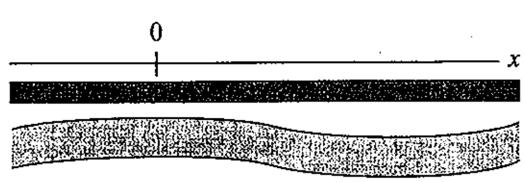
2 Kinematics

2.1 Quantitative Concept Building and Testing

EQUIPMENT

- Two constant-motion cars that move at different constant speeds (slower with one battery or faster with two batteries)
- Sugar packets or other small objects that won't bounce
- Metersticks
- Watch with a second hand
- Fan cart
- Metal track
- Motion detector
- Computer
- Stopwatch
- Pulleys
- Strings
- Small block of about 30–50 g

2.1.1 Observe and describe Imagine that you ride your bicycle along a straight path beside a river. A coordinate axis is shown above the path; see the illustration at the right.



activity continues ▶

The table indicates your position along the path at different clock readings.

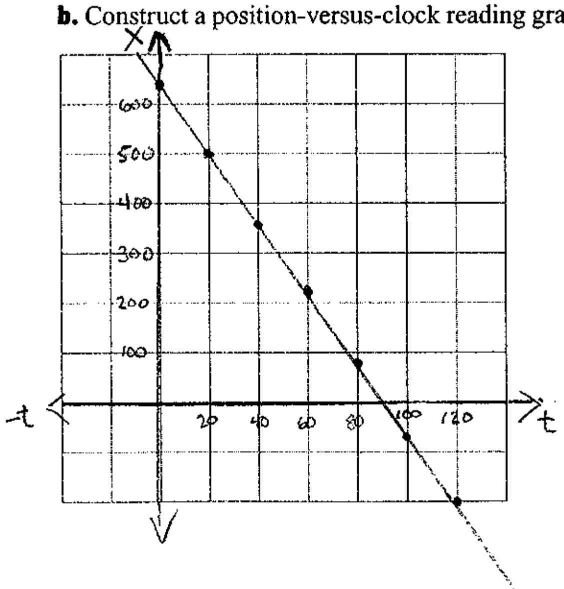
Clock reading t (s)	Position x (m)
$t_0 = 0$	$x_0 = 640$
$t_1 = 20$	$x_1 = 500$
$t_2 = 40$	$x_2 = 360$
$t_3 = 60$	$x_3 = 220$
$t_4 = 80$	$x_4 = 80$
$t_5 = 100$	$x_5 = -60$
$t_6 = 120$	$x_6 = -200$

a. Write everything you can about the bike ride and indicate any pattern in the data.

Every 20 seconds the position changed by 140 meters.

Total time: 120s
Total distance: 840m

b. Construct a position-versus-clock reading graph for the bike trip.



c. Write a function $x(t)$ of the graph in part b.

$X(t) = mt + b$

$X(t) = \left(-7 \frac{m}{s}\right)t + 640m$

d. What is the physical meaning of the slope of the function and the intercept on the vertical x axis? Explain what it means if these quantities are positive or negative.

- Slope describes how much the position changes per unit of time.
- X-intercept is the time when $X = 0$.

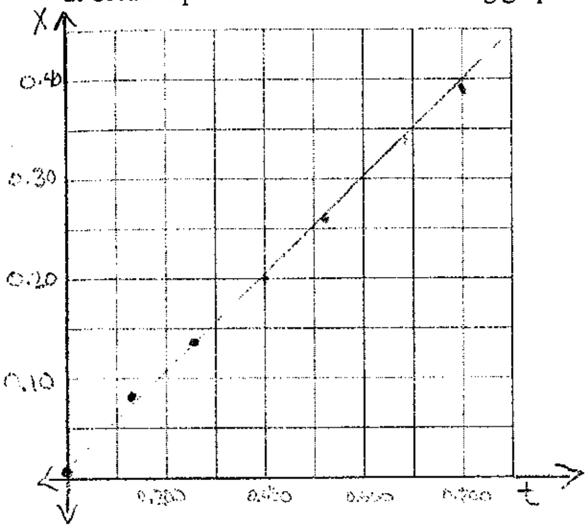
2.1.2 Observe and describe

In the table, we provide data describing a glider moving on an air track.

Clock reading t (s)	Position x (m)
$t_0 = 0.000$	$x_0 = 0.01$
$t_1 = 0.133$	$x_1 = 0.07$
$t_2 = 0.267$	$x_2 = 0.13$
$t_3 = 0.400$	$x_3 = 0.20$
$t_4 = 0.533$	$x_4 = 0.26$
$t_5 = 0.667$	$x_5 = 0.33$
$t_6 = 0.800$	$x_6 = 0.39$

activity continues ▶

a. Create a position-versus-clock reading graph using the data. Explain the meaning of the slope of the graph.



Slope describes how much the position changes per unit of time.

b. What common name could you use for the slope?

Velocity or speed

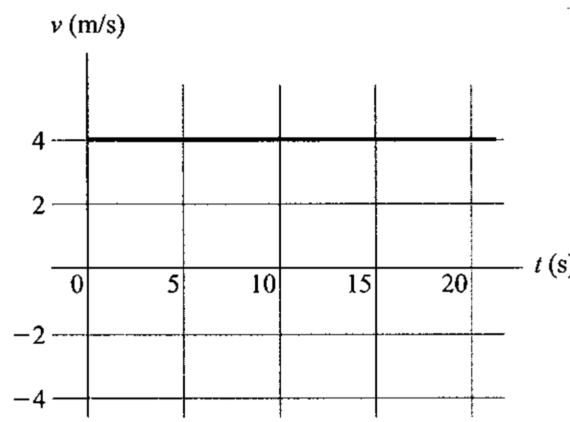
c. Use the graph from part a to write a mathematical function that gives the position of the glider as a function of the clock reading.

$X(t) = 0.01\,m + \left(0.45\,\frac{m}{s}\right)t$

d. Would the answers be different if data were taken for the back of the glider?

Yes, the initial position would be different therefore the x-axis intercept would be different.

2.1.3 Analyze The figure at the right shows a velocity-versus-clock reading graph that represents the motion of a bicycle modeled as a point particle moving along a straight bike path. The positive direction of the coordinate axis is toward the east.



a. Use the graph to estimate the bike's displacement from a clock reading of 10 s to a clock reading of 15 s. Explain.

$t = 10s \longrightarrow x = v \cdot t = 4\frac{m}{s} \cdot 10s = 40m$

$t = 15s \longrightarrow x = 4\frac{m}{s} \cdot 15s = 60m$

$d = x_f - x_i = 60m - 40m = 20m$

b. Use the graph to estimate its displacement from a clock reading of 0 s to 20 s.

$t = 0s \longrightarrow x = 0m$

$t = 20s \longrightarrow x = 80m$

$d = x_f - x_i = 80m - 0m = 80m$

activity continues ▶

c. Formulate a general rule for using a velocity-versus-clock reading graph to determine an object's displacement during some time interval if the object is moving at constant velocity.

$d = x_f - x_i = vt_f - vt_i = v(t_f - t_i) = v\Delta t$

also, d = area under curve during time interval.

2.1.4 Predict and test Gather two motorized cars.

a. For car *A*, use sugar packets, a meterstick, and a watch with a second hand to design an experiment to decide if the car moves with constant velocity. If it does, determine the magnitude of the velocity (the car's speed). Make a data table to record your measurements.

