

# Analog Integrated Circuit Design

*2<sup>nd</sup> Edition*

## **Chapter 1 Solutions**

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1.1 Arsenic is an n-type dopant.

Therefore,

$$n_n = N_D = 10^{25}/m^3$$

$$p_n = \frac{n_i^2}{N_D} = \frac{(1.1 \times 10^{16}/m^3 \times 2^{\frac{22}{11}})^2}{10^{25}/m^3} = \underline{193.6 \times 10^6/m^3}$$

$$n_n > p_n$$

∴ the resulting material is n-type.

$$1.2 \quad V_T = \frac{kT}{q} = \frac{1.38 \times 10^{-23} \text{ J/K} \cdot 311 \text{ K}}{1.602 \times 10^{-19}} = 26.8 \text{ mV}$$

The carrier concentration doubles with a  $11^\circ\text{C}$  Temperature increase.

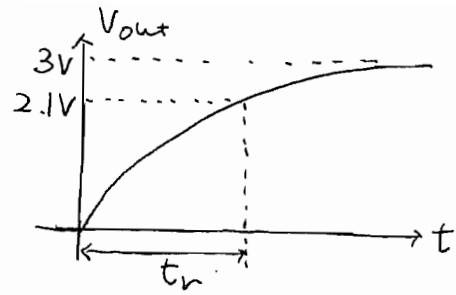
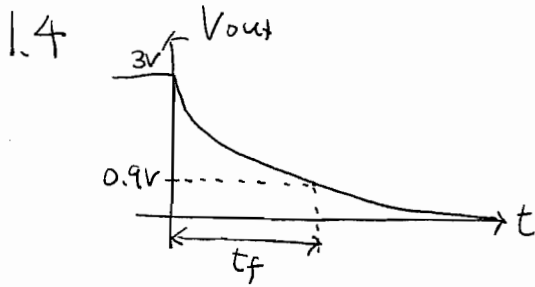
$$\begin{aligned} \Phi_0 &= V_T \ln \left( \frac{N_A N_D}{n_i^2} \right) \\ &= 26.8 \text{ mV} \cdot \ln \left( \frac{10^{25} \cdot 10^{22}}{(2 \times 1.1 \times 10^{16})^2} \right) \\ &= \underline{883 \text{ mV}} \end{aligned}$$

$$1.3 \quad Q^- = Q^+ \approx \left[ 2qk_s \epsilon_0 (\Phi_0 + V_R) N_D \right]^{\frac{1}{2}}$$

$$= (2 \times 1.602 \times 10^{-19} \text{ C} \cdot 11.8 \times 8.854 \times 10^{-12} \text{ F/m} (883 \text{ mV} + 3 \text{ V}) \\ \times 10^{22} / \text{m}^3)^{\frac{1}{2}}$$

$$= 1.14 \text{ mC/m}^2 = \underline{1.14 \text{ fC}/\mu\text{m}^2}$$

For  $10 \mu\text{m} \times 10 \mu\text{m} = 100 \mu\text{m}^2$ , 114 fC would present.



For  $t_f$ ,

$$3V \cdot e^{-\frac{t_f}{\tau_f}} = 0.9V$$

$$t_f = \tau_f \ln \frac{3V}{0.9V} \approx 1.2 \tau_f$$

$$\tau_f = R C_{j-av, fall}$$

$$C_{j-av, fall} = 2 C_{j0} \Phi_0 \frac{\sqrt{1 + \frac{V_2}{\Phi_0}} - \sqrt{1 + \frac{V_1}{\Phi_0}}}{V_2 - V_1}$$

$$= 2 C_{j0} \Phi_0 \frac{\sqrt{1 + \frac{3V}{0.9V}} - \sqrt{1 + \frac{0.9V}{0.9V}}}{3V - 0.9V}$$

