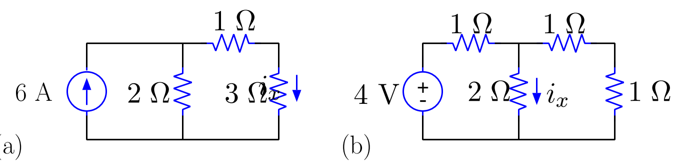
Chapter 2

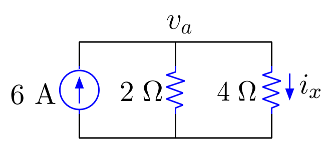
1. In the following circuits, determine i_x:



(a) (b)

Solution:

a) First we label the top left node as $v_a$and combine the resistors 1Ω and 3Ω as depicted in the figure:



Next we apply KCL at top node, and we find that

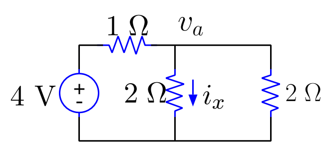
$$6\text{A} = \frac{v_a}{2\Omega} + \frac{v_a}{4\Omega} \Rightarrow v_a = 8\text{V},$$

and, using Ohm's law at the 4Ω resistor, we obtain

$$i_x = \frac{v_a}{4\Omega} = 2\text{A}.$$

Alternatively we could have used current division:

$$i_x = (6\text{A})\frac{2}{2+4} = 2\text{A}.$$

b) 

Applying KCL at node v_a, we obtain

$$\frac{v_a - 4\text{V}}{1} + \frac{v_a}{2} + \frac{v_a}{2} = 0 \Rightarrow v_a = 2\text{V}.$$

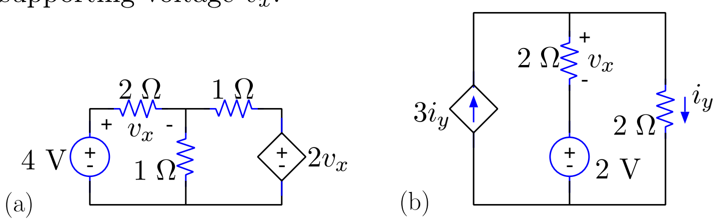
Alternatively using voltage division

$$v_a = (4\text{V})\frac{2 \parallel 2}{1 + 2 \parallel 2} = (4\text{V})\frac{1}{2} = 2\text{V}.$$

Finally using Ohm's law at the 2Ω resistor:

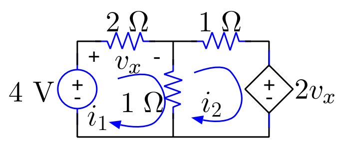
$$i_x = \frac{v_a}{2\Omega} = \frac{2\text{V}}{2\Omega} = 1\text{A}.$$

2. In the following circuits, determine v_x and the absorbed power in the elements supporting voltage v_x:



Solution:

a) First we assume the directions of the two loop currents to be clockwise:



The KVL equation for the first loop can be written as

$$4 - 2i_1 - (i_1 - i_2) = 0.$$

Likewise the KVL equation for the second loop is

$$-(i_2 - i_1) - i_2 - 2v_x = 0.$$

Note that applying Ohm's law in the 2Ω resistor, we can express v_x as

$$v_x = 2i_1.$$

Substituting $v_x = 2i_1$ in the second KVL equation and rearranging both KVL equations we obtain the following set of equations

$$\begin{aligned} -3i_1 + i_2 &= -4 \\ -3i_1 - 2i_2 &= 0. \end{aligned}$$

Their solution is

$$i_1 = \frac{8}{9}\text{A}, \text{ and } i_2 = -\frac{4}{3}\text{A}.$$

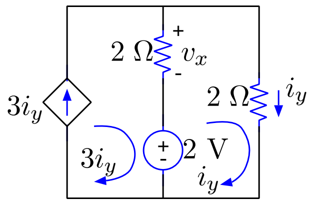
Consequently,

$$v_x = 2i_1 = \frac{16}{9}\text{A}.$$

The resistor carrying voltage v_x absorbs a power of

$$p = v_x i_1 = \frac{128}{81}\,\text{W} \approx 1.58\,\text{W}.$$

b) First we assign the currents of the two loops to be equal to $3i_y$ and i_y, since this values coincide with the current flowing out from the dependent current source and the current flowing through the resistor in the right branch.