Drop sugar packets at the position of the car for several clock readings. Use the meterstick to measure the position of the packets.

t (s)	x (m)
0	0
1	0.20
2	0.41
3	0.61
4	0.82
5	1.02

$x = 0 + (0.20\frac{m}{s})t$

$v = 0.20\frac{m}{s}$

b. For car *B*, use the same equipment as in part a to design an experiment to decide if this car moves with constant velocity. If it does, determine the magnitude of the velocity (the car's speed).

t (s)	x (m)
0	0
1	0.40
2	0.78
3	1.19
4	1.60
5	2.00
6	2.41

$x(t) = 0 + (0.40\frac{m}{s})t$

$v = 0.40\frac{m}{s}$

c. Fill in the table that follows to make your prediction of where the two cars will meet if you release them 2.0 m apart and moving straight toward each other. To make a prediction, use your knowledge of the cars' speeds gathered from parts a and b and the relationship between the initial state, velocity, time elapsed, and the final state of an object.

Experiment

- Write the position-versus-clock reading data for the car.
- Choose the origin and write $x(t)$ functions.

■ Car *A*

t (s)	x (m)
0	0
1	0.20
2	0.39
3	0.61
4	0.82
5	1.02

$$X(t) = 0 + (0.20\,\tfrac{m}{s})t$$

■ Car *B*

t (s)	x (m)
0	2.00
1	1.60
2	1.20
3	0.80
4	0.40
5	0.00

$$X(t) = 2.0 + (-0.40\,\tfrac{m}{s})t$$

- Predict where the cars will meet.
- Perform the experiment; record the meeting location and compare it to your prediction.

$$X_a = X_b$$

$$(0.20\,\tfrac{m}{s})t = 2.0\,m - 0.4\,\tfrac{m}{s}(t)$$

$$(0.60\,\tfrac{m}{s})t = 2.0\,m$$

$$t = 3.3\bar{3}\ \text{sec}$$

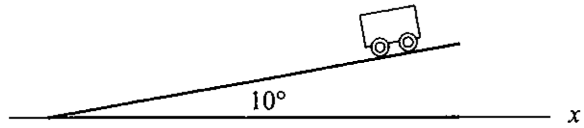
$$X_a = (0.20\,\tfrac{m}{s})(3.33\,s)$$

$$X = 0.666\,m$$

they meet at 0.65 m

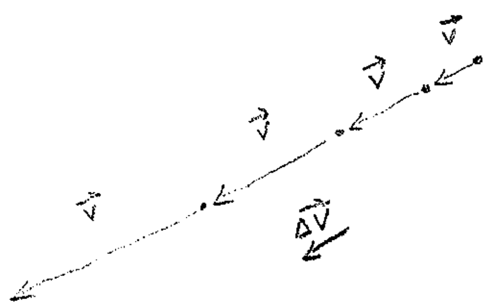
Very close to our predicted value

2.1.5 Observe and analyze You place a cart on a smooth metal track tilted at a 10° horizontal angle. The data table provided records the position of the front of the cart at different times. The x axis points along the track.

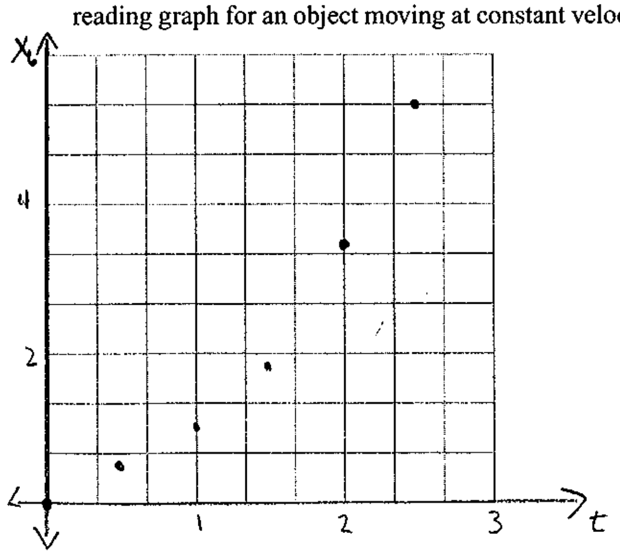


Clock reading t (s)	Position x (m)
$t_0 = 0.00$	$x_0 = 0.00$
$t_1 = 0.50$	$x_1 = 0.21$
$t_2 = 1.00$	$x_2 = 0.85$
$t_3 = 1.50$	$x_3 = 1.91$
$t_4 = 2.00$	$x_4 = 3.40$
$t_5 = 2.50$	$x_5 = 5.31$

a. Draw a motion diagram for the cart; consider the cart as a particle.



b. Draw a position-versus-clock reading graph for the cart. Discuss whether the graph resembles a position-versus-clock reading graph for an object moving at constant velocity.



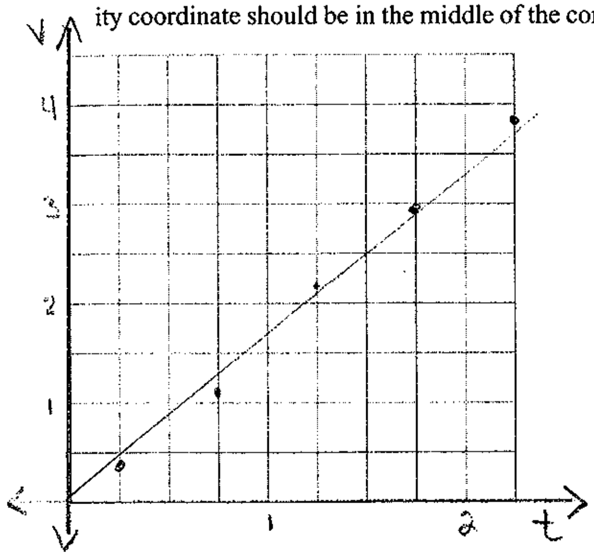
The graph is not a staight line. The cart moves faster and faster. The velocity is not constant.

c. Calculate the average velocity for the cart for each time interval and fill in the table that follows.

▪ Time interval $\Delta t = t_n - t_{n-1}$	▪ Displacement $\Delta x = x_n - x_{n-1}$	▪ Average velocity $\frac{\Delta x}{\Delta t}$
0.50	0.21	0.42
0.50	0.64	1.28
0.50	1.06	2.12
0.50	1.49	2.98
0.50	1.91	3.82

activity continues ▶

d. Plot this average velocity on a velocity-versus-clock reading graph. The clock reading coordinate for each average velocity coordinate should be in the middle of the corresponding time interval. Make a best-fit curve for your graph line.



e. Discuss the shape of the graph: How does the speed change as time elapses? Suggest a name for the slope of the graph.