$$= 8.58 \text{ fF}$$

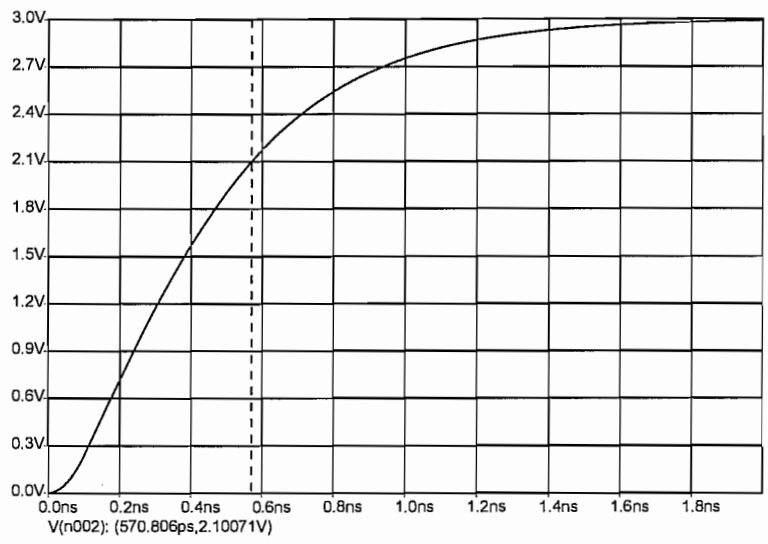
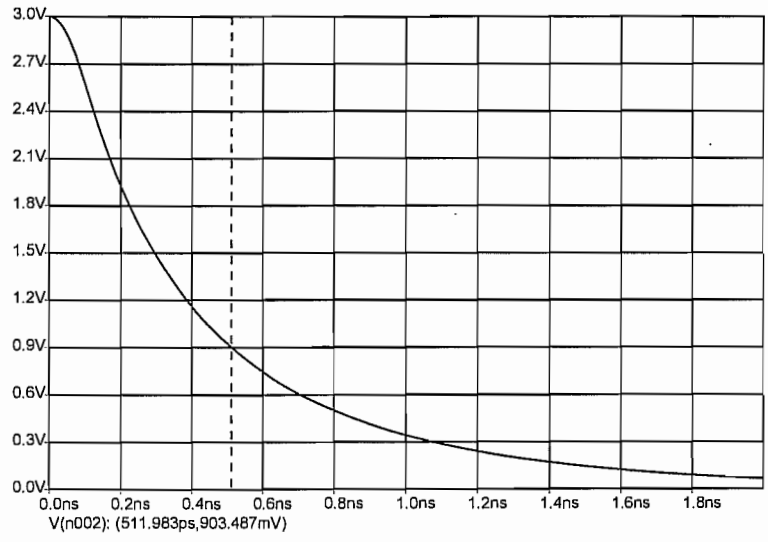
$$\tau_f = 369 \text{ ps}$$

$$\underline{t_f = 443 \text{ ps}}$$

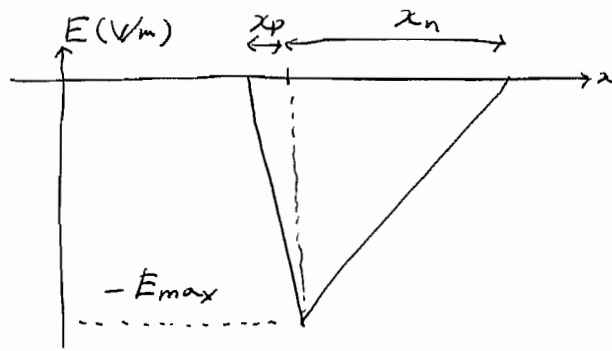
Similarly for  $t_r$ ,

$$C_{j-av, rise} = 10.6 \text{ fF}$$

$$\underline{t_r = 548 \text{ ps}}$$



1.5



$$\frac{E_{\max} (x_p + x_n)}{2} = V_R + \Phi_0$$

$$E_{\max} = \frac{2(V_R + \Phi_0)}{x_p + x_n} = \frac{2 \cdot 1.9}{0.50 \times 10^{-9} + 0.50 \times 10^{-6}}$$

$$= \underline{\underline{7.59 \text{ MV/m}}}$$

1.6 From 1.5,

$$E_{\max} = \frac{2(V_R + \Phi_0)}{x_n + x_p} \approx \frac{2(V_R + \Phi_0)}{x_n}$$

$$= \frac{2(V_R + \Phi_0)}{\sqrt{\frac{2k_s \epsilon_0 (V_R + \Phi_0)}{qN_D}}}$$

$$= \sqrt{\frac{2qN_D}{k_s \epsilon_0}} \cdot \sqrt{V_R + \Phi_0}$$

$$V_R = \frac{k_s \epsilon_0 E_{\max}^2}{2qN_D} - \Phi_0$$

$$= \frac{11.8 \times 8.854 \times 10^{-12} \cdot (3 \times 10^7)^2}{2 \times 1.6 \times 10^{-19} \times 10^{22}} - 0.9$$

$$= \underline{28.5V}$$



1.7 For  $N_A \ll N_D$ ,

$$C_j = \sqrt{\frac{q K_s \epsilon_0 N_A}{2(\Phi_0 + V_R)}} = \frac{30 \text{ fF}}{40 \mu\text{m}^2} = 750 \mu\text{F}/\text{m}^2$$

$$N_A = \frac{2 C_j^2 (\Phi_0 + V_R)}{q K_s \epsilon_0} = \frac{2 \cdot (750 \mu\text{F}/\text{m}^2) (0.9 \text{ V} + 1 \text{ V})}{1.6 \times 10^{-19} \text{ C} \cdot 11.8 \cdot 8.854 \times 10^{-12} \text{ F}/\text{m}}$$
$$= \underline{170 \times 10^{24} / \text{m}^3}$$

