3}{2}, \dfrac{-3+3}{2}\right)$ or $M_1(1, 0)$.

The midpoint of the line segment joining $P_3(-2, 3)$ and $P_4(2, -3)$ is $M_2\left(\dfrac{-2+2}{2}, \dfrac{3-3}{2}\right)$ or $M_2(0, 0)$.

The slope of the required line is $m = \dfrac{0-0}{1-0} = 0$, so an equation of the line is $y - 0 = 0(x - 0)$ or $y = 0$.

49. A line parallel to the x-axis has slope 0 and is of the form $y = b$. Because the line is 6 units below the axis, it passes through $(0, -6)$ and its equation is $y = -6$.

50. Because the required line is parallel to the line joining $(2, 4)$ and $(4, 7)$, it has slope $m = \dfrac{7-4}{4-2} = \dfrac{3}{2}$. We also know that the required line passes through the origin $(0, 0)$. Using the point-slope form of an equation of a line, we find $y - 0 = \frac{3}{2}(x - 0)$, or $y = \frac{3}{2}x$.

51. We use the point-slope form of an equation of a line to obtain $y - b = 0(x - a)$, or $y = b$.

52. Because the line is parallel to the x-axis, its slope is 0 and its equation has the form $y = b$. We know that the line passes through $(-3, 4)$, so the required equation is $y = 4$.

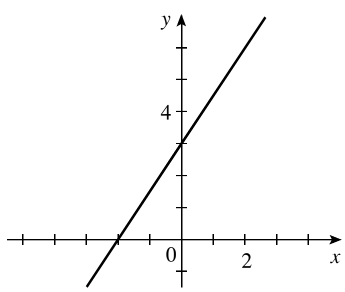
53. Because the required line is parallel to the line joining $(-3, 2)$ and $(6, 8)$, it has slope $m = \dfrac{8-2}{6-(-3)} = \dfrac{6}{9} = \dfrac{2}{3}$. We also know that the required line passes through $(-5, -4)$. Using the point-slope form of an equation of a line, we find $y - (-4) = \frac{2}{3}[x - (-5)]$, $y = \frac{2}{3}x + \frac{10}{3} - 4$, and finally $y = \frac{2}{3}x - \frac{2}{3}$.

54. Because the slope of the line is undefined, it has the form $x = a$. Furthermore, since the line passes through (a, b), the required equation is $x = a$.

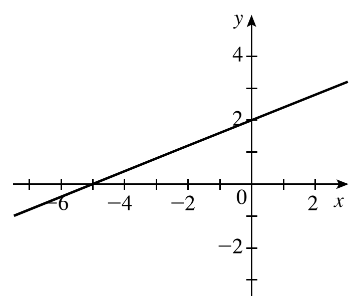
55. Because the point $(-3, 5)$ lies on the line $kx + 3y + 9 = 0$, it satisfies the equation. Substituting $x = -3$ and $y = 5$ into the equation gives $-3k + 15 + 9 = 0$, or $k = 8$.

56. Because the point $(2, -3)$ lies on the line $-2x + ky + 10 = 0$, it satisfies the equation. Substituting $x = 2$ and $y = -3$ into the equation gives $-2(2) + (-3)k + 10 = 0$, $-4 - 3k + 10 = 0$, $-3k = -6$, and finally $k = 2$.

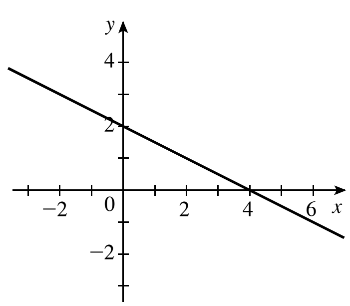
57. $3x - 2y + 6 = 0$. Setting $y = 0$, we have $3x + 6 = 0$ or $x = -2$, so the x-intercept is -2. Setting $x = 0$, we have $-2y + 6 = 0$ or $y = 3$, so the y-intercept is 3.



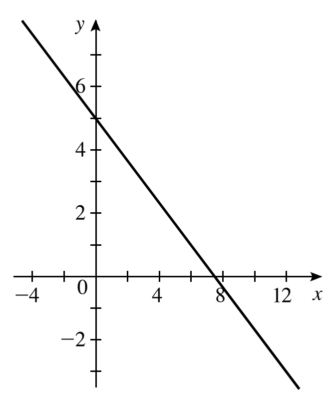
58. $2x - 5y + 10 = 0$. Setting $y = 0$, we have $2x + 10 = 0$ or $x = -5$, so the x-intercept is -5. Setting $x = 0$, we have $-5y + 10 = 0$ or $y = 2$, so the y-intercept is 2.



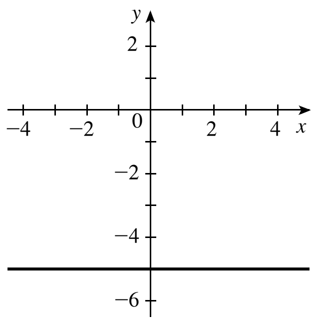
59. $x + 2y - 4 = 0$. Setting $y = 0$, we have $x - 4 = 0$ or $x = 4$, so the x-intercept is 4. Setting $x = 0$, we have $2y - 4 = 0$ or $y = 2$, so the y-intercept is 2.



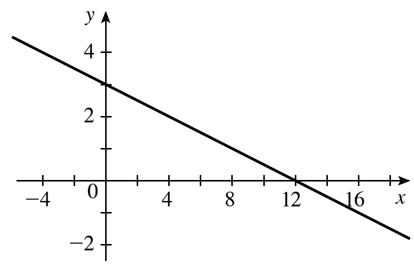
60. $2x + 3y - 15 = 0$. Setting $y = 0$, we have $2x - 15 = 0$, so the x-intercept is $\frac{15}{2}$. Setting $x = 0$, we have $3y - 15 = 0$, so the y-intercept is 5.



61. $y+5=0$. Setting $y=0$, we have $0+5=0$, which has no solution, so there is no x-intercept. Setting $x=0$, we have $y+5=0$ or $y=-5$, so the y-intercept is -5.



62. $-2x-8y+24=0$. Setting $y=0$, we have $-2x+24=0$ or $x=12$, so the x-intercept is 12. Setting $x=0$, we have $-8y+24=0$ or $y=3$, so the y-intercept is 3.



63. Because the line passes through the points $(a,0)$ and $(0,b)$, its slope is $m=\dfrac{b-0}{0-a}=-\dfrac{b}{a}$. Then, using the point-slope form of an equation of a line with the point $(a,0)$, we have $y-0=-\dfrac{b}{a}(x-a)$ or $y=-\dfrac{b}{a}x+b$, which may be written in the form $\dfrac{b}{a}x+y=b$. Multiplying this last equation by $\dfrac{1}{b}$, we have $\dfrac{x}{a}+\dfrac{y}{b}=1$.

64. Using the equation $\dfrac{x}{a}+\dfrac{y}{b}=1$ with $a=3$ and $b=4$, we have $\dfrac{x}{3}+\dfrac{y}{4}=1$. Then $4x+3y=12$, so $3y=12-4x$ and thus $y=-\frac{4}{3}x+4$.

65. Using the equation $\dfrac{x}{a}+\dfrac{y}{b}=1$ with $a=-2$ and $b=-4$, we have $-\dfrac{x}{2}-\dfrac{y}{4}=1$. Then $-4x-2y=8$, $2y=-8-4x$, and finally $y=-2x-4$.

66. Using the equation $\dfrac{x}{a}+\dfrac{y}{b}=1$ with $a=-\frac{1}{2}$ and $b=\frac{3}{4}$, we have $\dfrac{x}{-1/2}+\dfrac{y}{3/4}=1$, $\frac{3}{4}x-\frac{1}{2}y=\left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)$, $\frac{1}{2}y=-\frac{3}{4}x-\frac{3}{8}$, and finally $y=2\left(\frac{3}{4}x+\frac{3}{8}\right)=\frac{3}{2}x+\frac{3}{4}$.

67. Using the equation $\dfrac{x}{a}+\dfrac{y}{b}=1$ with $a=4$ and $b=-\frac{1}{2}$, we have $\dfrac{x}{4}+\dfrac{y}{-1/2}=1$, $-\frac{1}{4}x+2y=-1$, $2y=\frac{1}{4}x-1$, and so $y=\frac{1}{8}x-\frac{1}{2}$.

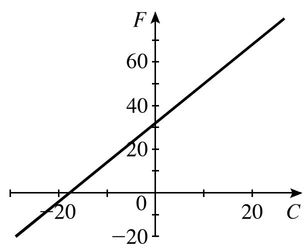
68. The slope of the line passing through A and B is $m=\dfrac{-2-7}{2-(-1)}=-\dfrac{9}{3}=-3$, and the slope of the line passing through B and C is $m=\dfrac{-9-(-2)}{5-2}=-\dfrac{7}{3}$. Because the slopes are not equal, the points do not lie on the same line.

69. The slope of the line passing through A and B is $m=\dfrac{7-1}{1-(-2)}=\dfrac{6}{3}=2$, and the slope of the line passing through B and C is $m=\dfrac{13-7}{4-1}=\dfrac{6}{3}=2$. Because the slopes are equal, the points lie on the same line.

70. The slope of the line L passing through $P_1(1.2, -9.04)$ and $P_2(2.3, -5.96)$ is $m = \dfrac{-5.96 - (-9.04)}{2.3 - 1.2} = 2.8$, so an equation of L is $y - (-9.04) = 2.8(x - 1.2)$ or $y = 2.8x - 12.4$.
Substituting $x = 4.8$ into this equation gives $y = 2.8(4.8) - 12.4 = 1.04$. This shows that the point $P_3(4.8, 1.04)$ lies on L. Next, substituting $x = 7.2$ into the equation gives $y = 2.8(7.2) - 12.4 = 7.76$, which shows that the point $P_4(7.2, 7.76)$ also lies on L. We conclude that John's claim is valid.

71. The slope of the line L passing through $P_1(1.8, -6.44)$ and $P_2(2.4, -5.72)$ is $m = \dfrac{-5.72 - (-6.44)}{2.4 - 1.8} = 1.2$, so an equation of L is $y - (-6.44) = 1.2(x - 1.8)$ or $y = 1.2x - 8.6$.
Substituting $x = 5.0$ into this equation gives $y = 1.2(5) - 8.6 = -2.6$. This shows that the point $P_3(5.0, -2.72)$ does not lie on L, and we conclude that Alison's claim is not valid.

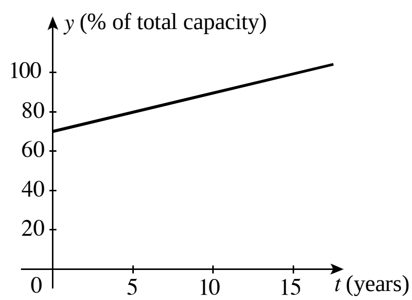
72. a.



b. The slope is $\frac{9}{5}$. It represents the change in °F per unit change in °C.

c. The F-intercept of the line is 32. It corresponds to 0°, so it is the freezing point in °F.

73. a.



b. The slope is 1.9467 and the y-intercept is 70.082.

c. The output is increasing at the rate of 1.9467% per year. The output at the beginning of 1990 was 70.082%.

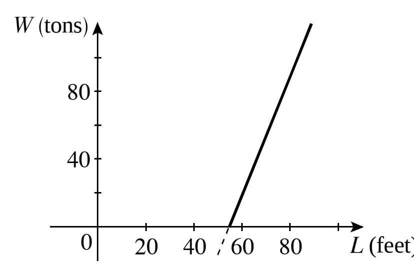
d. We solve the equation $1.9467t + 70.082 = 100$, obtaining $t \approx 15.37$. We conclude that the plants were generating at maximum capacity during April 2005.

74. a. $y = 0.0765x$ **b.** \$0.0765 **c.** $0.0765(65{,}000) = 4972.50$, or \$4972.50.

75. a. $y = 0.55x$ **b.** Solving the equation $1100 = 0.55x$ for x, we have $x = \dfrac{1100}{0.55} = 2000$.

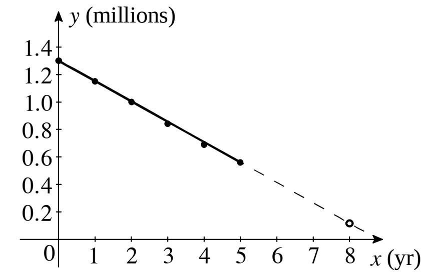
76. a. Substituting $L = 80$ into the given equation, we have $W = 3.51(80) - 192 = 280.8 - 192 = 88.8$, or 88.8 British tons.

b.



77. Using the points $(0, 0.68)$ and $(10, 0.80)$, we see that the slope of the required line is $m = \dfrac{0.80 - 0.68}{10 - 0} = \dfrac{0.12}{10} = 0.012$. Next, using the point-slope form of the equation of a line, we have $y - 0.68 = 0.012(t - 0)$ or $y = 0.012t + 0.68$. Therefore, when $t = 18$, we have $y = 0.012(18) + 0.68 = 0.896$, or 89.6%. That is, in 2008 women's wages were expected to be 89.6% of men's wages.

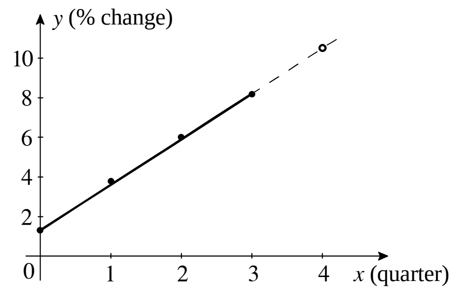
78. a, b.



c. The slope of L is $m = \dfrac{0.56 - 1.30}{5 - 0} = -0.148$, so an equation of L is $y - 1.3 = -0.148\,(x - 0)$ or $y = -0.148x + 1.3$.

d. The number of pay phones in 2012 is estimated to be $-0.148\,(8) + 1.3$, or approximately 116,000.

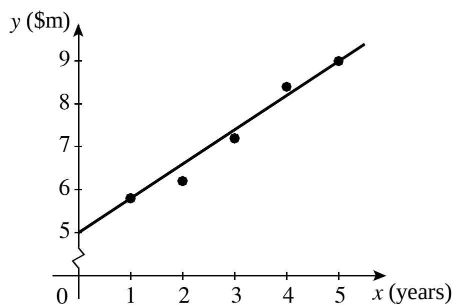
79. a, b.



c. The slope of L is $m = \dfrac{8.2 - 1.3}{3 - 0} = 2.3$, so an equation of L is $y - 1.3 = 2.3\,(x - 0)$ or $y = 2.3x + 1.3$.

d. The change in spending in the first quarter of 2014 is estimated to be $2.3\,(4) + 1.3$, or 10.5%.

80. a, b.

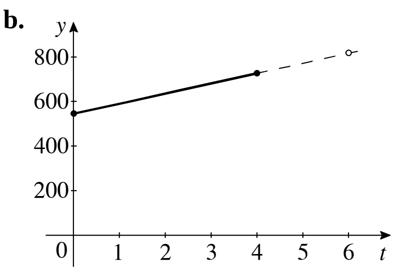


c. The slope of L is $m = \dfrac{9.0 - 5.8}{5 - 1} = \dfrac{3.2}{4} = 0.8$. Using the point-slope form of an equation of a line, we have $y - 5.8 = 0.8\,(x - 1) = 0.8x - 0.8$, or $y = 0.8x + 5$.

d. Using the equation from part (c) with $x = 9$, we have $y = 0.8\,(9) + 5 = 12.2$, or \$12.2 million.

81. a. The slope of the line L passing through $A\,(0, 545)$ and $B\,(4, 726)$ is $m = \dfrac{726 - 545}{4 - 0} = \dfrac{181}{4}$, so an equation of L is $y - 545 = \frac{181}{4}\,(t - 0)$ or $y = \frac{181}{4}t + 545$.

b.



c. The number of corporate fraud cases pending at the beginning of 2014 is estimated to be $\frac{181}{4}\,(6) + 545$, or approximately 817.

82. a. The slope of the line through $P_1\,(0, 27)$ and $P_2\,(1, 29)$ is $m_1 = \dfrac{29 - 27}{1 - 0} = 2$, which is equal to the slope of the line through $P_2\,(1, 29)$ and $P_3\,(2, 31)$, which is $m_2 = \dfrac{31 - 29}{1 - 0} = 2$. Thus, the three points lie on the line L.

b. The percentage is of moviegoers who use social media to chat about movies in 2014 is estimated to be $31 + 2\,(2)$, or 35%.

c. $y - 27 = 2\,(x - 0)$, so $y = 2x + 27$. The estimate for 2014 ($t = 4$) is $2\,(4) + 27 = 35$, as found in part (b).

83. True. The slope of the line is given by $-\frac{2}{4} = -\frac{1}{2}$.

84. True. The slope of the line $Ax + By + C = 0$ is $-\frac{A}{B}$. (Write it in slope-intercept form.) Similarly, the slope of the line $ax + by + c = 0$ is $-\frac{a}{b}$. They are parallel if and only if $-\frac{A}{B} = -\frac{a}{b}$, that is, if $Ab = aB$, or $Ab - aB = 0$.

85. False. Let the slope of L_1 be $m_1 > 0$. Then the slope of L_2 is $m_2 = -\frac{1}{m_1} < 0$.

86. True. The slope of the line $ax + by + c_1 = 0$ is $m_1 = -\frac{a}{b}$. The slope of the line $bx - ay + c_2 = 0$ is $m_2 = \frac{b}{a}$. Because $m_1 m_2 = -1$, the straight lines are indeed perpendicular.

87. True. Set $y = 0$ and we have $Ax + C = 0$ or $x = -C/A$, and this is where the line intersects the x-axis.

88. Yes. A straight line with slope zero ($m = 0$) is a horizontal line, whereas a straight line whose slope does not exist is a vertical line (m cannot be computed).

89. Writing each equation in the slope-intercept form, we have $y = -\frac{a_1}{b_1}x - \frac{c_1}{b_1}$ $(b_1 \neq 0)$ and $y = -\frac{a_2}{b_2}x - \frac{c_2}{b_2}$ $(b_2 \neq 0)$. Because two lines are parallel if and only if their slopes are equal, we see that the lines are parallel if and only if $-\frac{a_1}{b_1} = -\frac{a_2}{b_2}$, or $a_1 b_2 - b_1 a_2 = 0$.

90. The slope of L_1 is $m_1 = \frac{b-0}{1-0} = b$. The slope of L_2 is $m_2 = \frac{c-0}{1-0} = c$. Applying the Pythagorean theorem to ΔOAC and ΔOCB gives $(OA)^2 = 1^2 + b^2$ and $(OB)^2 = 1^2 + c^2$. Adding these equations and applying the Pythagorean theorem to ΔOBA gives $(AB)^2 = (OA)^2 + (OB)^2 = 1^2 + b^2 + 1^2 + c^2 = 2 + b^2 + c^2$. Also, $(AB)^2 = (b-c)^2$, so $(b-c)^2 = 2 + b^2 + c^2$, $b^2 - 2bc + c^2 = 2 + b^2 + c^2$, and $-2bc = 2$, $1 = -bc$. Finally, $m_1 m_2 = b \cdot c = bc = -1$, as was to be shown.

CHAPTER 1 Concept Review Questions page 48

1. ordered, abscissa or x-coordinate, ordinate or y-coordinate

2. a. x, y **b.** third

3. $\sqrt{(c-a)^2+(d-b)^2}$

4. $(x-a)^2+(y-b)^2=r^2$

5. a. $\dfrac{y_2-y_1}{x_2-x_1}$ **b.** undefined **c.** 0 **d.** positive

6. $m_1=m_2,\ m_1=-\dfrac{1}{m_2}$

7. a. $y-y_1=m\,(x-x_1)$, point-slope form **b.** $y=mx+b$, slope-intercept

8. a. $Ax+By+C=0$, where A and B are not both zero **b.** $-a/b$

CHAPTER 1 Review Exercises page 48

1. Adding x to both sides yields $3 \le 3x + 9$, $3x \ge -6$, or $x \ge -2$. We conclude that the solution set is $[-2, \infty)$.

2. $-2 \le 3x + 1 \le 7$ implies $-3 \le 3x \le 6$, or $-1 \le x \le 2$, and so the solution set is $[-1, 2]$.

3. The inequalities imply $x > 5$ or $x < -4$, so the solution set is $(-\infty, -4) \cup (5, \infty)$.

4. $2x^2 > 50$ is equivalent to $x^2 > 25$, so either $x > 5$ or $x < -5$ and the solution set is $(-\infty, -5) \cup (5, \infty)$.

5. $|-5+7| + |-2| = |2| + |-2| = 2 + 2 = 4.$

6. $\left|\dfrac{5-12}{-4-3}\right| = \dfrac{|5-12|}{|-7|} = \dfrac{|-7|}{7} = \dfrac{7}{7} = 1.$

7. $|2\pi - 6| - \pi = 2\pi - 6 - \pi = \pi - 6.$

8. $\left|\sqrt{3} - 4\right| + \left|4 - 2\sqrt{3}\right| = \left(4 - \sqrt{3}\right) + \left(4 - 2\sqrt{3}\right)$
$= 8 - 3\sqrt{3}.$

9. $\left(\dfrac{9}{4}\right)^{3/2} = \dfrac{9^{3/2}}{4^{3/2}} = \dfrac{27}{8}.$

10. $\dfrac{5^6}{5^4} = 5^{6-4} = 5^2 = 25.$

11. $(3 \cdot 4)^{-2} = 12^{-2} = \dfrac{1}{12^2} = \dfrac{1}{144}.$

12. $(-8)^{5/3} = \left(-8^{1/3}\right)^5 = (-2)^5 = -32.$

13. $\dfrac{\left(3 \cdot 2^{-3}\right)\left(4 \cdot 3^5\right)}{2 \cdot 9^3} = \dfrac{3 \cdot 2^{-3} \cdot 2^2 \cdot 3^5}{2 \cdot \left(3^2\right)^3} = \dfrac{2^{-1} \cdot 3^6}{2 \cdot 3^6} = \dfrac{1}{4}.$

14. $\dfrac{3\sqrt[3]{54}}{\sqrt[3]{18}} = \dfrac{3 \cdot \left(2 \cdot 3^3\right)^{1/3}}{\left(2 \cdot 3^2\right)^{1/3}} = \dfrac{3^2 \cdot 2^{1/3}}{2^{1/3} \cdot 3^{2/3}} = 3^{4/3} = 3\sqrt[3]{3}.$

15. $\dfrac{4\left(x^2+y\right)^3}{x^2+y} = 4\left(x^2+y\right)^2.$

16. $\dfrac{a^6 b^{-5}}{\left(a^3 b^{-2}\right)^{-3}} = \dfrac{a^6 b^{-5}}{a^{-9} b^6} = \dfrac{a^{15}}{b^{11}}.$

17. $\dfrac{\sqrt[4]{16x^5yz}}{\sqrt[4]{81xyz^5}} = \dfrac{\left(2^4x^5yz\right)^{1/4}}{\left(3^4xyz^5\right)^{1/4}} = \dfrac{2x^{5/4}y^{1/4}z^{1/4}}{3x^{1/4}y^{1/4}z^{5/4}} = \dfrac{2x}{3z}.$

18. $\left(2x^3\right)\left(-3x^{-2}\right)\left(\frac{1}{6}x^{-1/2}\right) = -x^{1/2}.$

19. $\left(\dfrac{3xy^2}{4x^3y}\right)^{-2}\left(\dfrac{3xy^3}{2x^2}\right)^3 = \left(\dfrac{3y}{4x^2}\right)^{-2}\left(\dfrac{3y^3}{2x}\right)^3 = \left(\dfrac{4x^2}{3y}\right)^2\left(\dfrac{3y^3}{2x}\right)^3 = \dfrac{\left(16x^4\right)\left(27y^9\right)}{\left(9y^2\right)\left(8x^3\right)} = 6xy^7.$

20. $\sqrt[3]{81x^5y^{10}}\sqrt[3]{9xy^2} = \sqrt[3]{\left(3^4x^5y^{10}\right)\left(3^2xy^2\right)} = \left(3^6x^6y^{12}\right)^{1/3} = 3^2x^2y^4 = 9x^2y^4.$

21. $-2\pi^2r^3 + 100\pi r^2 = -2\pi r^2(\pi r - 50).$

22. $2v^3w + 2vw^3 + 2u^2vw = 2vw\left(v^2 + w^2 + u^2\right).$

23. $16 - x^2 = 4^2 - x^2 = (4-x)(4+x).$

24. $12t^3 - 6t^2 - 18t = 6t\left(2t^2 - t - 3\right) = 6t(2t-3)(t+1).$

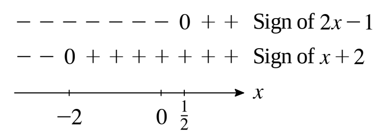
25. $8x^2 + 2x - 3 = (4x+3)(2x-1) = 0$, so $x = -\frac{3}{4}$ and $x = \frac{1}{2}$ are the roots of the equation.

26. $-6x^2 - 10x + 4 = 0$, $3x^2 + 5x - 2 = (3x-1)(x+2) = 0$, and so $x = -2$ or $x = \frac{1}{3}$.

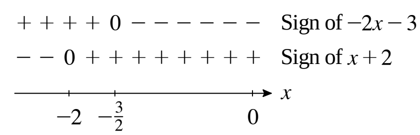
27. $-x^3 - 2x^2 + 3x = -x\left(x^2 + 2x - 3\right) = -x(x+3)(x-1) = 0$, and so the roots of the equation are $x = 0$, $x = -3$, and $x = 1$.

28. $2x^4 + x^2 = 1$. If we let $y = x^2$, we can write the equation as $2y^2 + y - 1 = (2y-1)(y+1) = 0$, giving $y = \frac{1}{2}$ or $y = -1$. We reject the second root since $y = x^2$ must be nonnegative. Therefore, $x^2 = \frac{1}{2}$, and so $x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$.

29. Factoring the given expression, we have $(2x-1)(x+2) \le 0$. From the sign diagram, we conclude that the given inequality is satisfied when $-2 \le x \le \frac{1}{2}$.



30. $\dfrac{1}{x+2} > 2$ gives $\dfrac{1}{x+2} - 2 > 0$, $\dfrac{1-2x-4}{x+2} > 0$, and finally $\dfrac{-2x-3}{x+2} > 0$. From the sign diagram, we see that the given inequality is satisfied when $-2 < x < -\frac{3}{2}$.



31. The given inequality is equivalent to $|2x-3| < 5$ or $-5 < 2x - 3 < 5$. Thus, $-2 < 2x < 8$, or $-1 < x < 4$.

32. The given equation implies that either $\dfrac{x+1}{x-1} = 5$ or $\dfrac{x+1}{x-1} = -5$. Solving the first equality, we have $x + 1 = 5(x-1) = 5x - 5$, $-4x = -6$, and $x = \frac{3}{2}$. Similarly, we solve the second equality and obtain $x + 1 = -5(x-1) = -5x + 5$, $6x = 4$, and $x = \frac{2}{3}$. Thus, the two values of x that satisfy the equation are $x = \frac{3}{2}$ and $x = \frac{2}{3}$.

33. We use the quadratic formula to solve the equation $x^2 - 2x - 5 = 0$. Here $a = 1$, $b = -2$, and $c = -5$, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)} = \frac{2 \pm \sqrt{24}}{2} = 1 \pm \sqrt{6}.$$

34. We use the quadratic formula to solve the equation $2x^2 + 8x + 7 = 0$. Here $a = 2$, $b = 8$, and $c = 7$, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4(2)(7)}}{4} = \frac{-8 \pm 2\sqrt{2}}{4} = -2 \pm \tfrac{1}{2}\sqrt{2}.$$

35. $$\frac{(t+6)(60) - (60t + 180)}{(t+6)^2} = \frac{60t + 360 - 60t - 180}{(t+6)^2} = \frac{180}{(t+6)^2}.$$

36. $$\frac{6x}{2\left(3x^2+2\right)} + \frac{1}{4(x+2)} = \frac{(6x)(2)(x+2) + \left(3x^2+2\right)}{4\left(3x^2+2\right)(x+2)} = \frac{12x^2 + 24x + 3x^2 + 2}{4\left(3x^2+2\right)(x+2)} = \frac{15x^2 + 24x + 2}{4\left(3x^2+2\right)(x+2)}.$$

37. $$\frac{2}{3}\left(\frac{4x}{2x^2-1}\right) + 3\left(\frac{3}{3x-1}\right) = \frac{8x}{3\left(2x^2-1\right)} + \frac{9}{3x-1} = \frac{8x(3x-1) + 27\left(2x^2-1\right)}{3\left(2x^2-1\right)(3x-1)} = \frac{78x^2 - 8x - 27}{3\left(2x^2-1\right)(3x-1)}.$$

38. $$\frac{-2x}{\sqrt{x+1}} + 4\sqrt{x+1} = \frac{-2x + 4(x+1)}{\sqrt{x+1}} = \frac{2(x+2)}{\sqrt{x+1}}.$$

39. $$\frac{\sqrt{x}-1}{x-1} = \frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{\left(\sqrt{x}\right)^2 - 1}{(x-1)\left(\sqrt{x}+1\right)} = \frac{x-1}{(x-1)\left(\sqrt{x}+1\right)} = \frac{1}{\sqrt{x}+1}.$$

40. $\dfrac{\sqrt{x}-1}{2\sqrt{x}} = \dfrac{\sqrt{x}-1}{2\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{x-\sqrt{x}}{2x}$.

41. The distance is $d = \sqrt{[1-(-2)]^2 + [-7-(-3)]^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$.

42. The distance is $d = \sqrt{(2-6)^2 + (6-9)^2} = \sqrt{16+9} = \sqrt{25} = 5$.

43. The distance is $d = \sqrt{\left(-\frac{1}{2}-\frac{1}{2}\right)^2 + \left(2\sqrt{3}-\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$.

44. An equation is $x = -2$.

45. An equation is $y = 4$.

46. The slope of L is $m = \dfrac{\frac{7}{2}-4}{3-(-2)} = -\dfrac{1}{10}$, and an equation of L is $y - 4 = -\frac{1}{10}[x-(-2)] = -\frac{1}{10}x - \frac{1}{5}$, or $y = -\frac{1}{10}x + \frac{19}{5}$. The general form of this equation is $x + 10y - 38 = 0$.

