Solutions Manual for

Advanced Mechanics of Materials and Applied Elasticity

Fifth Edition

Ansel C. Ugural
Saul K. Fenster

PRENTICE
HALL

Upper Saddle River, NJ • Boston • Indianapolis • San Francisco
New York • Toronto • Montreal • London • Munich • Paris • Madrid
Capetown • Sydney • Tokyo • Singapore • Mexico City

The author and publisher have taken care in the preparation of this book, but make no expressed or implied warranty of any kind and assume no responsibility for errors or omissions No liability is assumed for incidental or consequential damages in connection with or arising out of the use of the information or programs contained herein.

Visit us on the Web: InformIT.com/ph

Copyright © 2012 Pearson Education, Inc.

CONTENTS

NOTES TO THE INSTRUCTOR

The *Solutions Manual for Advanced Mechanics of Materials and Applied Elasticity, Fifth Edition* supplements the study of stress and deformation analyses developed in the book. The main objective of the manual is to provide efficient solutions for problems dealing with variously loaded members. This manual can also serve to guide the instructor in the assignments of problems, in grading these problems, and in preparing lecture materials as well as examination questions. Every effort has been made to have a solutions manual that can cut through the clutter and is as self-explanatory as possible, thus reducing the work on the instructor. It is written and class-tested by the author, Ansel Ugural.

As indicated in the book's *Preface*, the text is designed for the senior and/or first year graduate level courses in stress analysis. In order to accommodate courses of varying emphasis, considerably more material has been presented in the book than can be covered effectively in a single three-credit course. The instructor has the choice of assigning a variety of problems in each chapter. Answers to selected problems are given at the end of the text. A description of the topics covered is given in the introduction of each chapter throughout the text. It is hoped that the foregoing materials will help instructor in organizing his or her course to best fit the needs of his or her students.

Ansel C. Ugural
Holmdel, NJ

CHAPTER 1

SOLUTION (1.1)

We have
$$A = 50 \times 75 = 3.75(10^{-3}) \ m^2, \ \theta = 90^o - 40^o = 50^o, \text{ and } \sigma_x = P/A.$$

Equations (1.8), with $\theta = 50^o$:
$$\sigma_{x'} = 700(10^3) = \sigma_x \cos^2 50^o = 0.413\sigma_x = 110.18P$$

or
$$P = 6.35 \ kN$$
$$\left| \tau_{x'y'} \right| = 560(10^3)\sigma_x \sin 50^o \cos 50^o = 0.492\sigma_x = 131.2P$$

Solving
$$P = 4.27 \ kN = P_{all} \qquad \blacktriangleleft$$

SOLUTION (1.2)

Normal stress is
$$\sigma_x = \frac{P}{A} = \frac{125(10^3)}{0.05 \times 0.05} = 50 \ MPa$$
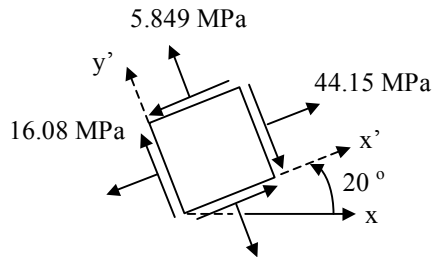
(a) Equations (1.11), with $\theta = 90^o - 70^o = 20^o$:
$$\sigma_{x'} = 50 \cos^2 20^o = 44.15 \ MPa$$
$$\tau_{x'y'} = -50 \sin 20^o \cos 20^o = -16.08 \ MPa \qquad \blacktriangleleft$$
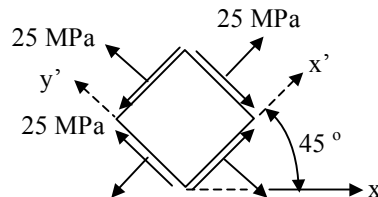$$\sigma_{y'} = 50 \cos^2(20^o + 90^o) = 5.849 \ MPa$$

(b) Equations (1.11), with $\theta = 45^o$:
$$\sigma_{x'} = 50 \cos^2 45^o = 25 \ MPa$$
$$\tau_{x'y'} = -50 \sin 45^o \cos 45^o = -25 \ MPa \qquad \blacktriangleleft$$
$$\sigma_{y'} = 50 \cos^2(45^o + 90^o) = 25 \ MPa$$