Now applying KVL to the right loop we obtain,

$$\begin{aligned} 2 + 2(3i_y - i_y) - 2i_y &= 0 \\ 2 + 2i_y &= 0 \\ i_y &= -1\text{A}. \end{aligned}$$

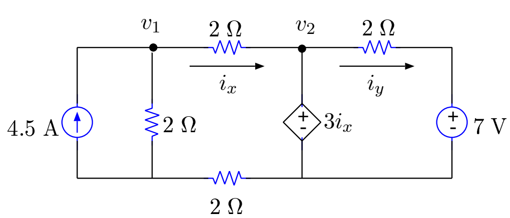
Next applying Ohm's law to the resistor in the middle branch, we have

$$\begin{aligned} v_x &= (2\Omega)(3i_y - i_y) \\ v_x &= -4\text{V}. \end{aligned}$$

Finally the absorbed power for the resistor is

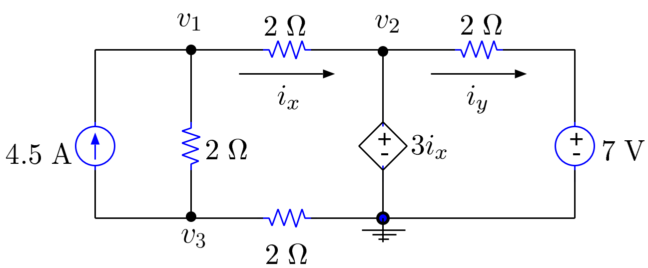
$$\begin{aligned} p &= v_x(3i_y - i_y) \\ p &= (-4\text{V})(-2\text{A}) \\ p &= 8\text{W}. \end{aligned}$$

3. In the circuit shown next, determine v_1, v_2, i_x, and i_y using the node voltage method. Notice that the reference node has not been marked in the circuit; therefore, you are free to select any one of the nodes in the circuit as the reference. The position of the reference (which should be shown in your solution) will influence the values obtained for v_1 and v_2 but not for i_x and i_y.



Solution:

We decide our reference to be in the bottom right node as depicted in the figure:



From the KCL equation at node (v_1), we obtain

$$\begin{aligned} \frac{v_1 - v_3}{2} + \frac{v_1 - v_2}{2} &= 4.5 \\ 2v_1 - v_2 - v_3 &= 9. \end{aligned}$$

Likewise from the KCL equation at node (v_3), we have

$$\begin{aligned} \frac{v_3}{2} + \frac{v_3 - v_1}{2} + 4.5 &= 0 \\ v_1 - 2v_3 &= 9. \end{aligned}$$

Note that Ohm's law between nodes v_1 and v_2 yields to

$$i_x = \frac{v_1 - v_2}{2}.$$

Now equalizing v_2 with the voltage from the dependent voltage source, we obtain

$$\begin{aligned} v_2 &= 3i_x \\ v_2 &= \frac{3}{2}(v_1 - v_2) \\ v_2 &= \frac{3}{5}v_1. \end{aligned}$$

Now substituting $v_2 = \frac{3}{5}v_1$ into the first KCL equation gives

$$\begin{aligned} 2v_1 - \frac{3}{5}v_1 - v_3 &= 9 \\ 7v_1 - 5v_3 &= 45. \end{aligned}$$

This last equation forms with the second KCL equation a set of two linear equations with two unknowns. Their solution is

$$v_1 = 5\text{V}, \text{ and } v_3 = -2\text{V}.$$

But, since $v_2 = \frac{3}{5}v_1$, then

$$v_2 = 3\text{V}.$$

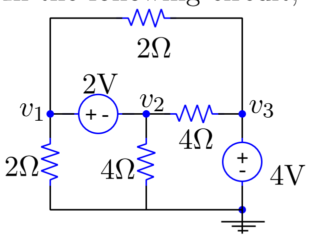
Finally applying Ohm's law we obtain

$$\begin{aligned} i_x &= \frac{v_1 - v_2}{2\Omega} = \frac{2\text{V}}{2\Omega} = 1\text{A} \\ i_y &= \frac{v_2 - 7\text{V}}{2\Omega} = \frac{-4\text{V}}{2\Omega} = -2\text{A}. \end{aligned}$$

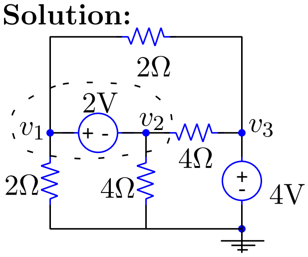
In the case of placing the ground at $v_3$then we obtain

$$\begin{aligned} v_1 &= 7\text{V} \\ v_2 &= 5\text{V} \\ i_x &= 1\text{A} \\ i_y &= -2\text{A}. \end{aligned}$$

4. In the following circuit, determine the node voltages v_1, v_2, and v_3:



Solution:



Since v_1 and v_2 are connected by only a voltage source, they makeup a "super-node". Then, the KCL equation at the super-node can be written as

$$\frac{v_3 - v_2}{4} = \frac{v_1 - v_3}{2} + \frac{v_1}{2} + \frac{v_2}{4}.$$

But from the figure we notice that

$$v_3 = 4\text{V}.$$

Substituting $v_3 = 4\text{V}$ into the KCL equation and rearranging we have

$$2v_1 + v_2 = 6.$$

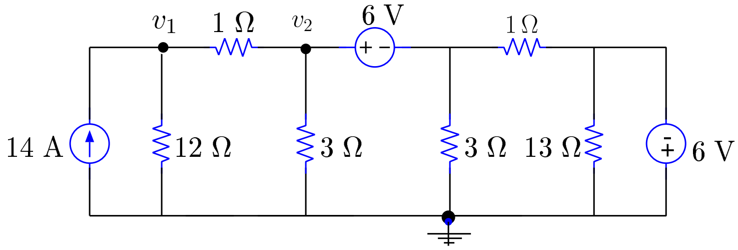
Also from the "super-node" we infer that

$$v_1 - v_2 = 2.$$

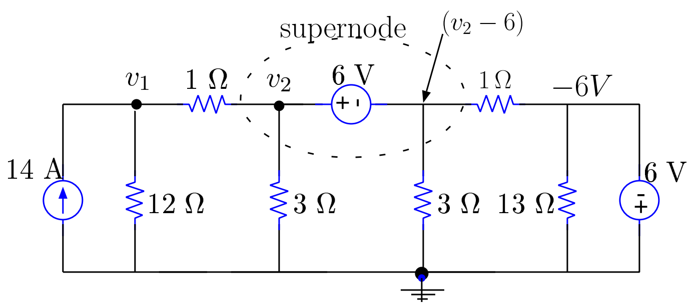
Finally solving these two last equations we obtain

$$\begin{aligned} v_1 &= \frac{8}{3}\text{V} \\ v_2 &= \frac{2}{3}\text{V}. \end{aligned}$$

5. In the following circuit determine node voltages v_1 and v_2:



Solution:



The KCL equation at node $v_1$can be written as:

$$\begin{aligned} \frac{v_1}{12} + \frac{v_1 - v_2}{1} &= 14 \\ \rightsquigarrow \; 13v_1 - 12v_2 &= 168. \end{aligned}$$

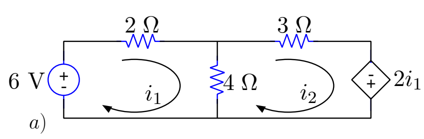
Also applying KCL at the super-node gives

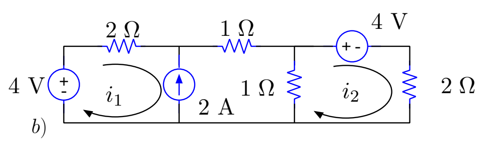
$$\begin{aligned} \frac{v_1 - v_2}{1} &= \frac{v_2}{3} + \frac{v_2 - 6}{3} + \frac{v_2 - 6 - (-6)}{1} \\ \rightsquigarrow \; 3v_1 - 8v_2 &= -6. \end{aligned}$$

Solving this last equation and the first KCL equation, we obtain

$$\begin{aligned} v_1 &= \frac{354}{17}\,\text{V} \approx 20.82\,\text{V} \\ v_2 &= \frac{291}{34}\,\text{V} \approx 8.56\,\text{V}. \end{aligned}$$

6. In the following circuits determine loop currents i_1 and i_2:





Solution:

a) Writing the KVL equation around loop 1 and simplifying, we have

$$\begin{aligned} 6 - 2i_1 - 4(i_1 - i_2) &= 0 \\ -6i_1 + 4i_2 &= -6. \end{aligned}$$

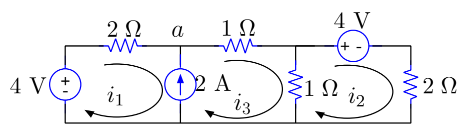
Likewise from KVL around loop 2, we have

$$\begin{aligned} 4(i_1 - i_2) - 3i_2 + 2i_1 &= 0 \\ 6i_1 \quad -7i_2 &= 0. \end{aligned}$$

Finally solving this two sets of equations, we obtain

$$\begin{aligned} i_1 &= \frac{7}{3}\text{A} \\ i_2 &= 2\text{A}. \end{aligned}$$

b) First we assume another loop current i_3 as depicted in the following figure:



Next, applying KVL around loop 1, we have

$$\begin{aligned} 4 - 2i_1 - i_3 - (i_3 - i_2) &= 0 \\ -2i_1 + i_2 - 2i_3 &= -4. \end{aligned}$$

Similarly the KVL around loop 2 gives us

$$\begin{aligned} (i_3 - i_2) - 4 - 2i_2 &= 0 \\ -3i_2 + i_3 &= 4. \end{aligned}$$

Now, the KCL equation at node "a" can be written as

$$i_1 + 2 = i_3.$$

Substituting $i_3 = i_1 + 2$ in the two KVL equations and rearranging, we obtain

$$4i_1 - i_2 = 0$$

and

$$i_1 - 3i_2 = 2.$$

Finally solving these two equations in two unknowns yields

$$i_1 = -\frac{2}{11}\text{A}$$

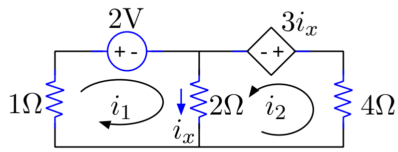
and

$$i_2 = -\frac{8}{11}\text{A}.$$

7.