$$\begin{aligned} 1.8 \quad I_D &= \mu_n C_{ox} \frac{W}{L} \left[ (V_{GS} - V_{th}) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \quad \text{in triode} \\ &= \mu_n C_{ox} \frac{W}{L} \left( V_{DS}^2 - \frac{1}{2} V_{DS}^2 \right) \quad \text{for } V_{eff} = V_{GS} - V_{th} = V_{DS} \\ &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{DS}^2 \end{aligned}$$

$$1.9 \quad I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{tn})^2 (1 + \lambda (V_{DS} - V_{eff}))$$

$$\lambda = \frac{k_{ds}}{2L \sqrt{V_{DS} - V_{eff} - \Phi_0}} \quad \text{where } k_{ds} = \sqrt{\frac{2k_s E_0}{q N_A}}$$

$$k_{ds} = \sqrt{\frac{2 \cdot 11.8 \cdot 8.854 \times 10^{-12}}{1.6 \times 10^{-19} \cdot 10^{23}}} = 114.3 \times 10^{-9} \text{ m}/\sqrt{\text{V}}$$

$$\lambda = \frac{114.3 \times 10^{-9}}{2 \cdot 0.5 \times 10^{-6} \sqrt{0.9}} = 120.5 \text{ mV}^{-1}$$

$$\begin{aligned} I_D \Big|_{V_{DS} = V_{eff}} &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{tn})^2 \\ &= \frac{1}{2} \cdot 270 \times 10^{-6} \cdot 10 (1 - 0.45)^2 \\ &= 408.4 \mu\text{A} \end{aligned}$$

$$\begin{aligned} \frac{\partial I_D}{\partial V_{DS}} &= \lambda \cdot \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{tn})^2 \\ &= \lambda I_D \Big|_{V_{DS} = V_{eff}} = 120.5 \text{ mV}^{-1} \cdot 408.4 \mu\text{A} \\ &= 49.2 \mu\text{A}/\text{V} \end{aligned}$$

$$\Delta V_{DS} \cdot \frac{\partial I_D}{\partial V_{DS}} = 0.3 \cdot 49.2 \times 10^{-6} = \frac{14.8}{\cancel{148}} \mu\text{A} = \Delta I_D$$

$$1.10 \quad \frac{\Delta V_{DS}}{r_{ds}} = \Delta I_D$$

$$r_{ds} = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{0.5 \text{ V}}{3 \mu\text{A}} = \underline{\underline{167 \text{ k}\Omega}}$$

1.11

$$\gamma = \sqrt{\frac{2qN_A k_s \epsilon_0}{C_{ox}}}$$

In Example 1.10,  $\gamma = 0.25 \sqrt{V}$  for  $N_A = 5 \times 10^{22} / m^3$

Since  $N_A = 10^{23}$  in this question,  $\gamma = \sqrt{2} \cdot 0.25 \sqrt{V}$   
 $= 0.354 \sqrt{V}$ .

$$\phi_F = \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right) = \frac{1.38 \times 10^{-23} \cdot 300}{1.6 \times 10^{-19}} \cdot \ln\left(\frac{10^{23}}{1.1 \times 10^{16}}\right)$$

$$= 0.415 \text{ V.}$$

$$V_{tn} = V_{th0} + \gamma (\sqrt{V_{SB} + |2\phi_F|} - \sqrt{|2\phi_F|})$$

$$= 0.45 + \frac{0.415}{0.354} (\sqrt{1 + 0.829} - \sqrt{0.829})$$

$$= 0.606 \text{ V}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{w}{L} (V_{GS} - V_{tn})^2 (1 + \lambda (V_{DS} - V_{eff}))$$

$$= \frac{1}{2} \cdot 270 \times 10^{-6} \cdot \frac{8}{0.6} \cdot (0.9 - 0.606)^2$$

$$= 156 \mu\text{A}$$

$$g_m = \frac{2I_D}{V_{eff}} = \frac{2 \cdot 156 \mu\text{A}}{0.9 - 0.606} = 1.06 \text{ mA/V}$$

$$k_{ds} = \sqrt{\frac{2k_s \epsilon_0}{qN_A}} = \sqrt{\frac{2 \cdot 11.8 \times 8.854 \times 10^{-12}}{1.6 \times 10^{-19} \cdot 10^{23}}}$$

$$= 114 \times 10^{-9} \text{ m} / \sqrt{V}$$

$$\lambda = \frac{k_{ds}}{2L \sqrt{V_{DS} - V_{eff} + \phi_0}} = \frac{114 \times 10^{-9}}{2 \cdot 0.6 \times 10^{-6} \sqrt{0.9}}$$

$$= 0.100$$

$$r_{ds} = \frac{1}{\lambda I_D} = \underline{64.0 \text{ k}\Omega}$$

$$g_s = \frac{r g_m}{2 \sqrt{V_{SB} + R_{\phi 1}}} = \frac{0.354 \cdot 1.06 \times 10^{-3}}{2 \cdot \sqrt{1 + 0.83}} = \underline{617 \mu A/V}$$

$$\begin{aligned}
 1.12 \quad C_{j0} &= \sqrt{\frac{q k_s \epsilon_0}{2 \Phi_0} \cdot \frac{N_D N_A}{N_D + N_A}} \\
 &= \sqrt{\frac{1.6 \times 10^{-19} \cdot 11.8 \times 8.854 \times 10^{-12}}{2 \cdot 0.9} \cdot \frac{10^{26} \cdot 10^{23}}{10^{26} + 10^{23}}} \\
 &= 963.2 \mu\text{F}/\text{m}^2
 \end{aligned}$$