47. The line passes through the points $(-2, 4)$ and $(3, 0)$, so its slope is $m = \dfrac{4-0}{-2-3} = -\dfrac{4}{5}$. An equation is $y - 0 = -\frac{4}{5}(x-3)$, or $y = -\frac{4}{5}x + \frac{12}{5}$.

48. Writing the given equation in the form $y = \frac{5}{2}x - 3$, we see that the slope of the given line is $\frac{5}{2}$. Thus, an equation is $y - 4 = \frac{5}{2}(x+2)$, or $y = \frac{5}{2}x + 9$. The general form of this equation is $5x - 2y + 18 = 0$.

49. Writing the given equation in the form $y = -\frac{4}{3}x + 2$, we see that the slope of the given line is $-\frac{4}{3}$. Therefore, the slope of the required line is $\frac{3}{4}$ and an equation of the line is $y - 4 = \frac{3}{4}(x+2)$ or $y = \frac{3}{4}x + \frac{11}{2}$.

50. Rewriting the given equation in slope-intercept form, we have $4y = -3x + 8$ or $y = -\frac{3}{4}x + 2$. We conclude that the slope of the required line is $-\frac{3}{4}$. Using the point-slope form of the equation of a line with the point $(2, 3)$ and slope $-\frac{3}{4}$, we obtain $y - 3 = -\frac{3}{4}(x-2)$, and so $y = -\frac{3}{4}x + \frac{6}{4} + 3 = -\frac{3}{4}x + \frac{9}{2}$. The general form of this equation is $3x + 4y - 18 = 0$.

51. The slope of the line joining the points $(-3, 4)$ and $(2, 1)$ is $m = \dfrac{1-4}{2-(-3)} = -\dfrac{3}{5}$. Using the point-slope form of the equation of a line with the point $(-1, 3)$ and slope $-\frac{3}{5}$, we have $y - 3 = -\frac{3}{5}[x-(-1)]$. Therefore, $y = -\frac{3}{5}(x+1) + 3 = -\frac{3}{5}x + \frac{12}{5}$.

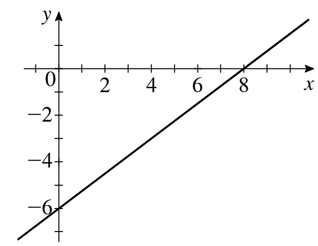
52. The slope of the line passing through $(-2, -4)$ and $(1, 5)$ is $m = \dfrac{5-(-4)}{1-(-2)} = \dfrac{9}{3} = 3$, so the required line is $y - (-2) = 3[x-(-3)]$. Simplifying, this is equivalent to $y + 2 = 3x + 9$, or $y = 3x + 7$.

53. Rewriting the given equation in the slope-intercept form $y = \frac{2}{3}x - 8$, we see that the slope of the line with this equation is $\frac{2}{3}$. The slope of a line perpendicular to this line is thus $-\frac{3}{2}$. Using the point-slope form of the equation of a line with the point $(-2, -4)$ and slope $-\frac{3}{2}$, we have $y - (-4) = -\frac{3}{2}[x-(-2)]$ or $y = -\frac{3}{2}x - 7$. The general form of this equation is $3x + 2y + 14 = 0$.

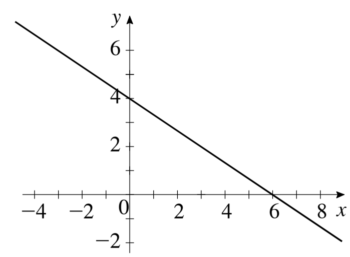
54. Substituting $x = -1$ and $y = -\frac{5}{4}$ into the left-hand side of the equation gives $6(-1) - 8\left(-\frac{5}{4}\right) - 16 = -12$. The equation is not satisfied, and so we conclude that the point $\left(-1, -\frac{5}{4}\right)$ does not lie on the line $6x - 8y - 16 = 0$.

55. Substituting $x = 2$ and $y = -4$ into the equation, we obtain $2(2) + k(-4) = -8$, so $-4k = -12$ and $k = 3$.

56. Setting $x = 0$ gives $y = -6$ as the y-intercept. Setting $y = 0$ gives $x = 8$ as the x-intercept. The graph of $3x - 4y = 24$ is shown.



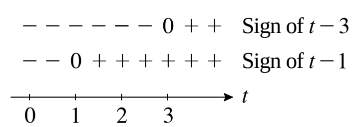
57. Using the point-slope form of an equation of a line, we have $y - 2 = -\frac{2}{3}(x - 3)$ or $y = -\frac{2}{3}x + 4$. If $y = 0$, then $x = 6$, and if $x = 0$, then $y = 4$. A sketch of the line is shown.



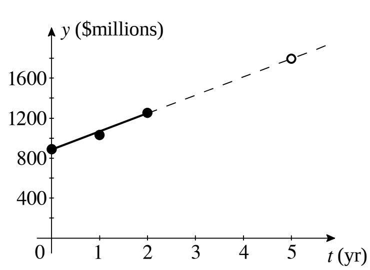
58. Simplifying $2(1.5C + 80) \leq 2(2.5C - 20)$, we obtain $1.5C + 80 \leq 2.5C - 20$, so $C \geq 100$ and the minimum cost is \$100.

59. $3(2R - 320) \leq 3R + 240$ gives $6R - 960 \leq 3R + 240$, $3R \leq 1200$ and finally $R \leq 400$. We conclude that the maximum revenue is \$400.

60. We solve the inequality $-16t^2 + 64t + 80 \geq 128$, obtaining $-16t^2 + 64t - 48 \geq 0$, $t^2 - 4t + 3 \leq 0$, and $(t - 3)(t - 1) \leq 0$. From the sign diagram, we see that the required solution is $[1, 3]$. Thus, the stone is 128 ft or higher off the ground between 1 and 3 seconds after it was thrown.



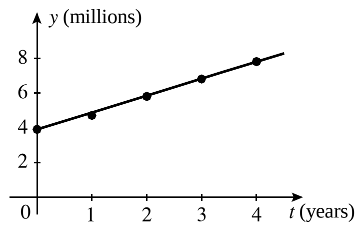
61. a, b.



c. The slope of L is $\dfrac{1251 - 887}{2 - 0} = 182$, so an equation of L is $y - 887 = 182(t - 0)$ or $y = 182t + 887$.

d. The amount consumers are projected to spend on Cyber Monday, 2014 ($t = 5$) is $182(5) + 887$, or \$1.797 billion.

62. a, b.



c. $P_1(0, 3.9)$ and $P_2(4, 7.8)$, so $m = \dfrac{7.8 - 3.9}{4 - 0} = \dfrac{3.9}{4} = 0.975$. Thus, $y - 3.9 = 0.975(t - 0)$, or $y = 0.975t + 3.9$.

d. If $t = 3$, then $y = 0.975(3) + 3.9 = 6.825$. Thus, the number of systems installed in 2005 (when $t = 3$) is 6,825,000, which is close to the projected value of 6.8 million.

CHAPTER 1 Before Moving On... page 50

1. a. $\left|\pi - 2\sqrt{3}\right| - \left|\sqrt{3} - \sqrt{2}\right| = -\left(\pi - 2\sqrt{3}\right) - \left(\sqrt{3} - \sqrt{2}\right) = \sqrt{3} + \sqrt{2} - \pi$.

b. $\left[\left(-\frac{1}{3}\right)^{-3}\right]^{1/3} = \left(-\frac{1}{3}\right)^{(-3)\left(\frac{1}{3}\right)} = \left(-\frac{1}{3}\right)^{-1} = -3$.

2. a. $\sqrt[3]{64x^6} \cdot \sqrt{9y^2x^6} = \left(4x^2\right)\left(3yx^3\right) = 12x^5y$.

b. $\left(\dfrac{a^{-3}}{b^{-4}}\right)^2 \left(\dfrac{b}{a}\right)^{-3} = \dfrac{a^{-6}}{b^{-8}} \cdot \dfrac{b^{-3}}{a^{-3}} = \dfrac{b^8}{a^6} \cdot \dfrac{a^3}{b^3} = \dfrac{b^5}{a^3}$.

3. a. $\dfrac{2x}{3\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{2x\sqrt{y}}{3y}$.

b. $\dfrac{x}{\sqrt{x} - 4} \cdot \dfrac{\sqrt{x} + 4}{\sqrt{x} + 4} = \dfrac{x\left(\sqrt{x} + 4\right)}{x - 16}$.

4. a. $\dfrac{\left(x^2 + 1\right)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(2x)}{\left(x^2 + 1\right)^2} = \dfrac{\frac{1}{2}x^{-1/2}\left[\left(x^2 + 1\right) - 4x^2\right]}{\left(x^2 + 1\right)^2} = \dfrac{1 - 3x^2}{2x^{1/2}\left(x^2 + 1\right)^2}$.

b. $-\dfrac{3x}{\sqrt{x + 2}} + 3\sqrt{x + 2} = \dfrac{-3x + 3(x + 2)}{\sqrt{x + 2}} = \dfrac{6}{\sqrt{x + 2}} = \dfrac{6\sqrt{x + 2}}{x + 2}$.

5. $\dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} \cdot \dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{x - y}{\left(\sqrt{x} - \sqrt{y}\right)^2}$.

6. a. $12x^3 - 10x^2 - 12x = 2x\left(6x^2 - 5x - 6\right) = 2x(2x - 3)(3x + 2)$.

b. $2bx - 2by + 3cx - 3cy = 2b(x - y) + 3c(x - y) = (2b + 3c)(x - y)$

7. a. $12x^2 - 9x - 3 = 0$, so $3\left(4x^2 - 3x - 1\right) = 0$ and $3(4x + 1)(x - 1) = 0$. Thus, $x = -\frac{1}{4}$ or $x = 1$.

b. $3x^2 - 5x + 1 = 0$. Using the quadratic formula with $a = 3$, $b = -5$, and $c = 1$, we have
$x = \dfrac{-(-5) \pm \sqrt{25 - 12}}{2(3)} = \dfrac{5 \pm \sqrt{13}}{6}$.

8. $d = \sqrt{[6 - (-2)]^2 + (8 - 4)^2} = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$.

9. $m = \dfrac{5 - (-2)}{4 - (-1)} = \dfrac{7}{5}$, so $y - (-2) = \frac{7}{5}[x - (-1)]$, $y + 2 = \frac{7}{5}x + \frac{7}{5}$, or $y = \frac{7}{5}x - \frac{3}{5}$.

10. $m = -\frac{1}{3}$ and $b = \frac{4}{3}$, so an equation is $y = -\frac{1}{3}x + \frac{4}{3}$.

CHAPTER 1 Explore & Discuss

Page 27

1. Let $P_1 = (2, 6)$ and $P_2 = (-4, 3)$. Then we have $x_1 = 2$, $y_1 = 6$, $x_2 = -4$, and $y_2 = 3$. Using Formula (1), we have $d = \sqrt{(-4-2)^2 + (3-6)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$, as obtained in Example 1.

2. Let $P_1\,(x_1, y_1)$ and $P_2\,(x_2, y_2)$ be any two points in the plane. Then the result follows from the equality $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Page 28

1. **a.** All points on and inside the circle with center (h, k) and radius r.

b. All points inside the circle with center (h, k) and radius r.

c. All points on and outside the circle with center (h, k) and radius r.

d. All points outside the circle with center (h, k) and radius r.

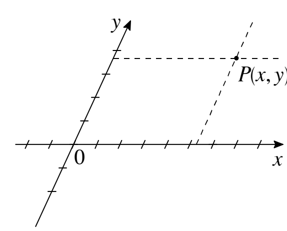
2. **a.** $y^2 = 4 - x^2$, and so $y = \pm\sqrt{4 - x^2}$.

b. **(i)** The upper semicircle with center at the origin and radius 2.

(ii) The lower semicircle with center at the origin and radius 2.

Page 29

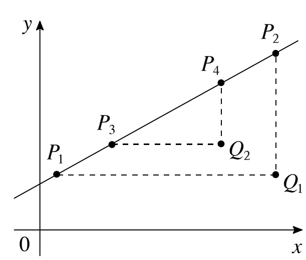
1. Let $P\,(x, y)$ be any point in the plane. Draw a line through P parallel to the y-axis and a line through P parallel to the x-axis (see the figure). The x-coordinate of P is the number corresponding to the point on the x-axis at which the line through P crosses the x-axis. Similarly, y is the number that corresponds to the point on the y-axis at which the line parallel to the x-axis crosses the y-axis. To show the converse, reverse the process.



2. You can use the Pythagorean Theorem in the Cartesian coordinate system. This greatly simplifies the computations.

Page 35

1. Refer to the accompanying figure. Observe that triangles $\Delta P_1 Q_1 P_2$ and $\Delta P_3 Q_2 P_4$ are similar. From this we conclude that $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_4 - y_3}{x_4 - x_3}$. Because P_3 and P_4 are arbitrary, the conclusion follows.



Page 39

1. We obtain a family of parallel lines each having slope m.

2. We obtain a family of straight lines all of which pass through the point $(0, b)$.

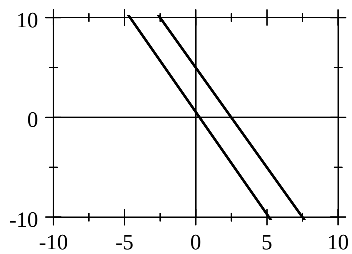
Page 40

1. In Example 11, we are told that the object is expected to appreciate in value at a given rate for the next five years, and the equation obtained in that example is based on this fact. Thus, the equation may not be used to predict the value of the object very much beyond five years from the date of purchase.

CHAPTER 1 Exploring with Technology

Page 38

1.

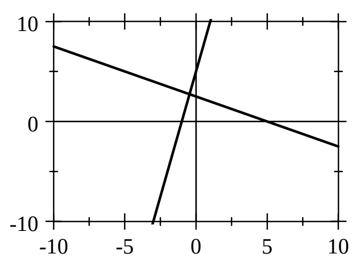


The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be parallel.

b. Writing each equation in the slope-intercept form gives $y = -2x + 5$ and $y = -\frac{41}{20}x + \frac{11}{20}$, from which we see that the slopes of L_1 and L_2 are -2 and $-\frac{41}{20} = -2.05$, respectively. This shows that L_1 and L_2 are not parallel.

2.



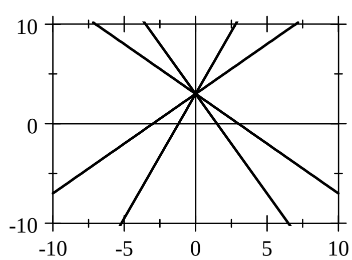
The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be perpendicular.

b. The slopes of L_1 and L_2 are $m_1 = -\frac{1}{2}$ and $m_2 = 5$, respectively. Because $m_1 = -\dfrac{1}{2} \neq -\dfrac{1}{5} = -\dfrac{1}{m_2}$, we see that L_1 and L_2 are not perpendicular.

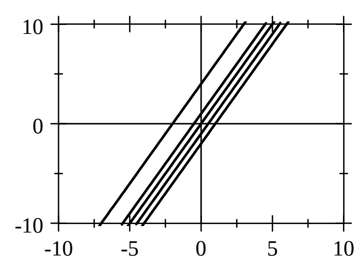
Page 39

1.



The straight lines with the given equations are shown in the figure. Changing the value of m in the equation $y = mx + b$ changes the slope of the line and thus rotates it.

2.



The straight lines of interest are shown in the figure. Changing the value of b in the equation $y = mx + b$ changes the y-intercept of the line and thus translates it (upward if $b > 0$ and downward if $b < 0$).

3. Changing both m and b in the equation $y = mx + b$ both rotates and translates the line.

2 FUNCTIONS, LIMITS, AND THE DERIVATIVE

2.1 Functions and Their Graphs

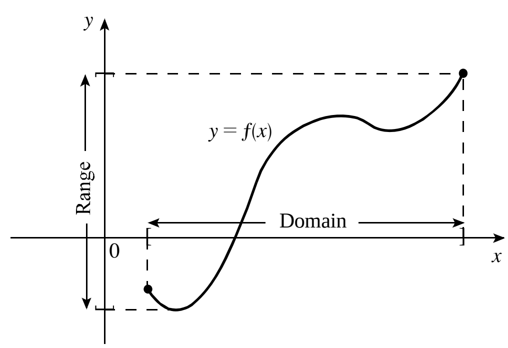
Concept Questions page 59

1. a. A function is a rule that associates with each element in a set A exactly one element in a set B.

b. The domain of a function f is the set of all elements x in the set such that $f(x)$ is an element in B. The range of f is the set of all elements $f(x)$ whenever x is an element in its domain.

c. An independent variable is a variable in the domain of a function f. The dependent variable is $y = f(x)$.

2. a. The graph of a function f is the set of all ordered pairs (x, y) such that $y = f(x)$, x being an element in the domain of f.



b. Use the vertical line test to determine if every vertical line intersects the curve in at most one point. If so, then the curve is the graph of a function.

3. a. Yes, every vertical line intersects the curve in at most one point.

b. No, a vertical line intersects the curve at more than one point.

c. No, a vertical line intersects the curve at more than one point.

d. Yes, every vertical line intersects the curve in at most one point.

4. The domain is $[1, 3) \cup [3, 5)$ and the range is $\left[\frac{1}{2}, 2\right) \cup (2, 4]$.

Exercises page 59

1. $f(x) = 5x + 6$. Therefore $f(3) = 5(3) + 6 = 21$, $f(-3) = 5(-3) + 6 = -9$, $f(a) = 5(a) + 6 = 5a + 6$, $f(-a) = 5(-a) + 6 = -5a + 6$, and $f(a+3) = 5(a+3) + 6 = 5a + 15 + 6 = 5a + 21$.

2. $f(x) = 4x - 3$. Therefore, $f(4) = 4(4) - 3 = 16 - 3 = 13$, $f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 3 = 1 - 3 = -2$, $f(0) = 4(0) - 3 = -3$, $f(a) = 4(a) - 3 = 4a - 3$, $f(a+1) = 4(a+1) - 3 = 4a + 1$.

3. $g(x) = 3x^2 - 6x - 3$, so $g(0) = 3(0) - 6(0) - 3 = -3$, $g(-1) = 3(-1)^2 - 6(-1) - 3 = 3 + 6 - 3 = 6$, $g(a) = 3(a)^2 - 6(a) - 3 = 3a^2 - 6a - 3$, $g(-a) = 3(-a)^2 - 6(-a) - 3 = 3a^2 + 6a - 3$, and $g(x+1) = 3(x+1)^2 - 6(x+1) - 3 = 3\left(x^2 + 2x + 1\right) - 6x - 6 - 3 = 3x^2 + 6x + 3 - 6x - 9 = 3x^2 - 6$.

4. $h(x) = x^3 - x^2 + x + 1$, so $h(-5) = (-5)^3 - (-5)^2 + (-5) + 1 = -125 - 25 - 5 + 1 = -154$, $h(0) = (0)^3 - (0)^2 + 0 + 1 = 1$, $h(a) = a^3 - (a)^2 + a + 1 = a^3 - a^2 + a + 1$, and $h(-a) = (-a)^3 - (-a)^2 + (-a) + 1 = -a^3 - a^2 - a + 1$.

5. $f(x) = 2x + 5$, so $f(a+h) = 2(a+h) + 5 = 2a + 2h + 5$, $f(-a) = 2(-a) + 5 = -2a + 5$, $f\left(a^2\right) = 2\left(a^2\right) + 5 = 2a^2 + 5$, $f(a - 2h) = 2(a - 2h) + 5 = 2a - 4h + 5$, and $f(2a - h) = 2(2a - h) + 5 = 4a - 2h + 5$

6. $g(x) = -x^2 + 2x$, $g(a+h) = -(a+h)^2 + 2(a+h) = -a^2 - 2ah - h^2 + 2a + 2h$, $g(-a) = -(-a)^2 + 2(-a) = -a^2 - 2a = -a(a+2)$, $g\left(\sqrt{a}\right) = -\left(\sqrt{a}\right)^2 + 2\left(\sqrt{a}\right) = -a + 2\sqrt{a}$, $a + g(a) = a - a^2 + 2a = -a^2 + 3a = -a(a-3)$, and $\dfrac{1}{g(a)} = \dfrac{1}{-a^2 + 2a} = -\dfrac{1}{a(a-2)}$.

7. $s(t) = \dfrac{2t}{t^2 - 1}$. Therefore, $s(4) = \dfrac{2(4)}{(4)^2 - 1} = \dfrac{8}{15}$, $s(0) = \dfrac{2(0)}{0^2 - 1} = 0$, $s(a) = \dfrac{2(a)}{a^2 - 1} = \dfrac{2a}{a^2 - 1}$; $s(2+a) = \dfrac{2(2+a)}{(2+a)^2 - 1} = \dfrac{2(2+a)}{a^2 + 4a + 4 - 1} = \dfrac{2(2+a)}{a^2 + 4a + 3}$, and $s(t+1) = \dfrac{2(t+1)}{(t+1)^2 - 1} = \dfrac{2(t+1)}{t^2 + 2t + 1 - 1} = \dfrac{2(t+1)}{t(t+2)}$.

8. $g(u) = (3u - 2)^{3/2}$. Therefore, $g(1) = [3(1) - 2]^{3/2} = (1)^{3/2} = 1$, $g(6) = [3(6) - 2]^{3/2} = 16^{3/2} = 4^3 = 64$, $g\left(\frac{11}{3}\right) = \left[3\left(\frac{11}{3}\right) - 2\right]^{3/2} = (9)^{3/2} = 27$, and $g(u+1) = [3(u+1) - 2]^{3/2} = (3u+1)^{3/2}$.

9. $f(t) = \dfrac{2t^2}{\sqrt{t-1}}$. Therefore, $f(2) = \dfrac{2\left(2^2\right)}{\sqrt{2-1}} = 8$, $f(a) = \dfrac{2a^2}{\sqrt{a-1}}$, $f(x+1) = \dfrac{2(x+1)^2}{\sqrt{(x+1)-1}} = \dfrac{2(x+1)^2}{\sqrt{x}}$, and $f(x-1) = \dfrac{2(x-1)^2}{\sqrt{(x-1)-1}} = \dfrac{2(x-1)^2}{\sqrt{x-2}}$.

10. $f(x) = 2 + 2\sqrt{5-x}$. Therefore, $f(-4) = 2 + 2\sqrt{5-(-4)} = 2 + 2\sqrt{9} = 2 + 2(3) = 8$, $f(1) = 2 + 2\sqrt{5-1} = 2 + 2\sqrt{4} = 2 + 4 = 6$, $f\left(\frac{11}{4}\right) = 2 + 2\left(5 - \frac{11}{4}\right)^{1/2} = 2 + 2\left(\frac{9}{4}\right)^{1/2} = 2 + 2\left(\frac{3}{2}\right) = 5$, and $f(x+5) = 2 + 2\sqrt{5-(x+5)} = 2 + 2\sqrt{-x}$.

11. Because $x = -2 \le 0$, we calculate $f(-2) = (-2)^2 + 1 = 4 + 1 = 5$. Because $x = 0 \le 0$, we calculate $f(0) = (0)^2 + 1 = 1$. Because $x = 1 > 0$, we calculate $f(1) = \sqrt{1} = 1$.

12. Because $x = -2 < 2$, $g(-2) = -\frac{1}{2}(-2) + 1 = 1 + 1 = 2$. Because $x = 0 < 2$, $g(0) = -\frac{1}{2}(0) + 1 = 0 + 1 = 1$. Because $x = 2 \ge 2$, $g(2) = \sqrt{2-2} = 0$. Because $x = 4 \ge 2$, $g(4) = \sqrt{4-2} = \sqrt{2}$.

13. Because $x = -1 < 1$, $f(-1) = -\frac{1}{2}(-1)^2 + 3 = \frac{5}{2}$. Because $x = 0 < 1$, $f(0) = -\frac{1}{2}(0)^2 + 3 = 3$. Because $x = 1 \ge 1$, $f(1) = 2\left(1^2\right) + 1 = 3$. Because $x = 2 \ge 1$, $f(2) = 2\left(2^2\right) + 1 = 9$.

14. Because $x = 0 \leq 1$, $f(0) = 2 + \sqrt{1-0} = 2 + 1 = 3$. Because $x = 1 \leq 1$, $f(1) = 2 + \sqrt{1-1} = 2 + 0 = 2$. Because $x = 2 > 1$, $f(2) = \dfrac{1}{1-2} = \dfrac{1}{-1} = -1$.

15. **a.** $f(0) = -2$.

b. **(i)** $f(x) = 3$ when $x \approx 2$. **(ii)** $f(x) = 0$ when $x = 1$.

c. $[0, 6]$

d. $[-2, 6]$

16. **a.** $f(7) = 3$. **b.** $x = 4$ and $x = 6$. **c.** $x = 2$; 0. **d.** $[-1, 9]$; $[-2, 6]$.

17. $g(2) = \sqrt{2^2 - 1} = \sqrt{3}$, so the point $\left(2, \sqrt{3}\right)$ lies on the graph of g.

18. $f(3) = \dfrac{3+1}{\sqrt{3^2+7}} + 2 = \dfrac{4}{\sqrt{16}} + 2 = \dfrac{4}{4} + 2 = 3$, so the point $(3, 3)$ lies on the graph of f.

19. $f(-2) = \dfrac{|-2-1|}{-2+1} = \dfrac{|-3|}{-1} = -3$, so the point $(-2, -3)$ does lie on the graph of f.

20. $h(-3) = \dfrac{|-3+1|}{(-3)^3+1} = \dfrac{2}{-27+1} = -\dfrac{2}{26} = -\dfrac{1}{13}$, so the point $\left(-3, -\frac{1}{13}\right)$ does lie on the graph of h.

21. Because the point $(1, 5)$ lies on the graph of f it satisfies the equation defining f. Thus, $f(1) = 2(1)^2 - 4(1) + c = 5$, or $c = 7$.

22. Because the point $(2, 4)$ lies on the graph of f it satisfies the equation defining f. Thus, $f(2) = 2\sqrt{9-(2)^2} + c = 4$, or $c = 4 - 2\sqrt{5}$.

23. Because $f(x)$ is a real number for any value of x, the domain of f is $(-\infty, \infty)$.

24. Because $f(x)$ is a real number for any value of x, the domain of f is $(-\infty, \infty)$.

25. $f(x)$ is not defined at $x = 0$ and so the domain of f is $(-\infty, 0) \cup (0, \infty)$.

26. $g(x)$ is not defined at $x = 1$ and so the domain of g is $(-\infty, 1) \cup (1, \infty)$.

27. $f(x)$ is a real number for all values of x. Note that $x^2 + 1 \geq 1$ for all x. Therefore, the domain of f is $(-\infty, \infty)$.

28. Because the square root of a number is defined for all real numbers greater than or equal to zero, we have $x - 5 \geq 0$ or $x \geq 5$, and the domain is $[5, \infty)$.

29. Because the square root of a number is defined for all real numbers greater than or equal to zero, we have $5 - x \geq 0$, or $-x \geq -5$ and so $x \leq 5$. (Recall that multiplying by -1 reverses the sign of an inequality.) Therefore, the domain of f is $(-\infty, 5]$.

30. Because $2x^2 + 3$ is always greater than zero, the domain of g is $(-\infty, \infty)$.

31. The denominator of f is zero when $x^2 - 1 = 0$, or $x = \pm 1$. Therefore, the domain of f is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

32. The denominator of f is equal to zero when $x^2 + x - 2 = (x+2)(x-1) = 0$; that is, when $x = -2$ or $x = 1$. Therefore, the domain of f is $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

33. f is defined when $x + 3 \geq 0$, that is, when $x \geq -3$. Therefore, the domain of f is $[-3, \infty)$.

34. g is defined when $x - 1 \geq 0$; that is when $x \geq 1$. Therefore, the domain of f is $[1, \infty)$.

35. The numerator is defined when $1 - x \geq 0$, $-x \geq -1$ or $x \leq 1$. Furthermore, the denominator is zero when $x = \pm 2$. Therefore, the domain is the set of all real numbers in $(-\infty, -2) \cup (-2, 1]$.

36. The numerator is defined when $x - 1 \geq 0$, or $x \geq 1$, and the denominator is zero when $x = -2$ and when $x = 3$. So the domain is $[1, 3) \cup (3, \infty)$.

37. a. The domain of f is the set of all real numbers.

b. $f(x) = x^2 - x - 6$, so

$f(-3) = (-3)^2 - (-3) - 6 = 9 + 3 - 6 = 6$,

$f(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 0$,

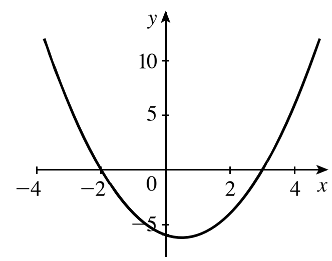
$f(-1) = (-1)^2 - (-1) - 6 = 1 + 1 - 6 = -4$,

$f(0) = (0)^2 - (0) - 6 = -6$,

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 6 = \frac{1}{4} - \frac{2}{4} - \frac{24}{4} = -\frac{25}{4}$, $f(1) = (1)^2 - 1 - 6 = -6$,

$f(2) = (2)^2 - 2 - 6 = 4 - 2 - 6 = -4$, and $f(3) = (3)^2 - 3 - 6 = 9 - 3 - 6 = 0$.

c.



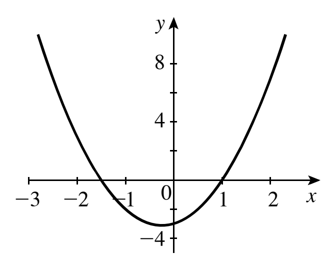
38. $f(x) = 2x^2 + x - 3$.

a. Because $f(x)$ is a real number for all values of x, the domain of f is $(-\infty, \infty)$.

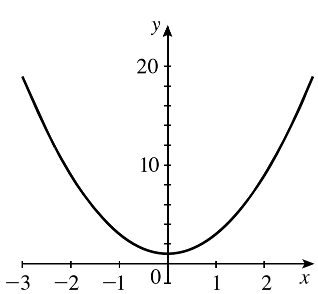
b.

x	-3	-2	-1	$-\frac{1}{2}$	0	1	2	3
y	12	3	-2	-3	-3	0	7	18

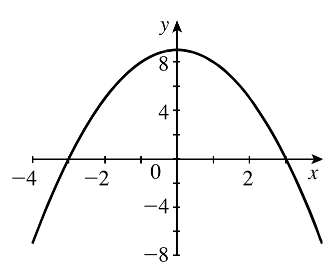
c.