The graph is a straight line so the velocity changes at a constant rate

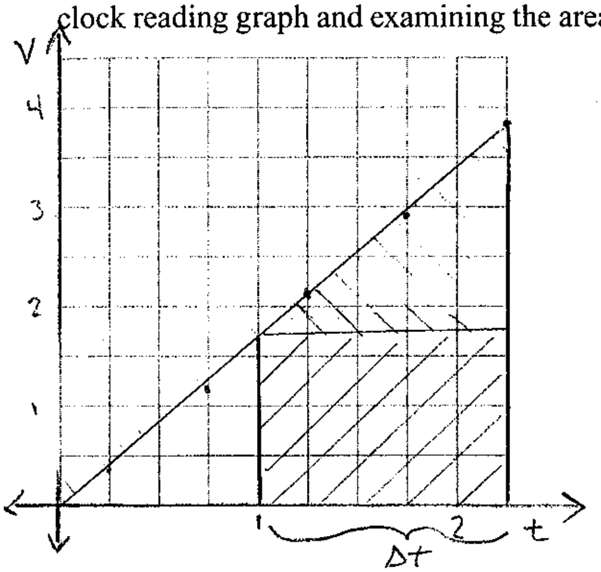
Suggested name for the slope : Acceleration.

f. Write an equation for the velocity as a function of a clock reading that is consistent with the graph line. Discuss how your equation would change if you started observing the cart when it was already moving down the same inclined plane. Discuss how the equation would change if the cart was slowing down instead of speeding up. How would it change if it was moving in a different direction?

$$V(t) = 0 + \left(1.7 \frac{m/s}{s}\right) t$$

- If we started observing after it started then the v-intercept would be greater than zero.
- If the cart were slowing down the slope would be negative.

2.1.6 Derive Use the method developed in Activity 2.1.3 to find a relationship between the displacement of the cart described in Activity 2.1.5 during some time interval, its velocity at the beginning of this interval, its acceleration, and the length of the time interval. Start by drawing a velocity-versus-clock reading graph and examining the area under the graph line.



the area under the curve during Δt is the displacement of the cart during Δt

d = area rectangle + area triangle

$$= V_i \Delta t + \frac{1}{2}\Delta t\,(V_f - V_i)$$

$$= V_i \Delta t + \frac{1}{2}\Delta t\, a\Delta t$$

$$= V_i \Delta t + \frac{1}{2} a \Delta t^2$$

2.1.7 Analyze Assemble a fan cart on a metal track, a motion detector connected to a computer, a meterstick, and a stopwatch. The cart is known to move with constant acceleration, but the acceleration is not known. Start the fan cart at rest on one end of the level track.

a. Measure the time interval needed for the cart to travel along the track.

$t = 2.2 \pm 0.1$ sec

$X = 1.50 \pm 0.01$ m

b. Use the concepts developed so far to determine the acceleration of the cart, assuming that it is constant.

$$\Delta X = V_i t + \frac{1}{2} a t^2$$

$$a = \frac{2X}{t^2} = \frac{2(1.5\,m)}{(2.2\,s)^2} = 0.62 \pm 0.2\ m/s^2$$

activity continues ▶

c. Then use the motion detector to measure the acceleration.

$a = 0.596\ m/s^2$

d. Compare the results and account for the differences. Think of the assumptions that you made and experimental uncertainties as a result of your equipment.

The difference between the measured and calculated values is within the limits of experimental uncertianties

Assumptions:

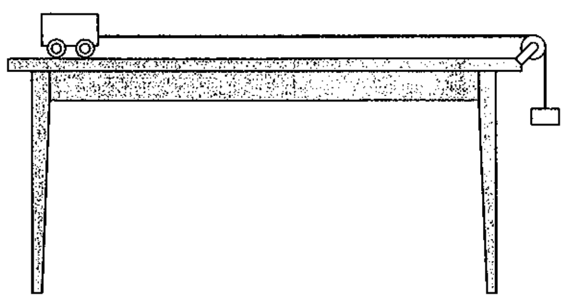
We assume is accelerating at a constant rate.

Uncertianties:

reading the clock ≃ 0.1 sec

reading the ruler ≃ 0.01 m

2.1.8 Predict and test Place a low-friction cart on a long, smooth table (or you can place a cart on a metal track). Assemble a motion detector connected to a computer, a pulley, a string going over the pulley connected to the cart on one end (see the illustration at the right) and to a small hanging block (an object of about 30–50 g) on the other end. Collect a meterstick and a watch with a second hand.



a. Use the motion detector to find the acceleration of the cart after the hanging block is released. Record it here.

Students can perform the experiment in the lab.

b. Now place a toy motorized car on the right end of the table (you now know its constant speed from Activity 2.1.4). Predict where the car and the cart will meet on the table if you face them directly opposite each other and then release them simultaneously. Use the position-versus-time relationship for constant-velocity motion and for constant-acceleration motion to help you make your prediction.

Motion model for cart : $X = \frac{1}{2}at^2$

Motion model for motorized car : $X = X_\ell - Vt$

Find collision point (x_c, t_c) using models of motion.

c. After you make your prediction, release the car and the cart and test your prediction. Do the results support your prediction in part b within experimental uncertainties? Explain carefully.

Students test prediction with experiment.

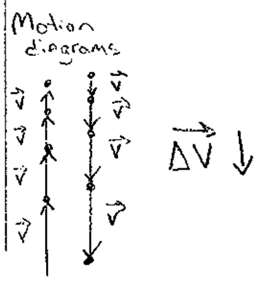
2.1.9 Analyze The data recorded in the completed table are a record of the up and down motion of the center of a ball thrown up into the air (the y axis points up).

Clock reading t (s)	Position y (m)
t_0 = 0.000	y_0 = 0.00
t_1 = 0.067	y_1 = 0.24
t_2 = 0.133	y_2 = 0.44
t_3 = 0.200	y_3 = 0.60
t_4 = 0.267	y_4 = 0.71
t_5 = 0.333	y_5 = 0.78
t_6 = 0.400	y_6 = 0.80
t_7 = 0.467	y_7 = 0.77
t_8 = 0.533	y_8 = 0.71
t_9 = 0.600	y_9 = 0.59
t_{10} = 0.667	y_{10} = 0.42
t_{11} = 0.733	y_{11} = 0.21
t_{12} = 0.800	y_{12} = –0.04

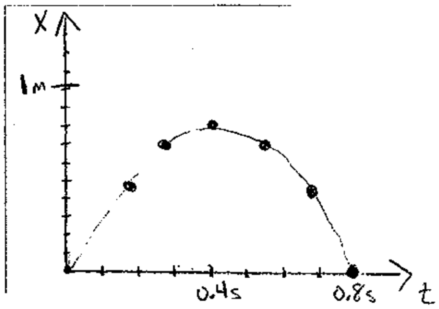
activity continues ▶

Fill in the table that follows.

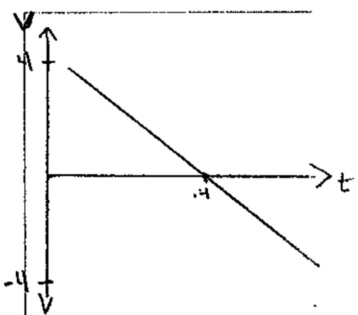
- Sketch a motion diagram for the ball modeled as a particle.



- Draw a position-versus-clock reading graph.



- Draw a velocity-versus-clock reading graph.



Find its slope. What do you call this slope?

Slope = -10.2 m/s²

this is the acceleration

$$\frac{V_f - V_i}{t_f - t_i} = \text{Slope}$$

- Use the velocity-versus-clock reading to determine the ball's acceleration at the top of its trajectory.