Since  $V_{sb} = V_{db} = 0 \text{ V}$ ,

$$\begin{aligned}
 C_{sb} &= A_s \cdot C_{j0} = 15 \times 10^{-12} \cdot 963.2 \times 10^{-6} \\
 &= \underline{14.4 \text{ fF}}
 \end{aligned}$$

$$\begin{aligned}
 C_{db} &= A_d \cdot C_{j0} = 15 \times 10^{-12} \cdot 963.2 \times 10^{-6} \\
 &= \underline{14.4 \text{ fF}}
 \end{aligned}$$

Since  $V_{ds} = 0 \text{ V}$ , the transistor is in triode assuming  $V_{gs} > V_{tn}$ . Then

$$\begin{aligned}
 C_{gs} &= \frac{1}{2} W L C_{ox} + W L_{ov} C_{ox} = C_{gd} \\
 &= \frac{1}{2} \cdot 7.5 \times 10^{-12} \cdot 4.5 \times 10^{-3} + 15 \times 10^{-6} \cdot 200 \times 10^{-12} \\
 &= \underline{19.9 \text{ fF}}
 \end{aligned}$$

1.13 Ignoring the body effect, the amount of charge injection is

$$\Delta Q = \frac{(V_{GS} - V_{th0})WL C_{ox}}{2} = (1.8V - 1.0V) \cdot 0.45\mu m \cdot 0.2\mu m \cdot 8.5 \text{ fF}/\mu m^2 / 2$$
$$= 1.19 \times 10^{-15} \text{ C}$$

$$\Delta V = \frac{\Delta Q}{C_L} = 1.19 \text{ mV}$$

$$\text{Therefore, } V_o = 1V - \Delta V = \underline{999 \text{ mV}}$$



1.14 For  $V_{in} 0.2V \rightarrow 0.4V$ ,

$$\begin{aligned} r_{on} \Big|_{V_{DS}=0.2V} &= \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th} - V_{DS})} \\ &= \frac{1}{270 \times 10^{-6} \cdot \frac{4}{0.2} (1.8 - 0.4 - 0.45 - 0.2)} \\ &= 247 \Omega \end{aligned}$$

$$\begin{aligned} r_{on} \Big|_{V_{DS}=\overset{0.0}{0V}} &= \frac{1}{270 \times 10^{-6} \cdot \frac{4}{0.2} (1.8 - 0.4 - 0.45)} \\ &= 194 \Omega \end{aligned}$$

$$r_{on-ave} = 221 \Omega$$

$$1 - e^{-\frac{t_n}{r_{on-ave} \cdot C_L}} = 0.99$$

$$\begin{aligned} t_r &= 4.61 r_{on-ave} \cdot C_L = 4.61 \times 221 \times 10^{-12} \\ &= \underline{1.02 \text{ ns}} \end{aligned}$$

For  $V_{in} 0.6V \rightarrow 0.8V$

$$r_{on} \Big|_{V_{DS}=0.2V} = 529 \Omega$$

$$r_{on} \Big|_{V_{DS}=0.0V} = 337 \Omega$$

$$r_{on-ave} = 433 \Omega$$

$$t_r = 4.61 \cdot r_{on-ave} \cdot C_L = \underline{2.00 \text{ ns}}$$

$$1.15 \quad \phi_F = \frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right) = \frac{1.38 \times 10^{-23} \cdot 300}{1.6 \times 10^{-19}} \ln \left( \frac{10^{23}}{1.1 \times 10^{16}} \right)$$

$$= 0.415 \text{ V}$$

$$\gamma = \frac{\sqrt{2qNAk_s E_0}}{C_{ox}} = \frac{\sqrt{2 \times 1.6 \times 10^{-19} \cdot 10^{23} \cdot 11.8 \times 8.854 \times 10^{-12}}}{8.5 \text{ mF/m}^2}$$

$$= 0.215 \sqrt{V}$$

$$V_{th} = V_{th0} + \gamma \left( \sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F} \right)$$

$$V_{th}|_{V_{SB}=0.4V} = 0.45 + 0.215 \left( \sqrt{0.4 + 0.83} - \sqrt{0.83} \right) \\ = 0.493 \text{ V}$$