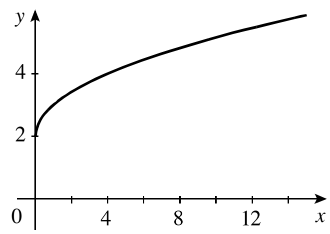
39. $f(x) = 2x^2 + 1$ has domain $(-\infty, \infty)$ and range $[1, \infty)$.



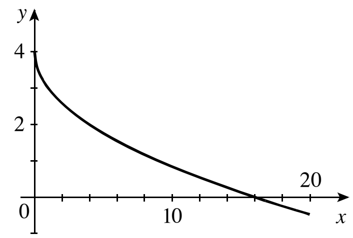
40. $f(x) = 9 - x^2$ has domain $(-\infty, \infty)$ and range $(-\infty, 9]$.



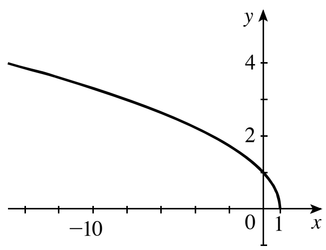
41. $f(x) = 2 + \sqrt{x}$ has domain $[0, \infty)$ and range $[2, \infty)$.



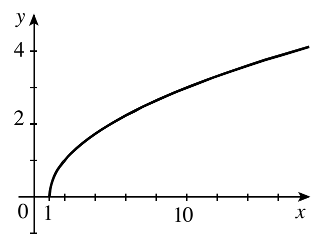
42. $g(x) = 4 - \sqrt{x}$ has domain $[0, \infty)$ and range $(-\infty, 4]$.



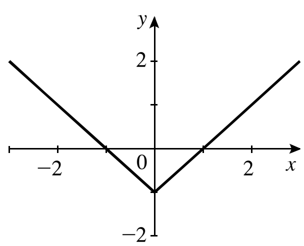
43. $f(x) = \sqrt{1 - x}$ has domain $(-\infty, 1]$ and range $[0, \infty)$



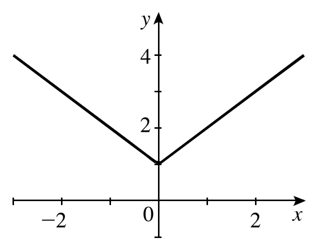
44. $f(x) = \sqrt{x - 1}$ has domain $(1, \infty)$ and range $[0, \infty)$.



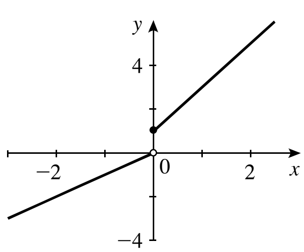
45. $f(x) = |x| - 1$ has domain $(-\infty, \infty)$ and range $[-1, \infty)$.



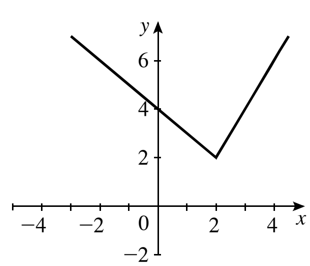
46. $f(x) = |x| + 1$ has domain $(-\infty, \infty)$ and range $[1, \infty)$



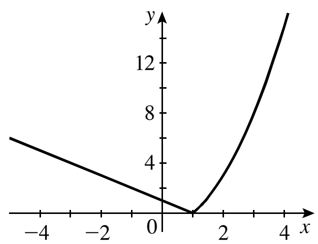
47. $f(x) = \begin{cases} x & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$ has domain $(-\infty, \infty)$ and range $(-\infty, 0) \cup [1, \infty)$.



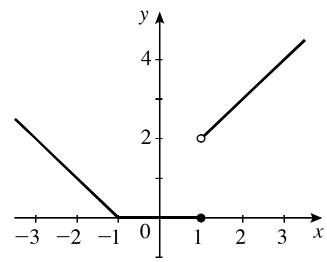
48. For $x < 2$, the graph of f is the half-line $y = 4 - x$. For $x \geq 2$, the graph of f is the half-line $y = 2x - 2$. f has domain $(-\infty, \infty)$ and range $[2, \infty)$.



49. If $x \leq 1$, the graph of f is the half-line $y = -x + 1$. For $x > 1$, we calculate a few points: $f(2) = 3$, $f(3) = 8$, and $f(4) = 15$. f has domain $(-\infty, \infty)$ and range $[0, \infty)$.



50. If $x < -1$ the graph of f is the half-line $y = -x - 1$. For $-1 \leq x \leq 1$, the graph consists of the line segment $y = 0$. For $x > 1$, the graph is the half-line $y = x + 1$. f has domain $(-\infty, \infty)$ and range $[0, \infty)$.



51. Each vertical line cuts the given graph at exactly one point, and so the graph represents y as a function of x.

52. Because the y-axis, which is a vertical line, intersects the graph at two points, the graph does not represent y as a function of x.

53. Because there is a vertical line that intersects the graph at three points, the graph does not represent y as a function of x.

54. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

55. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

56. The y-axis intersects the circle at *two* points, and this shows that the circle is not the graph of a function of x.

57. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

58. A vertical line containing a line segment comprising the graph cuts it at infinitely many points and so the graph does not define y as a function of x.

59. The circumference of a circle with a 5-inch radius is given by $C(5) = 2\pi(5) = 10\pi$, or 10π inches.

60. $V(2.1) = \frac{4}{3}\pi(2.1)^3 \approx 38.79$, $V(2) = \frac{4}{3}\pi(8) \approx 33.51$, and so $V(2.1) - V(2) = 38.79 - 33.51 = 5.28$ is the amount by which the volume of a sphere of radius 2.1 exceeds the volume of a sphere of radius 2.

61. $C(0) = 6$, or 6 billion dollars; $C(50) = 0.75(50) + 6 = 43.5$, or 43.5 billion dollars; and $C(100) = 0.75(100) + 6 = 81$, or 81 billion dollars.

62. The child should receive $D(4) = \frac{2}{25}(500)(4) = 160$, or 160 mg.

63. **a.** From $t = 0$ through $t = 5$, that is, from the beginning of 2001 until the end of 2005.

b. From $t = 5$ through $t = 9$, that is, from the beginning of 2006 until the end of 2010.

c. The average expenditures were the same at approximately $t = 5.2$, that is, in the year 2006. The level of expenditure on each service was approximately \$900.

64. a. The slope of the straight line passing through $(0, 0.61)$ and $(10, 0.59)$ is $m_1 = \frac{0.59 - 0.61}{10 - 0} = -0.002$.
Therefore, an equation of the straight line passing through the two points is $y - 0.61 = -0.002\,(t - 0)$ or $y = -0.002t + 0.61$. Next, the slope of the straight line passing through $(10, 0.59)$ and $(20, 0.60)$ is $m_2 = \frac{0.60 - 0.59}{20 - 10} = 0.001$, and so an equation of the straight line passing through the two points is $y - 0.59 = 0.001\,(t - 10)$ or $y = 0.001t + 0.58$. The slope of the straight line passing through $(20, 0.60)$ and $(30, 0.66)$ is $m_3 = \frac{0.66 - 0.60}{30 - 20} = 0.006$, and so an equation of the straight line passing through the two points is $y - 0.60 = 0.006\,(t - 20)$ or $y = 0.006t + 0.48$. The slope of the straight line passing through $(30, 0.66)$ and $(40.0, 0.78)$ is $m_4 = \frac{0.78 - 0.66}{40 - 30} = 0.012$, and so an equation of the straight line passing through the two points is $y = 0.012t + 0.30$. Therefore, a rule for f is $f(t) = \begin{cases} -0.002t + 0.61 & \text{if } 0 \le t \le 10 \\ 0.001t + 0.58 & \text{if } 10 < t \le 20 \\ 0.006t + 0.48 & \text{if } 20 < t \le 30 \\ 0.012t + 0.30 & \text{if } 30 < t \le 40 \end{cases}$

b. The gender gap was expanding between 1960 and 1970 and shrinking between 1970 and 2000.

c. The gender gap was expanding at the rate of $0.002/\text{yr}$ between 1960 and 1970, shrinking at the rate of $0.001/\text{yr}$ between 1970 and 1980, shrinking at the rate of $0.006/\text{yr}$ between 1980 and 1990, and shrinking at the rate of $0.012/\text{yr}$ between 1990 and 2000.

65. a. The slope of the straight line passing through the points $(0, 0.58)$ and $(20, 0.95)$ is $m_1 = \frac{0.95 - 0.58}{20 - 0} = 0.0185$, so an equation of the straight line passing through these two points is $y - 0.58 = 0.0185\,(t - 0)$ or $y = 0.0185t + 0.58$. Next, the slope of the straight line passing through the points $(20, 0.95)$ and $(30, 1.1)$ is $m_2 = \frac{1.1 - 0.95}{30 - 20} = 0.015$, so an equation of the straight line passing through the two points is $y - 0.95 = 0.015\,(t - 20)$ or $y = 0.015t + 0.65$. Therefore, a rule for f is
$$f(t) = \begin{cases} 0.0185t + 0.58 & \text{if } 0 \le t \le 20 \\ 0.015t + 0.65 & \text{if } 20 < t \le 30 \end{cases}$$

b. The ratios were changing at the rates of $0.0185/\text{yr}$ from 1960 through 1980 and $0.015/\text{yr}$ from 1980 through 1990.

c. The ratio was 1 when $t \approx 20.3$. This shows that the number of bachelor's degrees earned by women equaled the number earned by men for the first time around 1983.

66. a. $T(x) = 0.06x$

b. $T(200) = 0.06\,(200) = 12$, or \$12.00 and $T(5.65) = 0.06\,(5.65) = 0.34$, or \$0.34.

67. a. $I(x) = 1.053x$

b. $I(1520) = 1.053\,(1520) = 1600.56$, or \$1600.56.

68. a. The function is linear with y-intercept 1.44 and slope 0.058, so we have $f(t) = 0.058t + 1.44$, $0 \le t \le 9$.

b. The projected spending in 2018 will be $f(9) = 0.058\,(9) + 1.44 = 1.962$, or \$1.962 trillion.

69. $S(r) = 4\pi r^2$.

70. $\frac{4}{3}(\pi)(2r)^3 = \frac{4}{3}\pi 8r^3 = 8\left(\frac{4}{3}\pi r^3\right)$. Therefore, the volume of the tumor is increased by a factor of 8.

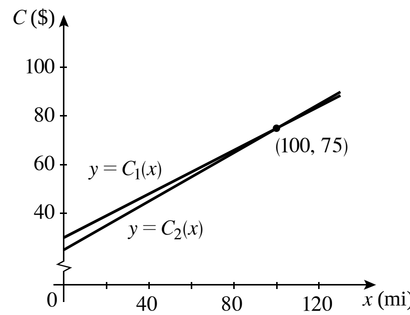
71. a. The median age was changing at the rate of 0.3 years/year.

b. The median age in 2011 was $M(11) = 0.3(11) + 37.9 = 41.2$ (years).

c. The median age in 2015 is projected to be $M(5) = 0.3(15) + 37.9 = 42.4$ (years).

72. a. The daily cost of leasing from Ace is $C_1(x) = 30 + 0.45x$, while the daily cost of leasing from Acme is $C_2(x) = 25 + 0.50x$, where x is the number of miles driven.

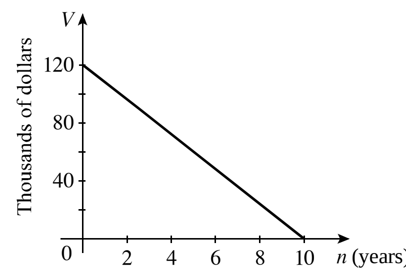
b.



c. The costs are the same when $C_1(x) = C_2(x)$, that is, when $30 + 0.45x = 25 + 0.50x$, $-0.05x = -5$, or $x = 100$. Because $C_1(70) = 30 + 0.45(70) = 61.5$ and $C_2(70) = 25 + 0.50(70) = 60$, and the customer plans to drive less than 70 miles, she should rent from Acme.

73. a. The graph of the function is a straight line passing through $(0, 120000)$ and $(10, 0)$. Its slope is $m = \dfrac{0 - 120{,}000}{10 - 0} = -12{,}000$. The required equation is $V = -12{,}000n + 120{,}000$.

b.



c. $V = -12{,}000(6) + 120{,}000 = 48{,}000$, or \$48,000.

d. This is given by the slope, that is, \$12,000 per year.

74. Here $V = -20{,}000n + 1{,}000{,}000$. The book value in 2010 is given by $V = -20{,}000(15) + 1{,}000{,}000$, or \$700,000. The book value in 2014 is given by $V = -20{,}000(19) + 1{,}000{,}000$, or \$620,000. The book value in 2019 is $V = -20{,}000(24) + 1{,}000{,}000$, or \$520,000.

75. a. The number of incidents in 2009 was $f(0) = 0.46$ (million).

b. The number of incidents in 2013 was $f(4) = 0.2\left(4^2\right) - 0.14(4) + 0.46 = 3.1$ (million).

76. a. The number of passengers in 1995 was $N(0) = 4.6$ (million).

b. The number of passengers in 2010 was $N(15) = 0.011(15)^2 + 0.521(15) + 4.6 = 14.89$ (million).

77. a. The life expectancy of a male whose current age is 65 is $f(65) = 0.0069502(65)^2 - 1.6357(65) + 93.76 \approx 16.80$, or approximately 16.8 years.

b. The life expectancy of a male whose current age is 75 is $f(75) = 0.0069502(75)^2 - 1.6357(75) + 93.76 \approx 10.18$, or approximately 10.18 years.

78. a. $N(t) = 0.00445t^2 + 0.2903t + 9.564$. $N(0) = 9.564$, or 9.6 million people; $N(12) = 0.00445(12)^2 + 0.2903(12) + 9.564 \approx 13.6884$, or approximately 13.7 million people.

b. $N(14) = 0.00445(14)^2 + 0.2903(14) + 9.564 \approx 14.5004$, or approximately 14.5 million people.

79. The projected number in 2030 is $P(20) = -0.0002083(20)^3 + 0.0157(20)^2 - 0.093(20) + 5.2 = 7.9536$, or approximately 8 million.

The projected number in 2050 is $P(40) = -0.0002083(40)^3 + 0.0157(40)^2 - 0.093(40) + 5.2 = 13.2688$, or approximately 13.3 million.

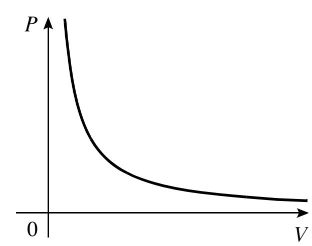
80. $N(t) = -t^3 + 6t^2 + 15t$. Between 8 a.m. and 9 a.m., the average worker can be expected to assemble $N(1) - N(0) = (-1 + 6 + 15) - 0 = 20$, or 20 walkie-talkies. Between 9 a.m. and 10 a.m., we expect that $N(2) - N(1) = \left[-2^3 + 6\left(2^2\right) + 15(2)\right] - (-1 + 6 + 15) = 46 - 20 = 26$, or 26 walkie-talkies can be assembled by the average worker.

81. When the proportion of popular votes won by the Democratic presidential candidate is 0.60, the proportion of seats in the House of Representatives won by Democratic candidates is given by

$$s(0.6) = \frac{(0.6)^3}{(0.6)^3 + (1 - 0.6)^3} = \frac{0.216}{0.216 + 0.064} = \frac{0.216}{0.280} \approx 0.77.$$

82. The amount spent in 2004 was $S(0) = 5.6$, or \$5.6 billion. The amount spent in 2008 was $S(4) = -0.03(4)^3 + 0.2(4)^2 + 0.23(4) + 5.6 = 7.8$, or \$7.8 billion.

83. The domain of the function f is the set of all real positive numbers where $V \neq 0$; that is, $(0, \infty)$.

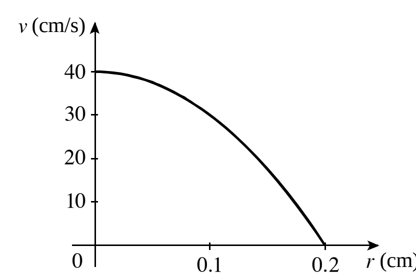


84. **a.** We require that $0.04 - r^2 \geq 0$ and $r \geq 0$. This is true if $0 \leq r \leq 0.2$. Therefore, the domain of v is $[0, 0.2]$.

b. We compute $v(0) = 1000\left[0.04 - (0)^2\right] = 1000(0.04) = 40$,

$$v(0.1) = 1000\left[0.04 - (0.1)^2\right] = 1000(0.04 - 0.01)$$
$$= 1000(0.03) = 30, \text{ and}$$
$$v(0.2) = 1000\left[0.04 - (0.2)^2\right] = 1000(0.04 - 0.04) = 0.$$

c.



d. As the distance r increases, the velocity of the blood decreases.

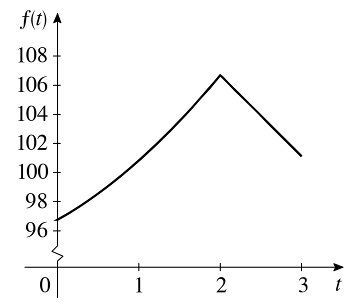
85. **a.** The assets at the beginning of 2002 were \$0.6 trillion. At the beginning of 2003, they were $f(1) = 0.6$, or \$0.6 trillion.

b. The assets at the beginning of 2005 were $f(3) = 0.6(3)^{0.43} \approx 0.96$, or \$0.96 trillion. At the beginning of 2007, they were $f(5) = 0.6(5)^{0.43} \approx 1.20$, or \$1.2 trillion.

86. a.

Year	2006	2007	2008
Rate	96.75	100.84	106.69

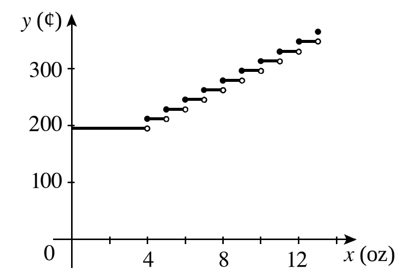
b.



87. a. The domain of f is $(0, 13]$.

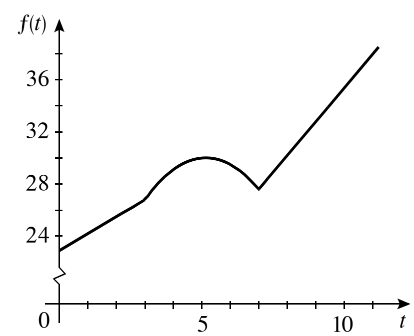
$$f(x) = \begin{cases} 1.95 & \text{if } 0 < x < 4 \\ 2.12 & \text{if } 4 \le x < 5 \\ 2.29 & \text{if } 5 \le x < 6 \\ 2.46 & \text{if } 6 \le x < 7 \\ 2.63 & \text{if } 7 \le x < 8 \\ 2.80 & \text{if } 8 \le x < 9 \\ 2.97 & \text{if } 9 \le x < 10 \\ 3.14 & \text{if } 10 \le x \le 11 \\ 3.31 & \text{if } 11 \le x < 12 \\ 3.48 & \text{if } 12 \le x < 13 \\ 3.65 & \text{if } x = 13 \end{cases}$$

b.



88. a. The median age of the U.S. population at the beginning of 1900 was $f(0) = 22.9$, or 22.9 years; at the beginning of 1950 it was $f(5) = -0.7(5)^2 + 7.2(5) + 11.5 = 30$, or 30 years; and at the beginning of 2000 it was $f(10) = 2.6(10) + 9.4 = 35.4$, or 35.4 years.

b.



89. a. The passenger ship travels a distance given by $14t$ miles east and the cargo ship travels a distance of $10(t-2)$ miles north. After two hours have passed, the distance between the two ships is given by $\sqrt{[10(t-2)]^2 + (14t)^2} = \sqrt{296t^2 - 400t + 400}$ miles, so $D(t) = \begin{cases} 14t & \text{if } 0 \le t \le 2 \\ 2\sqrt{74t^2 - 100t + 100} & \text{if } t > 2 \end{cases}$

b. Three hours after the cargo ship leaves port the value of t is 5. Therefore, $D = 2\sqrt{74(5)^2 - 100(5) + 100} \approx 76.16$, or 76.16 miles.

90. True, by definition of a function (page 52).

91. False. Take $f(x) = x^2$, $a = 1$, and $b = -1$. Then $f(1) = 1 = f(-1)$, but $a \ne b$.

92. False. Let $f(x) = x^2$, then take $a = 1$ and $b = 2$. Then $f(a) = f(1) = 1$, $f(b) = f(2) = 4$, and $f(a) + f(b) = 1 + 4 \ne f(a+b) = f(3) = 9$.

93. False. It intersects the graph of a function in at most one point.

94. True. We have $x + 2 \ge 0$ and $2 - x \ge 0$ simultaneously; that is $x \ge -2$ and $x \le 2$. These inequalities are satisfied if $-2 \le x \le 2$.

95. False. Take $f(x) = x^2$ and $k = 2$. Then $f(x) = (2x)^2 = 4x^2 \neq 2x^2 = 2f(x)$.

96. False. Take $f(x) = 2x + 3$ and $c = 2$. Then $f(2x + y) = 2(2x + y) + 3 = 4x + 2y + 3$, but $cf(x) + f(y) = 2(2x + 3) + (2y + 3) = 4x + 2y + 9 \neq f(2x + y)$.

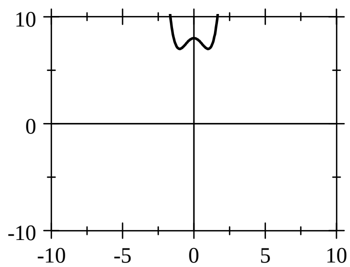
97. False. They are equal everywhere except at $x = 0$, where g is not defined.

98. False. The rule suggests that R takes on the values 0 and 1 when $x = 1$. This violates the uniqueness property that a function must possess.

Using Technology

page 68

1. a.



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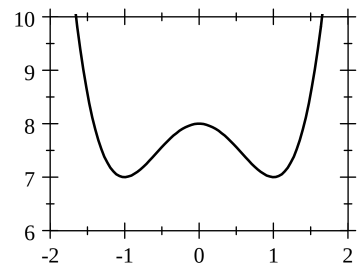
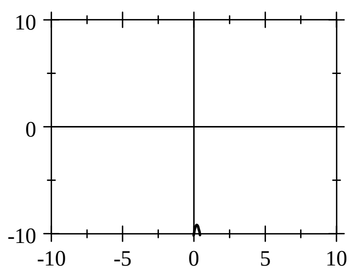


2. a.



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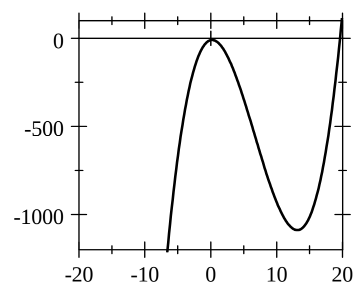
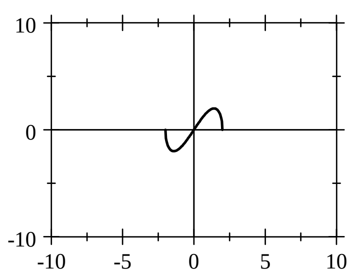


3. a.



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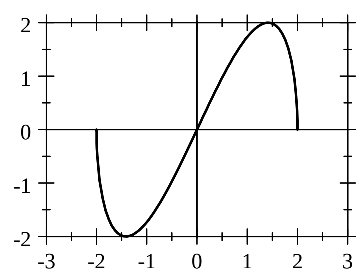
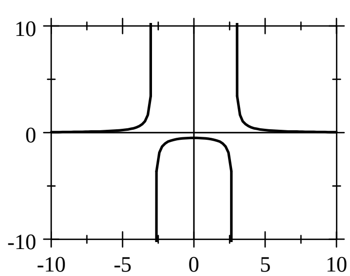


4. a.



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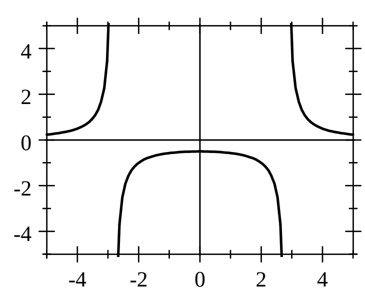

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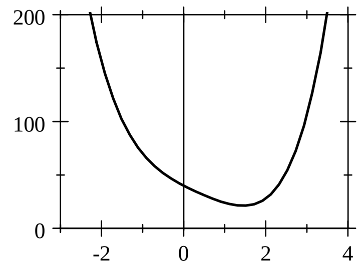

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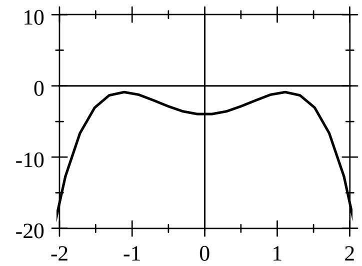

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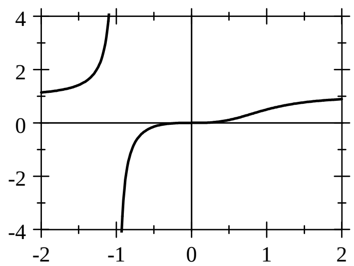

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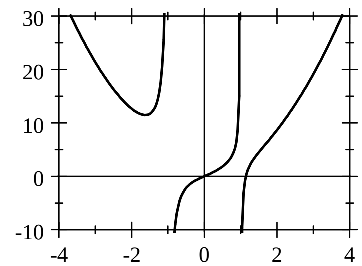


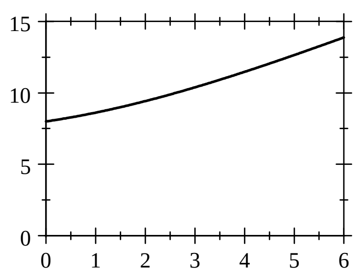
9. $f(2.145) \approx 18.5505$.

10. $f(1.28) \approx 17.3850$.

11. $f(2.41) \approx 4.1616$.

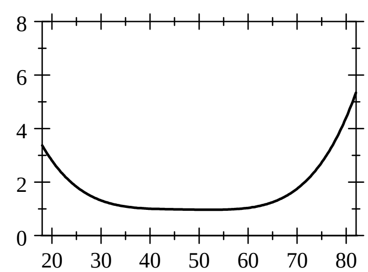
12. $f(0.62) \approx 1.7214$.

13. a.



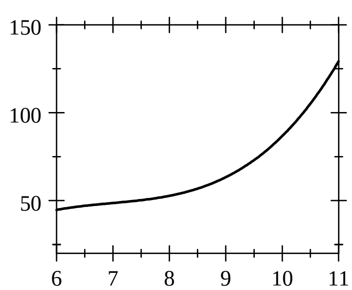
b. The amount spent in the year 2005 was $f(2) \approx 9.42$, or approximately \$9.4 billion. In 2009, it was $f(6) \approx 13.88$, or approximately \$13.9 billion.

14. a.



b. $f(18) = 3.3709$, $f(50) = 0.971$, and $f(80) = 4.4078$.

15. a.



b. $f(6) = 44.7$, $f(8) = 52.7$, and $f(11) = 129.2$.

2.2 The Algebra of Functions

Concept Questions

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1. **a.** $P(x_1) = R(x_1) - C(x_1)$ gives the profit if x_1 units are sold.

   b. $P(x_2) = R(x_2) - C(x_2)$. Because $P(x_2) < 0$, $|R(x_2) - C(x_2)| = -[R(x_2) - C(x_2)]$ gives the loss sustained if x_2 units are sold.

2. **a.** $(f+g)(x) = f(x) + g(x)$, $(f-g)(x) = f(x) - g(x)$, and $(fg)(x) = f(x)g(x)$; all have domain $A \cap B$. $(f/g)(x) = \dfrac{f(x)}{g(x)}$ has domain $A \cap B$ excluding $x \in A \cap B$ such that $g(x) = 0$.

   b. $(f+g)(2) = f(2) + g(2) = 3 + (-2) = 1$, $(f-g)(2) = f(2) - g(2) = 3 - (-2) = 5$, $(fg)(2) = f(2)g(2) = 3(-2) = -6$, and $(f/g)(2) = \dfrac{f(2)}{g(2)} = \dfrac{3}{-2} = -\dfrac{3}{2}$

3. **a.** $y = (f+g)(x) = f(x) + g(x)$

   b. $y = (f-g)(x) = f(x) - g(x)$

   c. $y = (fg)(x) = f(x)g(x)$

   d. $y = \left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

4. **a.** The domain of $(f \circ g)(x) = f(g(x))$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. The domain of $(g \circ f)(x) = g(f(x))$ is the set of all x in the domain of f such that $f(x)$ is in the domain of g.

   b. $(g \circ f)(2) = g(f(2)) = g(3) = 8$. We cannot calculate $(f \circ g)(3)$ because $(f \circ g)(3) = f(g(3)) = f(8)$, and we don't know the value of $f(8)$.

5. No. Let $A = (-\infty, \infty)$, $f(x) = x$, and $g(x) = \sqrt{x}$. Then $a = -1$ is in A, but $(g \circ f)(-1) = g(f(-1)) = g(-1) = \sqrt{-1}$ is not defined.

6. The required expression is $P = g(f(p))$.

Exercises

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1. $(f+g)(x) = f(x) + g(x) = \left(x^3 + 5\right) + \left(x^2 - 2\right) = x^3 + x^2 + 3$.

2. $(f-g)(x) = f(x) - g(x) = \left(x^3 + 5\right) - \left(x^2 - 2\right) = x^3 - x^2 + 7$.

3. $fg(x) = f(x)g(x) = \left(x^3 + 5\right)\left(x^2 - 2\right) = x^5 - 2x^3 + 5x^2 - 10$.

4. $gf(x) = g(x)f(x) = \left(x^2 - 2\right)\left(x^3 + 5\right) = x^5 - 2x^3 + 5x^2 - 10$.

5. $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^3 + 5}{x^2 - 2}$.