$a_{Top} = -10.2 \text{ m/s}^2$

This is the slope of the graph, it is the same all the way through the ball's motion

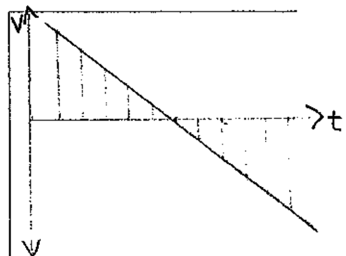
What is its velocity at the top?

$V_{top} = 0 \text{ m/s}$

Can you reconcile these two answers?

Yes, it is possible to have zero velocity and a non-zero acceleration. The ball's velocity is always decreasing, even after it passes through zero. As the graph shows, as the velocity pass through zero, the slope remains constant.

- Use the velocity-versus-clock reading to determine the distance that the ball traveled during the trip from clock reading 0.000 to 0.800 s.

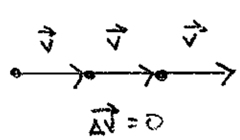
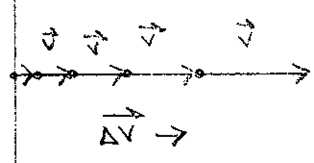
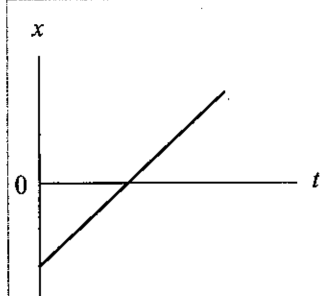
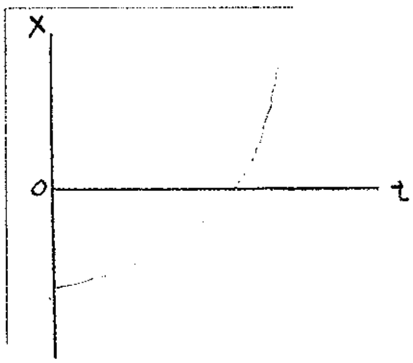


The shaded area indicates the distance traveled by the object. Since the two triangles are congruent, it is only necessary to calculate the area of one and double it.

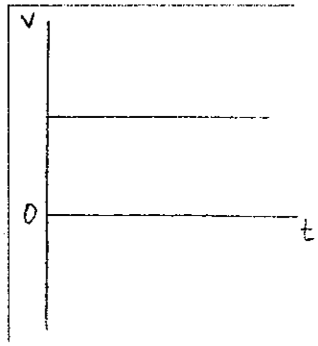
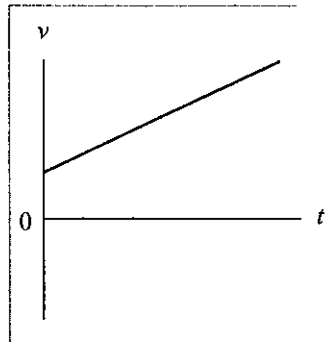
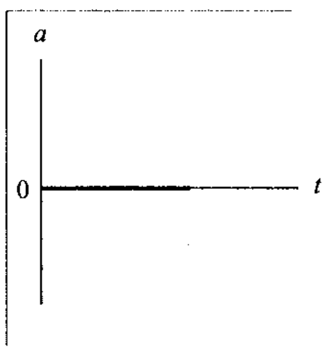
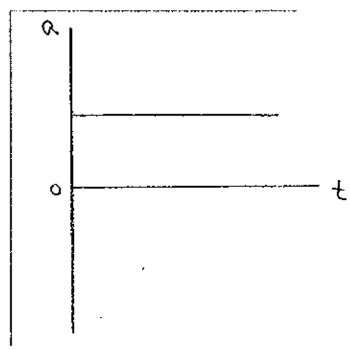
$$x = 2\left(\tfrac{1}{2}bh\right) = \cancel{2}\left(\cancel{\tfrac{1}{2}}(4.0\text{ m/s})(0.4\text{ s})\right)$$

$x = 1.6 \text{ m}$

2.1.10 Summarize Analyze the information in the table below and complete the empty cells to summarize your knowledge of motion with constant velocity and with constant acceleration, using different representations of motion.

▪ Describe the motion.	▪ Motion with constant velocity	▪ Motion with constant acceleration
▪ In words, providing an example	A motorized cart on a flat track	The object's velocity is increasing by the same amount every second—for example, a cart going down a smooth track tilted at an angle.
▪ With a motion diagram		
▪ With a graph of position-versus-clock reading		
▪ Mathematically as $x(t)$	$X = X_o + Vt$	$x = x_0 + v_0 + \frac{1}{2}at^2$

activity continues ▶

▪ Describe the motion.	▪ Motion with constant velocity	▪ Motion with constant acceleration
▪ With a graph of velocity-versus-clock reading		
▪ Mathematically as $v(t)$	$v(t) = \text{const}$	$v(t) = v_i + at$
▪ With a graph of acceleration-versus-clock reading		
▪ Mathematically as $a(t)$	$a(t) = 0$	$a(t) = \text{const}$

2.2 Quantitative Reasoning

EQUIPMENT

- ☐ Track
- ☐ Low-friction cart
- ☐ Stopwatch
- ☐ Meterstick
- ☐ Cotton balls

Problem-Solving Strategy: Kinematics Problems

PICTURE AND TRANSLATE

- Sketch the situation described in the problem.
- Include a coordinate system and indicate the origin on the sketch.
- Write the known and relevant information as physical quantities on the sketch.

SIMPLIFY

- Decide if you can consider a moving object as a particle.
- Decide whether you have enough information to consider this as motion with constant velocity or with constant acceleration.

REPRESENT PHYSICALLY

- Decide what physical representation is helpful to solve the problem.
- Draw a motion diagram or if appropriate an $x(t)$, $v(t)$, and/or $a(t)$ graph.

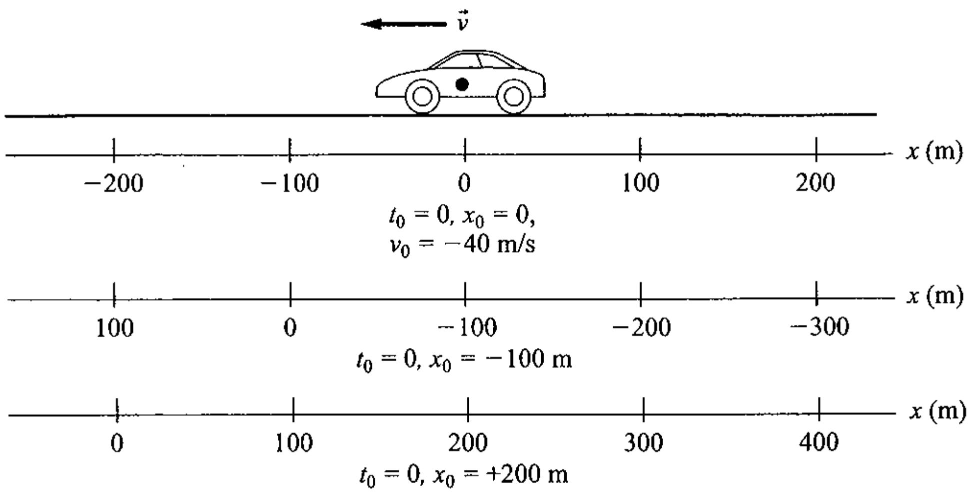
REPRESENT MATHEMATICALLY

- Use your physical representation to construct mathematical representations of the process. Be sure to consider the signs of the velocity and acceleration.