SOLUTION (1.3)

From Eq. (1.11a),

$$\sigma_x = \frac{\sigma_{x'}}{\cos^2\theta} = \frac{-75}{\cos^2 30^o} = -100 \; MPa$$
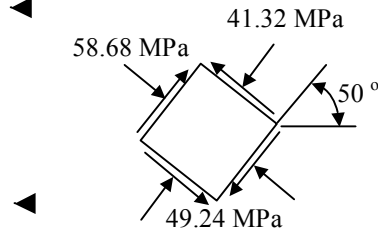
For $\theta = 50^o$, Eqs. (1.11) give then

$$\sigma_{x'} = -100\cos^2 50^o = -41.32 \; MPa$$

$$\tau_{x'y'} = -(-100)\sin 50^o \cos 50^o$$

$$= 49.24 \; MPa$$

Similarly, for $\theta = 140^o$:

$$\sigma_{x'} = -100\cos^2 140^o = -58.68 \; MPa$$

$$\tau_{x'y'} = -49.24 \; MPa$$

41.32 MPa

58.68 MPa

50°

49.24 MPa

SOLUTION (1.4)

Refer to Fig. 1.6c. Equations (1.11) by substituting the double angle-trigonometric relations, or Eqs. (1.18) with $\sigma_y = 0$ and $\tau_{xy} = 0$, become

$$\sigma_{x'} = \tfrac{1}{2}\sigma_x + \tfrac{1}{2}\sigma_x \cos 2\theta \qquad \text{and} \qquad \left|\tau_{x'y'}\right| = \tfrac{1}{2}\sigma_x \sin 2\theta$$

or

$$20 = \tfrac{P}{2A}(1 + \cos 2\theta) \qquad \text{and} \qquad 10 = \tfrac{P}{2A}\sin 2\theta$$

The foregoing lead to

$$2\sin 2\theta - \cos 2\theta = 1 \qquad\qquad\qquad\qquad\qquad (a)$$

By introducing trigonometric identities, Eq. (a) becomes

$$4\sin\theta \cos\theta - 2\cos^2\theta = 0 \quad \text{or} \quad \tan\theta = 1/2 \text{. Hence}$$

$$\theta = 26.56^o \qquad\qquad\qquad\qquad\qquad \blacktriangleleft$$

Thus,

$$20 = \tfrac{P}{2(1300)} = (1 + 0.6)$$

gives

$$P = 32.5 \; kN \qquad\qquad\qquad\qquad\qquad \blacktriangleleft$$

It can be shown that use of Mohr's circle yields readily the same result.

SOLUTION (1.5)

Equations (1.12):

$$\sigma_1 = \frac{P}{A} = \frac{-150(10^3)}{\dfrac{\pi}{4}(50)^2} = -76.4 \; MPa \qquad\qquad\qquad \blacktriangleleft$$

$$\tau_{max} = \frac{P}{2A} = 38.2 \; MPa \qquad\qquad\qquad\qquad \blacktriangleleft$$

SOLUTION (1.6)

Shaded transverse area:

$$A = 2at = 2(10)(75) = 1.5(10^3) \ mm^2$$

Metal is capable of supporting the load

$$P = \sigma A = 90(10^6)(1.5 \times 10^{-3}) = 135 \ kN$$

Apply Eqs. (1.11):

$$\sigma_{x'} = 25(10^6) = \frac{P}{1.5(10^{-3})}(\cos^2 55^o), \quad P = 114 \ kN$$

$$\tau_{x'y'} = 12(10^6) = -\frac{P}{1.5(10^{-3})}\sin 55^o \cos 55^o, \quad P = 38.3 \ kN$$

Thus,

$$P_{all} = 38.3 \ kN \qquad \blacktriangleleft$$

SOLUTION (1.7)

Use Eqs. (1.11):

$$\sigma_{x'} = 20(10^6) = \frac{P}{1.5(10^3)}(\cos^2 40^o), \quad P = 51.1 \ kN$$

$$\tau_{x'y'} = 8(10^6) = -\frac{P}{1.5(10^3)}\sin 40^o \cos 40^o, \quad P = 24.4 \ kN$$