6. $\dfrac{f-g}{h}(x) = \dfrac{f(x) - g(x)}{h(x)} = \dfrac{x^3 + 5 - \left(x^2 - 2\right)}{2x + 4} = \dfrac{x^3 - x^2 + 7}{2x + 4}$.

7. $\dfrac{fg}{h}(x) = \dfrac{f(x)g(x)}{h(x)} = \dfrac{\left(x^3 + 5\right)\left(x^2 - 2\right)}{2x + 4} = \dfrac{x^5 - 2x^3 + 5x^2 - 10}{2x + 4}$.

8. $fgh(x) = f(x)g(x)h(x) = \left(x^3+5\right)\left(x^2-2\right)(2x+4) = \left(x^5-2x^3+5x^2-10\right)(2x+4)$
$= 2x^6 - 4x^4 + 10x^3 - 20x + 4x^5 - 8x^3 + 20x^2 - 40 = 2x^6 + 4x^5 - 4x^4 + 2x^3 + 20x^2 - 20x - 40.$

9. $(f+g)(x) = f(x) + g(x) = x - 1 + \sqrt{x+1}.$

10. $(g-f)(x) = g(x) - f(x) = \sqrt{x+1} - (x-1) = \sqrt{x+1} - x + 1.$

11. $(fg)(x) = f(x)g(x) = (x-1)\sqrt{x+1}.$

12. $(gf)(x) = g(x)f(x) = \sqrt{x+1}(x-1).$

13. $\dfrac{g}{h}(x) = \dfrac{g(x)}{h(x)} = \dfrac{\sqrt{x+1}}{2x^3-1}.$

14. $\dfrac{h}{g}(x) = \dfrac{h(x)}{g(x)} = \dfrac{2x^3-1}{\sqrt{x+1}}.$

15. $\dfrac{fg}{h}(x) = \dfrac{(x-1)\left(\sqrt{x+1}\right)}{2x^3-1}.$

16. $\dfrac{fh}{g}(x) = \dfrac{(x-1)\left(2x^3-1\right)}{\sqrt{x+1}} = \dfrac{2x^4-2x^3-x+1}{\sqrt{x+1}}.$

17. $\dfrac{f-h}{g}(x) = \dfrac{x-1-\left(2x^3-1\right)}{\sqrt{x+1}} = \dfrac{x-2x^3}{\sqrt{x+1}}.$

18. $\dfrac{gh}{g-f}(x) = \dfrac{\sqrt{x+1}\left(2x^3-1\right)}{\sqrt{x+1}-(x-1)} = \dfrac{\sqrt{x+1}\left(2x^3-1\right)}{\sqrt{x+1}-x+1}.$

19. $(f+g)(x) = x^2+5+\sqrt{x}-2 = x^2+\sqrt{x}+3$, $(f-g)(x) = x^2+5-\left(\sqrt{x}-2\right) = x^2-\sqrt{x}+7$, $(fg)(x) = \left(x^2+5\right)\left(\sqrt{x}-2\right)$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2+5}{\sqrt{x}-2}.$

20. $(f+g)(x) = \sqrt{x-1}+x^3+1$, $(f-g)(x) = \sqrt{x-1}-x^3-1$, $(fg)(x) = \sqrt{x-1}\left(x^3+1\right)$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x-1}}{x^3+1}.$

21. $(f+g)(x) = \sqrt{x+3} + \dfrac{1}{x-1} = \dfrac{(x-1)\sqrt{x+3}+1}{x-1}$, $(f-g)(x) = \sqrt{x+3} - \dfrac{1}{x-1} = \dfrac{(x-1)\sqrt{x+3}-1}{x-1}$, $(fg)(x) = \sqrt{x+3}\left(\dfrac{1}{x-1}\right) = \dfrac{\sqrt{x+3}}{x-1}$, and $\left(\dfrac{f}{g}\right) = \sqrt{x+3}(x-1).$

22. $(f+g)(x) = \dfrac{1}{x^2+1} + \dfrac{1}{x^2-1} = \dfrac{x^2-1+x^2+1}{\left(x^2+1\right)\left(x^2-1\right)} = \dfrac{2x^2}{\left(x^2+1\right)\left(x^2-1\right)}$,
$(f-g)(x) = \dfrac{1}{x^2+1} - \dfrac{1}{x^2-1} = \dfrac{x^2-1-x^2-1}{\left(x^2+1\right)\left(x^2-1\right)} = -\dfrac{2}{\left(x^2+1\right)\left(x^2-1\right)}$, $(fg)(x) = \dfrac{1}{\left(x^2+1\right)\left(x^2-1\right)}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2-1}{x^2+1}.$

23. $(f+g)(x)=\dfrac{x+1}{x-1}+\dfrac{x+2}{x-2}=\dfrac{(x+1)(x-2)+(x+2)(x-1)}{(x-1)(x-2)}=\dfrac{x^2-x-2+x^2+x-2}{(x-1)(x-2)}$
$=\dfrac{2x^2-4}{(x-1)(x-2)}=\dfrac{2\left(x^2-2\right)}{(x-1)(x-2)}$,
$(f-g)(x)=\dfrac{x+1}{x-1}-\dfrac{x+2}{x-2}=\dfrac{(x+1)(x-2)-(x+2)(x-1)}{(x-1)(x-2)}=\dfrac{x^2-x-2-x^2-x+2}{(x-1)(x-2)}$
$=\dfrac{-2x}{(x-1)(x-2)}$,
$(fg)(x)=\dfrac{(x+1)(x+2)}{(x-1)(x-2)}$, and $\left(\dfrac{f}{g}\right)(x)=\dfrac{(x+1)(x-2)}{(x-1)(x+2)}$.

24. $(f+g)(x)=x^2+1+\sqrt{x+1}$, $(f-g)(x)=x^2+1-\sqrt{x+1}$, $(fg)(x)=\left(x^2+1\right)\sqrt{x+1}$, and $\left(\dfrac{f}{g}\right)(x)=\dfrac{x^2+1}{\sqrt{x+1}}$.

25. $(f\circ g)(x)=f(g(x))=f\left(x^2\right)=\left(x^2\right)^2+x^2+1=x^4+x^2+1$ and
$(g\circ f)(x)=g(f(x))=g\left(x^2+x+1\right)=\left(x^2+x+1\right)^2$.

26. $(f\circ g)(x)=f(g(x))=3\left[g(x)\right]^2+2g(x)+1=3(x+3)^2+2(x+3)+1=3x^2+20x+34$ and
$(g\circ f)(x)=g(f(x))=f(x)+3=3x^2+2x+1+3=3x^2+2x+4$.

27. $(f\circ g)(x)=f(g(x))=f\left(x^2-1\right)=\sqrt{x^2-1}+1$ and
$(g\circ f)(x)=g(f(x))=g\left(\sqrt{x}+1\right)=\left(\sqrt{x}+1\right)^2-1=x+2\sqrt{x}+1-1=x+2\sqrt{x}$.

28. $(f\circ g)(x)=f(g(x))=2\sqrt{g(x)}+3=2\sqrt{x^2+1}+3$ and
$(g\circ f)(x)=g(f(x))=\left[f(x)\right]^2+1=\left(2\sqrt{x}+3\right)^2+1=4x+12\sqrt{x}+10$.

29. $(f\circ g)(x)=f(g(x))=f\left(\dfrac{1}{x}\right)=\dfrac{1}{x}\div\left(\dfrac{1}{x^2}+1\right)=\dfrac{1}{x}\cdot\dfrac{x^2}{x^2+1}=\dfrac{x}{x^2+1}$ and
$(g\circ f)(x)=g(f(x))=g\left(\dfrac{x}{x^2+1}\right)=\dfrac{x^2+1}{x}$.

30. $(f\circ g)(x)=f(g(x))=f\left(\dfrac{1}{x-1}\right)=\sqrt{\dfrac{x}{x-1}}$ and
$(g\circ f)(x)=g(f(x))=g\left(\sqrt{x+1}\right)=\dfrac{1}{\sqrt{x+1}-1}\cdot\dfrac{\sqrt{x+1}+1}{\sqrt{x+1}+1}=\dfrac{\sqrt{x+1}+1}{x}$.

31. $h(2)=g(f(2))$. But $f(2)=2^2+2+1=7$, so $h(2)=g(7)=49$.

32. $h(2)=g(f(2))$. But $f(2)=\left(2^2-1\right)^{1/3}=3^{1/3}$, so $h(2)=g\left(3^{1/3}\right)=3\left(3^{1/3}\right)^3+1=3(3)+1=10$.

33. $h(2)=g(f(2))$. But $f(2)=\dfrac{1}{2(2)+1}=\dfrac{1}{5}$, so $h(2)=g\left(\dfrac{1}{5}\right)=\dfrac{1}{\sqrt{5}}=\dfrac{\sqrt{5}}{5}$.

34. $h(2)=g(f(2))$. But $f(2)=\dfrac{1}{2-1}=1$, so $g(1)=1^2+1=2$.

35. $f(x) = 2x^3 + x^2 + 1$, $g(x) = x^5$.

36. $f(x) = 3x^2 - 4$, $g(x) = x^{-3}$.

37. $f(x) = x^2 - 1$, $g(x) = \sqrt{x}$.

38. $f(x) = (2x - 3)$, $g(x) = x^{3/2}$.

39. $f(x) = x^2 - 1$, $g(x) = \dfrac{1}{x}$.

40. $f(x) = x^2 - 4$, $g(x) = \dfrac{1}{\sqrt{x}}$.

41. $f(x) = 3x^2 + 2$, $g(x) = \dfrac{1}{x^{3/2}}$.

42. $f(x) = \sqrt{2x+1}$, $g(x) = \dfrac{1}{x} + x$.

43. $f(a+h) - f(a) = [3(a+h) + 4] - (3a + 4) = 3a + 3h + 4 - 3a - 4 = 3h$.

44. $f(a+h) - f(a) = -\frac{1}{2}(a+h) + 3 - \left(-\frac{1}{2}a + 3\right) = -\frac{1}{2}a - \frac{1}{2}h + 3 + \frac{1}{2}a - 3 = -\frac{1}{2}h$.

45. $f(a+h) - f(a) = 4 - (a+h)^2 - \left(4 - a^2\right) = 4 - a^2 - 2ah - h^2 - 4 + a^2 = -2ah - h^2 = -h(2a+h)$.

46.
$$\begin{aligned} f(a+h) - f(a) &= \left[(a+h)^2 - 2(a+h) + 1\right] - \left(a^2 - 2a + 1\right) \\ &= a^2 + 2ah + h^2 - 2a - 2h + 1 - a^2 + 2a - 1 = h(2a + h - 2). \end{aligned}$$

47.
$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{\left[(a+h)^2 + 1\right] - \left(a^2 + 1\right)}{h} = \frac{a^2 + 2ah + h^2 + 1 - a^2 - 1}{h} = \frac{2ah + h^2}{h} \\ &= \frac{h(2a+h)}{h} = 2a + h. \end{aligned}$$

48.
$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{\left[2(a+h)^2 - (a+h) + 1\right] - \left(2a^2 - a + 1\right)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - a - h + 1 - 2a^2 + a - 1}{h} = \frac{4ah + 2h^2 - h}{h} = 4a + 2h - 1. \end{aligned}$$

49.
$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{\left[(a+h)^3 - (a+h)\right] - \left(a^3 - a\right)}{h} = \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a - h - a^3 + a}{h} \\ &= \frac{3a^2h + 3ah^2 + h^3 - h}{h} = 3a^2 + 3ah + h^2 - 1. \end{aligned}$$

50.
$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{\left[2(a+h)^3 - (a+h)^2 + 1\right] - \left(2a^3 - a^2 + 1\right)}{h} \\ &= \frac{2a^3 + 6a^2h + 6ah^2 + 2h^3 - a^2 - 2ah - h^2 + 1 - 2a^3 + a^2 - 1}{h} \\ &= \frac{6a^2h + 6ah^2 + 2h^3 - 2ah - h^2}{h} = 6a^2 + 6ah + 2h^2 - 2a - h. \end{aligned}$$

51.
$$\frac{f(a+h) - f(a)}{h} = \frac{\dfrac{1}{a+h} - \dfrac{1}{a}}{h} = \frac{\dfrac{a - (a+h)}{a(a+h)}}{h} = -\frac{1}{a(a+h)}.$$

52.
$$\frac{f(a+h) - f(a)}{h} = \frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} = \frac{(a+h) - a}{h\left(\sqrt{a+h} + \sqrt{a}\right)} = \frac{1}{\sqrt{a+h} + \sqrt{a}}.$$

53. $F(t)$ represents the total revenue for the two restaurants at time t.

54. $F(t)$ represents the net rate of growth of the species of whales in year t.

55. $f(t)g(t)$ represents the dollar value of Nancy's holdings at time t.

56. $f(t)/g(t)$ represents the unit cost of the commodity at time t.

57. $g \circ f$ is the function giving the amount of carbon monoxide pollution from cars in parts per million at time t.

58. $f \circ g$ is the function giving the revenue at time t.

59. $C(x) = 0.6x + 12{,}100$.

60. a. $h(t) = f(t) - g(t) = (3t + 69) - (-0.2t + 13.8) = 3.2t + 55.2$, $0 \leq t \leq 5$.

b. $f(5) = 3(5) + 69 = 84$, $g(5) = -0.2(5) + 13.8 = 12.8$, and $h(5) = 3.2(5) + 55.2 = 71.2$. Since $f(5) - g(5) = 84 - 12.8 = 71.2$, we see that $h(5)$ is indeed equal to $f(5) - g(5)$.

61. $$D(t) = (D_2 - D_1)(t) = D_2(t) - D_1(t) = \left(0.035t^2 + 0.21t + 0.24\right) - \left(0.0275t^2 + 0.081t + 0.07\right)$$
$$\approx 0.0075t^2 + 0.129t + 0.17.$$

The function D gives the difference in year t between the deficit without the \$160 million rescue package and the deficit with the rescue package.

62. a. $(g \circ f)(0) = g(f(0)) = g(0.64) = 26$, so the mortality rate of motorcyclists in the year 2000 was 26 per 100 million miles traveled.

b. $(g \circ f)(6) = g(f(6)) = g(0.51) = 42$, so the mortality rate of motorcyclists in 2006 was 42 per 100 million miles traveled.

c. Between 2000 and 2006, the percentage of motorcyclists wearing helmets had dropped from 64 to 51, and as a consequence, the mortality rate of motorcyclists had increased from 26 million miles traveled to 42 million miles traveled.

63. a. $(g \circ f)(1) = g(f(1)) = g(406) = 23$. So in 2002, the percentage of reported serious crimes that end in arrests or in the identification of suspects was 23.

b. $(g \circ f)(6) = g(f(6)) = g(326) = 18$. In 2007, 18% of reported serious crimes ended in arrests or in the identification of suspects.

c. Between 2002 and 2007, the total number of detectives had dropped from 406 to 326 and as a result, the percentage of reported serious crimes that ended in arrests or in the identification of suspects dropped from 23 to 18.

64. a. $C(x) = 0.000003x^3 - 0.03x^2 + 200x + 100{,}000$, so $C(2000) = 0.000003(2000)^3 - 0.03(2000)^2 + 200(2000) + 100{,}000 = 404{,}000$, or \$404,000.

b. $$P(x) = R(x) - C(x) = -0.1x^2 + 500x - \left(0.000003x^3 - 0.03x^2 + 200x + 100{,}000\right)$$
$$= -0.000003x^3 - 0.07x^2 + 300x - 100{,}000.$$

c. $P(1500) = -0.000003(1500)^3 - 0.07(1500)^2 + 300(1500) - 100{,}000 = 182{,}375$, or \$182,375.

65. **a.** $C(x) = V(x) + 20000 = 0.000001x^3 - 0.01x^2 + 50x + 20000 = 0.000001x^3 - 0.01x^2 + 50x + 20{,}000$.

b. $P(x) = R(x) - C(x) = -0.02x^2 + 150x - 0.000001x^3 + 0.01x^2 - 50x - 20{,}000$

$$= -0.000001x^3 - 0.01x^2 + 100x - 20{,}000.$$

c. $P(2000) = -0.000001(2000)^3 - 0.01(2000)^2 + 100(2000) - 20{,}000 = 132{,}000$, or \$132,000.

66. **a.** $D(t) = R(t) - S(t)$

$$= \left(0.023611t^3 - 0.19679t^2 + 0.34365t + 2.42\right) - \left(-0.015278t^3 + 0.11179t^2 + 0.02516t + 2.64\right)$$

$$= 0.038889t^3 - 0.30858t^2 + 0.31849t - 0.22,\ 0 \le t \le 6.$$

b. $S(3) = 3.309084$, $R(3) = 2.317337$, and $D(3) = -0.991747$, so the spending, revenue, and deficit are approximately \$3.31 trillion, \$2.32 trillion, and \$0.99 trillion, respectively.

c. Yes: $R(3) - S(3) = 2.317337 - 3.308841 = -0.991504 = D(3)$.

67. **a.** $h(t) = f(t) + g(t) = \left(4.389t^3 - 47.833t^2 + 374.49t + 2390\right) + \left(13.222t^3 - 132.524t^2 + 757.9t + 7481\right)$

$$= 17.611t^3 - 180.357t^2 + 1132.39t + 9871,\ 1 \le t \le 7.$$

b. $f(6) = 3862.976$ and $g(6) = 10{,}113.488$, so $f(6) + g(6) = 13{,}976.464$. The worker's contribution was approximately \$3862.98, the employer's contribution was approximately \$10,113.49, and the total contributions were approximately \$13,976.46.

c. $h(6) = 13{,}976 = f(6) + g(6)$, as expected.

68. **a.** $N(r(t)) = \dfrac{7}{1 + 0.02\left(\dfrac{5t + 75}{t + 10}\right)^2}$.

b. $N(r(0)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 0 + 75}{0 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{75}{10}\right)^2} \approx 3.29$, or 3.29 million units.

$N(r(12)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 12 + 75}{12 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{135}{22}\right)^2} \approx 3.99$, or 3.99 million units.

$N(r(18)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 18 + 75}{18 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{165}{28}\right)^2} \approx 4.13$, or 4.13 million units.

69. **a.** The occupancy rate at the beginning of January is $r(0) = \frac{10}{81}(0)^3 - \frac{10}{3}(0)^2 + \frac{200}{9}(0) + 55 = 55$, or 55%.
$r(5) = \frac{10}{81}(5)^3 - \frac{10}{3}(5)^2 + \frac{200}{9}(5) + 55 \approx 98.2$, or approximately 98.2%.

b. The monthly revenue at the beginning of January is $R(55) = -\frac{3}{5000}(55)^3 + \frac{9}{50}(55)^2 \approx 444.68$, or approximately \$444,700.
The monthly revenue at the beginning of June is $R(98.2) = -\frac{3}{5000}(98.2)^3 + \frac{9}{50}(98.2)^2 \approx 1167.6$, or approximately \$1,167,600.

70. $N(t) = 1.42 \cdot x(t) = \dfrac{1.42 \cdot 7(t+10)^2}{(t+10)^2 + 2(t+15)^2} = \dfrac{9.94(t+10)^2}{(t+10)^2 + 2(t+15)^2}$. The number of jobs created 6 months from now will be $N(6) = \dfrac{9.94(16)^2}{(16)^2 + 2(21)^2} \approx 2.24$, or approximately 2.24 million jobs. The number of jobs created 12 months from now will be $N(12) = \dfrac{9.94(22)^2}{(22)^2 + 2(27)^2} \approx 2.48$, or approximately 2.48 million jobs.

71. a. $s = f + g + h = (f + g) + h = f + (g + h)$. This suggests we define the sum s by $s(x) = (f + g + h)(x) = f(x) + g(x) + h(x)$.

b. Let f, g, and h define the revenue (in dollars) in week t of three branches of a store. Then its total revenue (in dollars) in week t is $s(t) = (f + g + h)(t) = f(t) + g(t) + h(t)$.

72. a. $(h \circ g \circ f)(x) = h(g(f(x)))$

b. Let t denote time. Suppose f gives the number of people at time t in a town, g gives the number of cars as a function of the number of people in the town, and H gives the amount of carbon monoxide in the atmosphere. Then $(h \circ g \circ f)(t) = h(g(f(t)))$ gives the amount of carbon monoxide in the atmosphere at time t.

73. True. $(f + g)(x) = f(x) + g(x) = g(x) + f(x) = (g + f)(x)$.

74. False. Let $f(x) = x + 2$ and $g(x) = \sqrt{x}$. Then $(g \circ f)(x) = \sqrt{x+2}$ is defined at $x = -1$, But $(f \circ g)(x) = \sqrt{x} + 2$ is not defined at $x = -1$.

75. False. Take $f(x) = \sqrt{x}$ and $g(x) = x + 1$. Then $(g \circ f)(x) = \sqrt{x} + 1$, but $(f \circ g)(x) = \sqrt{x+1}$.

76. False. Take $f(x) = x + 1$. Then $(f \circ f)(x) = f(f(x)) = x + 2$, but $f^2(x) = [f(x)]^2 = (x+1)^2 = x^2 + 2x + 1$.

77. True. $(h \circ (g \circ f))(x) = h((g \circ f)(x)) = h(g(f(x)))$ and $((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$.

78. False. Take $h(x) = \sqrt{x}$, $g(x) = x$, and $f(x) = x^2$. Then $(h \circ (g + f))(x) = h(x + x^2) = \sqrt{x + x^2} \neq ((h \circ g) + (h \circ f))(x) = h(g(x)) + h(f(x)) = \sqrt{x} + \sqrt{x^2}$.

2.3 Functions and Mathematical Models

Concept Questions

page 88

1. See page 78 of the text. Answers will vary.

2. a. $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$, where $a_n \neq 0$ and n is a positive integer. An example is $P(x) = 4x^3 - 3x^2 + 2$.

b. $R(x) = \dfrac{P(x)}{Q(x)}$, where P and Q are polynomials with $Q(x) \neq 0$. An example is $R(x) = \dfrac{3x^4 - 2x^2 + 1}{x^2 + 3x + 5}$.

3. a. A demand function $p = D(x)$ gives the relationship between the unit price of a commodity p and the quantity x demanded. A supply function $p = S(x)$ gives the relationship between the unit price of a commodity p and the quantity x the supplier will make available in the marketplace.

b. Market equilibrium occurs when the quantity produced is equal to the quantity demanded. To find the market equilibrium, we solve the equations $p = D(x)$ and $p = S(x)$ simultaneously.

Exercises page 88

1. Yes. $2x + 3y = 6$ and so $y = -\frac{2}{3}x + 2$.

2. Yes. $4y = 2x + 7$ and so $y = \frac{1}{2}x + \frac{7}{4}$.

3. Yes. $2y = x + 4$ and so $y = \frac{1}{2}x + 2$.

4. Yes. $3y = 2x - 8$ and so $y = \frac{2}{3}x - \frac{8}{3}$.

5. Yes. $4y = 2x + 9$ and so $y = \frac{1}{2}x + \frac{9}{4}$.

6. Yes. $6y = 3x + 7$ and so $y = \frac{1}{2}x + \frac{7}{6}$.

7. No, because of the term x^2.

8. No, because of the term $\sqrt{x}$.

9. f is a polynomial function in x of degree 6.

10. f is a rational function.

11. Expanding $G(x) = 2\left(x^2 - 3\right)^3$, we have $G(x) = 2x^6 - 18x^4 + 54x^2 - 54$, and we conclude that G is a polynomial function of degree 6 in x.

12. We can write $H(x) = \dfrac{2}{x^3} + \dfrac{5}{x^2} + 6 = \dfrac{2 + 5x + 6x^3}{x^3}$ and conclude that H is a rational function.

13. f is neither a polynomial nor a rational function.

14. f is a rational function.

15. $f(0) = 2$ gives $f(0) = m(0) + b = b = 2$. Next, $f(3) = -1$ gives $f(3) = m(3) + b = -1$. Substituting $b = 2$ in this last equation, we have $3m + 2 = -1$, or $3m = -3$, and therefore, $m = -1$ and $b = 2$.

16. $f(2) = 4$ gives $f(2) = 2m + b = 4$. We also know that $m = -1$. Therefore, we have $2(-1) + b = 4$ and so $b = 6$.

17. a. $C(x) = 8x + 40{,}000$.

b. $R(x) = 12x$.

c. $P(x) = R(x) - C(x) = 12x - (8x + 40{,}000) = 4x - 40{,}000$.

d. $P(8000) = 4(8000) - 40{,}000 = -8000$, or a loss of \$8000. $P(12{,}000) = 4(12{,}000) - 40{,}000 = 8000$, or a profit of \$8000.

18. a. $C(x) = 14x + 100{,}000$.

b. $R(x) = 20x$.

c. $P(x) = R(x) - C(x) = 20x - (14x + 100{,}000) = 6x - 100{,}000$.

d. $P(12{,}000) = 6(12{,}000) - 100{,}000 = -28{,}000$, or a loss of \$28,000. $P(20{,}000) = 6(20{,}000) - 100{,}000 = 20{,}000$, or a profit of \$20,000.

19. The individual's disposable income is $D = (1 - 0.28) \cdot 60{,}000 = 43{,}200$, or \$43,200.

20. The child should receive $D(0.4) = \dfrac{(0.4)(500)}{1.7} \approx 117.65$, or approximately 118 mg.

21. The child should receive $D(4) = \left(\dfrac{4 + 1}{24}\right)(500) \approx 104.17$, or approximately 104 mg.

22. a. The graph of f passes through the points $P_1(0, 17.5)$ and $P_2(10, 10.3)$. Its slope is $\dfrac{10.3 - 17.5}{10 - 0} = -0.72$. An equation of the line is $y - 17.5 = -0.72(t - 0)$ or $y = -0.72t + 17.5$, so the linear function is $f(t) = -0.72t + 17.5$.

b. The rate was decreasing at 0.72% per year.

c. The percentage of high school students who drink and drive at the beginning of 2014 is projected to be $f(13) = -0.72(13) + 17.5 = 8.14$, or 8.14%.

23. a. The slope of the graph of f is a line with slope -13.2 passing through the point $(0, 400)$, so an equation of the line is $y - 400 = -13.2(t - 0)$ or $y = -13.2t + 400$, and the required function is $f(t) = -13.2t + 400$.

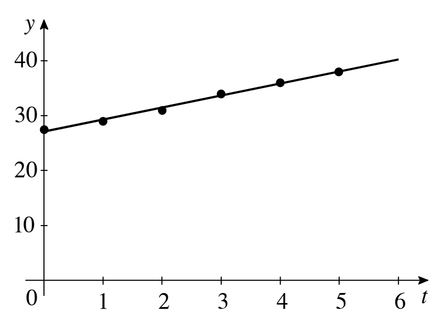
b. The emissions cap is projected to be $f(2) = -13.2(2) + 400 = 373.6$, or 373.6 million metric tons of carbon dioxide equivalent.

24. a. The graph of f is a line through the points $P_1(0, 0.7)$ and $P_2(20, 1.2)$, so it has slope $\dfrac{1.2 - 0.7}{20 - 0} = 0.025$. Its equation is $y - 0.7 = 0.025(t - 0)$ or $y = 0.025t + 0.7$. The required function is thus $f(t) = 0.025t + 0.7$.

b. The projected annual rate of growth is the slope of the graph of f, that is, 0.025 billion per year, or 25 million per year.

c. The projected number of boardings per year in 2022 is $f(10) = 0.025(10) + 0.7 = 0.95$, or 950 million boardings per year.

25. a.



b. The projected revenue in 2010 is projected to be $f(6) = 2.19(6) + 27.12 = 40.26$, or \$40.26 billion.

c. The rate of increase is the slope of the graph of f, that is, 2.19 (billion dollars per year).

26. Two hours after starting work, the average worker will be assembling at the rate of $f(2) = -\frac{3}{2}(2)^2 + 6(2) + 10 = 16$, or 16 phones per hour.

27. $P(28) = -\frac{1}{8}(28)^2 + 7(28) + 30 = 128$, or \$128,000.

28. a. The amount paid out in 2010 was $S(0) = 0.72$, or \$0.72 trillion (or \$720 billion).

b. The amount paid out in 2030 is projected to be $S(3) = 0.1375(3)^2 + 0.5185(3) + 0.72 = 3.513$, or \$3.513 trillion.

29. a. The average time spent per day in 2009 was $f(0) = 21.76$ (minutes).

b. The average time spent per day in 2013 is projected to be $f(4) = 2.25(4)^2 + 13.41(4) + 21.76 = 111.4$ (minutes).

30. a. The GDP in 2011 was $G(0) = 15$, or \$15 trillion.

b. The projected GDP in 2015 is $G(4) = 0.064(4)^2 + 0.473(4) + 15.0 = 17.916$, or \$17.196 trillion.

31. a. The GDP per capita in 2000 was $f(10) = 1.86251(10)^2 - 28.08043(10) + 884 = 789.4467$, or \$789.45.

b. The GDP per capita in 2030 is projected to be $f(40) = 1.86251(40)^2 - 28.08043(40) + 884 = 2740.7988$, or \$2740.80.

32. **a.** The number of enterprise IM accounts in 2006 is given by $N(0) = 59.7$, or 59.7 million.

b. The number of enterprise IM accounts in 2010, assuming a continuing trend, is given by $N(4) = 2.96(4)^2 + 11.37(4) + 59.7 = 152.54$ million.

33. $S(6) = 0.73(6)^2 + 15.8(6) + 2.7 = 123.78$ million kilowatt-hr. $S(8) = 0.73(8)^2 + 15.8(8) + 2.7 = 175.82$ million kilowatt-hr.

34. The U.S. public debt in 2005 was $f(0) = 8.246$, or \$8.246 trillion. The public debt in 2008 was $f(3) = -0.03817(3)^3 + 0.4571(3)^2 - 0.1976(3) + 8.246 = 10.73651$, or approximately \$10.74 trillion.

35. The percentage who expected to work past age 65 in 1991 was $f(0) = 11$, or 11%. The percentage in 2013 was $f(22) = 0.004545(22)^3 - 0.1113(22)^2 + 1.385(22) + 11 = 35.99596$, or approximately 36%.

36. $N(0) = 0.7$ per 100 million vehicle miles driven. $N(7) = 0.0336(7)^3 - 0.118(7)^2 + 0.215(7) + 0.7 = 7.9478$ per 100 million vehicle miles driven.