SOLVE AND EVALUATE

- Solve the equations and evaluate the results to see if they are reasonable. To assess for reasonableness, check the units, limiting cases, and the magnitude of the unknown quantity and make sure they make intuitive sense.

2.2.1 Represent and reason A car moves west at 40 m/s on a straight highway. The car's motion at the time we start describing it is presented in the illustration below using three different reference frames. Note the description in the first reference frame. Answer the following questions.

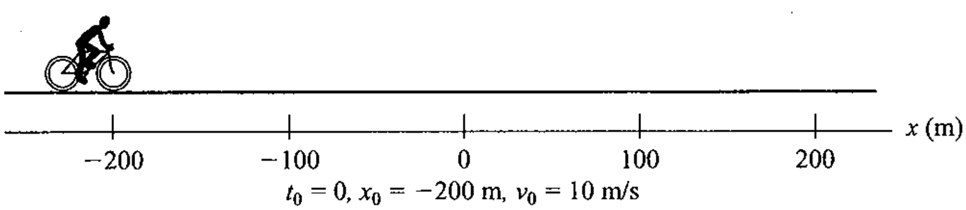


a. What is the car's velocity in the two other reference frames?

Reference frame 2: V = 40 m/s

Reference frame 3: V = -40 m/s

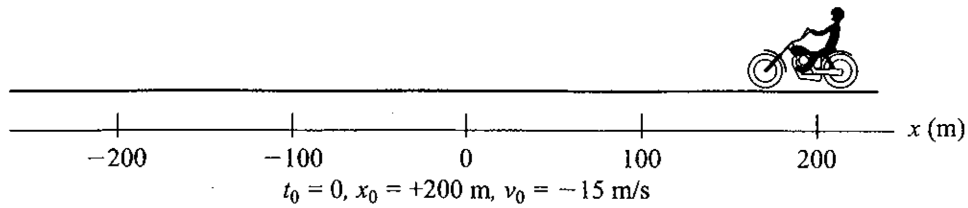
b. A bicycle moves east at constant speed on the same straight road. The bicycle's motion is described in the illustration below at one particular time using one coordinate system. Describe the bicycle's location and velocity at the same time, using the last two coordinate systems shown in the illustration for part a.



Reference frame 2: X_o = 100 m , V = -10 m/s

Reference frame 3: X_o = 0 , V = 10 m/s

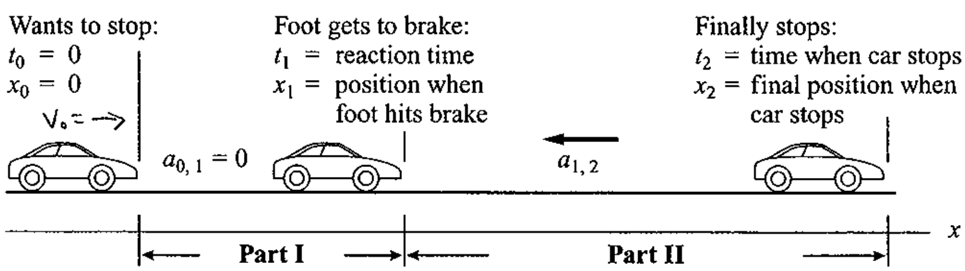
c. A motorcyclist rides west on the same road at constant speed. Her motion is depicted in the illustration below at one particular time using one coordinate system. Describe her location and velocity at the same time, using the last two coordinate systems given in part a.



rf 2: $X_o = -300$ m , $V_1 = 15$ m/s

rf3: $X_o = 400$ m , $V_0 = -15$ m/s

2.2.2 Picture and translate The driver of a car moving east at speed v_0 sees a red light in front of him and hits the brakes after a short reaction-time interval. The car slows down at a rate of $a_{1,\,2}$. A typical reaction time is 0.8 s. The situation is represented in the picture below. Note the details in the illustration.



a. Where is the origin of the reference frame?

$t = 0$

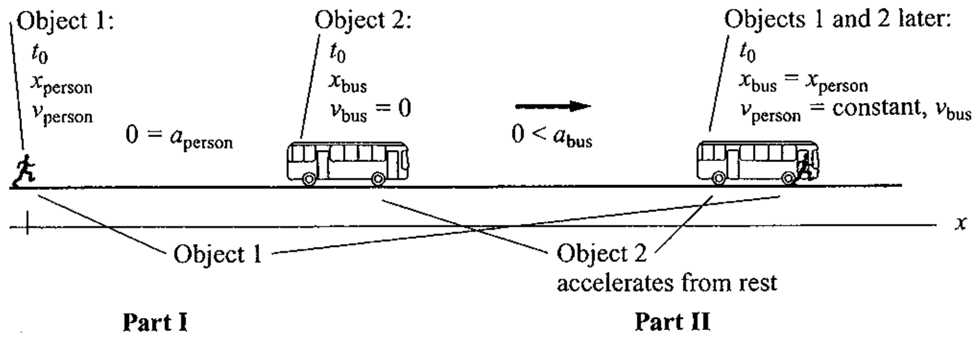
$X = 0$

b. What information given in the problem is missing from the illustration? Add it to the illustration.

Initial velocity is not indicated on drawing

2.2.3 Picture and translate Assume Jabari is running at a constant speed v_0, trying to catch a bus that starts at rest and moves with the acceleration a_{bus}.

a. The illustration below represents when and where Jabari catches the bus. Note the details in the illustration.

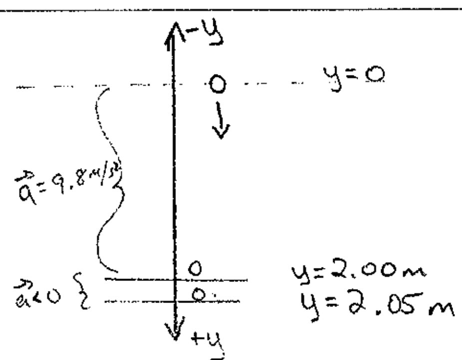


b. What is the reference frame used in the illustration?

The reference frame of the bus is used for the illustration.

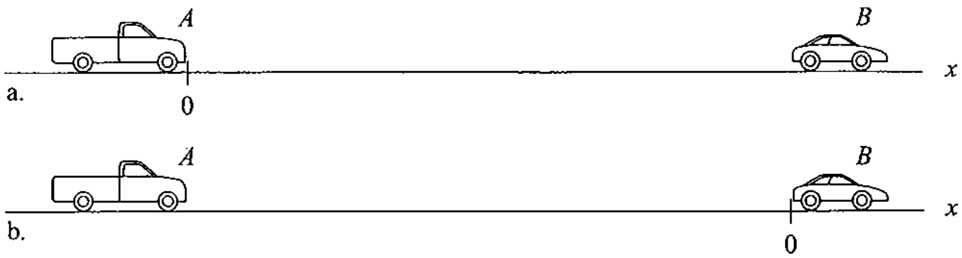
c. What information given in the problem is missing from the illustration? Add it to the illustration.