$$V_{th}|_{V_{SB}=0.8V} = 0.529 \text{ V}$$

For  $V_{in} = 0.2V \rightarrow 0.4V$

$$r_{on}|_{V_{DS}=0.2V} = \frac{1}{270 \times 10^{-6} \cdot \frac{4}{0.2} (1.8 - 0.4 - 0.493 - 0.2)} \\ = 262 \Omega$$

$$r_{on}|_{V_{DS}=0V} = \frac{1}{270 \times 10^{-6} \cdot \frac{4}{0.2} (1.8 - 0.4 - 0.493)} \\ = 204 \Omega$$

$$r_{on-ave} = 233 \Omega$$

$$t_r = 4.61 r_{on-ave} \cdot C_L = \underline{1.07 \text{ ns}}$$

For  $V_{in} = 0.6V \rightarrow 0.8V$

$$r_{on}|_{V_{DS}=0.2V} = 683 \Omega$$

$$r_{on}|_{V_{DS}=0V} = 393 \Omega$$

$$r_{on-ave} = 538 \Omega$$

$$t_r = 4.61 r_{on-ave} \cdot C_L = \underline{2.48 \text{ ns}}$$

1.16	Node	WL	WL Cox	$\Delta Q$
	0.8 $\mu\text{m}$	12.8 $\mu\text{m}^2$	23.04 fF	2.3 fC
	0.35 $\mu\text{m}$	2.45 $\mu\text{m}^2$	11.03 fF	1.1 fC
	0.18 $\mu\text{m}$	0.648 $\mu\text{m}^2$	5.508 fF	0.55 fC
	45 nm	0.0405 $\mu\text{m}^2$	1.01 fF	0.10 fC

$$\frac{W}{L} = 20 \Rightarrow WL = 20L^2$$

1.19

$$r_{ds} \approx \frac{1}{\lambda I_D}$$

$$\lambda = \frac{0.16 \mu\text{m}/\text{V}}{0.4 \mu\text{m}} = 0.4/\text{V}$$

$$g_m = \frac{2I_D}{V_{eff}}$$

$$A_i = g_m r_{ds} = \frac{2I_D}{V_{eff}} \cdot \frac{1}{\lambda I_D} = \frac{2}{\lambda V_{eff}}$$

$$V_{eff} = \frac{2}{\lambda A_i} = \frac{2}{0.4 \cdot 10} = 0.5 \text{ V}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{eff}$$

$$W = \frac{g_m \cdot L}{\mu_n C_{ox} V_{eff}} = \frac{0.5 \times 10^{-3} \cdot 0.4 \times 10^{-6}}{190 \times 10^{-6} \cdot 0.5}$$

$$= \underline{2.11 \mu\text{m}}$$

$$1.18 \quad V_{\text{eff}} = \frac{2}{A_i \cdot \lambda} \quad \text{from 1.17.}$$

$$\lambda = \frac{0.08}{0.2} = 0.4 \text{ V}$$

$$V_{\text{eff}} = \frac{2}{10 \cdot 0.4} = 0.5 \text{ V.}$$

$$W = \frac{g_m \cdot L}{\mu_n C_{ox} \cdot V_{\text{eff}}} \quad \text{from 1.17}$$

$$= \frac{0.5 \times 10^{-3} \cdot 0.2 \times 10^{-6}}{270 \times 10^{-6} \cdot 0.5}$$

$$= \underline{\underline{0.74 \mu\text{m}}}$$

1.19

$$r_{ds} \approx \frac{1}{\lambda I_D} = \frac{L}{\lambda \cdot L \cdot I_D}$$

$$L = \lambda \cdot L \cdot I_D \cdot r_{ds}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$

$$W = \frac{2 I_D \cdot L}{\mu_n C_{ox} (V_{GS} - V_{TH})^2}$$

For  $0.35 \mu\text{m}$ ,

$$L = 0.16 \times 10^{-6} \cdot 0.2 \times 10^{-3} \cdot 20 \times 10^3$$

$$= \underline{0.64 \mu\text{m}}$$

$$W = \frac{2 \cdot 0.2 \times 10^{-3} \cdot 0.64 \times 10^{-6}}{190 \times 10^{-6} \cdot (0.25)^2}$$

$$= \underline{21.6 \mu\text{m}}$$

For  $0.18 \mu\text{m}$ 

$$L = 0.08 \cdot 0.2 \times 10^{-3} \times 20 \times 10^3 \times 10^{-6}$$

$$= \underline{0.32 \mu\text{m}}$$

$$W = \frac{2 \cdot 0.2 \times 10^{-3} \cdot 0.32 \times 10^{-6}}{270 \times 10^{-6} \cdot (0.25)^2}$$

$$= \underline{7.59 \mu\text{m}}$$

1.20 Assuming the minimum L of 0.35  $\mu\text{m}$ ,

$$\lambda = \frac{2 \cdot L}{L} = \frac{0.16}{0.35} = 0.457 / \text{V}$$

$$r_{ds} = \frac{1}{\lambda \cdot I_D} = \frac{1}{0.35 \times 10^{-3} \cdot 0.457} = 6.25 \text{ k}\Omega$$