37. **a.** Total global mobile data traffic in 2009 was $f(0) = 0.06$, or 60,000 terabytes.

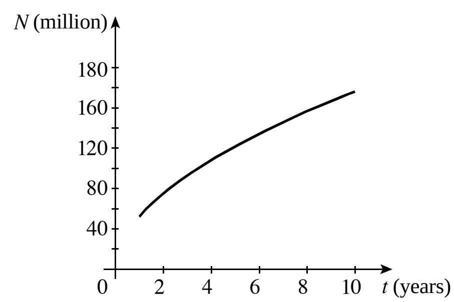
b. The total in 2014 will be $f(5) = 0.021(5)^3 + 0.015(5)^2 + 0.12(5) + 0.06 = 3.66$, or 3.66 million terabytes.

38. Here $Y = 0.06$, $D = 0.2$, and $R = 0.05$, so the leveraged return is $L = \dfrac{0.06 - (1 - 0.2)(0.05)}{0.2} = 0.1$, or 10%.

39. **a.** We first construct a table.

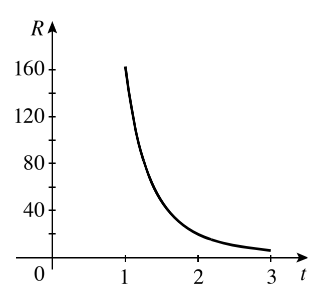
t	$N(t)$
1	52
2	75
3	93
4	109
5	122

t	$N(t)$
6	135
7	146
8	157
9	167
10	177



b. The number of viewers in 2012 is given by $N(10) = 52(10)^{0.531} \approx 176.61$, or approximately 177 million viewers.

40. **a.**



$R(1) = 162.8(1)^{-3.025} = 162.8$, $R(2) = 162.8(2)^{-3.025} \approx 20.0$, and $R(3) = 162.8(3)^{-3.025} \approx 5.9$.

b. The infant mortality rates in 1900, 1950, and 2000 are 162.8, 20.0, and 5.9 per 1000 live births, respectively.

41. $N(5) = 0.0018425(10)^{2.5} \approx 0.58265$, or approximately 0.583 million. $N(13) = 0.0018425(18)^{2.5} \approx 2.5327$, or approximately 2.5327 million.

42. **a.** $S(0) = 4.3(0 + 2)^{0.94} \approx 8.24967$, or approximately \$8.25 billion.

b. $S(8) = 4.3(8 + 2)^{0.94} \approx 37.45$, or approximately \$37.45 billion.

43. a. We are given that $f(1) = 5240$ and $f(4) = 8680$. This leads to the system of equations $a + b = 5240$, $11a + b = 8680$. Solving, we find $a = 344$ and $b = 4896$.

b. From part (a), we have $f(t) = 344t + 4896$, so the approximate per capita costs in 2005 were $f(5) = 344(5) + 4896 = 6616$, or \$6616.

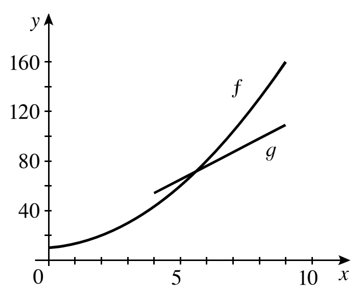
44. a. The given data imply that $R(40) = 50$, that is, $\dfrac{100(40)}{b+40} = 50$, so $50(b+40) = 4000$, or $b = 40$. Therefore, the required response function is $R(x) = \dfrac{100x}{40+x}$.

b. The response will be $R(60) = \dfrac{100(60)}{40+60} = 60$, or approximately 60 percent.

45. a. $f(0) = 6.85$, $g(0) = 16.58$. Because $g(0) > f(0)$, we see that more film cameras were sold in 2001 (when $t = 0$).

b. We solve the equation $f(t) = g(t)$, that is, $3.05t + 6.85 = -1.85t + 16.58$, so $4.9t = 9.73$ and $t = 1.99 \approx 2$. So sales of digital cameras first exceed those of film cameras in approximately 2003.

46. a.



b. $5x^2 + 5x + 30 = 33x + 30$, so $5x^2 - 28x = 0$, $x(5x - 28) = 0$, and $x = 0$ or $x = \frac{28}{5} = 5.6$, representing 5.6 mi/h. $g(x) = 11(5.6) + 10 = 71.6$, or 71.6 mL/lb/min.

c. The oxygen consumption of the walker is greater than that of the runner.

47. a. We are given that $T = aN + b$ where a and b are constants to be determined. The given conditions imply that $70 = 120a + b$ and $80 = 160a + b$. Subtracting the first equation from the second gives $10 = 40a$, or $a = \frac{1}{4}$. Substituting this value of a into the first equation gives $70 = 120\left(\frac{1}{4}\right) + b$, or $b = 40$. Therefore, $T = \frac{1}{4}N + 40$.

b. Solving the equation in part (a) for N, we find $\frac{1}{4}N = T - 40$, or $N = f(t) = 4T - 160$. When $T = 102$, we find $N = 4(102) - 160 = 248$, or 248 times per minute.

48. a. $f(0) = 3173$ gives $c = 3173$, $f(4) = 6132$ gives $16a + 4b + c = 6132$, and $f(6) = 7864$ gives $36a + 6b + c = 1864$. Solving, we find $a \approx 21.0417$, $b \approx 655.5833$, and $c = 3173$.

b. From part (a), we have $f(t) = 21.0417t^2 + 655.5833t + 3173$, so the number of farmers' markets in 2014 is projected to be $f(8) = 21.0417(8)^2 + 655.5833(8) + 3173 = 9764.3352$, or approximately 9764.

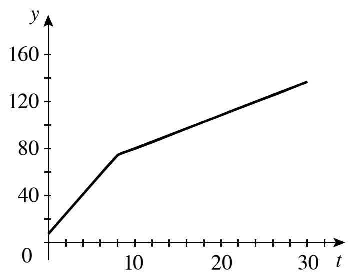
49. a. We have $f(0) = c = 1547$, $f(2) = 4a + 2b + c = 1802$, and $f(4) = 16a + 4b + c = 2403$. Solving this system of equations gives $a = 43.25$, $b = 41$, and $c = 1547$.

b. From part (a), we have $f(t) = 43.25t^2 + 41t + 1547$, so the number of craft-beer breweries in 2014 is projected to be $f(6) = 43.25(6)^2 + 41(6) + 1547 = 3350$.

50. The slope of the line is $m = \dfrac{S-C}{n}$. Therefore, an equation of the line is $y - C = \dfrac{S-C}{n}(t-0)$. Letting $y = V(t)$, we have $V(t) = C - \dfrac{C-S}{n}t$.

51. Using the formula given in Exercise 50, we have $V(2) = 100{,}000 - \frac{100{,}000 - 30{,}000}{5}(2) = 100{,}000 - \frac{70{,}000}{5}(2) = 72{,}000$, or \$72,000.

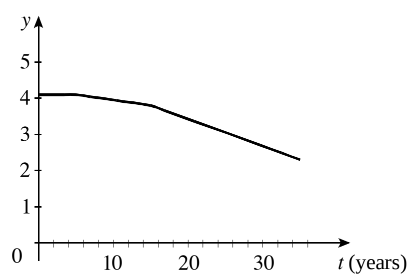
52. a.



b. $f(0) = 8.37(0) + 7.44 = 7.44$, or \$7.44/kilo.
$f(20) = 2.84(20) + 51.68 = 108.48$, or \$108.48/kilo.

53. The total cost by 2011 is given by $f(1) = 5$, or \$5 billion. The total cost by 2015 is given by $f(5) = -0.5278\left(5^3\right) + 3.012\left(5^2\right) + 49.23(5) - 103.29 = 152.185$, or approximately \$152 billion.

54. a.

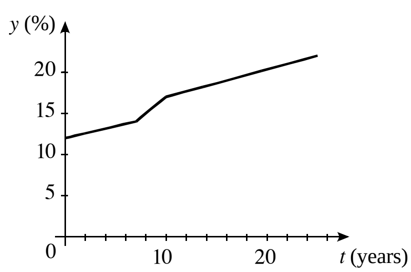


b. At the beginning of 2005, the ratio will be $f(10) = -0.03(10) + 4.25 = 3.95$. At the beginning of 2020, the ratio will be $f(25) = -0.075(25) + 4.925 = 3.05$.

c. The ratio is constant from 1995 to 2000.

d. The decline of the ratio is greatest from 2010 through 2030. It is $\frac{f(35) - f(15)}{35 - 15} = \frac{2.3 - 3.8}{20} = -0.075$.

55. a.

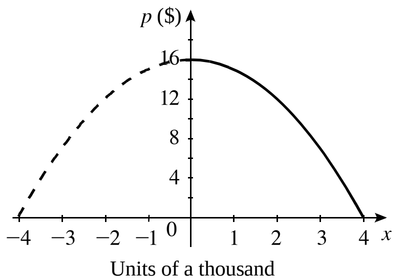


b. $f(5) = \frac{2}{7}(5) + 12 = \frac{10}{7} + 12 \approx 13.43$, or approximately 13.43%. $f(25) = \frac{1}{3}(25) + \frac{41}{3} = 22$, or 22%.

56. a. $f(0) = 5.6$ and $g(0) = 22.5$. Because $g(0) > f(0)$, we conclude that more VCRs than DVD players were sold in 2001.

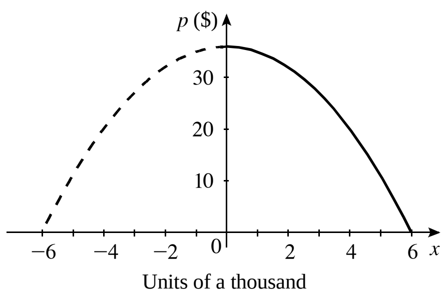
b. We solve the equations $f(t) = g(t)$ over each of the subintervals. $5.6 + 5.6t = -9.6t + 22.5$ for $0 \le t \le 1$. We solve to find $15.2t = 16.9$, so $t \approx 1.11$. This is outside the range for t, so we reject it. $5.6 + 5.6t = -0.5t + 13.4$ for $1 < t \le 2$, so $6.1t = 7.8$, and thus $t \approx 1.28$. So sales of DVD players first exceed those of VCRs at $t \approx 1.3$, or in early 2002.

57. a.



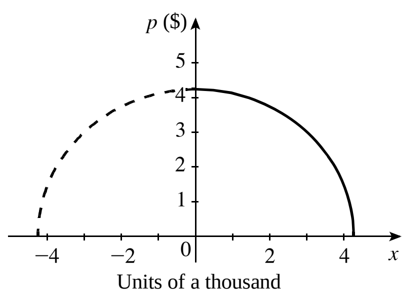
b. If $p = 7$, we have $7 = -x^2 + 16$, or $x^2 = 9$, so that $x = \pm 3$. Therefore, the quantity demanded when the unit price is $7 is 3000 units.

58. a.



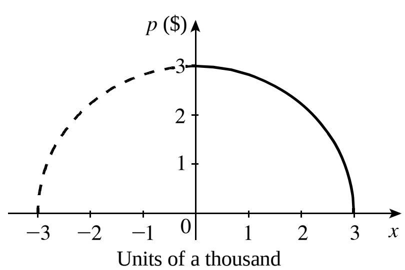
b. If $p = 11$, we have $11 = -x^2 + 36$, or $x^2 = 25$, so that $x = \pm 5$. Therefore, the quantity demanded when the unit price is $11 is 5000 units.

59. a.



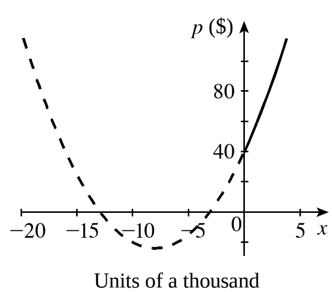
b. If $p = 3$, then $3 = \sqrt{18 - x^2}$, and $9 = 18 - x^2$, so that $x^2 = 9$ and $x = \pm\, 3$. Therefore, the quantity demanded when the unit price is $3 is 3000 units.

60. a.



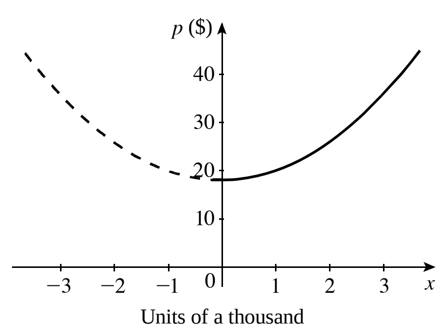
b. If $p = 2$, then $2 = \sqrt{9 - x^2}$, and $4 = 9 - x^2$, so that $x^2 = 5$ and and $x = \pm\sqrt{5}$, or $x \approx \pm 2.236$, Therefore, the quantity demanded when the unit price is $2 is approximately 2236 units.

61. a.



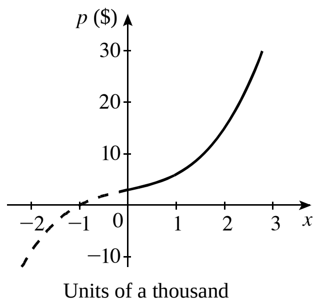
b. If $x = 2$, then $p = 2^2 + 16\,(2) + 40 = 76$, or $76.

62. a.



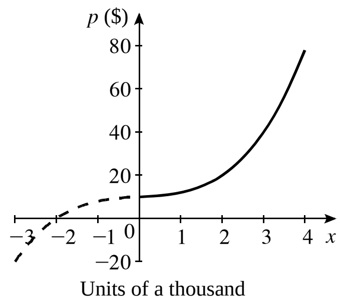
b. If $x = 2$, then $p = 2\,(2)^2 + 18 = 26$, or $26.

63. a.



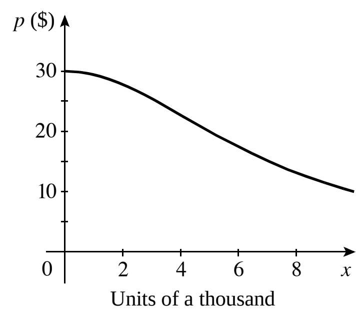
b. $p = 2^3 + 2(2) + 3 = 15$, or \$15.

64. a.



b. $p = 2^3 + 2 + 10 = 20$, or \$20.

65. a.

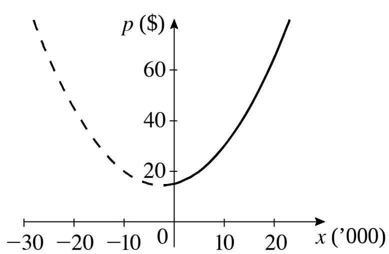


b. Substituting $x = 10$ into the demand function, we have

$$p = \frac{30}{0.02(10)^2 + 1} = \frac{30}{3} = 10, \text{ or } \$10.$$

66. Substituting $x = 6$ and $p = 8$ into the given equation gives $8 = \sqrt{-36a + b}$, or $-36a + b = 64$. Next, substituting $x = 8$ and $p = 6$ into the equation gives $6 = \sqrt{-64a + b}$, or $-64a + b = 36$. Solving the system $\begin{cases} -36a + b = 64 \\ -64a + b = 36 \end{cases}$ for a and b, we find $a = 1$ and $b = 100$. Therefore the demand equation is $p = \sqrt{-x^2 + 100}$. When the unit price is set at \$7.50, we have $7.5 = \sqrt{-x^2 + 100}$, or $56.25 = -x^2 + 100$ from which we deduce that $x \approx \pm 6.614$. Thus, the quantity demanded is approximately 6614 units.

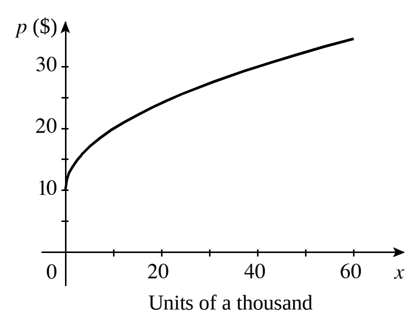
67. a.



b. If $x = 5$, then

$$p = 0.1(5)^2 + 0.5(5) + 15 = 20, \text{ or } \$20.$$

68. Substituting $x = 10{,}000$ and $p = 20$ into the given equation yields $20 = a\sqrt{10{,}000} + b = 100a + b$. Next, substituting $x = 62{,}500$ and $p = 35$ into the equation yields $35 = a\sqrt{62{,}500} + b = 250a + b$. Subtracting the first equation from the second yields $15 = 150a$, or $a = \frac{1}{10}$. Substituting this value of a into the first equation gives $b = 10$. Therefore, the required equation is $p = \frac{1}{10}\sqrt{x} + 10$. Substituting $x = 40{,}000$ into the supply equation yields $p = \frac{1}{10}\sqrt{40{,}000} + 10 = 30$, or \$30.



69. **a.** We solve the system of equations $p = cx + d$ and $p = ax + b$. Substituting the first equation into the second gives $cx + d = ax + d$, so $(c - a)x = b - d$ and $x = \dfrac{b-d}{c-a}$. Because $a < 0$ and $c > 0$, $c - a \neq 0$ and x is well-defined. Substituting this value of x into the second equation, we obtain $p = a\left(\dfrac{b-d}{c-a}\right) + b = \dfrac{ab - ad + bc - ab}{c-a} = \dfrac{bc-ad}{c-a}$. Therefore, the equilibrium quantity is $\dfrac{b-d}{c-a}$ and the equilibrium price is $\dfrac{bc-ad}{c-a}$.

b. If c is increased, the denominator in the expression for x increases and so x gets smaller. At the same time, the first term in the first equation for p decreases and so p gets larger. This analysis shows that if the unit price for producing the product is increased then the equilibrium quantity decreases while the equilibrium price increases.

c. If b is decreased, the numerator of the expression for x decreases while the denominator stays the same. Therefore, x decreases. The expression for p also shows that p decreases. This analysis shows that if the (theoretical) upper bound for the unit price of a commodity is lowered, then both the equilibrium quantity and the equilibrium price drop.

70. We solve the system of equations $p = -x^2 - 2x + 100$ and $p = 8x + 25$. Thus, $-x^2 - 2x + 100 = 8x + 25$, or $x^2 + 10x - 75 = 0$. Factoring this equation, we have $(x + 15)(x - 5) = 0$. Therefore, $x = -15$ or $x = 5$. Rejecting the negative root, we have $x = 5$, and the corresponding value of p is $p = 8(5) + 25 = 65$. We conclude that the equilibrium quantity is 5000 and the equilibrium price is \$65.

71. We solve the equation $-2x^2 + 80 = 15x + 30$, or $2x^2 + 15x - 50 = 0$ for x. Thus, $(2x - 5)(x + 10) = 0$, and so $x = \frac{5}{2}$ or $x = -10$. Rejecting the negative root, we have $x = \frac{5}{2}$. The corresponding value of p is $p = -2\left(\frac{5}{2}\right)^2 + 80 = 67.5$. We conclude that the equilibrium quantity is 2500 and the equilibrium price is \$67.50.

72. We solve the system $\begin{cases} p = 60 - 2x^2 \\ p = x^2 + 9x + 30 \end{cases}$ Equating the right-hand sides, we have $x^2 + 9x + 30 = 60 - 2x^2$, so $3x^2 + 9x - 30 = 0$, $x^2 + 3x - 10 = 0$, and $(x + 5)(x - 2) = 0$, giving $x = -5$ or $x = 2$. We take $x = 2$. The corresponding value of p is 52, so the equilibrium quantity is 2000 and the equilibrium price is \$52.

73. Solving both equations for x, we have $x = -\frac{11}{3}p + 22$ and $x = 2p^2 + p - 10$. Equating the right-hand sides, we have $-\frac{11}{3}p + 22 = 2p^2 + p - 10$, or $-11p + 66 = 6p^2 + 3p - 30$, and so $6p^2 + 14p - 96 = 0$. Dividing this last equation by 2 and then factoring, we have $(3p + 16)(p - 3) = 0$, so $p = 3$ is the only valid solution. The corresponding value of x is $2(3)^2 + 3 - 10 = 11$. We conclude that the equilibrium quantity is 11,000 and the equilibrium price is \$3.

74. Equating the right-hand sides of the two equations, we have $0.1x^2 + 2x + 20 = -0.1x^2 - x + 40$, so $0.2x^2 + 3x - 20 = 0$, $2x^2 + 30x - 200 = 0$, $x^2 + 15x - 100 = 0$, and $(x + 20)(x - 5) = 0$. Therefore the only valid solution is $x = 5$. Substituting $x = 5$ into the first equation gives $p = -0.1(25) - 5 + 40 = 32.5$. Therefore, the equilibrium quantity is 500 tents (x is measured in hundreds) and the equilibrium price is \$32.50.

75. Equating the right-hand sides of the two equations, we have $144 - x^2 = 48 + \frac{1}{2}x^2$, so $288 - 2x^2 = 96 + x^2$, $3x^2 = 192$, and $x^2 = 64$. Therefore, $x = \pm 8$. We take $x = 8$, and the corresponding value of p is $144 - 8^2 = 80$. We conclude that the equilibrium quantity is 8000 tires and the equilibrium price is \$80.

76. Because there is 80 feet of fencing available, $2x + 2y = 80$, so $x + y = 40$ and $y = 40 - x$. Then the area of the garden is given by $f = xy = x(40 - x) = 40x - x^2$. The domain of f is $[0, 40]$.

77. The area of Juanita's garden is 250 ft^2. Therefore $xy = 250$ and $y = \dfrac{250}{x}$. The amount of fencing needed is given by $2x + 2y$. Therefore, $f = 2x + 2\left(\dfrac{250}{x}\right) = 2x + \dfrac{500}{x}$. The domain of f is $x > 0$.

78. The volume of the box is given by area of the base times the height of the box. Thus, $V = f(x) = (15 - 2x)(8 - 2x)x$.

79. Because the volume of the box is the area of the base times the height of the box, we have $V = x^2 y = 20$. Thus, we have $y = \dfrac{20}{x^2}$. Next, the amount of material used in constructing the box is given by the area of the base of the box, plus the area of the four sides, plus the area of the top of the box; that is, $A = x^2 + 4xy + x^2$. Then, the cost of constructing the box is given by $f(x) = 0.30x^2 + 0.40x \cdot \dfrac{20}{x^2} + 0.20x^2 = 0.5x^2 + \dfrac{8}{x}$, where $f(x)$ is measured in dollars and $f(x) > 0$.

80. Because the perimeter of a circle is $2\pi r$, we know that the perimeter of the semicircle is πx. Next, the perimeter of the rectangular portion of the window is given by $2y + 2x$, so the perimeter of the Norman window is $\pi x + 2y + 2x$ and $\pi x + 2y + 2x = 28$, or $y = \frac{1}{2}(28 - \pi x - 2x)$. Because the area of the window is given by $2xy + \frac{1}{2}\pi x^2$, we see that $A = 2xy + \frac{1}{2}\pi x^2$. Substituting the value of y found earlier, we see that

$$A = f(x) = x(28 - \pi x - 2x) + \tfrac{1}{2}\pi x^2 = \tfrac{1}{2}\pi x^2 + 28x - \pi x^2 - 2x^2 = 28x - \tfrac{\pi}{2}x^2 - 2x^2$$
$$= 28x - \left(\frac{\pi}{2} + 2\right)x^2.$$

81. The average yield of the apple orchard is 36 bushels/tree when the density is 22 trees/acre. Let x be the unit increase in tree density beyond 22. Then the yield of the apple orchard in bushels/acre is given by $(22 + x)(36 - 2x)$.

82. $xy = 50$ and so $y = \dfrac{50}{x}$. The area of the printed page is $A = (x - 1)(y - 2) = (x - 1)\left(\dfrac{50}{x} - 2\right) = -2x + 52 - \dfrac{50}{x}$, so the required function is $f(x) = -2x + 52 - \dfrac{50}{x}$. We must have $x > 0$, $x - 1 \geq 0$, and and $\dfrac{50}{x} - 2 \geq 2$. The last inequality is solved as follows: $\dfrac{50}{x} \geq 4$, so $\dfrac{x}{50} \leq \dfrac{1}{4}$, so $x \leq \frac{50}{4} = \frac{25}{2}$. Thus, the domain is $\left[1, \frac{25}{2}\right]$.

83. **a.** Let x denote the number of bottles sold beyond 10,000 bottles. Then $P(x) = (10{,}000 + x)(5 - 0.0002x) = -0.0002x^2 + 3x + 50{,}000$.

b. He can expect a profit of $P(6000) = -0.0002\left(6000^2\right) + 3(6000) + 50{,}000 = 60{,}800$, or \$60,800.

84. **a.** Let x denote the number of people beyond 20 who sign up for the cruise. Then the revenue is $R(x) = (20 + x)(600 - 4x) = -4x^2 + 520x + 12{,}000$.

b. $R(40) = -4\left(40^2\right) + 520(40) + 12{,}000 = 26{,}400$, or \$26,400.

c. $R(60) = -4\left(60^2\right) + 520(60) + 12{,}000 = 28{,}800$, or \$28,800.

85. False. $f(x) = 3x^{3/4} + x^{1/2} + 1$ is not a polynomial function. The powers of x must be nonnegative integers.

86. True. If $P(x)$ is a polynomial function, then $P(x) = \dfrac{P(x)}{1}$ and so it is a rational function. The converse is false. For example, $R(x) = \dfrac{x+1}{x-1}$ is a rational function that is not a polynomial.

87. False. $f(x) = x^{1/2}$ is not defined for negative values of x.

88. False. A power function has the form x^r, where r is a real number.

Using Technology page 98

1. $(-3.0414, 0.1503)$, $(3.0414, 7.4497)$.

2. $(-5.3852, 9.8007)$, $(5.3852, -4.2007)$.

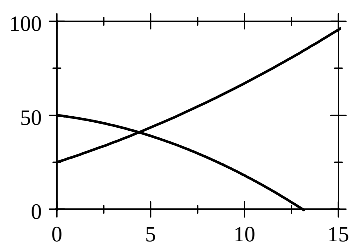
3. $(-2.3371, 2.4117)$, $(6.0514, -2.5015)$.

4. $(-2.5863, -0.3585)$, $(6.1863, -4.5694)$.

5. $(-1.0219, -6.3461)$, $(1.2414, -1.5931)$, and $(5.7805, 7.9391)$.

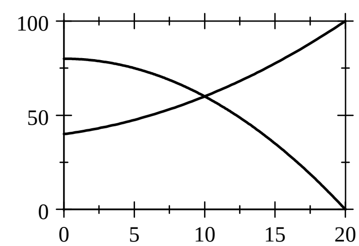
6. $(-0.0484, 2.0609)$, $(2.0823, 2.8986)$, and $(4.9661, 1.1405)$.

7. a.



b. 438 wall clocks; $40.92.

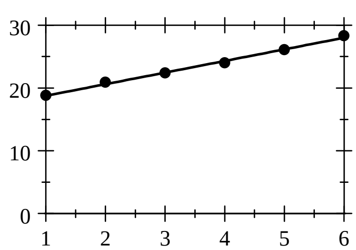
8. a.



b. 1000 cameras; $60.00.

9. a. $f(t) = 1.85t + 16.9$.

b.



c.

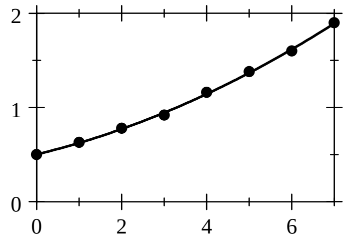
t	y
1	18.8
2	20.6
3	22.5
4	24.3
5	26.2
6	28.0

These values are close to the given data.

d. $f(8) = 1.85(8) + 16.9 = 31.7$ gallons.

10. a. $f(t) = 0.0128t^2 + 0.109t + 0.50.$

b.



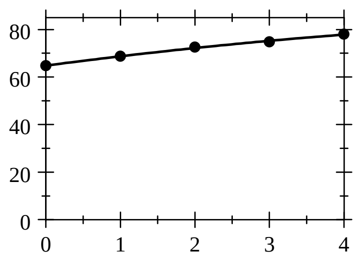
c.

t	y
0	0.50
3	0.94
6	1.61
7	1.89

These values are close to the given data.

11. a. $f(t) = -0.221t^2 + 4.14t + 64.8.$

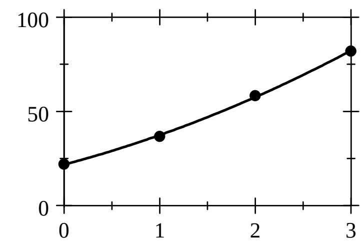
b.



c. 77.8 million

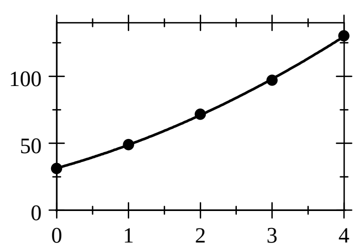
12. a. $f(t) = 2.25x^2 + 13.41x + 21.76.$

b.



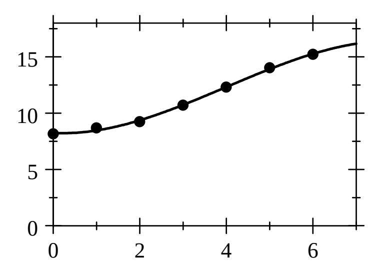
13. a. $f(t) = 2.4t^2 + 15t + 31.4.$

b.



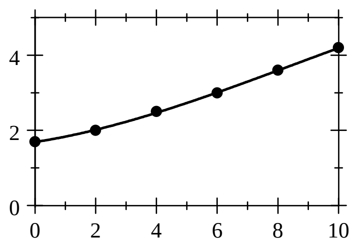
14. a. $f(t) = -0.038167t^3 + 0.45713t^2 - 0.19758t + 8.2457.$

b.



15. a. $f(t) = -0.00081t^3 + 0.0206t^2 + 0.125t + 1.69.$

b.



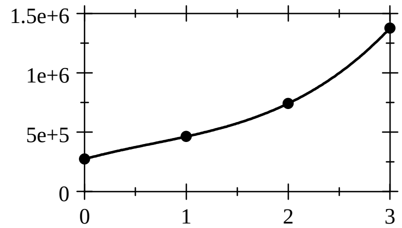
c.

t	y
1	1.8
5	2.7
10	4.2

The revenues were \$1.8 trillion in 2001, \$2.7 trillion in 2005, and \$4.2 trillion in 2010.