2.2.4 Represent and reason An apple falls from a tree, starting a distance of 2.0 m from the top of the grass. While falling, the apple has a downward acceleration of 9.8 m/s². As the apple sinks into the grass, its speed decreases, until it finally stops after sinking 0.050 m into the grass. Represent this situation with a picture, using the techniques from Activities 2.2.1 through 2.2.3.



Notice the positive y-direction was chosen to be down. The signs of the acceleration vectors are consistent with this choice

2.2.5 Represent and reason Truck A, starting at rest, moves faster and faster toward the east so that its speed increases by 3.0 m/s each second. At the same time that truck A begins to move, car B, 200 m east of truck A and moving at speed 16 m/s toward the east, begins to slow down by 1.0 m/s each second.



Fill in the table that follows, using a different description for each coordinate axis shown.

	▪ Truck A (coordinate system a)	▪ Car B (coordinate system a)
▪ Indicate the value of the initial clock reading.	$t_0 = 0$	$t_0 = 0$
	▪ Truck A (coordinate system b)	▪ Car B (coordinate system b)
	$t_0 = 0$	$t_0 = 0$

activity continues ▶

- Indicate the value of the initial position.

- Truck A (coordinate system a)

$X_o = 0$

- Car B (coordinate system a)

$X_o = 200\,m$

- Truck A (coordinate system b)

$X_A = -200\,m$

- Car B (coordinate system b)

$X_b = 0$

- Indicate the value of the initial velocity.

- Truck A (coordinate system a)

$V_A = 0$

- Car B (coordinate system a)

$V_b = 16\,m/s$

- Truck A (coordinate system b)

$V_A = 0$

- Car B (coordinate system b)

$V_b = 16\,m/s$

- Indicate the value of the acceleration.

- Truck A (coordinate system a)

$a_A = 3 m/s^2$

- Car B (coordinate system a)

$a_b = -1 m/s^2$

- Truck A (coordinate system b)

$a_a = 3 m/s^2$

- Car B (coordinate system b)

$a_b = -1 m/s^2$

- Write equations that can be used to determine the position and velocity of the vehicle at any given clock reading in the future.

- Truck A (coordinate system a)

$X_A(t) = \frac{1}{2}(3\frac{m}{s^2})t^2$

$= \frac{3}{2}t^2$

- Car B (coordinate system a)

$X_b = (200m) + (16\frac{m}{s})t + \frac{1}{2}(-1\frac{m}{s^2})t^2$

- Truck A (coordinate system b)

$X(t) = (-200m) + \frac{1}{2}(3 m/s^2)t^2$

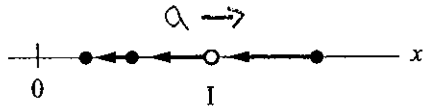
- Car B (coordinate system b)

$X_b(t) = (16\frac{m}{s})t + \frac{1}{2}(-1\frac{m}{s^2})t^2$

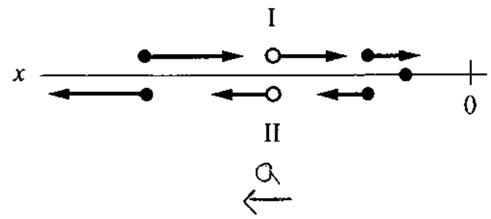
$X_b = (16\frac{m}{s})t - \frac{1}{2}t^2$

2.2.6 Represent and reason The motion diagrams in the illustrations represent the motion of different objects modeled as particles. The arrows are velocity arrows.

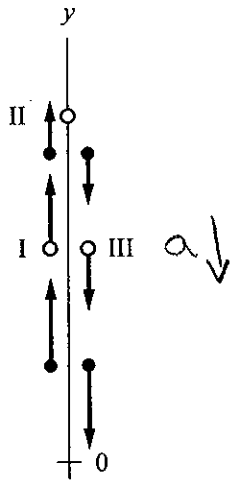
a.



b.



c.



A different coordinate axis is provided in each situation. An open circle indicates the location of interest.

a. Draw acceleration arrows on each diagram above.

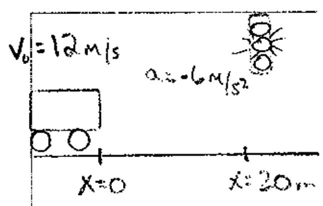
b. Fill in the table that follows. Be sure to make your choices relative to the coordinate axis shown in each motion diagram.

▪ Describe the motion in words.	▪ Determine the sign (+, 0, or –) of the position.	▪ Determine the sign (+, 0, or –) of the velocity.	▪ Determine the sign (+, 0, or –) of the acceleration.
a. Slowing down in the negative direction	*Location I:* x +	*Location I:* v –	*Location I:* a +
b. Slowing down in the negative direction, comes to a stop then speeds up in the positive direction	*Location I:* x + *Location II:* x +	*Location I:* v – *Location II:* v +	*Location I:* a + *Location II:* a +

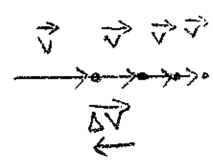
▪ Describe the motion in words.	▪ Determine the sign (+, 0, or –) of the position.	▪ Determine the sign (+, 0, or –) of the velocity.	▪ Determine the sign (+, 0, or –) of the acceleration.
c. Slowing down in the positive direction, comes to a stop then speeds up in the negative direction	*Location I:* y (+) *Location II:* y (+) *Location III:* y (+)	*Location I:* v (+) *Location II:* v = 0 *Location III:* v (–)	*Location I:* a (–) *Location II:* a (–) *Location III:* a (–)

2.2.7 Represent and reason A stoplight turns yellow when you are 20 m from the edge of the intersection. Your car is traveling at 12 m/s. After you hit the brakes, your car's speed decreases at a rate of 6.0 m/s each second. Fill in the table that follows. (Ignore the reaction time needed to bring your foot from the floor to the brake pedal.)

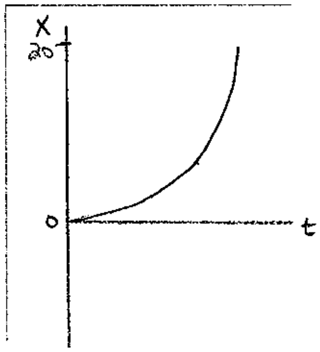
▪ Sketch the situation.



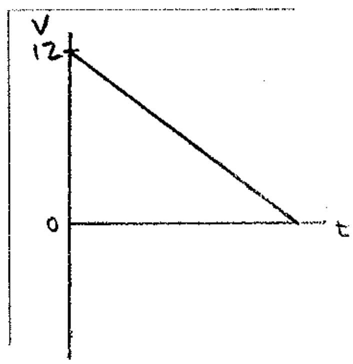
▪ Draw a motion diagram.



▪ Draw an $x(t)$ graph.



▪ Draw a $v(t)$ graph.



