

Advanced Digital Design with the Verilog HDL, Second Edition

Michael D. Ciletti

Prentice Hall, Pearson Education, 2011

Problem 2-1

Recall that a minterm is a cube in which every variable appears.

A Boolean expression in SOP form is canonical if every cube in the expression has a unique representation in which all of the literals are in complemented or uncomplemented form.

$$F(a, b, c) = \sum m(1, 3, 5, 7)$$

Canonical SOP form:

$$F(a, b, c) = a'b'c + a'bc + ab'c + abc$$

Also:

K-map for F:

		bc			
		00	01	11	10
a	0	0 m ₀	1 m ₁	1 m ₃	0 m ₂
	1	0 m ₄	1 m ₅	1 m ₇	0 m ₆

K-map reduction: $F = c$

$$F' = m_0 + m_2 + m_4 + m_6$$

$$F' = a'b'c' + a'bc' + ab'c' + abc'$$

$$F = (a'b'c' + a'bc' + ab'c' + abc')$$

$$F = (a'b'c')' (a'bc')' (ab'c')' (abc')$$

Canonical POS form:

$$F = (a + b + c)(a + b' + c)(a' + b + c)(a' + b' + c) = c \text{ (Prove by expansion)}$$

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Problem 2-2

$$F(a, b, c, d) = \prod M(0, 1, 2, 3, 4, 5, 12)$$

$$F = (a + b + c + d)(a + b + c + d')(a + b + c' + d)(a + b' + c + d)(a + b' + c + d)(a + b' + c + d')(a' + b' + c + d)$$

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Problem 2-3

$$F(a, b, c) = a'b + c \text{ (Sum of cubes)}$$

		bc			
a		00	01	11	10
0		0 m0	1 m1	1 m3	1 m2
1		0 m4	1 m5	1 m7	0 m6

$$F(a, b, c) = m1 + m2 + m3 + m5 + m7$$

$$F(a, b, c) = a'b'c + a'bc' + a'bc + ab'c + abc \text{ (Sum of minterms)}$$

Problem 2-4

$$F(a, b, c, d) = a'bcd' + a'bcd + a'b'c'd' + a'b'c'd = m_6 + m_7 + m_0 + m_1$$

$$F(a, b, c, d) = \sum m(0, 1, 6, 7) \quad (\text{Sum of minterms})$$

The K-map of F is given below:

		cd			
		00	01	11	10
ab	00	1 m ₀	1 m ₁		
	01			1 m ₇	1 m ₆
	11				
	10				

$$F = a'b'c' + a'bc$$

$$F'(a, b, c, d) = m_2 + m_3 + m_4 + m_5 + m_8 + m_9 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14} + m_{15}$$

The K-map of F' is given below:

		cd			
		00	01	11	10
ab	00			1 m ₃	1 m ₂
	01	1 m ₄	1 m ₅		
	11	1 m ₁₂	1 m ₁₃	1 m ₁₅	1 m ₁₄
	10	1 m ₈	1 m ₉	1 m ₁₁	1 m ₁₀

Reduce the K-map to get: $F' = b'c + bc' + a$

$$F = (b'c + bc' + a)' = (b'c)'(bc')'(a)'$$

$$F = a'bc + a'b'c' \quad (\text{agrees with K-map for } F \text{ given above})$$

Using the sum of minterms expression for F, we get the product of maxterms for :

$$F' = a'b'cd' + a'b'cd + a'bc'd' + a'bc'd + a'bc'd + ab'c'd' + ab'c'd + ab'cd' + ab'cd + abc'd' + abc'd + abcd' + abcd$$

F =

$$\begin{aligned} &(a + b + c' + d)(a + b + c' + d')(a + b' + c + d) \\ &(a + b' + c + d')(a + b' + c + d)(a' + b + c + d) \\ &(a' + b + c + d')(a' + b + c' + d)a' + b + c' + d' \\ &(a' + b' + c + d)(a' + b' + c + d')(a' + b' + c' + d) \\ &(a' + b' + c' + d') \end{aligned}$$