a) For the next circuit, obtain two independent equations in terms of loop-currents i_1 and i_2 and simplify them to the form

$$\begin{aligned} Ai_1 + Bi_2 &= E \\ Ci_1 + Di_2 &= F, \end{aligned}$$

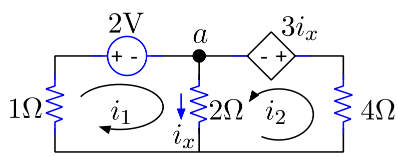


b) Express the previous equations in the matrix form

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} E \\ F \end{pmatrix}$$

and use matrix inversion or *Cramer's rule* to solve for i_1 and i_2.

Solution:



a) Applying KCL at node "a", gives

$$i_x = i_1 + i_2.$$

Now, from KVL around loop 1, we have

$$2 + 2(i_1 + i_2) + i_1 = 0,$$

Likewise applying KVL around loop 2, gives

$$3i_x + 2i_x + 4i_2 = 0.$$

Replacing i_x from the KCL equation into the second KVL yields

$$5(i_1 + i_2) + 4i_2 = 0.$$

Now rearranging this last equation and the first KVL we obtain

$$3i_1 + 2i_2 = -2$$

and

$$5i_1 + 9i_2 = 0.$$

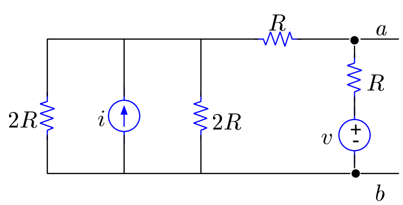
b) Expressing the last two equations in matrix form and using matrix inversion, we obtain

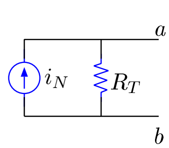
$$
\begin{aligned}
\begin{pmatrix} 3 & 2 \\ 5 & 9 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} &= \begin{pmatrix} -2 \\ 0 \end{pmatrix} \\
\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} &= \begin{pmatrix} 3 & 2 \\ 5 & 9 \end{pmatrix}^{-1} \begin{pmatrix} -2 \\ 0 \end{pmatrix} \\
\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} &= \frac{\begin{pmatrix} 9 & -2 \\ -5 & 3 \end{pmatrix}}{|3 \times 9 - 2 \times 5|} \begin{pmatrix} -2 \\ 0 \end{pmatrix} \\
\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} &= \begin{pmatrix} -18/17 \\ 10/17 \end{pmatrix}.
\end{aligned}
$$

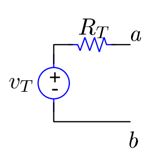
8. By a sequence of resistor combinations and source transformations, the next circuit shown can be simplified to its Norton (bottom left) and Thevenin (bottom right) equivalents between nodes a and b. Show that

$$i_N = \frac{i}{2} + \frac{v}{R} \quad \text{and} \quad R_T = \frac{2}{3}R$$

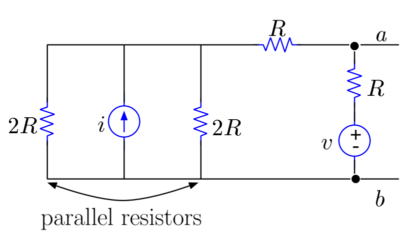
and obtain the expression for v_T.



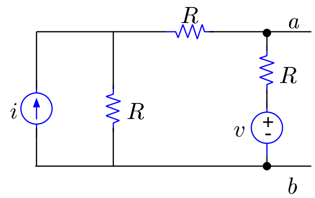




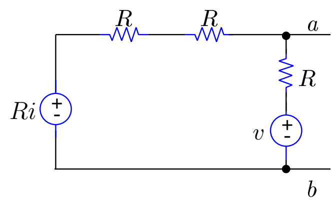
Solution:



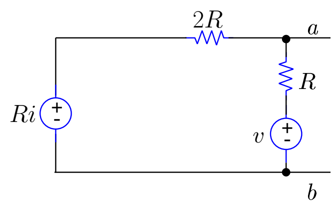
First we combine the two resistors in parallel:



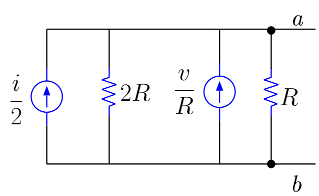
Afterwards we apply source transformation:



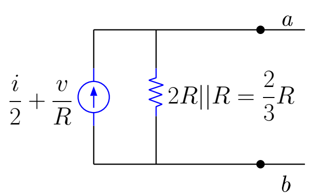
Next the two resistors in series can be combined as follows:



We now apply two source transformations, to have:



Finally combining the two resistors and current sources in parallel, we obtain:

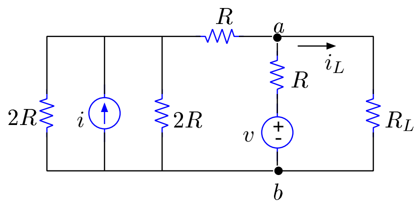


From this figure we can infer the following values for the Thevenin and Norton equivalents:

$$\begin{aligned} i_N &= \frac{i}{2} + \frac{v}{R} \ , \ \ R_T = \frac{2}{3}R \\ v_T &= i_N R_T \\ v_T &= \frac{2}{3}R(\frac{i}{2} + \frac{v}{R}). \end{aligned}$$

9. In the following circuit it is observed that for $i = 0$ and $v = 1$ V, $i_L = \frac{1}{2}$ A, while

for $i = 1$ A and $v = 0$, $i_L = \frac{1}{4}$ A.



a) Determine i_L when $i = 4$ A and $v = 2$ V (you do not need R and R_L to answer this part; just make use of the results of Problem 8).

b) Determine the values of resistances R and R_L.

c) Is it possible to change the value of R_L in order to increase the power absorbed in R_L when $i = 4$ A and $v = 2$ V? Explain.

Solution:

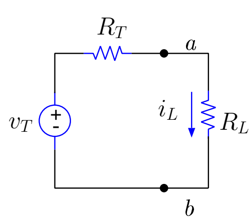
a) $v = 1$ contributes $\frac{1}{2}$A to i_L
$i = 1$ contributes $\frac{1}{4}$A to i_L
Using superposition we have

$$i_L = \frac{1}{2} \times v + \frac{1}{4} \times i.$$

Now, substituting $v = 2$V & $i = 4$A in this last equation yields

$$i_L = \frac{1}{2} \times 2 + \frac{1}{4} \times 4 = 2\text{A}.$$

b) Using the result from exercise 8, we have



Where

$$v_T = \frac{2}{3}R(\frac{i}{2} + \frac{v}{R})$$

and

$$R_T = \frac{2}{3}R.$$

Applying KVL, we can write the following equation

$$v_T = i_L(R_T + R_L).$$

Substituting the expressions for v_T and R_T into the KVL equation yields

$$\frac{2}{3}R(\frac{i}{2}+\frac{v}{R}) = i_L(\frac{2}{3}R+R_L).$$

Now substituting $i=0,\ v=1,\ i_L=\frac{1}{2}$ into this last equation, we have

$$\frac{2}{3}R(\frac{1}{R}) = \frac{1}{2}(\frac{2}{3}R+R_L),$$

which simplifies to

$$2R+3R_L = 4.$$

Similarly we substitute $i=1,\ v=0,\ i_L=\frac{1}{4}$ and obtain

$$\frac{2}{3}R(\frac{1}{2}) = \frac{1}{4}(\frac{2}{3}R+R_L),$$

or, equivalently,

$$-2R+3R_L = 0.$$

The solution from this two last simplified equations gives

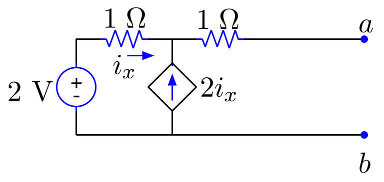
$$\begin{aligned} R_L &= \frac{2}{3}\Omega \\ R &= 1\Omega. \end{aligned}$$

c) Calculating the equivalent resistor:

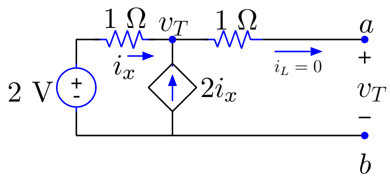
$$R_T = \frac{2}{3}R = \frac{2}{3}\Omega$$

Notice that the absorbed load power is already at it's maximum ($R_L = R_T$). Hence changing R_L will only decrease the power absorbed.

10. In the following circuit, find the open-circuit voltage and the short-circuit current between nodes a to b and determine the Thevenin and Norton equivalent of the network between nodes a and b.



Solution:



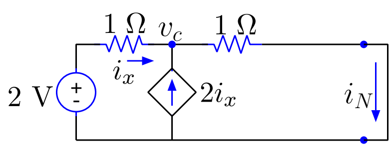
Applying KCL at node v_T we obtain

$$\begin{aligned} i_x + 2i_x &= 0 \\ i_x &= 0. \end{aligned}$$

That means no current flowing through the resistor, consequently no voltage drop. Hence,

$$v_T = 2\text{V}.$$

To get the Norton current we analyze the following figure.