$$A_i = g_m r_{ds}$$

$$g_m = \frac{A_i}{r_{ds}} = 5.6 \text{ mA/V.}$$

$$g_m = \sqrt{2 I_D \mu_n C_{ox} \frac{W}{L}}$$

$$W = \frac{g_m^2 \cdot L}{2 I_D \mu_n C_{ox}} = \underline{82.5 \mu\text{m}}$$

1. 21

$$g_m = \sqrt{2 I_{DQ} \mu_n C_{ox} \frac{W}{L}}$$

$$\frac{W}{L} = \frac{g_m^2}{2 I_{DQ} \mu_n C_{ox}} = \frac{(2.2 \times 10^{-3})^2}{2 \cdot 0.25 \times 10^{-3} \times 270 \times 10^{-6}}$$

$$= \underline{35.9}$$



1, 22

$$f_{3dB} = \frac{1}{2\pi r_{on} C_L}$$

$$r_{on} = \frac{1}{2\pi C_L f_{3dB}} = \frac{1}{2\pi \cdot 1 \times 10^{-12} \times 250 \times 10^6}$$

$$= 637 \Omega.$$

$$r_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th} - V_{DS})}$$

$$\approx \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})} \quad \text{for a small } V_{DS}.$$

$$W = \frac{L}{\mu_n C_{ox} (V_{GS} - V_{th}) r_{on}}$$

$$= \frac{0.35 \times 10^{-6}}{190 \times 10^{-6} (1.8 - 0.3 - 0.57) \cdot 637}$$

$$= \underline{3.11 \mu m}$$

$$W L C_{ox} = 3.11 \mu m \cdot 0.35 \mu m \cdot 4.5 \text{ fF}/\mu m^2$$

$$= \underline{4.90 \text{ fF}}$$

If  $V_{th}$  increases by 70 mV,

$$W = \frac{0.35 \times 10^{-6}}{190 \times 10^{-6} (1.8 - 0.3 - 0.64) \cdot 637}$$

$$= \underline{3.36 \mu m}$$

$$W L C_{ox} = 3.36 \mu m \cdot 0.35 \mu m \cdot 4.5 \text{ fF}/\mu m^2$$

$$= \underline{5.30 \text{ fF}}$$

1.23 From 1.22,

$$\begin{aligned}W &= \frac{L}{\mu_n C_{ox} (V_{GS} - V_{tn}) r_{on}} \\&= \frac{0.18 \mu\text{m}}{270 \times 10^{-6} (1.8 - 0.45) \cdot 637} \\&= \underline{0.997 \mu\text{m}}\end{aligned}$$

$$\begin{aligned}WL C_{ox} &= 0.997 \mu\text{m} \cdot 0.18 \mu\text{m} \cdot 8.5 \text{ fF}/\mu\text{m}^2 \\&= \underline{1.53 \text{ fF}}\end{aligned}$$

If  $V_{tn}$  increases by 70mV,

$$\begin{aligned}W &= \frac{0.18 \times 10^{-6}}{270 \times 10^{-6} (1.8 - 0.3 - 0.52) \cdot 637} \\&= \underline{1.07 \mu\text{m}}\end{aligned}$$

$$\begin{aligned}WL C_{ox} &= 1.07 \mu\text{m} \cdot 0.18 \mu\text{m} \cdot 8.5 \text{ fF}/\mu\text{m}^2 \\&= \underline{1.63 \text{ fF}}\end{aligned}$$

1.24 In strong inversion with very high  $V_{os}$ ,

$$I_D \approx \frac{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{eff}^2}{\theta V_{eff}} = \frac{1}{2} \frac{1}{\theta} \mu_n C_{ox} \frac{W}{L} V_{eff}$$

$$\log I_D = \log\left(\frac{1}{2} \frac{1}{\theta} \mu_n C_{ox} \frac{W}{L}\right) + \log V_{eff}$$

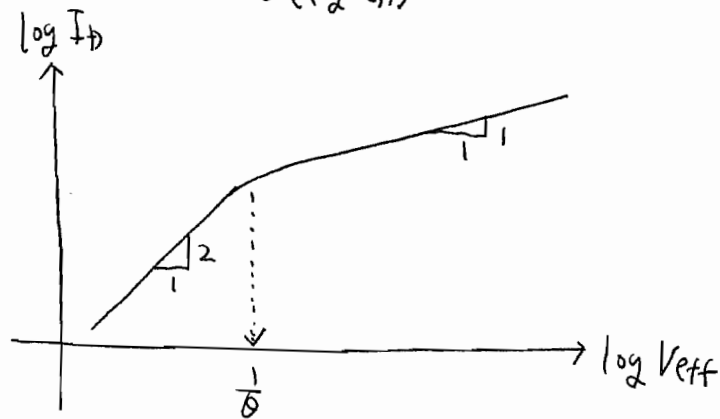
Therefore,  $\frac{\partial(\log I_D)}{\partial(\log V_{eff})} = 1$  in strong inversion.