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Problem 2-5

$$G(a, b, c, d) = (a'bcd' + a'bcd + a'b'c'd' + a'b'c'd)'$$

$$G'(a, b, c, d) = a'bcd' + a'bcd + a'b'c'd' + a'b'c'd$$

K-map for G':

		cd			
		00	01	11	10
ab	00	1 m0	1 m1	0 m3	0 m2
	01	0 m4	0 m5	1 m7	1 m6
	11	0 m12	0 m13	0 m15	0 m14
	10	0 m8	0 m9	0 m11	0 m10

$$G(a, b, c) = \Sigma m(2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15)$$

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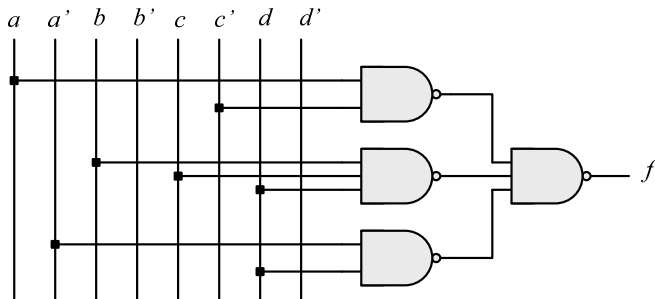
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Problem 2-6

$$f = ac' + bcd + a'd$$

$$f' = (ac')' (bcd)' (a'd)'$$

$$f = [(ac')' (bcd)' (a'd)']'$$



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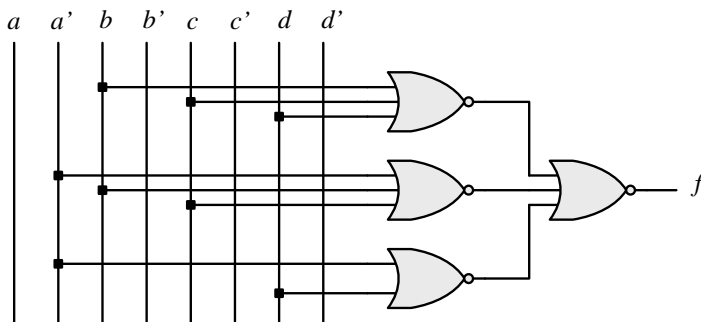
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Problem 2-7

$$f = (b + c + d)(a' + b + c)(a' + d)$$

$$f' = (b + c + d)' + (a' + b + c)' + (a' + d)'$$

$$f = [(b + c + d)' + (a' + b + c)' + (a' + d)']'$$



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Problem 2-8

$$(a) (ab' + a'b)' = (ab')'(a'b)' = (a' + b)(a + b') = a'b' + ab$$

$$(b) (b + (cd' + e)a')' = (b)' [cd' + e)a']' = b'[(c' + d) e' + a]$$

$$(c) [(a' + b + c)(b' + c')(a + c)]' = ab'c' + bc + a'c'$$

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Problem 2-9

(a) $F = a + a'b = a + b$

(b) $F = a(a' + b) = ab$

(c) $F = ac + bc' + ab = ac + bc'$

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Problem 2-10a

$$F(a, b, c) = \sum m(0, 2, 4, 5, 6)$$

a	bc			
	00	01	11	10
0	1 m0	0 m1	0 m3	1 m2
1	1 m4	1 m5	0 m7	1 m6

$$F(a, b, c) = \sum m(0, 2, 4, 5, 6) = ab' + c'$$

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Problem 2-10b

$$F(a, b, c) = \Sigma m(2, 3, 4, 5)$$

		bc			
		00	01	11	10
a	0	0 m0	0 m1	1 m3	1 m2
	1	1 m4	1 m5	0 m7	0 m6

$$F(a, b, c) = \Sigma m(2, 3, 4, 5) = ab' + a'b = a \oplus b$$

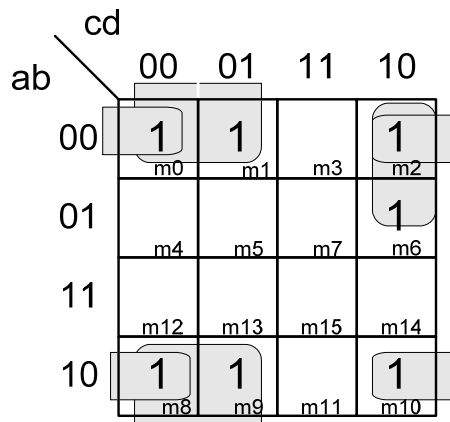
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Problem 2-10e