▪ Write $x(t)$ and $v(t)$ expressions.

$$x(t) = 0 + \left(12\tfrac{m}{s}\right)t + \tfrac{1}{2}\left(-6\tfrac{m}{s^2}\right)t^2$$

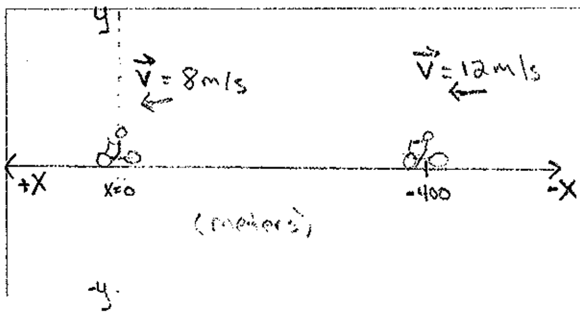
$$v(t) = \left(12\tfrac{m}{s}\right) + \left(-6\tfrac{m}{s^2}\right)t$$

2.2.8 Represent and reason You ride your bike west at a speed of 8.0 m/s. Your friend, 400 m east of you, is riding her bike west at a speed of 12 m/s.

a. Fill in the table that follows. (Consider the bikes as particles.)

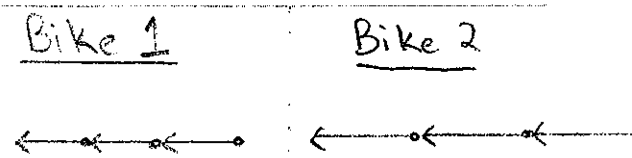
PICTURE AND TRANSLATE

- Draw a sketch of the initial situation and choose a coordinate system to describe the motion of both bikes.



REPRESENT PHYSICALLY

- Draw a motion diagram for each bike.



REPRESENT MATHEMATICALLY

- Construct equations that describe the positions of each bicycle as a function of time.

Bike 1

$X(t) = X_0 + vt$

$X(t) = (8\frac{m}{s})t$

Bike 2

$X(t) = X_0 + vt$

$X(t) = -400_m + (12\frac{m}{s})t$

SOLVE AND EVALUATE

- Use the equations to determine when the bicycles are at the same position. Does your result make intuitive sense?

$X_1 = X_2$

$(8\frac{m}{s})t = -400_m + (12\frac{m}{s})t$

$400_m = (12\frac{m}{s})t - (8\frac{m}{s})t$

$400_m = (4\frac{m}{s})t$

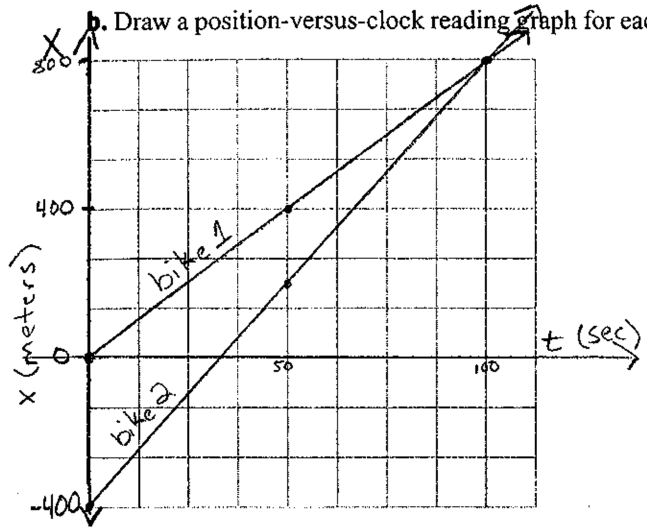
$\frac{400\,m}{4\frac{m}{s}} = t$

$t = 100\,sec$

Yes, because my friend is traveling 4 m/s faster it should take 100 seconds to make up the 400 meter difference.

After you fill in the table, complete the following:

b. Draw a position-versus-clock reading graph for each bicycle using the same set of axes.



c. Are the slopes of the two lines and their initial values consistent with the actual motion and the coordinate system used to describe the motion?

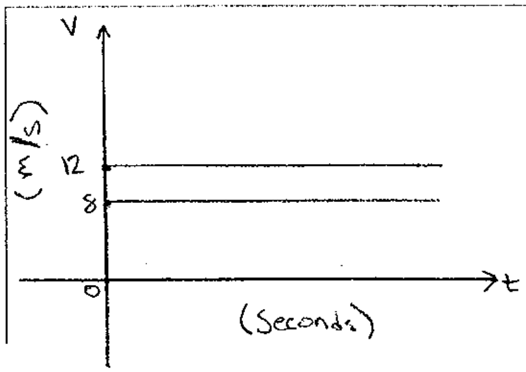
yes

d. Does the appearance of the graph correspond to the calculated answer for the time when the bicycles are at the same position? Explain.

yes, the lines intersect at the point (100, 800) so the clock reading at the time of the intersection is 100 seconds.

activity continues ▶

e. Draw a velocity-versus-time graph with lines representing the velocity of each bicycle.



f. Are the signs consistent with the word description of the motion?

Yes, I made positive velocity because the motion is in the positive x-direction.

2.2.9 Represent and reason An object moves horizontally. The equations below represent its motion mathematically. Fill in the table that follows to indicate actual motion that these two equations *together* might describe.

a. $v = +20\ \text{m/s} + (-2\ \text{m/s}^2)t$

b. $x = -200\ \text{m} + (+20\ \text{m/s})t + (1/2)(-2\ \text{m/s}^2)t^2$

▪ Describe the motion in words.

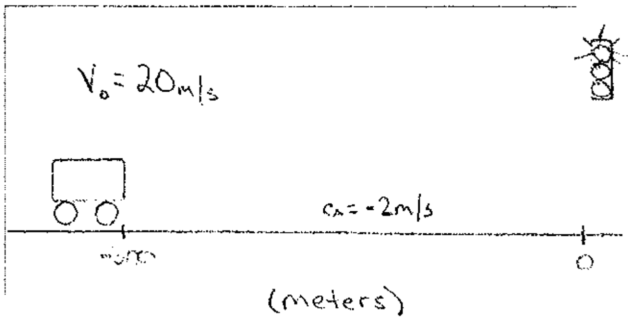
$v_0 = 20$ m/s

$a = -2$ m/s^2

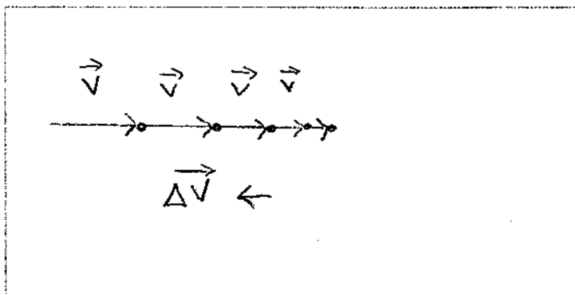
$x_0 = -200$ m

A car is approaching a traffic light travelling at 20 m/s. 200 meters before the light it begins to slow down at a rate of 2 m/s^2.

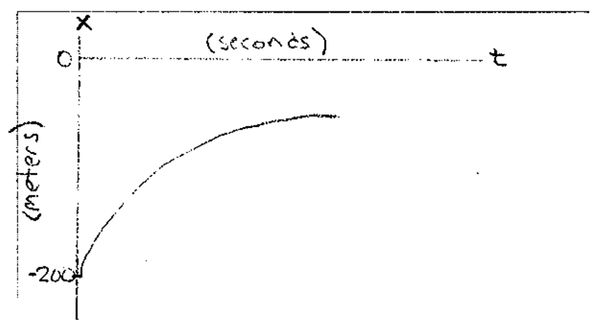
- Sketch the process.