Without mobility degradation,

$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{eff}^2$$

$$\log I_D = \log\left(\frac{1}{2} \mu_n C_{ox} \frac{W}{L}\right) + 2 \log V_{eff}$$

Therefore,  $\frac{\partial(\log I_D)}{\partial(\log V_{eff})} = 2$  without mobility degradation.



At  $V_{eff} = \frac{1}{\theta}$ ,  $1 < \frac{\partial I_D}{\partial V_{eff}} < 2$ .

1.25 In weak inversion,

$$g_m = \frac{q I_D}{n k T}$$

$$r_{ds} = \frac{1}{\lambda I_D}$$

$$A_i = g_m \cdot r_{ds} = \frac{q}{\lambda n k T}$$

In active mode,

$$g_m = \frac{2 I_D}{V_{eff}}$$

$$r_o = \frac{1}{\lambda I_D}$$

$$A_i = g_m \cdot r_{ds} = \frac{2}{\lambda V_{eff}}$$

In strong mobility degradation,

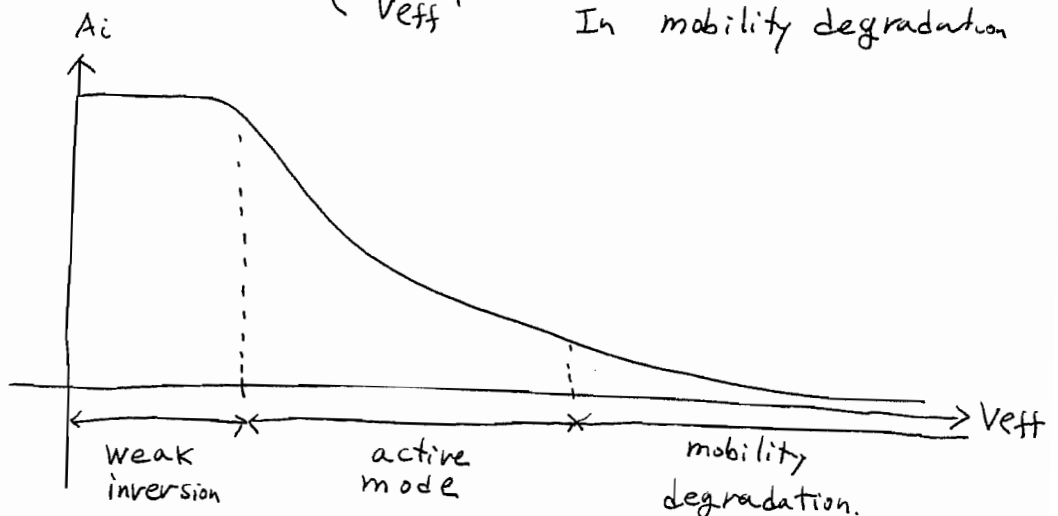
$$g_m = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{1}{\theta}$$

$$r_{ds} = \frac{1}{\lambda I_D} = \frac{1}{\lambda \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{eff} \frac{1}{\theta}}$$

$$A_i = g_m \cdot r_{ds} = \frac{1}{\lambda V_{eff}}$$

To summarize,

$$A_i \propto \begin{cases} 1 & \text{In weak inversion} \\ \frac{2}{V_{eff}} & \text{In active mode} \\ \frac{1}{V_{eff}} & \text{In mobility degradation} \end{cases}$$



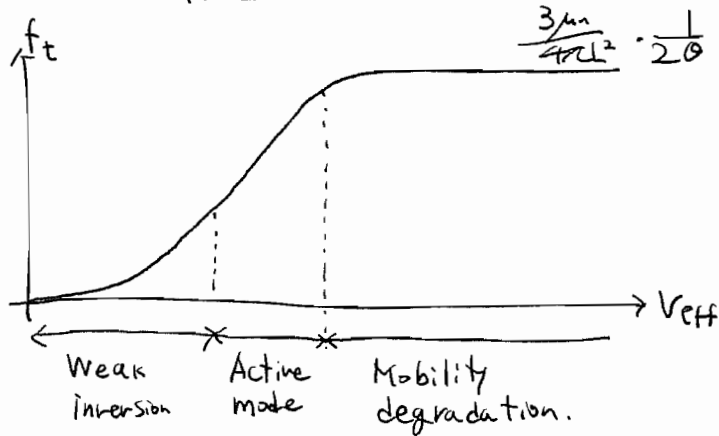
$$\begin{aligned}
 1.26 \quad f_t &= \frac{g_m}{2\pi(C_{gd} + C_{gs})} \approx \frac{g_m}{2\pi C_{gs}} \\
 &= \frac{\frac{g_{ID}}{nKT}}{2\pi\left(\frac{2}{3}WL C_{ox}\right)} = \frac{\frac{3g}{nKT} \cdot (n-1)\mu_n C_{ox} \frac{W}{L} \left(\frac{KT}{q}\right)^2 e^{qV_{eff}/nKT}}{4\pi WL C_{ox}} \\
 &= \frac{3 \frac{n-1}{n} \frac{KT}{q} e^{\frac{qV_{eff}}{nKT}}}{4\pi L^2} \quad \text{in weak inversion}
 \end{aligned}$$