Thus,

$$P_{all} = 24.4 \ kN \qquad \blacktriangleleft$$

SOLUTION (1.8)

$$A = 15 \times 30 = 450 \ mm^2$$

Apply Eqs. (1.11):

$$\sigma_{x'} = \frac{120(10^3)}{450 \times 10^{-6}}(\cos^2 40^o) = 156 \ MPa$$

$$\tau_{x'y'} = -\frac{120(10^3)}{450 \times 10^{-6}}\sin 40^o \cos 40^o = -131 \ MPa \qquad \blacktriangleleft$$

SOLUTION (1.9)

We have $A = 450(10^{-6}) \ m^2$.   Use Eqs. (1.11):

$$\sigma_{x'} = \frac{-100(10^3)}{450 \times 10^{-6}}(\cos^2 60^o) = -55.6 \ MPa$$

$$\tau_{x'y'} = -\frac{-100(10^3)}{450 \times 10^{-6}}\sin 60^o \cos 60^o = 96.2 \ MPa \qquad \blacktriangleleft$$

SOLUTION (1.10)

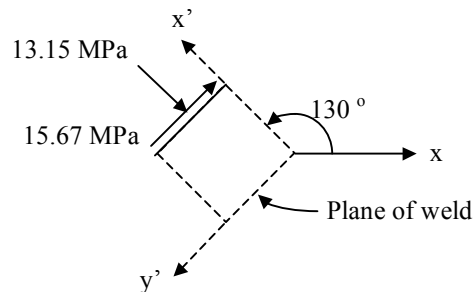$$\theta = 40^o + 90^o = 130^o$$

$$\sigma_x = \frac{P}{A} = -\frac{150(10^3)}{\pi(0.08^2 - 0.07^2)} = -31.83 \; MPa$$

Equations (1.11):

$$\sigma_{x'} = -31.83\cos^2 130^o = -13.15 \; MPa \qquad \blacktriangleleft$$

$$\tau_{x'y'} = 31.83\sin 130^o \cos 130^o = -15.67 \; MPa \qquad \blacktriangleleft$$

SOLUTION (1.11)

Use Eqs. (1.14),

$$(2x) + (-2xy) + (x) + F_x = 0$$

$$(-y^2) + (-2yz + x) + (0) + F_y = 0$$

$$(z - 4xy) + (0) + (-2z) + F_z = 0$$

Solving, we have (in MN/m^3):

$$F_x = -3x + 2xy \qquad F_y = -x + y^2 + 2xz \qquad F_z = 4xy + z \qquad \text{(a)} \; \blacktriangleleft$$

Substituting x=-0.01 m, y=0.03 m, and z=0.06 m, Eqs. (a) yield the following values

$$F_x = 29.4 \; kN/m^3 \qquad F_y = 14.5 \; kN/m^3 \qquad F_z = 58.8 \; kN/m^3$$

Resultant body force is thus

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = 67.32 \; kN/m^3 \qquad \blacktriangleleft$$

SOLUTION (1.12)

Equations (1.14):

$$-2c_1 y - 2c_1 y + 0 + 0 = 0, \qquad 4c_1 y \neq 0$$

$$0 + c_3 z + 0 + 0 = 0, \qquad c_3 z \neq 0$$

$$0 + 0 + 0 + 0 = 0$$

No. Eqs. (1.14) are not satisfied. $\qquad \blacktriangleleft$

_____

SOLUTION (1.13)
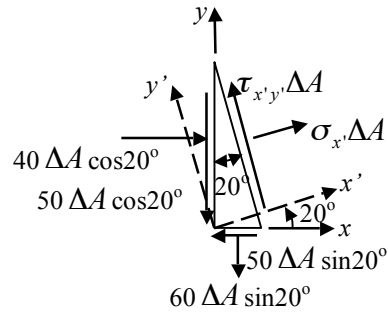
   (a) No. Eqs. (1.14) are not satisfied.   ◄

   (b) Yes. Eqs. (1.14) are satisfied.   ◄

_____

SOLUTION (1.14)

Eqs. (1.14) for the given stress field yield:
$$F_x = F_y = F_z = 0$$
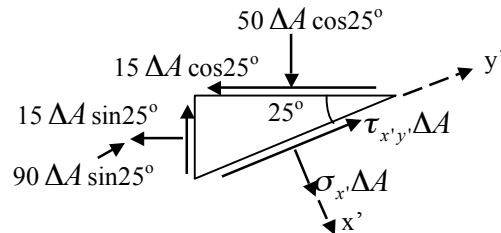   ◄