KCL at v_c:

$$\begin{aligned} i_x + 2i_x &= i_N \\ i_N &= 3i_x. \end{aligned}$$

KVL around outer loop:

$$\begin{aligned} 2 &= (1)i_x + (1)i_N \\ i_x &= 2 - i_N. \end{aligned}$$

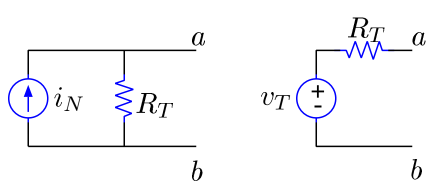
Replacing i_x in the KCL equation with $2 - i_N$ we find that

$$\begin{aligned} i_N &= 3(2 - i_N) \\ i_N &= \frac{3}{2}\text{A}. \end{aligned}$$

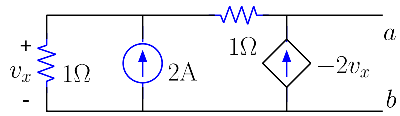
Obtaining the equivalent resistor:

$$R_T = \frac{v_T}{i_N} = \frac{2\text{V}}{3/2\text{A}} = \frac{4}{3}\Omega.$$

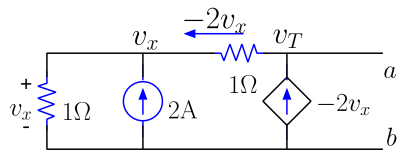
Norton and Thevenin equivalent circuits:



11. Determine the Thevenin equivalent of the following network between nodes a and b, and then determine the available power of the network:



Solution:



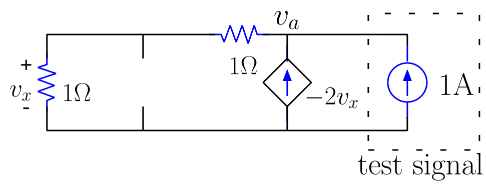
KCL at node v_x:

$$\begin{aligned} 2 + (-2v_x) &= \frac{v_x}{1} \\ v_x &= \frac{2}{3}\text{V}. \end{aligned}$$

KVL around outer loop:

$$\begin{aligned} v_T &= v_x + (-2v_x) \times (1) \\ v_T &= -v_x = -\frac{2}{3}\text{V}. \end{aligned}$$

To find R_T set the independent source to zero and add a test signal:



KCL at v_a:

$$1 - 2v_x = \frac{v_a}{2}$$

Voltage divider:

$$v_x = \frac{1}{2}v_a.$$

Replacing v_x in the KCL equation with $\frac{1}{2}v_a$, we find that

$$\begin{aligned} 1 - 2(\frac{1}{2}v_a) &= \frac{1}{2}v_a \\ v_a &= \frac{2}{3}\text{V}. \end{aligned}$$

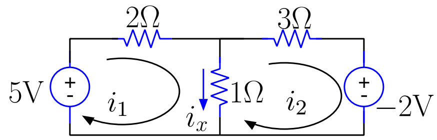
Hence,

$$R_T = \frac{v_a}{1\text{A}} = \frac{2}{3}\Omega.$$

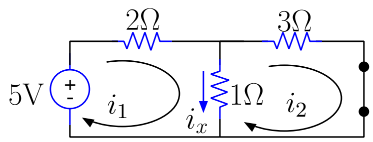
Therefore, the available power is:

$$p_a = \frac{v_T^2}{4R_T} = \frac{(-\frac{2}{3})^2}{4 \times \frac{2}{3}}$$
$$p_a = \frac{1}{6}\text{W}.$$

12. Determine i_x in Figure 2.11b using source suppression followed by superposition.
Solution:



First, suppressing the -2V source:



The KVL equations for loop 1 and 2 simplify to

$$\begin{aligned} 5 &= 3i_1 - i_2 \\ 0 &= -i_1 + 4i_2. \end{aligned}$$

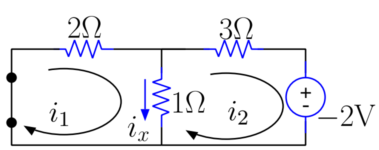
Their solution is

$$\begin{aligned} i_1 &= \frac{20}{11}\text{A} \\ i_2 &= \frac{5}{11}\text{A}. \end{aligned}$$

Consequently from the KCL equation for the top node,

$$i_x = i_1 - i_2 = \frac{15}{11}\text{A}.$$

Next, we suppress the 5V source



The KVL equations for loop 1 and 2 simplify to

$$\begin{aligned} 0 &= 3i_1 - i_2 \\ 2 &= -i_1 + 4i_2. \end{aligned}$$

Solving this equations yields

$$i_1 = \frac{2}{11}\text{A}$$
$$i_2 = \frac{6}{11}\text{A},$$

and, consequently from the KCL equation for the top node, we have

$$i_x = i_1 - i_2 = -\frac{4}{11}\text{A}.$$

Finally, according to the superposition principle, we obtain

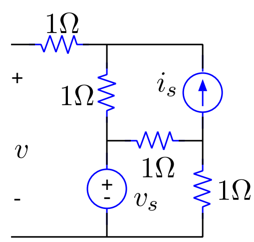
$$i_x = \frac{15}{11}\text{A} + (-\frac{4}{11})\text{A} = 1\text{A}.$$

13. In the next circuit, do the following:

    a) Determine v when $i_s = 0$.