Under mobility degradation,

$$\begin{aligned}
 f_t &= \frac{g_m}{2\pi C_{gs}} = \frac{\frac{1}{2}\mu_n C_{ox} \frac{W}{L} \frac{1}{\theta}}{2\pi \cdot \frac{2}{3}WL C_{ox}} \\
 &= \frac{3\mu_n}{4\pi L^2} \cdot \frac{1}{2\theta}
 \end{aligned}$$

In active mode,

$$f_t = \frac{3\mu_n V_{eff}}{4\pi L^2} \quad \text{as shown in (1.117).}$$



1. 28

a)  $30\mu\text{m} \rightarrow I_D \propto W$  if other parameters are the same

b)  $10\mu\text{m} \rightarrow g_m \propto W$  if other parameters are the same

c)  $3\mu\text{m} \rightarrow r_{ds} \propto \frac{1}{W}$  if other parameters are the same.

1.29

$$\frac{4\text{k}\Omega}{1\text{k}\Omega/\text{sq}} \cdot \frac{1\mu\text{m}^2}{\text{sq}} \cdot \frac{0.4\text{fF}}{1\mu\text{m}^2} = \frac{1.6}{\text{fF}}$$

$$\tau = 4\text{k}\Omega \cdot 1.6\text{fF} = 6.4\text{ps}$$

$$\frac{4\text{k}\Omega}{1\text{k}\Omega/\text{sq}} \cdot \frac{0.16\mu\text{m}^2}{\text{sq}} \cdot \frac{0.4\text{fF}}{1\mu\text{m}^2} = \underline{0.26\text{fF}}$$

$$\tau = 4\text{k}\Omega \cdot 0.26\text{fF} = 1.04\text{ps}$$

The time constants are a lot smaller than that of Example 1.20.

$$\begin{aligned}
 1.30 \quad C_{j0} &= \sqrt{\frac{q k_s \epsilon_0}{2 \Phi_0} \cdot \frac{N_A N_D}{N_A + N_D}} \\
 &= \sqrt{\frac{1.6 \times 10^{-19} \cdot 11.8 \times 8.854 \times 10^{-12}}{2 \cdot 0.9} \cdot \frac{10^{23} \cdot 10^{26}}{10^{23} + 10^{26}}} \\
 &= 963.2 \mu\text{F}/\text{m}^2
 \end{aligned}$$

If we assume  $0.3 \text{ pF} = A \cdot C_{j0}$ ,

$$A = 311.5 \mu\text{m}^2$$

For  $0.2 \text{ pF}$ , we need  $\frac{2}{3} = \frac{1}{\sqrt{1 + \frac{V_R}{\Phi_0}}}$

$$\left(\frac{2}{3}\right)^2 = \frac{1}{1 + \frac{V_R}{\Phi_0}}$$

$$V_R = 1.125 \text{ V.}$$

$$\therefore \underline{\underline{0 \text{ V} < V_R < 1.125 \text{ V}}} \quad \text{with } A = 311.5 \mu\text{m}^2$$



$$1.31 \quad \text{Parallel-plate cap} : \frac{1\text{pF}}{7\text{fF}/\mu\text{m}^2} = \underline{143\mu\text{m}^2}$$

$$\text{Sidewall cap} : \frac{1\text{pF}}{10\text{fF}/\mu\text{m}^2} = \underline{100\mu\text{m}^2}$$

$$\text{MOS cap} : \frac{1\text{pF}}{25\text{fF}/\mu\text{m}^2} = \underline{40\mu\text{m}^2}$$

$$1.32 \quad C_{\text{MOS(ON)}} = 2WL_{\text{ov}}C_{\text{ox}} + WL C_{\text{ox}}$$

$$C_{\text{MOS(OFF)}} = 2WL_{\text{ov}}C_{\text{ox}}$$

$$\frac{C_{\text{MOS(ON)}}}{C_{\text{MOS(OFF)}}} = \frac{2WL_{\text{ov}}C_{\text{ox}} + WL C_{\text{ox}}}{2WL_{\text{ov}}C_{\text{ox}}}$$

$$= 1 + \frac{L C_{\text{ox}}}{2L_{\text{ov}}C_{\text{ox}}}$$

$$= 1 + \frac{0.18 \mu\text{m} \cdot 8.5 \text{fF}/\mu\text{m}^2}{2 \cdot 0.35 \text{fF}/\mu\text{m}}$$

$$= 3.19$$

219% change from minimum to maximum