_____

SOLUTION (1.15)

$$\sum F_{x'} = 0: \quad \sigma_{x'}\Delta A + 40\cos^2 20^o - 60\Delta A \sin^2 20^o$$
$$-2(50\Delta A \sin 20^o \cos 20^o) = 0$$
$$\sigma_{x'} = -35.32 + 7.02 + 32.14 = 3.84 \ MPa \qquad ◄$$

$$\sum F_{y'} = 0: \quad \tau_{x'y'}\Delta A - 40\Delta A \sin 20^o \cos 20^o$$
$$-60\Delta A \sin 20^o \cos 20^o - 50\Delta A \cos^2 20^o$$
$$+50\Delta A \sin^2 20^o = 0$$
$$\tau_{x'y'} = 12.86 + 19.28 + 44.15 - 5.85 = 70.4 \ MPa \qquad ◄$$

_____

SOLUTION (1.16)

$$\sum F_{x'} = 0: \quad \sigma_{x'}\Delta A + 50\Delta A \cos^2 25^o$$
$$-90\Delta A \sin^2 25^o - 2(15\Delta A \sin 25^o \cos 25^o) = 0$$
$$\sigma_{x'} = -41.7 + 16.07 + 11.49 = -12.9 \ MPa \qquad ◄$$

                                                            (CONT.)

_____

1.16 (CONT.)

$$\sum F_{y'} = 0: \quad \tau_{x'y'}\Delta A - 50\Delta A \sin 25^{o}\cos 25^{o}$$

$$-90\Delta A \sin 25^{o}\cos 25^{o} - 15\Delta A \cos^{2}25^{o} + 15\Delta A \sin^{2}25^{o} = 0$$

$$\tau_{x'y'} = 19.15 + 34.47 + 12.32 - 2.68 = 63.3 \; MPa \quad \blacktriangleleft$$
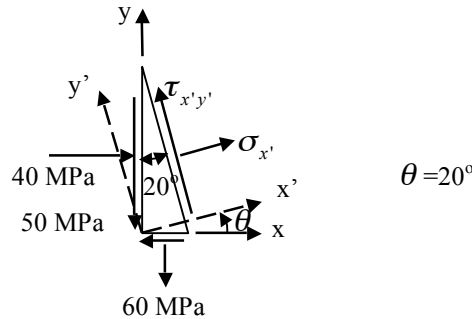
SOLUTION (1.17)

$$\sigma_{x'} = \frac{1}{2}(-40+60) + \frac{1}{2}(-40-60)\cos 40^{o} + 50\sin 40^{o}$$

$$= 10 - 38.3 + 32.1 = -3.8 \; MPa \quad \blacktriangleleft$$

$$\tau_{x'y'} = -\frac{1}{2}(-40-60)\sin 40^{o} + 50\cos 40^{o}$$

$$= 32.14 + 38.3 = 70.4 \; MPa \quad \blacktriangleleft$$
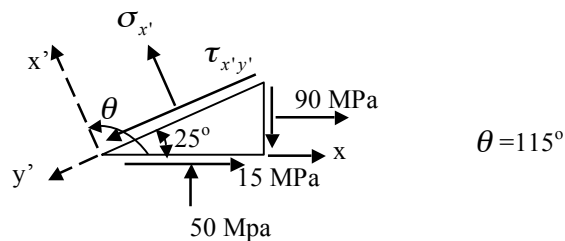
SOLUTION (1.18)

$$\sigma_{x'} = \frac{1}{2}(90-50) + \frac{1}{2}(90+50)\cos 230^{o} - 15\sin 230^{o}$$

$$= 20 - 45 + 11.5 = -13.5 \; MPa \quad \blacktriangleleft$$

$$\tau_{x'y'} = -\frac{1}{2}(90+50)\sin 230^{o} - 15\cos 230^{o}$$

$$= 53.62 + 9.64 = 63.3 \; MPa \quad \blacktriangleleft$$

SOLUTION (1.19)

Transform from $\theta = 40^o$ to $\theta = 0$. For convenience in computations, Let