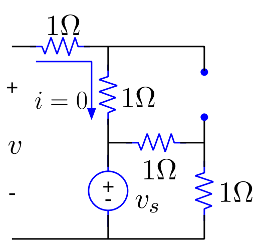
    b) Determine v when $v_s = 0$.

    c) When $v_s = 4\,\text{V}$ and $i_s = 2\,\text{A}$ what is the value of v and what is the available power of the network? Hint: make use of the results of parts (a) and (b) and the superposition method.



Solution:

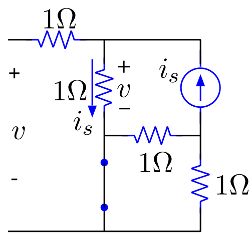
a) When $i_s = 0$



Since no current runs through the top branch we can infer that

$$v = v_s.$$

b) When $v_s = 0$



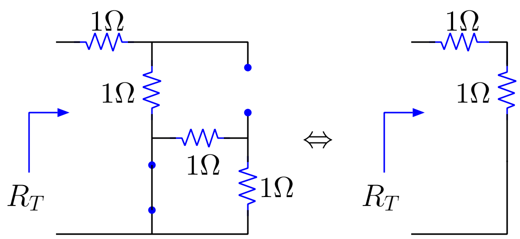
From the graph is evident that

$$v = (1)i_s = i_s.$$

c) Using superposition we find the Thevenin voltage,

$$v_T = v = v_s + i_s \Longleftrightarrow 4 + 2 = 6\text{V}.$$

To find the equivalent resistor we suppress all the independent voltages as depicted in this figure:



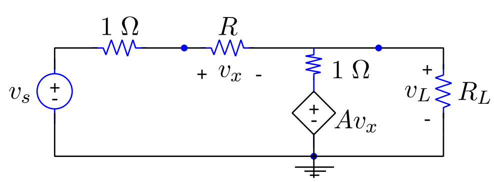
From the graph, we obtain

$$R_T = 2\Omega.$$

Therefore, the available power is

$$\begin{aligned} p_a &= \frac{v_T^2}{4R_T} = \frac{(6)^2}{4 \times 2} \\ p_a &= 4.5\text{W}. \end{aligned}$$

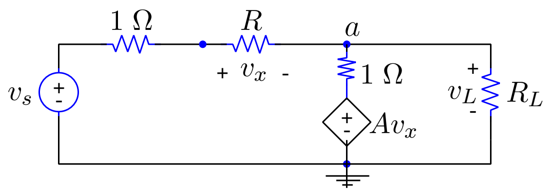
14. Consider the following circuit:



a) Determine v_L given that $v_s = 1$ V, $R = 1\,\text{k}\Omega$, $R_L = 0.1\,\Omega$, and $A = 100$.

b) Find an approximate expression for v_L which is valid when $R \gg 1\,\Omega$, $R_L \approx 1\,\Omega$, and $A \gg 1$.

Solution:



a) The KCL equation for node "a" can be written as

$$\begin{aligned}\frac{v_L}{R_L} + \frac{v_L - Av_x}{1} &= \frac{v_x}{R} \\ (1 + \frac{1}{R_L})v_L &= (A + \frac{1}{R})v_x \ .\end{aligned}$$

Also, from voltage division in the left branch, we find

$$\begin{aligned}v_x &= (v_s - v_L)\frac{R}{1+R} \\ v_x &= \frac{v_s - v_L}{1 + \frac{1}{R}}.\end{aligned}$$

Now, replacing v_x in the KCL equation with $\frac{v_s - v_L}{1+\frac{1}{R}}$, we obtain

$$(1 + \frac{1}{R_L})v_L = (A + \frac{1}{R})\frac{v_s - v_L}{1 + \frac{1}{R}}.$$

But $R \gg 1\,\Omega$, then $\frac{1}{R} \ll 1$. Therefore,

$$\begin{aligned}(1 + \frac{1}{R_L})v_L &\approx A(v_s - v_L) \\ (1 + \frac{1}{R_L} + A)v_L &\approx Av_s.\end{aligned}$$

Finally substituting $A = 100$, $v_s = 1\,\text{V}$, and $R_L = 0.1\,\Omega$ into this equation we obtain

$$v_L \approx \frac{100}{111}\text{V} \approx 0.9\text{V} \ .$$

b) Previously when considering $R \gg 1\,\Omega$, we obtained

$$(1 + \frac{1}{R_L} + A)v_L \approx Av_s \ .$$

Now considering $R_L \approx 1\,\Omega$, we find that

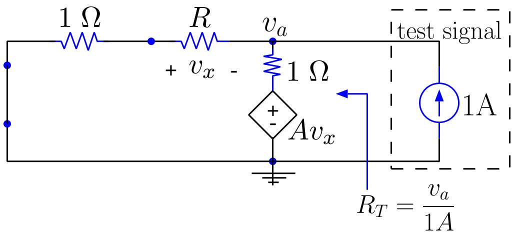
$$v_L \approx \frac{A}{2+A}v_s \ .$$

But since $A \gg 1$,

$$v_L \approx v_s.$$

15. Determine the Thevenin resistance R_T of the network to the left of R_L in the circuit shown in Problem 14. What is the approximate expression for R_T if $R \gg 1\,\Omega$ and $A \gg 1$?