16. a. $y = 44{,}560t^3 - 89{,}394t^2 + 234{,}633t + 273{,}288.$

b.

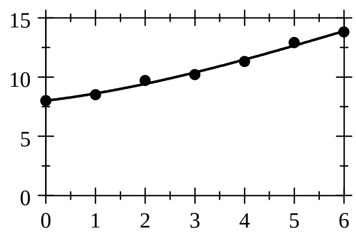


c.

t	$f(t)$
0	273,288
1	463,087
2	741,458
3	1,375,761

17. a. $f(t) = -0.0056t^3 + 0.112t^2 + 0.51t + 8.$

b.

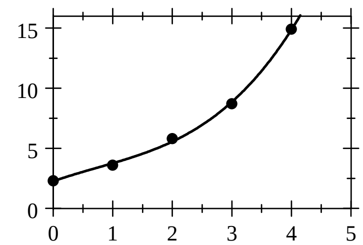


c.

t	0	3	6
$f(t)$	8	10.4	13.9

18. a. $f(t) = 0.2t^3 - 0.45t^2 + 1.75t + 2.26.$

b.

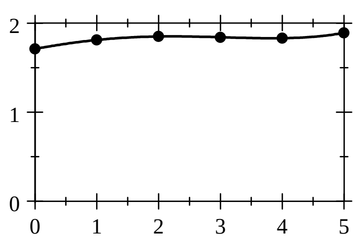


c.

t	0	1	2	3	4
$f(t)$	2.3	3.8	5.6	8.9	14.9

19. a. $f(t) = 0.00125t^4 + 0.0051t^3 - 0.0243t^2 + 0.129t + 1.71.$

b.



c.

t	0	1	2	3	4	5
$f(t)$	1.71	1.81	1.85	1.84	1.83	1.89

d. The average amount of nicotine in 2005 is $f(6) = 2.128$, or approximately 2.13 mg/cigarette.

20. $A(t) = 0.000008140t^4 - 0.00043833t^3 - 0.0001305t^2 + 0.02202t + 2.612.$

2.4 Limits

Concept Questions

page 115

1. The values of $f(x)$ can be made as close to 3 as we please by taking x sufficiently close to $x = 2$.

2. a. Nothing. Whether $f(3)$ is defined or not does not depend on $\lim_{x\to 3} f(x)$.

b. Nothing. $\lim_{x\to 2} f(x)$ has nothing to do with the value of f at $x = 2$.

3. a. $\lim_{x\to 4} \sqrt{x}\left(2x^2+1\right) = \lim_{x\to 4}\left(\sqrt{x}\right)\lim_{x\to 4}\left(2x^2+1\right)$ (Rule 4)

$= \sqrt{4}\left[2(4)^2+1\right]$ (Rules 1 and 3)

$= 66$

b. $\lim_{x\to 1}\left(\dfrac{2x^2+x+5}{x^4+1}\right)^{3/2} = \left(\lim_{x\to 1}\dfrac{2x^2+x+5}{x^4+1}\right)^{3/2}$ (Rule 1)

$= \left(\dfrac{2+1+5}{1+1}\right)^{3/2}$ (Rules 2, 3, and 5)

$= 4^{3/2} = 8$

4. A limit that has the form $\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{0}{0}$. For example, $\lim_{x\to 3}\dfrac{x^2-9}{x-3}$.

5. $\lim_{x\to\infty} f(x) = L$ means $f(x)$ can be made as close to L as we please by taking x sufficiently large. $\lim_{x\to-\infty} f(x) = M$ means $f(x)$ can be made as close to M as we please by taking negative x as large as we please in absolute value.

Exercises page 115

1. $\lim_{x\to -2} f(x) = 3.$

2. $\lim_{x\to 1} f(x) = 2.$

3. $\lim_{x\to 3} f(x) = 3.$

4. $\lim_{x\to 1} f(x)$ does not exist. If we consider any value of x to the right of $x = 1$, we find that $f(x) = 3$. On the other hand, if we consider values of x to the left of $x = 1$, $f(x) \le 1.5$, so that $f(x)$ does not approach a fixed number as x approaches 1.

5. $\lim_{x\to -2} f(x) = 3.$

6. $\lim_{x\to -2} f(x) = 3.$

7. The limit does not exist. If we consider any value of x to the right of $x = -2$, $f(x) \le 2$. If we consider values of x to the left of $x = -2$, $f(x) \ge -2$. Because $f(x)$ does not approach any one number as x approaches $x = -2$, we conclude that the limit does not exist.

8. The limit does not exist.

9.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	4.61	4.9601	4.9960	5.004	5.0401	5.41

$\lim_{x\to 2}\left(x^2+1\right) = 5.$

10.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.62	0.9602	0.996002	1.004002	1.0402	1.42

$\lim_{x\to 1}(2x^2 - 1) = 1.$

11.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	-1	-1	-1	1	1	1

The limit does not exist.

12.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	-1	-1	-1	1	1	1

The limit does not exist.

13.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	100	10,000	1,000,000	1,000,000	10,000	100

The limit does not exist.

14.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	-10	-100	-1000	1000	100	10

The limit does not exist.

15.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	2.9	2.99	2.999	3.001	3.01	3.1

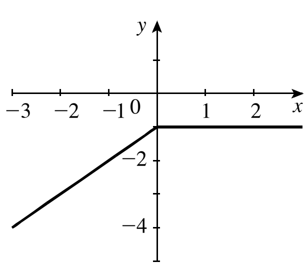
$\lim_{x\to 1}\frac{x^2 + x - 2}{x - 1} = 3.$

16.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	1	1	1	1	1	1

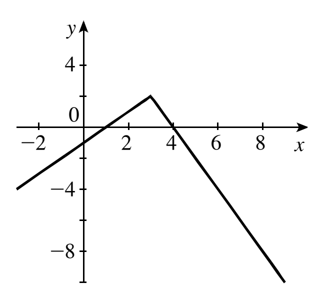
$\lim_{x\to 1}\frac{x - 1}{x - 1} = 1.$

17.



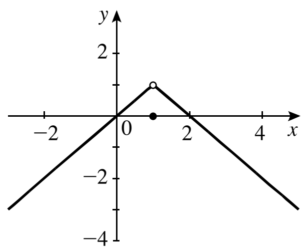
$\lim_{x\to 0} f(x) = -1.$

18.



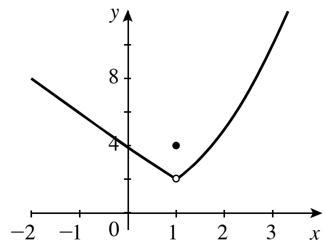
$\lim_{x\to 3} f(x) = 2.$

19.



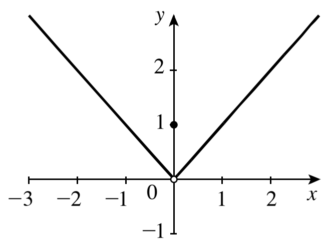
$\lim_{x\to 1} f(x) = 1.$

20.



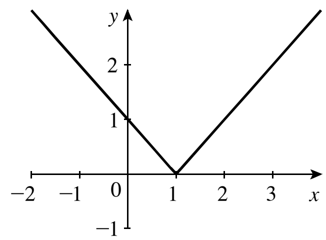
$\lim_{x\to 1} f(x) = 2.$

21.



$\lim_{x\to 0} f(x) = 0.$

22.



$\lim_{x\to 1} f(x) = 0.$

23. $\lim_{x\to 2} 3 = 3.$

24. $\lim_{x\to -2} -3 = -3.$

25. $\lim_{x\to 3} x = 3.$

26. $\lim_{x\to -2} -3x = -3(-2) = 6.$

27. $\lim_{x\to 1} \left(1 - 2x^2\right) = 1 - 2(1)^2 = -1.$

28. $\lim_{t\to 3} \left(4t^2 - 2t + 1\right) = 4(3)^2 - 2(3) + 1 = 31.$

29. $\lim_{x\to 1} \left(2x^3 - 3x^2 + x + 2\right) = 2(1)^3 - 3(1)^2 + 1 + 2 = 2.$

30. $\lim_{x\to 0} \left(4x^5 - 20x^2 + 2x + 1\right) = 4(0)^5 - 20(0)^2 + 2(0) + 1 = 1.$

31. $\lim_{s\to 0} \left(2s^2 - 1\right)(2s + 4) = (-1)(4) = -4.$

32. $\lim_{x\to 2} \left(x^2 + 1\right)\left(x^2 - 4\right) = \left(2^2 + 1\right)\left(2^2 - 4\right) = 0.$

33. $\lim_{x\to 2} \dfrac{2x + 1}{x + 2} = \dfrac{2(2) + 1}{2 + 2} = \dfrac{5}{4}.$

34. $\lim_{x\to 1} \dfrac{x^3 + 1}{2x^3 + 2} = \dfrac{1^3 + 1}{2\left(1^3\right) + 2} = \dfrac{2}{4} = \dfrac{1}{2}.$

35. $\lim_{x\to 2} \sqrt{x + 2} = \sqrt{2 + 2} = 2.$

36. $\lim_{x\to -2} \sqrt[3]{5x + 2} = \sqrt[3]{5(-2) + 2} = \sqrt[3]{-8} = -2.$

37. $\lim_{x\to -3} \sqrt{2x^4 + x^2} = \sqrt{2(-3)^4 + (-3)^2} = \sqrt{162 + 9} = \sqrt{171} = 3\sqrt{19}.$

38. $\lim_{x\to 2} \sqrt{\dfrac{2x^3 + 4}{x^2 + 1}} = \sqrt{\dfrac{2(8) + 4}{4 + 1}} = 2.$

39. $\lim_{x\to -1} \dfrac{\sqrt{x^2 + 8}}{2x + 4} = \dfrac{\sqrt{(-1)^2 + 8}}{2(-1) + 4} = \dfrac{\sqrt{9}}{2} = \dfrac{3}{2}.$

40. $\lim_{x\to 3} \dfrac{x\sqrt{x^2 + 7}}{2x - \sqrt{2x + 3}} = \dfrac{3\sqrt{3^2 + 7}}{2(3) - \sqrt{2(3) + 3}} = \dfrac{12}{3} = 4.$

41. $\lim_{x\to a}\left[f(x)-g(x)\right]=\lim_{x\to a}f(x)-\lim_{x\to a}g(x)$
$=3-4=-1.$

42. $\lim_{x\to a}2f(x)=2(3)=6.$

43. $\lim_{x\to a}\left[2f(x)-3g(x)\right]=\lim_{x\to a}2f(x)-\lim_{x\to a}3g(x)$
$=2(3)-3(4)=-6.$

44. $\lim_{x\to a}\left[f(x)g(x)\right]=\lim_{x\to a}f(x)\cdot\lim_{x\to a}g(x)=3\cdot4$
$=12.$

45. $\lim_{x\to a}\sqrt{g(x)}=\lim_{x\to a}\sqrt{4}=2.$

46. $\lim_{x\to a}\sqrt[3]{5f(x)+3g(x)}=\sqrt[3]{5(3)+3(4)}=\sqrt[3]{27}=3.$

47. $\lim_{x\to a}\dfrac{2f(x)-g(x)}{f(x)g(x)}=\dfrac{2(3)-(4)}{(3)(4)}=\dfrac{2}{12}=\dfrac{1}{6}.$

48. $\lim_{x\to a}\dfrac{g(x)-f(x)}{f(x)+\sqrt{g(x)}}=\dfrac{4-3}{3+2}=\dfrac{1}{5}.$

49. $\lim_{x\to1}\dfrac{x^2-1}{x-1}=\lim_{x\to1}\dfrac{(x-1)(x+1)}{x-1}=\lim_{x\to1}(x+1)$
$=1+1=2.$

50. $\lim_{x\to-2}\dfrac{x^2-4}{x+2}=\lim_{x\to-2}\dfrac{(x-2)(x+2)}{x+2}$
$=\lim_{x\to-2}(x-2)=-2-2=-4.$

51. $\lim_{x\to0}\dfrac{x^2-x}{x}=\lim_{x\to0}\dfrac{x(x-1)}{x}=\lim_{x\to0}(x-1)$
$=0-1=-1.$

52. $\lim_{x\to0}\dfrac{2x^2-3x}{x}=\lim_{x\to0}\dfrac{x(2x-3)}{x}=\lim_{x\to0}(2x-3)$
$=-3.$

53. $\lim_{x\to-5}\dfrac{x^2-25}{x+5}=\lim_{x\to-5}\dfrac{(x+5)(x-5)}{x+5}$
$=\lim_{x\to-5}(x-5)=-10.$

54. $\lim_{b\to-3}\dfrac{b+1}{b+3}$ does not exist.

55. $\lim_{x\to1}\dfrac{x}{x-1}$ does not exist.

56. $\lim_{x\to2}\dfrac{x+2}{x-2}$ does not exist.

57. $\lim_{x\to-2}\dfrac{x^2-x-6}{x^2+x-2}=\lim_{x\to-2}\dfrac{(x-3)(x+2)}{(x+2)(x-1)}=\lim_{x\to-2}\dfrac{x-3}{x-1}=\dfrac{-2-3}{-2-1}=\dfrac{5}{3}.$

58. $\lim_{z\to2}\dfrac{z^3-8}{z-2}=\lim_{z\to2}\dfrac{(z-2)\left(z^2+2z+4\right)}{z-2}=\lim_{z\to2}\left(z^2+2z+4\right)=2^2+2(2)+4=12.$

59. $\lim_{x\to1}\dfrac{\sqrt{x}-1}{x-1}=\lim_{x\to1}\dfrac{\sqrt{x}-1}{x-1}\cdot\dfrac{\sqrt{x}+1}{\sqrt{x}+1}=\lim_{x\to1}\dfrac{x-1}{(x-1)\left(\sqrt{x}+1\right)}=\lim_{x\to1}\dfrac{1}{\sqrt{x}+1}=\dfrac{1}{2}.$

60. $\lim_{x\to4}\dfrac{x-4}{\sqrt{x}-2}=\lim_{x\to4}\dfrac{x-4}{\sqrt{x}-2}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2}=\lim_{x\to4}\sqrt{x}+2=2+2=4.$

61. $\lim_{x\to1}\dfrac{x-1}{x^3+x^2-2x}=\lim_{x\to1}\dfrac{x-1}{x(x-1)(x+2)}=\lim_{x\to1}\dfrac{1}{x(x+2)}=\dfrac{1}{3}.$

62. $\lim_{x\to-2}\dfrac{4-x^2}{2x^2+x^3}=\lim_{x\to-2}\dfrac{(2-x)(2+x)}{x^2(2+x)}=\lim_{x\to-2}\dfrac{2-x}{x^2}=\dfrac{2-(-2)}{(-2)^2}=1.$

63. $\lim_{x\to\infty}f(x)=\infty$ (does not exist) and $\lim_{x\to-\infty}f(x)=\infty$ (does not exist).

64. $\lim_{x\to\infty} f(x) = \infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

65. $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$.

66. $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = 1$.

67. $\lim_{x\to\infty} f(x) = -\infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

68. $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = \infty$ (does not exist).

69. $f(x) = \dfrac{1}{x^2+1}$.

x	1	10	100	1000
$f(x)$	0.5	0.009901	0.0001	0.000001

x	-1	-10	-100	-1000
$f(x)$	0.5	0.009901	0.0001	0.000001

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$.

70. $f(x) = \dfrac{2x}{x+1}$.

x	1	10	100	1000
$f(x)$	1	1.818	1.980	1.998

x	-5	-10	-100	-1000
$f(x)$	2.5	2.222	2.020	2.002

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 2$.

71. $f(x) = 3x^3 - x^2 + 10$.

x	1	5	10	100	1000
$f(x)$	12	360	2910	2.99×10^6	2.999×10^9

x	-1	-5	-10	-100	-1000
$f(x)$	6	-390	-3090	-3.01×10^6	-3.0×10^9

$\lim_{x\to\infty} f(x) = \infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

72. $f(x) = \dfrac{|x|}{x}$

x	1	10	100
$f(x)$	1	1	1

x	-1	-10	-100
$f(x)$	-1	-1	-1

$\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = -1$.

73. $\displaystyle\lim_{x\to\infty} \frac{3x+2}{x-5} = \lim_{x\to\infty} \frac{3+\frac{2}{x}}{1-\frac{5}{x}} = \frac{3}{1} = 3.$

74. $\lim\limits_{x\to-\infty} \dfrac{4x^2-1}{x+2} = \lim\limits_{x\to-\infty} \dfrac{4x-\frac{1}{x}}{1+\frac{2}{x}} = -\infty$; that is, the limit does not exist.

75. $\lim\limits_{x\to-\infty} \dfrac{3x^3+x^2+1}{x^3+1} = \lim\limits_{x\to-\infty} \dfrac{3+\frac{1}{x}+\frac{1}{x^3}}{1+\frac{1}{x^3}} = 3.$

76. $\lim\limits_{x\to\infty} \dfrac{2x^2+3x+1}{x^4-x^2} = \lim\limits_{x\to\infty} \dfrac{\frac{2}{x^2}+\frac{3}{x^3}+\frac{1}{x^4}}{1-\frac{1}{x^2}} = 0.$

77. $\lim\limits_{x\to-\infty} \dfrac{x^4+1}{x^3-1} = \lim\limits_{x\to-\infty} \dfrac{x+\frac{1}{x^3}}{1-\frac{1}{x^3}} = -\infty$; that is, the limit does not exist.

78. $\lim\limits_{x\to\infty} \dfrac{4x^4-3x^2+1}{2x^4+x^3+x^2+x+1} = \lim\limits_{x\to\infty} \dfrac{4-\frac{3}{x^2}+\frac{1}{x^4}}{2+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}} = 2.$

79. $\lim\limits_{x\to\infty} \dfrac{x^5-x^3+x-1}{x^6+2x^2+1} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}-\frac{1}{x^3}+\frac{1}{x^5}-\frac{1}{x^6}}{1+\frac{2}{x^4}+\frac{1}{x^6}} = 0.$

80. $\lim\limits_{x\to\infty} \dfrac{2x^2-1}{x^3+x^2+1} = \lim\limits_{x\to\infty} \dfrac{\frac{2}{x}-\frac{1}{x^3}}{1+\frac{1}{x}+\frac{1}{x^3}} = 0.$

81. a. The cost of removing 50% of the pollutant is $C(50) = \dfrac{0.5(50)}{100-50} = 0.5$, or \$500,000. Similarly, we find that the cost of removing 60%, 70%, 80%, 90%, and 95% of the pollutant is \$750,000, \$1,166,667, \$2,000,000, \$4,500,000, and \$9,500,000, respectively.

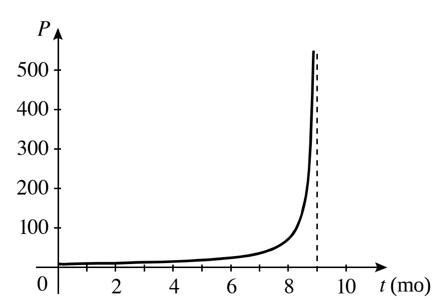
b. $\lim\limits_{x\to100} \dfrac{0.5x}{100-x} = \infty$, which means that the cost of removing the pollutant increases without bound if we wish to remove almost all of the pollutant.

82. a. The number present initially is given by $P(0) = \dfrac{72}{9-0} = 8.$

b. As t approaches 9 (remember that $0 < t < 9$), the denominator approaches 0 while the numerator remains constant at 72. Therefore, $P(t)$ gets larger and larger. Thus,

$$\lim_{t\to9} P(t) = \lim_{t\to9} \frac{72}{9-t} = \infty.$$

c.



83. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty} 2.2 + \frac{2500}{x} = 2.2$, or \$2.20 per DVD. In the long run, the average cost of producing x DVDs approaches \$2.20/disc.

84. $\lim_{t\to\infty} C(t) = \lim_{t\to\infty} \frac{0.2t}{t^2+1} = \lim_{t\to\infty} \frac{\frac{0.2}{t}}{1+\frac{1}{t^2}} = 0$, which says that the concentration of drug in the bloodstream eventually decreases to zero.

85. a. $T(1) = \frac{120}{1+4} = 24$, or \$24 million. $T(2) = \frac{120(2)^2}{8} = 60$, or \$60 million. $T(3) = \frac{120(3)^2}{13} = 83.1$, or \$83.1 million.

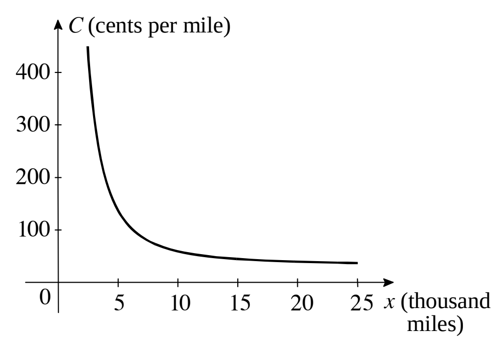
b. In the long run, the movie will gross $\lim_{x\to\infty} \frac{120x^2}{x^2+4} = \lim_{x\to\infty} \frac{120}{1+\frac{4}{x^2}} = 120$, or \$120 million.

86. a. The current population is $P(0) = \frac{200}{40} = 5$, or 5000.

b. The population in the long run will be $\lim_{t\to\infty} \frac{25t^2+125t+200}{t^2+5t+40} = \lim_{t\to\infty} \frac{25+\frac{125}{t}+\frac{200}{t^2}}{1+\frac{5}{t}+\frac{40}{t^2}} = 25$, or 25,000.

87. a. The average cost of driving 5000 miles per year is $C(5) = \frac{2410}{5^{1.95}} + 32.8 \approx 137.28$, or 137.3 cents per mile. Similarly, we see that the average costs of driving 10, 15, 20, and 25 thousand miles per year are 59.8, 45.1, 39.8, and 37.3 cents per mile, respectively.

b.



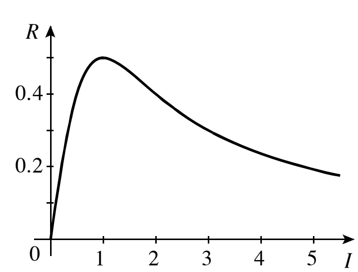
c. It approaches 32.8 cents per mile.

88. a. $R(I) = \frac{I}{1+I^2}$

I	0	1	2	3	4	5
$R(I)$	0	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{4}{17}$	$\frac{5}{26}$

b. $\lim_{I\to\infty} R(I) = \lim_{I\to\infty} \frac{I}{1+I^2} = 0$.

c.



89. False. Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ Then $\lim_{x\to 0} f(x) = 1$, but $f(1)$ is not defined.

90. True.

91. True. Division by zero is not permitted.

92. False. Let $f(x) = (x-3)^2$ and $g(x) = x-3$. Then $\lim_{x\to 3} f(x) = 0$ and $\lim_{x\to 3} g(x) = 0$, but $\lim_{x\to 3} \dfrac{f(x)}{g(x)} = \lim_{x\to 3} \dfrac{(x-3)^2}{x-3} = \lim_{x\to 3} (x-3) = 0$.

93. True. Each limit in the sum exists. Therefore, $\lim_{x\to 2} \left(\dfrac{x}{x+1} + \dfrac{3}{x-1}\right) = \lim_{x\to 2} \dfrac{x}{x+1} + \lim_{x\to 2} \dfrac{3}{x-1} = \dfrac{2}{3} + \dfrac{3}{1} = \dfrac{11}{3}$.

94. False. Neither of the limits $\lim_{x\to 1} \dfrac{2x}{x-1}$ and $\lim_{x\to 1} \dfrac{2}{x-1}$ exists.

95. $\lim_{x\to\infty} \dfrac{ax}{x+b} = \lim_{x\to\infty} \dfrac{a}{1+\frac{b}{x}} = a$. As the amount of substrate becomes very large, the initial speed approaches the constant a moles per liter per second.

96. Consider the functions $f(x) = 1/x$ and $g(x) = -1/x$. Observe that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} \left[f(x) + g(x)\right] = \lim_{x\to 0} 0 = 0$. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

97. Consider the functions $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x \geq 0 \end{cases}$ Then $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} \left[f(x)\, g(x)\right] = \lim_{x\to 0} (-1) = -1$. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

98. Take $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{1}{x^2}$, and $a = 0$. Then $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} \dfrac{f(x)}{g(x)} = \lim_{x\to 0} \dfrac{1}{x} \cdot \dfrac{x^2}{1} = \lim_{x\to 0} x = a$ exists. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

Using Technology

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1. 5

2. 11

3. 3

4. 0

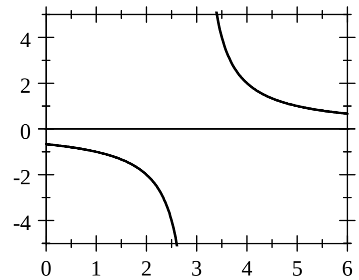
5. $\frac{2}{3}$

6. $\frac{10}{11}$

7. $e^2 \approx 7.38906$

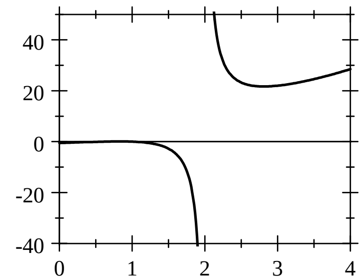
8. $\ln 2 \approx 0.693147$

9.



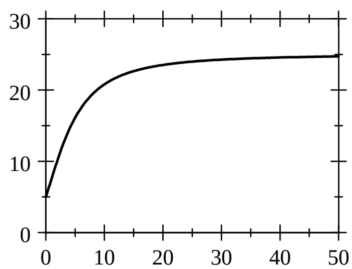
From the graph we see that $f(x)$ does not approach any finite number as x approaches 3.

10.



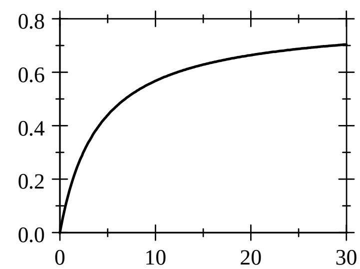
From the graph, we see that $f(x)$ does not approach any finite number as x approaches 2.

11. a.



b. $\lim_{t\to\infty} \dfrac{25t^2 + 125t + 200}{t^2 + 5t + 40} = 25$, so in the long run the population will approach 25,000.

12. a.



b. $\lim_{t\to\infty} \dfrac{0.8t}{t + 4.1} = \lim_{t\to\infty} \dfrac{0.8}{1 + \dfrac{4.1}{t}} = 0.8.$

2.5 One-Sided Limits and Continuity

Concept Questions

page 129

1. $\lim_{x\to 3^-} f(x) = 2$ means $f(x)$ can be made as close to 2 as we please by taking x sufficiently close to but to the left of $x = 3$. $\lim_{x\to 3^+} f(x) = 4$ means $f(x)$ can be made as close to 4 as we please by taking x sufficiently close to but to the right of $x = 3$.

2. a. $\lim_{x\to 1} f(x)$ does not exist because the left- and right-hand limits at $x = 1$ are different.

b. Nothing, because the existence or value of f at $x = 1$ does not depend on the existence (or nonexistence) of the left- or right-hand, or two-sided, limits of f.

3. a. f is continuous at a if $\lim_{x\to a} f(x) = f(a)$.

b. f is continuous on an interval I if f is continuous at each point in I.

4. $f(a) = L = M$.

5. a. f is continuous because the plane does not suddenly jump from one point to another.

b. f is continuous.

c. f is discontinuous because the fare “jumps”after the cab has covered a certain distance or after a certain amount of time has elapsed.

d. f is discontinuous because the rates "jump"by a certain amount (up or down) when it is adjusted at certain times.

6. Refer to page 127 in the text. Answers will vary.

Exercises page 130

1. $\lim_{x\to 2^-} f(x) = 3$ and $\lim_{x\to 2^+} f(x) = 2$, so $\lim_{x\to 2} f(x)$ does not exist.

2. $\lim_{x\to 3^-} f(x) = 3$ and $\lim_{x\to 3^+} f(x) = 5$, so $\lim_{x\to 3} f(x)$ does not exist.

3. $\lim_{x\to -1^-} f(x) = \infty$ and $\lim_{x\to -1^+} f(x) = 2$, so $\lim_{x\to -1} f(x)$ does not exist.

4. $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 3$, so $\lim_{x\to 1} f(x) = 3$.

5. $\lim_{x\to 1^-} f(x) = 0$ and $\lim_{x\to 1^+} f(x) = 2$, so $\lim_{x\to 1} f(x)$ does not exist.

6. $\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = \infty$, so $\lim_{x\to 0} f(x)$ does not exist.

7. $\lim_{x\to 0^-} f(x) = -2$ and $\lim_{x\to 0^+} f(x) = 2$, so $\lim_{x\to 0} f(x)$ does not exist.

8. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 2$.

9. True. **10.** True. **11.** True. **12.** True. **13.** False. **14.** True.

15. True. **16.** True. **17.** False. **18.** True. **19.** True. **20.** False

21. $\lim_{x\to 1^+} (2x+4) = 6$.

22. $\lim_{x\to 1^-} (3x-4) = -1$.

23. $\lim_{x\to 2^-} \frac{x-3}{x+2} = \frac{2-3}{2+2} = -\frac{1}{4}$.

24. $\lim_{x\to 1^+} \frac{x+2}{x+1} = \frac{1+2}{1+1} = \frac{3}{2}$.

25. $\lim_{x\to 0^+} \frac{1}{x}$ does not exist because $\frac{1}{x} \to \infty$ as $x \to 0$ from the right.

26. $\lim_{x\to 0^-} \frac{1}{x} = \infty$; that is, the limit does not exist.

27. $\lim_{x\to 0^+} \frac{x-1}{x^2+1} = \frac{-1}{1} = -1$.

28. $\lim_{x\to 2^+} \frac{x+1}{x^2-2x+3} = \frac{2+1}{4-4+3} = 1$.

29. $\lim_{x\to 0^+} \sqrt{x} = \sqrt{\lim_{x\to 0^+} x} = 0$.

30. $\lim_{x\to 2^+} 2\sqrt{x-2} = 2\cdot 0 = 0$.

31. $\lim_{x\to -2^+} \left(2x+\sqrt{2+x}\right) = \lim_{x\to -2^+} 2x + \lim_{x\to -2^+} \sqrt{2+x} = -4+0 = -4$.