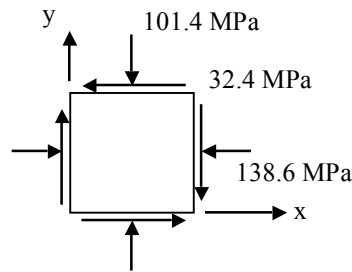$$\sigma_x = -160 \; MPa, \qquad \sigma_y = -80 \; MPa, \qquad \tau_{xy} = 40 \; MPa \text{ and } \theta = -40^o$$

Then

$$\sigma_{x'} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{1}{2}(-160 - 80) + \frac{1}{2}(-160 + 80)\cos(-80^o) + 40\sin(-80^o)$$

$$= -138.6 \; MPa \qquad \blacktriangleleft$$

$$\tau_{x'y'} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{1}{2}(-160 + 80)\sin(-80^o) + 40\cos(-80^o)$$

$$= -32.4 \; MPa \qquad \blacktriangleleft$$

So $\qquad \sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = -160 - 80 + 138.6 = -101.4 \; MPa \qquad \blacktriangleleft$
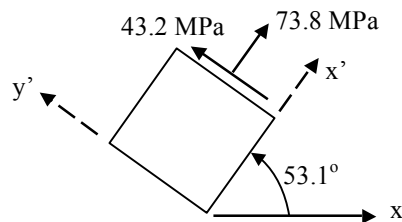
For $\theta = 0^o$:

___

SOLUTION (1.20)

$$\theta = \tan^{-1}\frac{4}{3} = 53.1^o$$

$$\sigma_{x'} = \frac{45 + 90}{2} + \frac{45 - 90}{2}\cos 106.2^o$$

$$= 67.5 + 6.28 = 73.8 \; MPa \qquad \blacktriangleleft$$

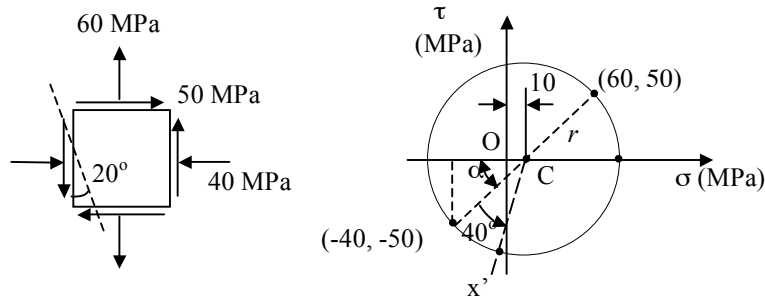$$\tau_{x'y'} = -\frac{45 - 90}{2}\sin 106.2^o = 43.2 \; MPa$$

___

SOLUTION (1.21)

$$\tau_{xy} = 0 \qquad \theta = 70^o$$

(a) $\quad \tau_{x'y'} = -30 = -\dfrac{\sigma - 60}{2}\sin 140^o \qquad \sigma = 153.3 \; MPa$ ◄

(b) $\quad \sigma_{x'} = 80 = \dfrac{\sigma + 60}{2} + \dfrac{\sigma - 60}{2}\cos 140^o \qquad \sigma = 231 \; MPa$ ◄

SOLUTION (1.22)

$$\alpha = \tan^{-1}\frac{50}{60} = 39.8^o$$
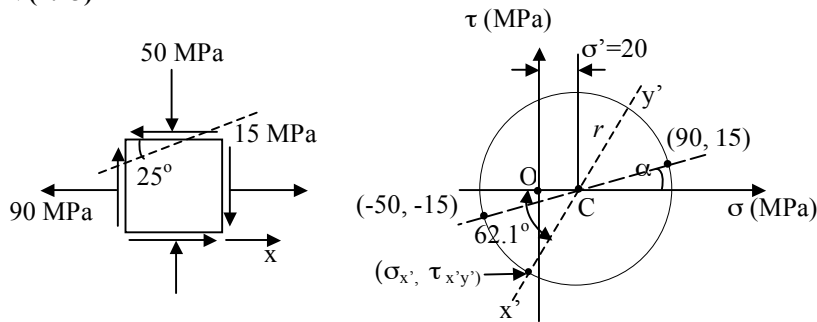
$$r = (60^2 + 50^2)^{\frac{1}{2}} = 78.1$$

$$\tau_{x'y'} = \sin 79.8^o(78.1) = 76.9 \; MPa$$

$$\sigma_{x'} = \cos 79.8^o(78.1) = -13.83 \; MPa$$ ◄

Sketch of results is as shown in solution of Prob. 1.15.