Solution:



Suppressing the independent source v_s and connecting a 1A test signal we get the Thevenin resistance by measuring the Voltage v_a.
The KCL equation for node $v_a$can be written as

$$\begin{aligned} 1 + \frac{v_x}{R} &= \frac{v_a - Av_x}{1} \\ v_a &= 1 + (A + \frac{1}{R})v_x \,. \end{aligned}$$

From voltage division in the left branch, we find

$$\begin{aligned} -v_x &= v_a \frac{R}{1+R} \\ v_x &= \frac{-v_a}{1+\frac{1}{R}} \,. \end{aligned}$$

Now, replacing v_x in the KCL equation with $\frac{-v_a}{1+\frac{1}{R}}$, we have that

$$v_a = 1 + (A + \frac{1}{R})\frac{(-v_a)}{(1+\frac{1}{R})} \,.$$

But, since $R \gg 1 \;\Rightarrow\; \frac{1}{R} \ll 1$, then this equation simplifies to

$$\begin{aligned} v_a &\approx 1 + A(-v_a) \\ v_a &\approx \frac{1}{1+A} \,. \end{aligned}$$

Finally since $A \gg 1$, the equivalent resistance obtained is approximately

$$R_T = v_a \approx 0.$$

16. For (a) through (e), assume that $A = 3 - j3$, $B = -1 - j1$, and $C = 5e^{-j\frac{\pi}{3}}$:

a) Let $D = AB$. Express D in exponential form.
b) Let $E = A/B$. Express E in rectangular form.
c) Let $F = \frac{B}{C}$. Express F in exponential form.
d) Let $G = (CD)^*$ where * denotes complex conjugation. Express G in rectangular and exponential forms.
e) Let $H = (A + C)^*$. Determine $|H|$ and $\angle H$, the magnitude and angle of H.

Solution:

a) First we write A and B in exponential form:

$$\begin{aligned} A &= 3 - j3 = 3\sqrt{2}e^{-j\frac{\pi}{4}} \\ B &= -1 - j1 = \sqrt{2}e^{j\frac{5\pi}{4}}. \end{aligned}$$

Then, multiplying magnitudes and adding arguments, we obtain

$$\begin{aligned} D &= AB = 3\sqrt{2}e^{-j\frac{\pi}{4}}\sqrt{2}e^{j\frac{5\pi}{4}} \\ D &= 6e^{j\pi} = -6. \end{aligned}$$

b) Evaluating the division in exponential form and then converting to rectangular form:

$$\begin{aligned} E &= A/B = \frac{3\sqrt{2}}{\sqrt{2}}e^{j(-\frac{\pi}{4}-\frac{5\pi}{4})} \\ E &= 3e^{-j\frac{3\pi}{2}} = 3e^{j\frac{\pi}{2}} = j3. \end{aligned}$$

c) Evaluating in exponential form:

$$\begin{aligned} F &= B/C = \frac{\sqrt{2}}{5}e^{j(\frac{5\pi}{4}-(-\frac{\pi}{3}))} \\ F &= \frac{\sqrt{2}}{5}e^{j\frac{19}{12}\pi} = \frac{\sqrt{2}}{5}e^{-j\frac{5}{12}\pi}. \end{aligned}$$

d) Multiplying in exponential form, and then applying Euler's identity:

$$\begin{aligned} G &= (CD)^* = (5e^{-j\frac{\pi}{3}}6e^{j\pi})^* \\ G &= (30e^{j\frac{2}{3}\pi})^* = 30e^{-j\frac{2}{3}\pi} \\ G &= 30(\cos(-\frac{2\pi}{3}) + j\sin(-\frac{2\pi}{3})) \\ G &= 30(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) = -15 - j15\sqrt{3}. \end{aligned}$$

e) Expressing C in rectangular form gives

$$\begin{aligned} C &= 5(\cos(-\frac{\pi}{3}) + j\sin(-\frac{\pi}{3})) \\ C &= \frac{5}{2} - j\frac{5\sqrt{3}}{2}\,. \end{aligned}$$

Consequently,

$$\begin{aligned} H &= (A + C)^* \\ H &= (3 + \frac{5}{2} - j(3 + \frac{5\sqrt{3}}{2}))^* \\ H &= \frac{11}{2} + j\frac{6 + 5\sqrt{3}}{2} \\ |H| &= \sqrt{(^{11}/_{2})^2 + (\frac{6 + 5\sqrt{3}}{2})^2} \approx 9.1641 \\ \angle H &= \arctan(\frac{6 + 5\sqrt{3}}{11}) \approx 53.1181^\circ. \end{aligned}$$