(e) $F = a'b'c' + b'cd' + a'bcd' + ab'c'$



$F = b'c' + b'd' + a'cd'$

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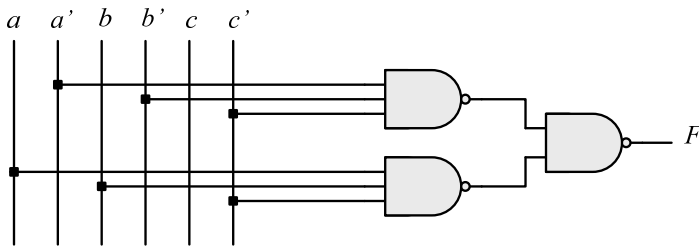
Problem 2-11

$$F(a, b, c) = \sum m(0, 6)$$

$$F(a, b, c) = a'b'c' + abc'$$

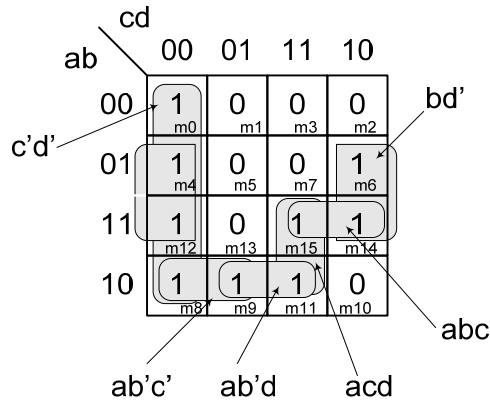
$$F' = [a'b'c' + abc']' = (a'b'c')(abc)'$$

$$F = [(a'b'c')(abc)']'$$



Problem 2-12

Karnaugh Map for $f = \sum m(0, 4, 6, 8, 9, 11, 12, 14, 15)$



1. Prime implicants are implicants that do not imply any other implicant

Answer: $c'd'$, $ab'c'$, $ab'd$, acd , abc , bd'

2. Essential prime implicants are prime implicants that cannot be covered by a set of other implicants:

Answer: $c'd'$, bd'

3. A minimal expression consists of the set of essential prime implicants together with other implicants that cover the function:

Answer:

$$f = c'd' + bd' + ab'd + abc$$

$$f = c'd' + bd' + ab'd + acd$$

$$f = c'd' + bd' + ab'c' + acd$$

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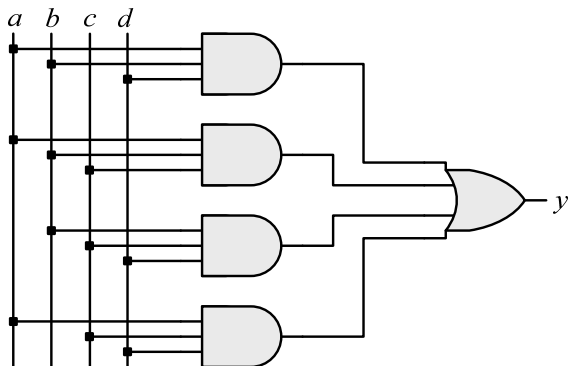
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Problem 2-13

a b c d y
0 0 0 0
0 0 0 1
0 0 1 0
0 0 1 1
0 1 0 0
0 1 0 1
0 1 1 0
0 1 1 1

1 0 0 0
1 0 0 1
1 0 1 0
1 0 1 1
1 1 0 0
1 1 0 1
1 1 1 0
1 1 1 1

$$y = abd + abc + bcd + acd$$



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Problem 2-14

a	b	c	d	y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1

1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$y = abd + abc + bcd + acd = abcd + abc'd + abcd' + a'bcd + ab'cd$$