SOLUTION (1.23)
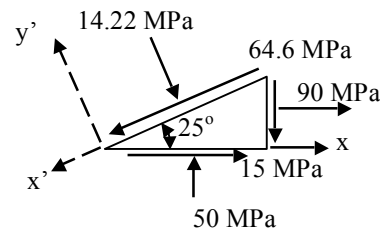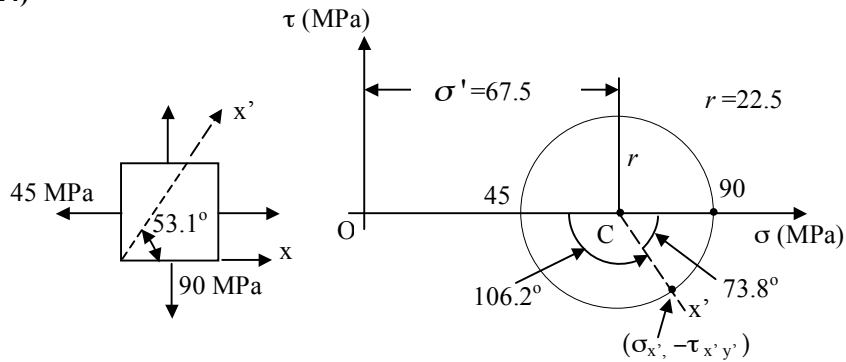

$$\alpha = \tan^{-1}\frac{15}{70} = 12.1^o$$

$$r = (15^2 + 70^2)^{\frac{1}{2}} = 73.14$$

$$\tau_{x'y'} = 73.14\sin 62.1^o = 64.6 \; MPa$$

$$\sigma_{x'} = -73.14\cos 62.1^o + 20$$
$$\quad\;\; = -14.22 \; MPa$$ ◄

SOLUTION (1.24)

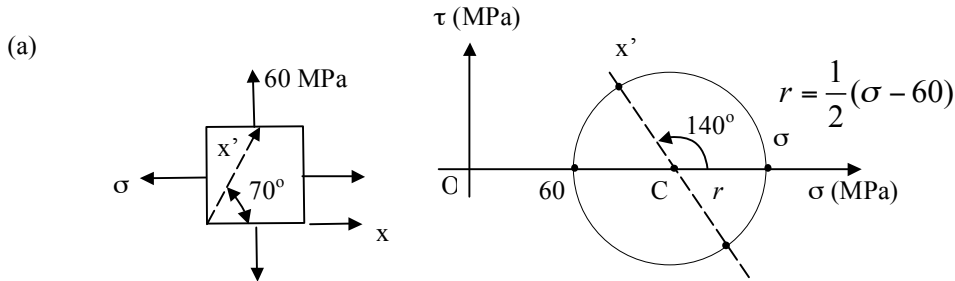
$$\tau_{x'y'} = 22.5\sin 73.8^o = 21.6\ MPa$$

$$\sigma_{x'} = 67.5 + 22.5\cos 73.8^o = 73.8\ MPa$$

Sketch of results is as shown in solution of Prob. 1.20.

SOLUTION (1.25)

(a)

$$\tau_{x'y'} = -30 = \frac{\sigma - 60}{2}\sin 40^o;\qquad \sigma = 153.3\ MPa$$

(b) $\quad \sigma_{x'} = 80 = 60 + \dfrac{\sigma - 60}{2}(1 - \cos 40^o)$

$$\sigma = 231\ MPa$$

SOLUTION (1.26)

(a) From Mohr's circle, Fig. (a):

$$\sigma_1 = 121\ MPa \qquad \sigma_2 = -71\ MPa \qquad \tau_{max} = 96\ MPa$$

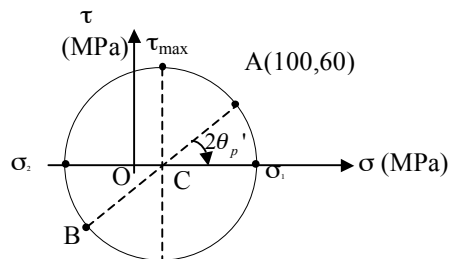$$\theta_p' = -19.3^o \qquad \theta_s' = 25.7^o$$

Figure (a)

(CONT.)

1.26 (CONT.)