32. $\lim_{x\to -5^+} x\left(1+\sqrt{5+x}\right) = -5\left[1+\sqrt{5+(-5)}\right] = -5$.

33. $\lim_{x\to 1^-} \frac{1+x}{1-x} = \infty$, that is, the limit does not exist.

34. $\lim_{x\to 1^+} \frac{1+x}{1-x} = -\infty$.

35. $\lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^-} \frac{(x+2)(x-2)}{x-2} = \lim_{x\to 2^-} (x+2) = 4$.

36. $\lim_{x\to -3^+} \frac{\sqrt{x+3}}{x^2+1} = \frac{0}{10} = 0$.

37. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0$ and $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2x = 0$.

38. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (2x+3) = 3$ and $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x+1) = 1$.

39. The function is discontinuous at $x = 0$. Conditions 2 and 3 are violated.

40. The function is not continuous because condition 3 for continuity is not satisfied.

41. The function is continuous everywhere.

42. The function is continuous everywhere.

43. The function is discontinuous at $x = 0$. Condition 3 is violated.

44. The function is not continuous at $x = -1$ because condition 3 for continuity is violated.

45. f is continuous for all values of x.

46. f is continuous for all values of x.

47. f is continuous for all values of x. Note that $x^2 + 1 \geq 1 > 0$.

48. f is continuous for all values of x. Note that $2x^2 + 1 \geq 1 > 0$.

49. f is discontinuous at $x = \frac{1}{2}$, where the denominator is 0. Thus, f is continuous on $\left(-\infty, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \infty\right)$.

50. f is discontinuous at $x = 1$, where the denominator is 0. Thus, f is continuous on $(-\infty, 1)$ and $(1, \infty)$.

51. Observe that $x^2 + x - 2 = (x+2)(x-1) = 0$ if $x = -2$ or $x = 1$, so f is discontinuous at these values of x. Thus, f is continuous on $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$.

52. Observe that $x^2 + 2x - 3 = (x+3)(x-1) = 0$ if $x = -3$ or $x = 1$, so, f is discontinuous at these values of x. Thus, f is continuous on $(-\infty, -3)$, $(-3, 1)$, and $(1, \infty)$.

53. f is continuous everywhere since all three conditions are satisfied.

54. f is continuous everywhere since all three conditions are satisfied.

55. f is continuous everywhere since all three conditions are satisfied.

56. f is not defined at $x = 1$ and is discontinuous there. It is continuous everywhere else.

57. Because the denominator $x^2 - 1 = (x-1)(x+1) = 0$ if $x = -1$ or 1, we see that f is discontinuous at -1 and 1.

58. The function f is not defined at $x = 1$ and $x = 2$. Therefore, f is discontinuous at 1 and 2.

59. Because $x^2 - 3x + 2 = (x - 2)(x - 1) = 0$ if $x = 1$ or 2, we see that the denominator is zero at these points and so f is discontinuous at these numbers.

60. The denominator of the function f is equal to zero when $x^2 - 2x = x(x - 2) = 0$; that is, when $x = 0$ or $x = 2$. Therefore, f is discontinuous at $x = 0$ and $x = 2$.

61. The function f is discontinuous at $x = 4, 5, 6, \ldots, 13$ because the limit of f does not exist at these points.

62. f is discontinuous at $t = 20$, 40, and 60. When $t = 0$, the inventory stands at 750 reams. The level drops to about 200 reams by the twentieth day at which time a new order of 500 reams arrives to replenish the supply. A similar interpretation holds for the other values of t.

63. Having made steady progress up to $x = x_1$, Michael's progress comes to a standstill at that point. Then at $x = x_2$ a sudden breakthrough occurs and he then continues to solve the problem.

64. The total deposits of Franklin make a jump at each of these points as the deposits of the ailing institutions become a part of the total deposits of the parent company.

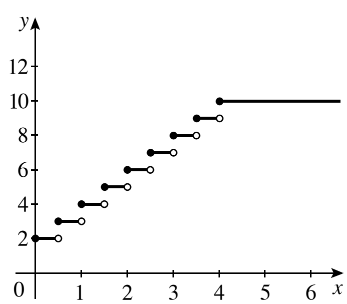
65. Conditions 2 and 3 are not satisfied at any of these points.

66. The function P is discontinuous at $t = 12$, 16, and 28. At $t = 12$, the prime interest rate jumped from $3\frac{1}{2}$% to 4%, at $t = 16$ it jumped to $4\frac{1}{2}$%, and at $t = 28$ it jumped back down to 4%.

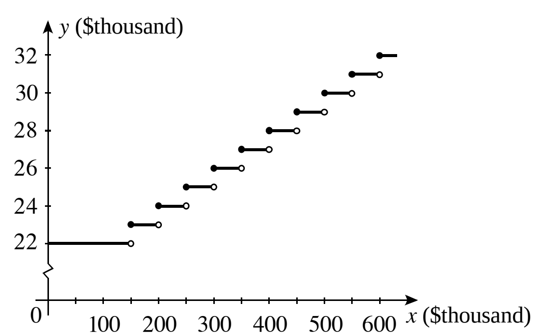
67.

$$f(x) = \begin{cases} 2 & \text{if } 0 < x \leq \frac{1}{2} \\ 3 & \text{if } \frac{1}{2} < x \leq 1 \\ \vdots & \qquad \vdots \\ 10 & \text{if } 4\frac{1}{2} < x \leq 5 \end{cases}$$

f is discontinuous at $x = \frac{1}{2}, 1, 1\frac{1}{2}, \ldots, 4$.

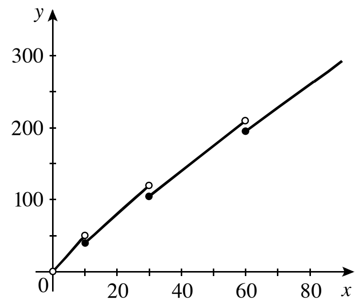


68.



f is discontinuous at $x = 150{,}000$, at $x = 200{,}000$, at $x = 250{,}000$, and so on.

69.



C is discontinuous at $x = 0$, 10, 30, and 60.

70. a. $\lim_{v \to u^+} \dfrac{aLv^3}{v-u} = \infty$. This reflects the fact that when the speed of the fish is very close to that of the current, the energy expended by the fish will be enormous.

b. $\lim_{v \to \infty} \dfrac{aLv^3}{v-u} = \infty$. This says that if the speed of the fish increases greatly, so does the amount of energy required to swim a distance of L ft.

71. a. $\lim_{t \to 0^+} S(t) = \lim_{t \to 0^+} \dfrac{a}{t} + b = \infty$. As the time taken to excite the tissue is made shorter and shorter, the electric current gets stronger and stronger.

b. $\lim_{t \to \infty} \dfrac{a}{t} + b = b$. As the time taken to excite the tissue is made longer and longer, the electric current gets weaker and weaker and approaches b.

72. a. $\lim_{D \to 0^+} L = \lim_{D \to 0^+} \dfrac{Y - (1-D) R}{D} = \infty$, so if the investor puts down next to nothing to secure the loan, the leverage approaches infinity.

b. $\lim_{D \to 1} L = \lim_{D \to 1} \dfrac{Y - (1-D) R}{D} = Y$, so if the investor puts down all of the money to secure the loan, the leverage is equal to the yield.

73. We require that $f(1) = 1 + 2 = 3 = \lim_{x \to 1^+} kx^2 = k$, so $k = 3$.

74. Because $\lim_{x \to -2} \dfrac{x^2 - 4}{x+2} = \lim_{x \to -2} \dfrac{(x-2)(x+2)}{x+2} = \lim_{x \to -2} (x-2) = -4$, we define $f(-2) = k = -4$, that is, take $k = -4$.

75. a. f is a polynomial of degree 2 and is therefore continuous everywhere, including the interval $[1, 3]$.

b. $f(1) = 3$ and $f(3) = -1$ and so f must have at least one zero in $(1, 3)$.

76. a. f is a polynomial of degree 3 and is thus continuous everywhere.

b. $f(0) = 14$ and $f(1) = -23$ and so f has at least one zero in $(0, 1)$.

77. a. f is a polynomial of degree 3 and is therefore continuous on $[-1, 1]$.

b. $f(-1) = (-1)^3 - 2(-1)^2 + 3(-1) + 2 = -1 - 2 - 3 + 2 = -4$ and $f(1) = 1 - 2 + 3 + 2 = 4$. Because $f(-1)$ and $f(1)$ have opposite signs, we see that f has at least one zero in $(-1, 1)$.

78. f is continuous on $[14, 16]$, $f(14) = 2(14)^{5/3} - 5(14)^{4/3} \approx -6.06$, and $f(16) = 2(16)^{5/3} - 5(16)^{4/3} \approx 1.60$. Thus, f has at least one zero in $(14, 16)$.

79. $f(0) = 6$, $f(3) = 3$, and f is continuous on $[0, 3]$. Thus, the Intermediate Value Theorem guarantees that there is at least one value of x for which $f(x) = 4$. Solving $f(x) = x^2 - 4x + 6 = 4$, we find $x^2 - 4x + 2 = 0$. Using the quadratic formula, we find that $x = 2 \pm \sqrt{2}$. Because $2 + \sqrt{2}$ does not lie in $[0, 3]$, we see that $x = 2 - \sqrt{2} \approx 0.59$.

80. Because $f(-1) = 3$, $f(4) = 13$, and f is continuous on $[-1, 4]$, the Intermediate Value Theorem guarantees that there is at least one value of x for which $f(x) = 7$ because $3 < 7 < 13$. Solving $f(x) = x^2 - x + 1 = 7$, we find $x^2 - x - 6 = (x-3)(x+2) = 0$, that is, $x = -2$ or 3. Because -2 does not lie in $[-1, 4]$, the required solution is 3.

81. $x^5 + 2x - 7 = 0$

Step	Interval in which a root lies
1	$(1, 2)$
2	$(1, 1.5)$
3	$(1.25, 1.5)$
4	$(1.25, 1.375)$
5	$(1.3125, 1.375)$
6	$(1.3125, 1.34375)$
7	$(1.328125, 1.34375)$
8	$(1.3359375, 1.34375)$

We see that a root is approximately 1.34.

82. $x^3 - x + 1 = 0$

Step	Interval in which a root lies
1	$(-2, -1)$
2	$(-1.5, -1)$
3	$(-1.5, -1.25)$
4	$(-1.375, -1.25)$
5	$(-1.375, -1.3125)$
6	$(-1.34375, -1.3125)$
7	$(-1.328125, -1.3125)$
8	$(-1.328125, -1.3203125)$
9	$(-1.32421875, -1.3203125)$

We see that a root is approximately -1.32.

83. **a.** $h(0) = 4 + 64(0) - 16(0) = 4$ and $h(2) = 4 + 64(2) - 16(4) = 68$.

b. The function h is continuous on $[0, 2]$. Furthermore, the number 32 lies between 4 and 68. Therefore, the Intermediate Value Theorem guarantees that there is at least one value of t in $(0, 2]$ such that $h(t) = 32$, that is, Joan must see the ball at least once during the time the ball is in the air.

c. We solve $h(t) = 4 + 64t - 16t^2 = 32$, obtaining $16t^2 - 64t + 28 = 0$, $4t^2 - 16t + 7 = 0$, and $(2t-1)(2t-7) = 0$. Thus, $t = \frac{1}{2}$ or $t = \frac{7}{2}$. Joan sees the ball on its way up half a second after it was thrown and again 3 seconds later when it is on its way down. Note that the ball hits the ground when $t \approx 4.06$, but Joan sees it approximately half a second before it hits the ground.

84. **a.** $f(0) = 100\left(\dfrac{0+0+100}{0+0+100}\right) = 100$ and $f(10) = 100\left(\dfrac{100+100+100}{100+200+100}\right) = \dfrac{30{,}000}{400} = 75$.

b. Because 80 lies between 75 and 100 and f is continuous on $[75, 100]$, we conclude that there exists some t in $[0, 10]$ such that $f(t) = 80$.

c. We solve $f(t) = 80$; that is, $100\left[\dfrac{t^2+10t+100}{t^2+20t+100}\right] = 80$, obtaining $5\left(t^2+10t+100\right) = 4\left(t^2+20t+100\right)$, and $t^2 - 30t + 100 = 0$. Thus, $t = \dfrac{30 \pm \sqrt{900-400}}{2} = \dfrac{30 \pm \sqrt{500}}{2} \approx 3.82$ or 26.18. Because 26.18 lies outside the interval of interest, we reject it. Thus, the oxygen content is 80% approximately 3.82 seconds after the organic waste has been dumped into the pond.

85. False. Take $f(x) = \begin{cases} -1 & \text{if } x < 2 \\ 4 & \text{if } x = 2 \\ 1 & \text{if } x > 2 \end{cases}$ Then $f(2) = 4$, but $\lim_{x\to 2} f(x)$ does not exist.

86. False. Take $f(x) = \begin{cases} x+3 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ Then $\lim_{x\to 0} f(x) = 3$, but $f(1)$ is not defined.

87. False. Consider $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$ Then $\lim_{x\to 2^+} f(x) = f(2) = 3$, but $\lim_{x\to 2^-} f(x) = 0$.

88. False. Consider $f(x) = \begin{cases} 2 & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ 4 & \text{if } x \geq 3 \end{cases}$ Then $\lim_{x\to 3^-} f(x) = 2$ and $\lim_{x\to 3^+} f(x) = 4$, so $\lim_{x\to 3} f(x)$ does not exist.

89. False. Consider $f(x) = \begin{cases} 2 & \text{if } x < 5 \\ 3 & \text{if } x > 5 \end{cases}$ Then $f(5)$ is not defined, but $\lim_{x\to 5^-} f(x) = 2$.

90. False. Consider the function $f(x) = x^2 - 1$ on the interval $[-2, 2]$. Here $f(-2) = f(2) = 3$, but f has zeros at $x = -1$ and $x = 1$.

91. False. Let $f(x) = \begin{cases} x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ Then $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$, but $f(0) = 1$.

92. False. Let $f(x) = x$ and let $g(x) = \begin{cases} x & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ Then $\lim_{x\to 1} f(x) = 1 = L$, $g(1) = 2 = M$, $\lim_{x\to 1} g(x) = 1$, and $\lim_{x\to 1} f(x)\, g(x) = \left[\lim_{x\to 1} f(x)\right]\left[\lim_{x\to 1} g(x)\right] = (1)(1) = 1 \neq 2 = LM$.

93. False. Let $f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ Then f is continuous for all $x \neq 0$ and $f(0) = 0$, but $\lim_{x\to 0} f(x)$ does not exist.

94. False. Consider $f(x) = \begin{cases} -1 & \text{if } -1 \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq 1 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } -1 \leq x \leq 0 \\ -1 & \text{if } 0 < x \leq 1 \end{cases}$

95. False. Consider $f(x) = \begin{cases} -1 & \text{if } -1 \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq 1 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } -1 \leq x \leq 0 \\ -1 & \text{if } 0 < x \leq 1 \end{cases}$

96. False. Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = x^2$.

97. False. Consider $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1 & \text{if } x < 0 \\ x-1 & \text{if } x \geq 0 \end{cases}$

98. The statement is false. The Intermediate Value Theorem says that there is at least one number c in $[a, b]$ such that $f(c) = M$ if M is a number between $f(a)$ and $f(b)$.

99. **a.** f is a rational function whose denominator is never zero, and so it is continuous for all values of x.

b. Because the numerator x^2 is nonnegative and the denominator is $x^2 + 1 \geq 1$ for all values of x, we see that $f(x)$ is nonnegative for all values of x.

c. $f(0) = \dfrac{0}{0+1} = \dfrac{0}{1} = 0$, and so f has a zero at $x = 0$. This does not contradict Theorem 5.

100. **a.** Both $g(x) = x$ and $h(x) = \sqrt{1-x^2}$ are continuous on $[-1, 1]$ and so $f(x) = x - \sqrt{1-x^2}$ is continuous on $[-1, 1]$.

b. $f(-1) = -1$ and $f(1) = 1$, and so f has at least one zero in $(-1, 1)$.

c. Solving $f(x) = 0$, we have $x = \sqrt{1-x^2}$, $x^2 = 1 - x^2$, and $2x^2 = 1$, so $x = \frac{\pm\sqrt{2}}{2}$.

101. **a.** **(i)** Repeated use of Property 3 shows that $g(x) = x^n = \underbrace{x \cdot x \cdot \cdots \cdot x}_{n \text{ times}}$ is a continuous function, since $f(x) = x$ is continuous by Property 1.

(ii) Properties 1 and 5 combine to show that $c \cdot x^n$ is continuous using the results of part (a)(i).

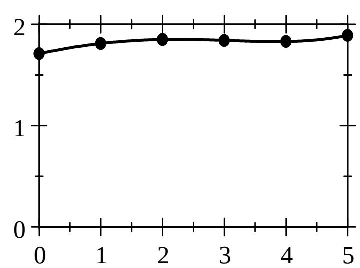
(iii) Each of the terms of $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$ is continuous and so Property 4 implies that p is continuous.

b. Property 6 now shows that $R(x) = \dfrac{p(x)}{q(x)}$ is continuous if $q(a) \neq 0$, since p and q are continuous at $x = a$.

102. Consider the function f defined by $f(x) = \begin{cases} -1 & \text{if } -1 \leq x < 0 \\ 1 & \text{if } 0 \leq x < 1 \end{cases}$ Then $f(-1) = -1$ and $f(1) = 1$, but if we take the number $\frac{1}{2}$, which lies between $y = -1$ and $y = 1$, there is no value of x such that $f(x) = \frac{1}{2}$.

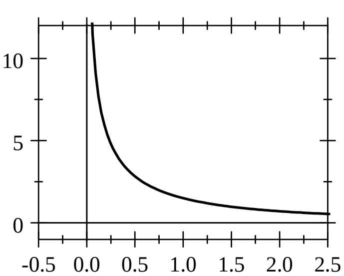
Using Technology page 136

1.



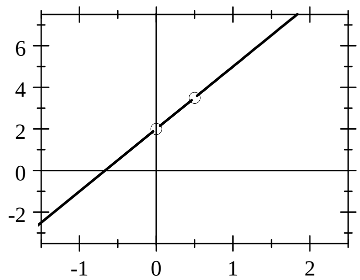
The function is discontinuous at $x = 0$ and $x = 1$.

2.



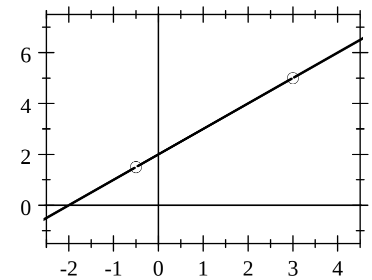
The function is undefined for $x \leq 0$.

3.



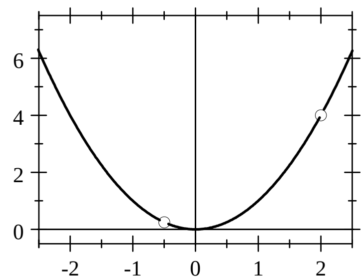
The function is discontinuous at $x = 0$ and $\frac{1}{2}$.

4.



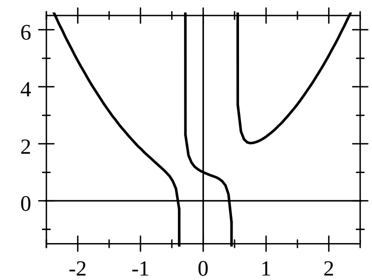
The function is discontinuous at $x = -\frac{1}{2}$ and 3.

5.



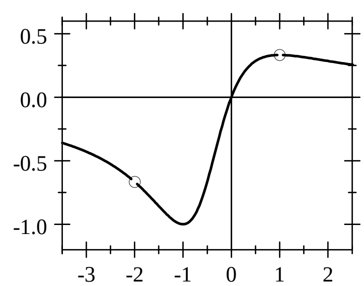
The function is discontinuous at $x = -\frac{1}{2}$ and 2.

6.



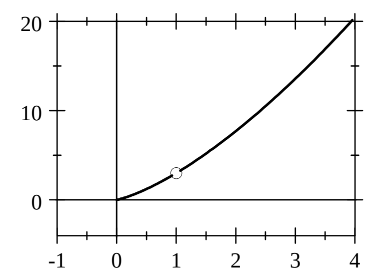
The function is discontinuous at $x = -\frac{1}{3}$ and $\frac{1}{2}$.

7.



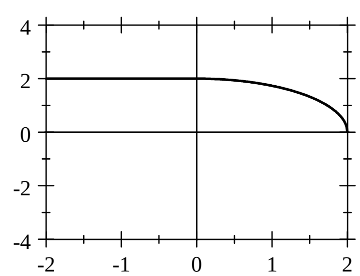
The function is discontinuous at $x = -2$ and 1.

8.



The function is discontinuous only at $x = -1$ and 1. Part of the graph is missing because some graphing calculators cannot evaluate cube roots of negative numbers.

9.



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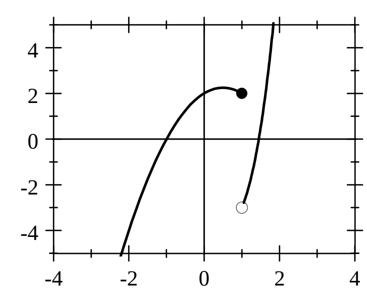

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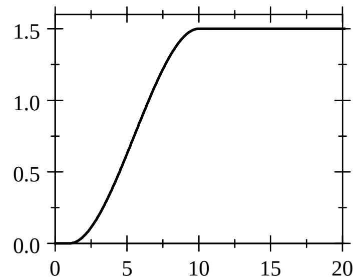
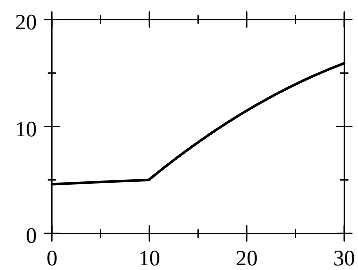


12.



2.6 The Derivative

Concept Questions

page 148

1. a. $m = \dfrac{f(2+h) - f(2)}{h}$

b. The slope of the tangent line is $\displaystyle\lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$.

2. a. The average rate of change is $\dfrac{f(2+h) - f(2)}{h}$.

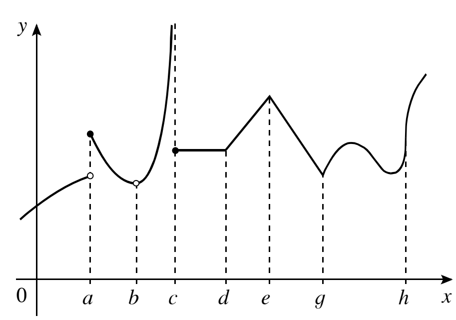
b. The instantaneous rate of change of f at 2 is $\displaystyle\lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$.

c. The expression for the slope of the secant line is the same as that for the average rate of change. The expression for the slope of the tangent line is the same as that for the instantaneous rate of change.

3. a. The expression $\dfrac{f(x+h) - f(x)}{h}$ gives (i) the slope of the secant line passing through the points $(x, f(x))$ and $(x+h, f(x+h))$, and (ii) the average rate of change of f over the interval $[x, x+h]$.

b. The expression $\displaystyle\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ gives (i) the slope of the tangent line to the graph of f at the point $(x, f(x))$, and (ii) the instantaneous rate of change of f at x.

4. Loosely speaking, a function f does not have a derivative at a if the graph of f does not have a tangent line at a, or if the tangent line does exist, but is vertical. In the figure, the function fails to be differentiable at $x = a$, b, and c because it is discontinuous at each of these numbers. The derivative of the function does not exist at $x = d$, e, and g because it has a kink at each point on the graph corresponding to these numbers. Finally, the function is not differentiable at $x = h$ because the tangent line is vertical at $(h, f(h))$.



5. a. $C(500)$ gives the total cost incurred in producing 500 units of the product.

b. $C'(500)$ gives the rate of change of the total cost function when the production level is 500 units.

6. a. $P(5)$ gives the population of the city (in thousands) when $t = 5$.

b. $P'(5)$ gives the rate of change of the city's population (in thousands/year) when $t = 5$.

Exercises page 149

1. The rate of change of the average infant's weight when $t = 3$ is $\frac{7.5}{5}$, or 1.5 lb/month. The rate of change of the average infant's weight when $t = 18$ is $\frac{3.5}{6}$, or approximately 0.58 lb/month. The average rate of change over the infant's first year of life is $\frac{22.5 - 7.5}{12}$, or 1.25 lb/month.

2. The rate at which the wood grown is changing at the beginning of the 10th year is $\frac{4}{12}$, or $\frac{1}{3}$ cubic meter per hectare per year. At the beginning of the 30th year, it is $\frac{10}{8}$, or 1.25 cubic meters per hectare per year.

3. The rate of change of the percentage of households watching television at 4 p.m. is $\frac{12.3}{4}$, or approximately 3.1 percent per hour. The rate at 11 p.m. is $\frac{-42.3}{2} = -21.15$, that is, it is dropping off at the rate of 21.15 percent per hour.

4. The rate of change of the crop yield when the density is 200 aphids per bean stem is $\frac{-500}{300}$, a decrease of approximately 1.7 kg/4000 m^2 per aphid per bean stem. The rate of change when the density is 800 aphids per bean stem is $\frac{-150}{300}$, a decrease of approximately 0.5 kg/4000 m^2 per aphid per bean stem.

5. a. Car A is travelling faster than Car B at t_1 because the slope of the tangent line to the graph of f is greater than the slope of the tangent line to the graph of g at t_1.

b. Their speed is the same because the slope of the tangent lines are the same at t_2.

c. Car B is travelling faster than Car A.

d. They have both covered the same distance and are once again side by side at t_3.

6. a. At t_1, the velocity of Car A is greater than that of Car B because $f(t_1) > g(t_1)$. However, Car B has greater acceleration because the slope of the tangent line to the graph of g is increasing, whereas the slope of the tangent line to f is decreasing as you move across t_1.

b. Both cars have the same velocity at t_2, but the acceleration of Car B is greater than that of Car A because the slope of the tangent line to the graph of g is increasing, whereas the slope of the tangent line to the graph of f is decreasing as you move across t_2.

7. a. P_2 is decreasing faster at t_1 because the slope of the tangent line to the graph of g at t_1 is greater than the slope of the tangent line to the graph of f at t_1.

b. P_1 is decreasing faster than P_2 at t_2.

c. Bactericide B is more effective in the short run, but bactericide A is more effective in the long run.

8. a. The revenue of the established department store is decreasing at the slowest rate at $t = 0$.

b. The revenue of the established department store is decreasing at the fastest rate at t_3.

c. The revenue of the discount store first overtakes that of the established store at t_1.

d. The revenue of the discount store is increasing at the fastest rate at t_2 because the slope of the tangent line to the graph of f is greatest at the point $(t_2, f(t_2))$.

9. $f(x) = 13$.

Step 1 $f(x+h) = 13$.

Step 2 $f(x+h) - f(x) = 13 - 13 = 0$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{0}{h} = 0$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 0 = 0$.

10. $f(x) = -6$.

Step 1 $f(x+h) = -6$.

Step 2 $f(x+h) - f(x) = -6 - (-6) = 0$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{0}{h} = 0$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 0 = 0$.

11. $f(x) = 2x + 7$.

Step 1 $f(x+h) = 2(x+h) + 7$.

Step 2 $f(x+h) - f(x) = 2(x+h) + 7 - (2x+7) = 2h$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2h}{h} = 2$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 2 = 2$.

12. $f(x) = 8 - 4x$.

Step 1 $f(x+h) = 8 - 4(x+h) = 8 - 4x - 4h$.

Step 2 $f(x+h) - f(x) = (8 - 4x - 4h) - (8 - 4x) = -4h$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = -\dfrac{4h}{h} = -4$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (-4) = -4$.

13. $f(x) = 3x^2$.

Step 1 $f(x+h) = 3(x+h)^2 = 3x^2 + 6xh + 3h^2$.

Step 2 $f(x+h) - f(x) = \left(3x^2 + 6xh + 3h^2\right) - 3x^2 = 6xh + 3h^2 = h(6x + 3h)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(6x+3h)}{h} = 6x + 3h$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (6x + 3h) = 6x$.

14. $f(x) = -\frac{1}{2}x^2$.

Step 1 $f(x+h) = -\frac{1}{2}(x+h)^2$.

Step 2 $f(x+h) - f(x) = -\frac{1}{2}x^2 - xh - \frac{1}{2}h^2 + \frac{1}{2}x^2 = -h\left(x + \frac{1}{2}h\right)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-h\left(x + \frac{1}{2}h\right)}{h} = -\left(x + \frac{1}{2}h\right)$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} -\left(x + \frac{1}{2}h\right) = -x$.

15. $f(x) = -x^2 + 3x$.

Step 1 $f(x+h) = -(x+h)^2 + 3(x+h) = -x^2 - 2xh - h^2 + 3x + 3h$.

Step 2 $f(x+h) - f(x) = \left(-x^2 - 2xh - h^2 + 3x + 3h\right) - \left(-x^2 + 3x\right) = -2xh - h^2 + 3h$

$$= h(-2x - h + 3).$$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(-2x - h + 3)}{h} = -2x - h + 3$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} (-2x - h + 3) = -2x + 3$.

16. $f(x) = 2x^2 + 5x$.

Step 1 $f(x+h) = 2(x+h)^2 + 5(x+h) = 2x^2 + 4xh + 2h^2 + 5x + 5h$.

Step 2 $f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 + 5x + 5h - 2x^2 - 5x = h(4x + 2h + 5)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(4x + 2h + 5)}{h} = 4x + 2h + 5$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} (4x + 2h + 5) = 4x + 5$.

17. $f(x) = 2x + 7$.

Step 1 $f(x+h) = 2(x+h) + 7 = 2x + 2h + 7$.

Step 2 $f(x+h) - f(x) = 2x + 2h + 7 - 2x - 7 = 2h$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2h}{h} = 2$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} 2 = 2$.

Therefore, $f'(x) = 2$. In particular, the slope at $x = 2$ is 2. Therefore, an equation of the tangent line is $y - 11 = 2(x - 2)$ or $y = 2x + 7$.