By applying Eq. (1.20):
$$\sigma_{1,2} = \frac{50}{2} \pm \left[\frac{22,500}{4} + 36000\right] = 25 \pm 96$$

or
$$\sigma_1 = 121 \; MPa \qquad \sigma_2 = -71 \; MPa \qquad \blacktriangleleft$$

Using Eq. (1.19):
$$\tan 2\theta_p = -\frac{12}{15} = -0.8 \qquad \blacktriangleleft$$
$$\theta_p' = -19.3^o \qquad \theta_s' = 25.7^o$$

(b) From Mohr's circle, Fig. (b):
$$\sigma_1 = 200 \; MPa \qquad \sigma_2 = -50 \; MPa \qquad \tau_{max} = 125 \; MPa$$
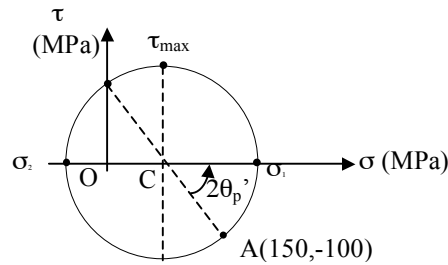$$\theta_p' = 26.55^o \qquad \theta_s' = 71.55^o \qquad \blacktriangleleft$$

Figure (b)

Through the use of Eq. (1.20),
$$\sigma_{1,2} = 75 \pm \left[\frac{22,500}{4} + 10,000\right] = 75 \pm 125$$

or
$$\sigma_1 = 200 \; MPa \qquad \sigma_2 = -50 \; MPa \qquad \blacktriangleleft$$

Using Eq. (1.19), $\tan 2\theta_p = 4/3$:
$$\theta_p' = 26.55^o \qquad \theta_s' = 71.55^o \qquad \blacktriangleleft$$

SOLUTION (1.27)

Referring to Mohr's circle, Fig. 1.15:
$$\sigma_{x'} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta \qquad \text{(a)}$$
$$\sigma_{y'} = \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$
$$\tau_{x'y'} = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta \qquad \text{(b)}$$

From Eqs. (a),
$$\sigma_{x'} + \sigma_{y'} = \sigma_1 + \sigma_2 \qquad \blacktriangleleft$$

By using $\cos^2 2\theta + \sin^2 2\theta = 1$, and Eqs. (a) and (b), we have
$$\sigma_{x'} \cdot \sigma_{y'} - \tau_{x'y'}^2 = \sigma_1 \cdot \sigma_2 = const. \qquad \blacktriangleleft$$
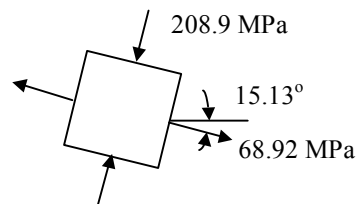

SOLUDION (1.28)

We have

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-70)}{50-(190)} = -0.583$$

$$2\theta_p = -30.26^o \qquad \text{and } \theta_p = -15.13^o$$

Equations (1.18):

$$\sigma_{x'} = \frac{50-190}{2} + \frac{50+190}{2}\cos(-30.26^o) - 70\sin(-30.26^o)$$

$$= -70 + 103.65 + 35.275 = 68.92 \ MPa = \sigma_1 \qquad \blacktriangleleft$$

$$\sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = -208.9 \ MPa = \sigma_2$$

SOLUTION (1.29)

$$\tau_{max} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

Substituting the given values

$$140^2 = \left(\frac{60+100}{2}\right)^2 + \tau_{xy}^2$$

or

$$\tau_{xy,max} = 114.19 \ MPa \qquad \blacktriangleleft$$

SOLUTION (1.30)

Transform from $\theta = 60^o$ to $\theta = 0^o$ with $\sigma_{x'} = -20 \ MPa$, $\sigma_{y'} = 60 \ MPa$,

$\tau_{x'y'} = -22 \ MPa$, and $\theta = -60^o$. Use Eqs. (1.18):

$$\sigma_x = \frac{-20+60}{2} + \frac{-20-60}{2}\cos 2(-60^o) - 22\sin 2(-60^o) = 59 \ MPa$$

$$\sigma_y = \sigma_{x'} + \sigma_{y'} - \sigma_x = -19 \ MPa \qquad \blacktriangleleft$$

$$\tau_{xy} = -23.6 \ MPa$$