18. $f(x) = -3x + 4$. First, we find $f'(x) = -3$ using the four-step process. Thus, the slope of the tangent line is $f'(-1) = -3$ and an equation is $y - 7 = -3(x + 1)$ or $y = -3x + 4$.

19. $f(x) = 3x^2$.We first compute $f'(x) = 6x$ (see Exercise 13). Because the slope of the tangent line is $f'(1) = 6$, we use the point-slope form of the equation of a line and find that an equation is $y - 3 = 6(x - 1)$, or $y = 6x - 3$.

20. $f(x) = 3x - x^2$.

Step 1 $f(x+h) = 3(x+h) - (x+h)^2 = 3x + 3h - x^2 - 2xh - h^2$.

Step 2 $f(x+h) - f(x) = 3x + 3h - x^2 - 2xh - h^2 - 3x + x^2 = 3h - 2xh - h^2 = h(3 - 2x - h)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(3 - 2x - h)}{h} = 3 - 2x - h$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} (3 - 2x - h) = 3 - 2x$.

Therefore, $f'(x) = 3 - 2x$. In particular, $f'(-2) = 3 - 2(-2) = 7$. Using the point-slope form of an equation of a line, we find $y + 10 = 7(x + 2)$, or $y = 7x + 4$.

21. $f(x) = -1/x$. We first compute $f'(x)$ using the four-step process:

Step 1 $f(x+h) = -\dfrac{1}{x+h}$.

Step 2 $f(x+h) - f(x) = -\dfrac{1}{x+h} + \dfrac{1}{x} = \dfrac{-x + (x+h)}{x(x+h)} = \dfrac{h}{x(x+h)}$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{h}{x(x+h)}}{h} = \dfrac{1}{x(x+h)}$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} \dfrac{1}{x(x+h)} = \dfrac{1}{x^2}$.

The slope of the tangent line is $f'(3) = \frac{1}{9}$. Therefore, an equation is $y - \left(-\frac{1}{3}\right) = \frac{1}{9}(x-3)$, or $y = \frac{1}{9}x - \frac{2}{3}$.

22. $f(x) = \dfrac{3}{2x}$. First use the four-step process to find $f'(x) = -\dfrac{3}{2x^2}$. (This is similar to Exercise 21.) The slope of the tangent line is $f'(1) = -\frac{3}{2}$. Therefore, an equation is $y - \frac{3}{2} = -\frac{3}{2}(x-1)$ or $y = -\frac{3}{2}x + 3$.

23. a. $f(x) = 2x^2 + 1$.

Step 1 $f(x+h) = 2(x+h)^2 + 1 = 2x^2 + 4xh + 2h^2 + 1$.

Step 2 $f(x+h) - f(x) = \left(2x^2 + 4xh + 2h^2 + 1\right) - \left(2x^2 + 1\right)$

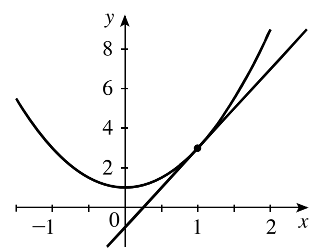
$$= 4xh + 2h^2 = h(4x + 2h).$$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(4x+2h)}{h} = 4x + 2h$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} (4x + 2h) = 4x$.

b. The slope of the tangent line is $f'(1) = 4(1) = 4$. Therefore, an equation is $y - 3 = 4(x-1)$ or $y = 4x - 1$.

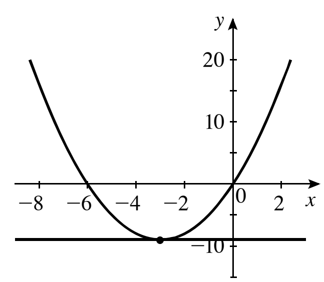
c.



24. a. $f(x) = x^2 + 6x$. Using the four-step process, we find that $f'(x) = 2x + 6$.

b. At a point on the graph of f where the tangent line to the curve is horizontal, $f'(x) = 0$. Then $2x + 6 = 0$, or $x = -3$. Therefore, $y = f(-3) = (-3)^2 + 6(-3) = -9$. The required point is $(-3, -9)$.

c.



25. a. $f(x) = x^2 - 2x + 1$. We use the four-step process:

Step 1 $f(x+h) = (x+h)^2 - 2(x+h) + 1 = x^2 + 2xh + h^2 - 2x - 2h + 1$.

Step 2 $f(x+h) - f(x) = \left(x^2 + 2xh + h^2 - 2x - 2h + 1\right) - \left(x^2 - 2x + 1\right) = 2xh + h^2 - 2h$

$$= h(2x + h - 2).$$

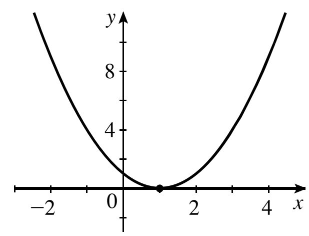
Step 3 $\dfrac{f(x+h)-f(x)}{h}=\dfrac{h(2x+h-2)}{h}=2x+h-2.$

Step 4 $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to0}(2x+h-2)$

$=2x-2.$

b. At a point on the graph of f where the tangent line to the curve is horizontal, $f'(x)=0$. Then $2x-2=0$, or $x=1$. Because $f(1)=1-2+1=0$, we see that the required point is $(1,0)$.

c.



d. It is changing at the rate of 0 units per unit change in x.

26. a. $f(x)=\dfrac{1}{x-1}.$

Step 1 $f(x+h)=\dfrac{1}{(x+h)-1}=\dfrac{1}{x+h-1}.$

Step 2 $f(x+h)-f(x)=\dfrac{1}{x+h-1}-\dfrac{1}{x-1}=\dfrac{x-1-(x+h-1)}{(x+h-1)(x-1)}=-\dfrac{h}{(x+h-1)(x-1)}.$

Step 3 $\dfrac{f(x+h)-f(x)}{h}=-\dfrac{1}{(x+h-1)(x-1)}.$

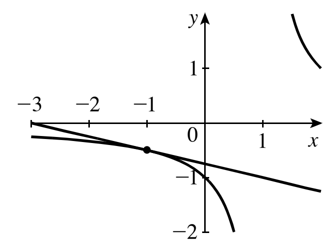
Step 4 $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to0}-\dfrac{1}{(x+h-1)(x-1)}=-\dfrac{1}{(x-1)^2}.$

b. The slope is $f'(-1)=-\frac{1}{4}$, so, an equation is

$y-\left(-\frac{1}{2}\right)=-\frac{1}{4}(x+1)$ or $y=-\frac{1}{4}x-\frac{3}{4}.$

c.



27. a. $f(x)=x^2+x$, so $\dfrac{f(3)-f(2)}{3-2}=\dfrac{(3^2+3)-(2^2+2)}{1}=6,$

$\dfrac{f(2.5)-f(2)}{2.5-2}=\dfrac{(2.5^2+2.5)-(2^2+2)}{0.5}=5.5$, and $\dfrac{f(2.1)-f(2)}{2.1-2}=\dfrac{(2.1^2+2.1)-(2^2+2)}{0.1}=5.1.$

b. We first compute $f'(x)$ using the four-step process.

Step 1 $f(x+h)=(x+h)^2+(x+h)=x^2+2xh+h^2+x+h.$

Step 2 $f(x+h)-f(x)=\left(x^2+2xh+h^2+x+h\right)-\left(x^2+x\right)=2xh+h^2+h=h(2x+h+1).$

Step 3 $\dfrac{f(x+h)-f(x)}{h}=\dfrac{h(2x+h+1)}{h}=2x+h+1.$

Step 4 $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to0}(2x+h+1)=2x+1.$

The instantaneous rate of change of y at $x=2$ is $f'(2)=2(2)+1$, or 5 units per unit change in x.

c. The results of part (a) suggest that the average rates of change of f at $x=2$ approach 5 as the interval $[2,2+h]$ gets smaller and smaller ($h=1$, 0.5, and 0.1). This number is the instantaneous rate of change of f at $x=2$ as computed in part (b).

28. a. $f(x)=x^2-4x$, so $\dfrac{f(4)-f(3)}{4-3}=\dfrac{(16-16)-(9-12)}{1}=3,$

$\dfrac{f(3.5)-f(3)}{3.5-3}=\dfrac{(12.25-14)-(9-12)}{0.5}=2.5$, and $\dfrac{f(3.1)-f(3)}{3.1-3}=\dfrac{(9.61-12.4)-(9-12)}{0.1}=2.1.$

b. We first compute $f'(x)$ using the four-step process:

Step 1 $f(x+h) = (x+h)^2 - 4(x+h) = x^2 + 2xh + h^2 - 4x - 4h$.

Step 2 $f(x+h) - f(x) = \left(x^2 + 2xh + h^2 - 4x - 4h\right) - \left(x^2 - 4x\right) = 2xh + h^2 - 4h = h(2x + h - 4)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(2x + h - 4)}{h} = 2x + h - 4$.

Step 4 $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (2x + h - 4) = 2x - 4$.

The instantaneous rate of change of y at $x = 3$ is $f'(3) = 6 - 4 = 2$, or 2 units per unit change in x.

c. The results of part (a) suggest that the average rates of change of f over smaller and smaller intervals containing $x = 3$ approach the instantaneous rate of change of 2 units per unit change in x obtained in part (b).

29. a. $f(t) = 2t^2 + 48t$. The average velocity of the car over the time interval $[20, 21]$ is $\dfrac{f(21) - f(20)}{21 - 20} = \dfrac{\left[2(21)^2 + 48(21)\right] - \left[2(20)^2 + 48(20)\right]}{1} = 130\ \dfrac{\text{ft}}{\text{s}}$. Its average velocity over $[20, 20.1]$ is $\dfrac{f(20.1) - f(20)}{20.1 - 20} = \dfrac{\left[2(20.1)^2 + 48(20.1)\right] - \left[2(20)^2 + 48(20)\right]}{0.1} = 128.2\ \dfrac{\text{ft}}{\text{s}}$. Its average velocity over $[20, 20.01]$ is $\dfrac{f(20.01) - f(20)}{20.01 - 20} = \dfrac{\left[2(20.01)^2 + 48(20.01)\right] - \left[2(20)^2 + 48(20)\right]}{0.01} = 128.02\ \dfrac{\text{ft}}{\text{s}}$.

b. We first compute $f'(t)$ using the four-step process.

Step 1 $f(t+h) = 2(t+h)^2 + 48(t+h) = 2t^2 + 4th + 2h^2 + 48t + 48h$.

Step 2 $f(t+h) - f(t) = \left(2t^2 + 4th + 2h^2 + 48t + 48h\right) - \left(2t^2 + 48t\right) = 4th + 2h^2 + 48h$

$= h(4t + 2h + 48)$.

Step 3 $\dfrac{f(t+h) - f(t)}{h} = \dfrac{h(4t + 2h + 48)}{h} = 4t + 2h + 48$.

Step 4 $f'(t) = \lim\limits_{t \to 0} \dfrac{f(t+h) - f(t)}{h} = \lim\limits_{t \to 0} (4t + 2h + 48) = 4t + 48$.

The instantaneous velocity of the car at $t = 20$ is $f'(20) = 4(20) + 48$, or 128 ft/s.

c. Our results show that the average velocities do approach the instantaneous velocity as the intervals over which they are computed decreases.

30. a. The average velocity of the ball over the time interval $[2, 3]$ is $\dfrac{s(3) - s(2)}{3 - 2} = \dfrac{\left[128(3) - 16(3)^2\right] - \left[128(2) - 16(2)^2\right]}{1} = 48$, or 48 ft/s. Over the time interval $[2, 2.5]$, it is $\dfrac{s(2.5) - s(2)}{2.5 - 2} = \dfrac{\left[128(2.5) - 16(2.5)^2\right] - \left[128(2) - 16(2)^2\right]}{0.5} = 56$, or 56 ft/s. Over the time interval $[2, 2.1]$, it is $\dfrac{s(2.1) - s(2)}{2.1 - 2} = \dfrac{\left[128(2.1) - 16(2.1)^2\right] - \left[128(2) - 16(2)^2\right]}{0.1} = 62.4$, or 62.4 ft/s.

b. Using the four-step process, we find that the instantaneous velocity of the ball at any time t is given by $v(t) = 128 - 32t$. In particular, the velocity of the ball at $t = 2$ is $v(2) = 128 - 32(2) = 64$, or 64 ft/s.

c. At $t = 5$, $v(5) = 128 - 32(5) = -32$, so the speed of the ball at $t = 5$ is 32 ft/s and it is falling.

d. The ball hits the ground when $s(t) = 0$, that is, when $128t - 16t^2 = 0$, whence $t(128 - 16t) = 0$, so $t = 0$ or $t = 8$. Thus, it will hit the ground when $t = 8$.

31. a. We solve the equation $16t^2 = 400$ and find $t = 5$, which is the time it takes the screwdriver to reach the ground.

b. The average velocity over the time interval [0, 5] is $\dfrac{f(5)-f(0)}{5-0}=\dfrac{16(25)-0}{5}=80$, or 80 ft/s.

c. The velocity of the screwdriver at time t is

$$v(t)=\lim_{h\to 0}\frac{f(t+h)-f(t)}{h}=\lim_{h\to 0}\frac{16(t+h)^2-16t^2}{h}=\lim_{h\to 0}\frac{16t^2+32th+16h^2-16t^2}{h}$$
$$=\lim_{h\to 0}\frac{(32t+16h)h}{h}=32t.$$

In particular, the velocity of the screwdriver when it hits the ground (at $t=5$) is $v(5)=32(5)=160$, or 160 ft/s.

32. a. We write $f(t)=\frac{1}{2}t^2+\frac{1}{2}t$. The height after 40 seconds is $f(40)=\frac{1}{2}(40)^2+\frac{1}{2}(40)=820$.

b. Its average velocity over the time interval [0, 40] is $\dfrac{f(40)-f(0)}{40-0}=\dfrac{820-0}{40}=20.5$, or 20.5 ft/s.

c. Its velocity at time t is

$$v(t)=\lim_{h\to 0}\frac{f(t+h)-f(t)}{h}=\lim_{h\to 0}\frac{\frac{1}{2}(t+h)^2+\frac{1}{2}(t+h)-\left(\frac{1}{2}t^2+\frac{1}{2}t\right)}{h}$$
$$=\lim_{h\to 0}\frac{\frac{1}{2}t^2+th+\frac{1}{2}h^2+\frac{1}{2}t+\frac{1}{2}h-\frac{1}{2}t^2-\frac{1}{2}t}{h}=\lim_{h\to 0}\frac{th+\frac{1}{2}h^2+\frac{1}{2}h}{h}=\lim_{h\to 0}\left(t+\frac{1}{2}h+\frac{1}{2}\right)=t+\frac{1}{2}.$$

In particular, the velocity at the end of 40 seconds is $v(40)=40+\frac{1}{2}$, or $40\frac{1}{2}$ ft/s.

33. a. We write $V=f(p)=\dfrac{1}{p}$. The average rate of change of V is $\dfrac{f(3)-f(2)}{3-2}=\dfrac{\frac{1}{3}-\frac{1}{2}}{1}=-\dfrac{1}{6}$, a decrease of $\frac{1}{6}$ liter/atmosphere.

b. $V'(t)=\lim_{h\to 0}\dfrac{\dfrac{f(p+h)-f(p)}{h}}{h}=\lim_{h\to 0}\dfrac{\dfrac{1}{p+h}-\dfrac{1}{p}}{h}=\lim_{h\to 0}\dfrac{p-(p+h)}{hp(p+h)}=\lim_{h\to 0}-\dfrac{1}{p(p+h)}=-\dfrac{1}{p^2}$. In particular, the rate of change of V when $p=2$ is $V'(2)=-\dfrac{1}{2^2}$, a decrease of $\frac{1}{4}$ liter/atmosphere.

34. $C(x)=-10x^2+300x+130$.

a. Using the four-step process, we find

$$C'(x)=\lim_{h\to 0}\frac{C(x+h)-C(x)}{h}=\lim_{h\to 0}\frac{h(-20x-10h+300)}{h}=-20x+300.$$

b. The rate of change is $C'(10)=-20(10)+300=100$, or \$100/surfboard.

35. a. $P(x)=-\frac{1}{3}x^2+7x+30$. Using the four-step process, we find that

$$P'(x)=\lim_{h\to 0}\frac{P(x+h)-P(x)}{h}=\lim_{h\to 0}\frac{-\frac{1}{3}\left(x^2+2xh+h^2\right)+7x+7h+30-\left(-\frac{1}{3}x^2+7x+30\right)}{h}$$
$$=\lim_{h\to 0}\frac{-\frac{2}{3}xh-\frac{1}{3}h^2+7h}{h}=\lim_{h\to 0}\left(-\frac{2}{3}x-\frac{1}{3}h+7\right)=-\frac{2}{3}x+7.$$

b. $P'(10)=-\frac{2}{3}(10)+7\approx 0.333$, or approximately \$333 per \$1000 spent on advertising.
$P'(30)=-\frac{2}{3}(30)+7=-13$, a decrease of \$13,000 per \$1000 spent on advertising.

36. a. $f(x) = -0.1x^2 - x + 40$, so $\dfrac{f(5.05) - f(5)}{5.05 - 5} = \dfrac{\left[-0.1(5.05)^2 - 5.05 + 40\right] - \left[-0.1(5)^2 - 5 + 40\right]}{0.05} = -2.005$, or approximately $-\$2.01$ per 1000 tents. $\dfrac{f(5.01) - f(5)}{5.01 - 5} = \dfrac{\left[-0.1(5.01)^2 - 5.01 + 40\right] - \left[-0.1(5)^2 - 5 + 40\right]}{0.01} = -2.001$, or approximately $-\$2.00$ per 1000 tents.

b. We compute $f'(x)$ using the four-step process, obtaining $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \dfrac{h(-0.2x - 0.1h - 1)}{h} = \lim\limits_{h \to 0}(-0.2x - 0.1h - 1) = -0.2x - 1$. The rate of change of the unit price if $x = 5000$ is $f'(5) = -0.2(5) - 1 = -2$, a decrease of \$2 per 1000 tents.

37. $N(t) = t^2 + 2t + 50$. We first compute $N'(t)$ using the four-step process.

Step 1 $N(t+h) = (t+h)^2 + 2(t+h) + 50 = t^2 + 2th + h^2 + 2t + 2h + 50$.

Step 2 $N(t+h) - N(t) = \left(t^2 + 2th + h^2 + 2t + 2h + 50\right) - \left(t^2 + 2t + 50\right) = 2th + h^2 + 2h = h(2t + h + 2)$.

Step 3 $\dfrac{N(t+h) - N(t)}{h} = 2t + h + 2$.

Step 4 $N'(t) = \lim\limits_{h \to 0}(2t + h + 2) = 2t + 2$.

The rate of change of the country's GNP two years from now is $N'(2) = 2(2) + 2 = 6$, or \$6 billion/yr. The rate of change four years from now is $N'(4) = 2(4) + 2 = 10$, or \$10 billion/yr.

38. $f(t) = 3t^2 + 2t + 1$. Using the four-step process, we obtain $f'(t) = \lim\limits_{t \to 0} \dfrac{f(t+h) - f(t)}{h} = \lim\limits_{t \to 0} \dfrac{h(6t + 3h + 2)}{h} = \lim\limits_{t \to 0}(6t + 3h + 2) = 6t + 2$. Next, $f'(10) = 6(10) + 2 = 62$, and we conclude that the rate of bacterial growth at $t = 10$ is 62 bacteria per minute.

39. a. $f'(h)$ gives the instantaneous rate of change of the temperature with respect to height at a given height h, in $^\circ$F per foot.

b. Because the temperature decreases as the altitude increases, the sign of $f'(h)$ is negative.

c. Because $f'(1000) = -0.05$, the change in the air temperature as the altitude changes from 1000 ft to 1001 ft is approximately -0.05° F.

40. a. $\dfrac{f(b) - f(a)}{b - a}$ measures the average rate of change in revenue as the advertising expenditure changes from a thousand dollars to b thousand dollars. The units of measurement are thousands of dollars per thousands of dollars.

b. $f'(x)$ gives the instantaneous rate of change in the revenue when x thousand dollars is spent on advertising. It is measured in thousands of dollars per thousands of dollars.

c. Because $f'(20) \cdot (21 - 20) = 3 \cdot 1 = 3$, the approximate change in revenue is \$3000.

41. $\dfrac{f(a+h) - f(a)}{h}$ gives the average rate of change of the seal population over the time interval $[a, a+h]$. $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ gives the instantaneous rate of change of the seal population at $x = a$.

42. $\dfrac{f(a+h) - f(a)}{h}$ gives the average rate of change of the prime interest rate over the time interval $[a, a+h]$. $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ gives the instantaneous rate of change of the prime interest rate at $x = a$.

43. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the country's industrial production over the time interval $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the country's industrial production at $x=a$.

44. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the cost incurred in producing the commodity over the production level $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the cost of producing the commodity at $x=a$.

45. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the atmospheric pressure over the altitudes $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the atmospheric pressure with respect to altitude at $x=a$.

46. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the fuel economy of a car over the speeds $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the fuel economy at $x=a$.

47. **a.** f has a limit at $x=a$.

b. f is not continuous at $x=a$ because $f(a)$ is not defined.

c. f is not differentiable at $x=a$ because it is not continuous there.

48. **a.** f has a limit at $x=a$.

b. f is continuous at $x=a$.

c. f is differentiable at $x=a$.

49. **a.** f has a limit at $x=a$.

b. f is continuous at $x=a$.

c. f is not differentiable at $x=a$ because f has a kink at the point $x=a$.

50. **a.** f does not have a limit at $x=a$ because the left-hand and right-hand limits are not equal.

b. f is not continuous at $x=a$ because the limit does not exist there.

c. f is not differentiable at $x=a$ because it is not continuous there.

51. **a.** f does not have a limit at $x=a$ because it is unbounded in the neighborhood of a.

b. f is not continuous at $x=a$.

c. f is not differentiable at $x=a$ because it is not continuous there.

52. **a.** f does not have a limit at $x=a$ because the left-hand and right-hand limits are not equal.

b. f is not continuous at $x=a$ because the limit does not exist there.

c. f is not differentiable at $x=a$ because it is not continuous there.

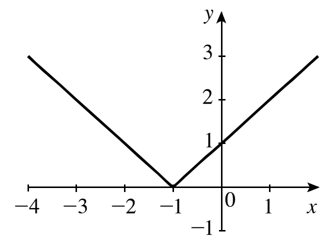
53. $s(t)=-0.1t^3+2t^2+24t$. Our computations yield the following results: 32.1, 30.939, 30.814, 30.8014, 30.8001, and 30.8000. The motorcycle's instantaneous velocity at $t=2$ is approximately 30.8 ft/s.

54. $C(x) = 0.000002x^3 + 5x + 400$. Our computations yield the following results: 5.060602, 5.06006002, 5.060006, 5.0600006, and 5.0600001. The rate of change of the total cost function when the level of production is 100 cases a day is approximately \$5.06.

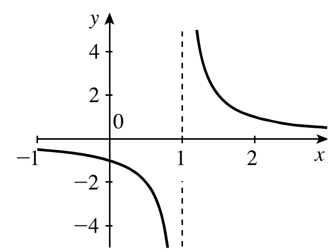
55. False. Let $f(x) = |x|$. Then f is continuous at $x = 0$, but is not differentiable there.

56. True. If g is differentiable at $x = a$, then it is continuous there. Therefore, the product fg is continuous, and so $\lim_{x\to a} f(x)g(x) = \left[\lim_{x\to a} f(x)\right]\left[\lim_{x\to a} g(x)\right] = f(a)g(a)$.

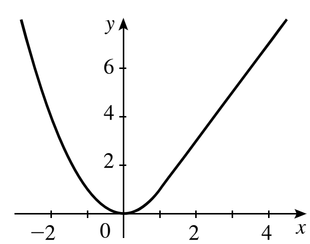
57. Observe that the graph of f has a kink at $x = -1$. We have $\dfrac{f(-1+h) - f(-1)}{h} = 1$ if $h > 0$, and -1 if $h < 0$, so that $\lim_{h\to 0} \dfrac{f(-1+h) - f(-1)}{h}$ does not exist.



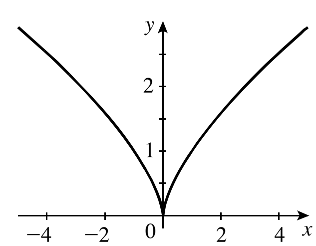
58. f does not have a derivative at $x = 1$ because it is not continuous there.



59. For continuity, we require that $f(1) = 1 = \lim_{x\to 1^+} (ax + b) = a + b$, or $a + b = 1$. Next, using the four-step process, we have $f'(x) = \begin{cases} 2x & \text{if } x < 1 \\ a & \text{if } x > 1 \end{cases}$ In order that the derivative exist at $x = 1$, we require that $\lim_{x\to 1^-} 2x = \lim_{x\to 1^+} a$, or $2 = a$. Therefore, $b = -1$ and so $f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$



60. f is continuous at $x = 0$, but $f'(0)$ does not exist because the graph of f has a vertical tangent line at $x = 0$.



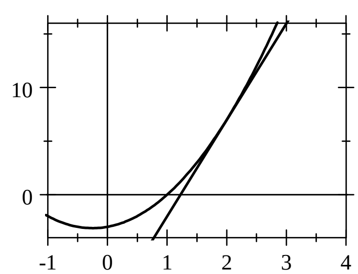
61. We have $f(x) = x$ if $x > 0$ and $f(x) = -x$ if $x < 0$. Therefore, when $x > 0$, $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{x+h-x}{h} = \lim_{h\to 0} \frac{h}{h} = 1$, and when $x < 0$, $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{-x-h-(-x)}{h} = \lim_{h\to 0} \frac{-h}{h} = -1$. Because the right-hand limit does not equal the left-hand limit, we conclude that $\lim_{h\to 0} f(x)$ does not exist.

62. From $f(x) - f(a) = \left[\frac{f(x) - f(a)}{x-a}\right](x-a)$, we see that $\lim_{x\to a} [f(x) - f(a)] = \lim_{x\to a} \left[\frac{f(x) - f(a)}{x-a}\right] \lim_{x\to a} (x-a) = f'(a) \cdot 0 = 0$, and so $\lim_{x\to a} f(x) = f(a)$. This shows that f is continuous at $x = a$.

Using Technology page 155

1. a. 9

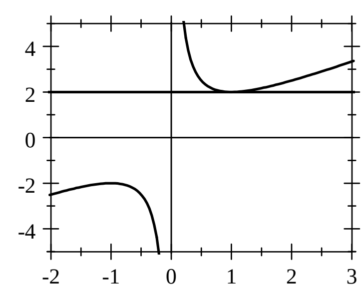
b.



c. $y = 9x - 11$

2. a. 0

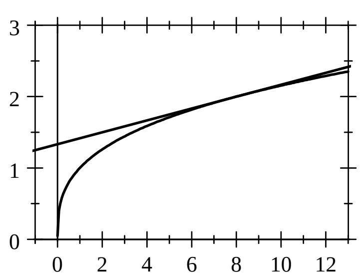
b.



c. $y = 2$

3. a. 0.083

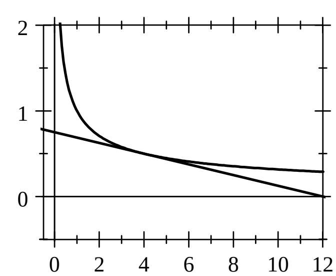
b.



c. $y = \frac{1}{12}x + \frac{4}{3}$

4. a. -0.0625

b.



c. $y = -\frac{1}{16}x + \frac{3}{4}$

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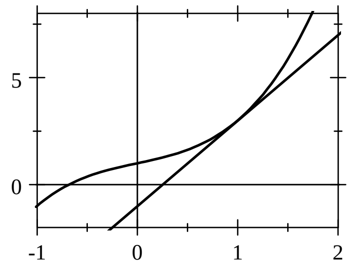
5. a. 4

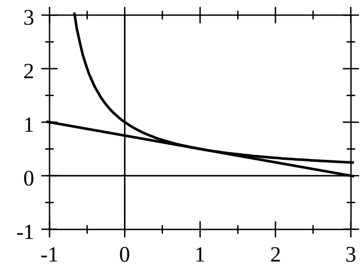
b.



c. $y = 4x - 1$

6. a. -0.25

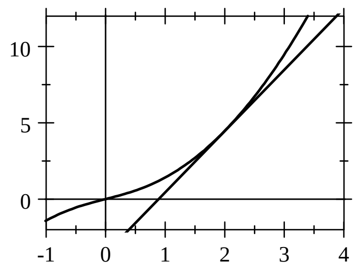
b.



c. $y = -\frac{1}{4}x + \frac{3}{4}$

7. a. 4.02

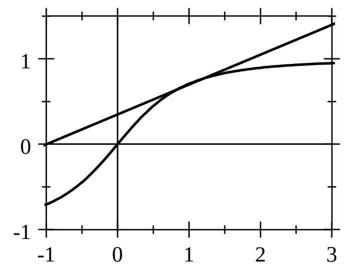
b.



c. $y = 4.02x - 3.57$

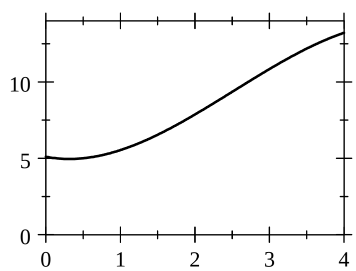
8. a. 0.35

b.



c. $y = 0.35x + 0.35$

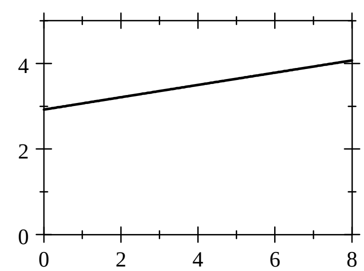
9. a.



b. $f'(3) = 2.8826$ (million per decade)

10. a. $S(t) = -0.000114719t^2 + 0.144618t + 2.92202$

b.



c. \$3.786 billion

d. \$143 million/yr

CHAPTER 2 Concept Review Questions page 156

1. domain, range, B

2. domain, $f(x)$, vertical, point

3. $f(x) \pm g(x)$, $f(x)g(x)$, $\dfrac{f(x)}{g(x)}$, $A \cap B$, $A \cap B$, 0

4. $g(f(x))$, f, $f(x)$, g

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