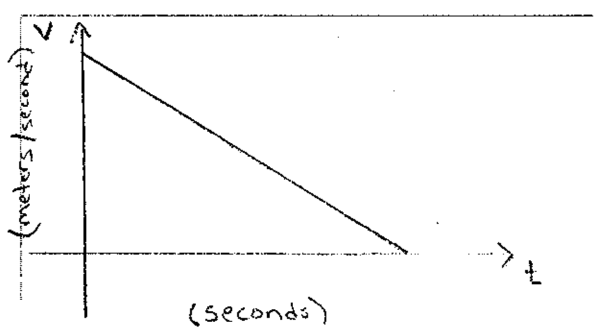
- Draw a motion diagram.



- Draw a position-versus-time graph.



- Draw a velocity-versus-time graph.



- Determine when and where the object will stop.

$V = 20 m/s - 2 \frac{m}{s^2} t$

$0 = 20 \frac{m}{s} - 2 \frac{m}{s^2} t$

$t = \frac{20}{2} = 10 \text{ sec}$

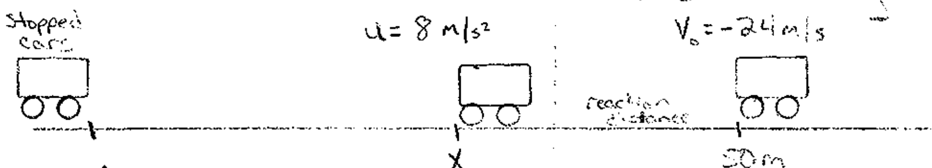
$X = -200m + (20 \frac{m}{s})t + \frac{1}{2}(-2 \frac{m}{s^2})t^2$

$X = -200m + (20 \frac{m}{s}) 10s - 1 \frac{m}{s^2} (10s)^2$

$X = -100m$

2.2.10 Regular problem While traveling in your car at 24 m/s, you find that traffic has stopped 50 m in front of you. Will you smash into the back of the car stopped in front of you? Your reaction time is 0.80 s and the magnitude of the car acceleration is 8.0 m/s² after the brakes have been applied. List all the assumptions you make.

Brake problem into two parts, distance traveled during reaction time, and the distance travelled while braking



Assumptions
1) Car is modeled as a point particle
2) Acceleration is constant during slow down phase
3) brake is depressed instantly.

Phase 1

$X_1 = X_i + Vt$

$X_1 = 50m - (24\frac{m}{s})(0.8s)$

$X_1 = 30.8m$

because I am slowing at $8\frac{m}{s^2}$ it should take me 3 seconds to go from -24 m/s to 0 m/s

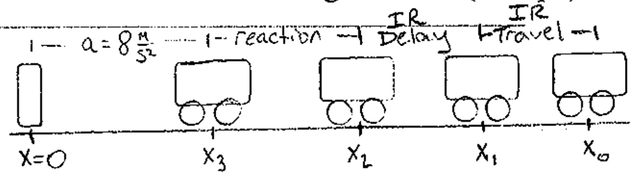
Phase 2

$X_{stop} = X_1 + Vt + \frac{1}{2}at^2$

$X_{stop} = 30.8m - (24\frac{m}{s})(3s) + \frac{1}{2}(8\frac{m}{s^2})(3)^2$

$X_{stop} = -5.2\ m$ ~ this is 5 meters to the left of the stopped cars, I will crash!

2.2.11 Regular problem Your summer job is with an automobile accident avoidance research team. The system they are investigating sends out infrared radiation from an avoidance device in the car. The infrared signal reflects from an object in front of the car and returns to the avoidance device. The device measures the time delay and is able to process the signal in 0.010 s. If a hazard is indicated, the signal causes the car's brakes to be applied in another 0.060 s. The car can slow to a stop with an acceleration of 8.0 m/s². Determine the shortest distance to a stationary object on the freeway that can be avoided with this device if the car is traveling at 28 m/s (63 mph). Infrared radiation travels at a speed of 3.0 × 10⁸ m/s.



To solve, break problem into 4 parts
Solve for the stopping distance X_3 and work backwards to X_0.

① Solve for X_3

$V_f^2 = V_0^2 + 2AX$

$0 = (-28\frac{m}{s})^2 + 2(8\frac{m}{s^2})X_3$

$\frac{784\frac{m^2}{s^2}}{16\frac{m}{s}} = X_3$

$X_3 = 49m$

② Solve for X_2

$X_3 = X_2 + Vt$

$49m = X_2 - (28\frac{m}{s})(0.06s)$

$X_2 = 50.68\ m$

③ Solve for X_1

$X_2 = X_1 + Vt$

$50.68m = X_1 - (28\frac{m}{s})(0.01s)$

$X_1 = 50.96$

④ Solve for X_0

Assume: IR signal travels $2X_1$

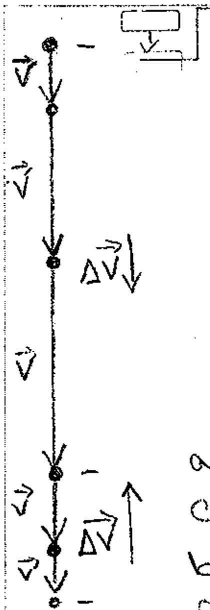
$t_{IR} = \frac{2X_1}{V_{IR}} = \frac{2(50.96m)}{3\times10^8 m/s} = 3.4\times10^{-7}s$

$X_1 = X_0 + Vt_{IR}$

$(50.96m) = X_0 + (-28\frac{m}{s})(3.4\times10^{-7}s)$

$X_0 = 50.96$

2.2.12 Regular problem During a heated conversation, you step backward off a 1.2-m ledge. You land stiff-legged and stop in 0.020 m. Determine your speed just before reaching the floor and your acceleration while stopping. What assumptions did you make? Can you model yourself as a particle to solve this problem?



① Speed before reaching floor

$$V_f^2 = 2(9.8\tfrac{m}{s^2})(1.2m) = 23.52$$

$$V_f = 4.85 \tfrac{m}{s}$$

② Acceleration while stopping

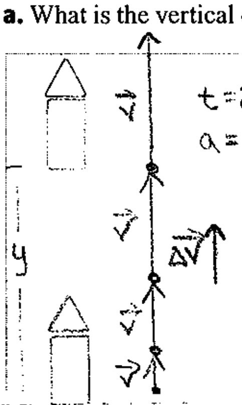
$$A = \frac{0-(4.85\frac{m}{s})^2}{2(0.02m)} = -588\ m/s^2$$

Assumptions:
- Start from rest
- The object is modeled as a point particle.

I found the acceleration while stopping to be almost 60 times g. This acceleration would surely cause injuries. In reality, your body's joints would bend allowing to acceleration to occur over a longer period of time.

2.2.13 Regular problem The fuel in a bottle rocket burns for 2.0 s. While burning, the rocket moves upward with an acceleration of 30 m/s².

a. What is the vertical distance traveled while the fuel is still burning, and how fast is it traveling at the end of the burn?



$t = 2s$
$a = 30 m/s^2$

Assume $V_0 = 0$

$$y = \tfrac{1}{2}(30 m/s^2)(2)^2 = 60m$$

$$V_f^2 = 2(30\tfrac{m}{s^2})(60m) = 3600\tfrac{m^2}{s^2}$$

$$V_f = 60\tfrac{m}{s}$$

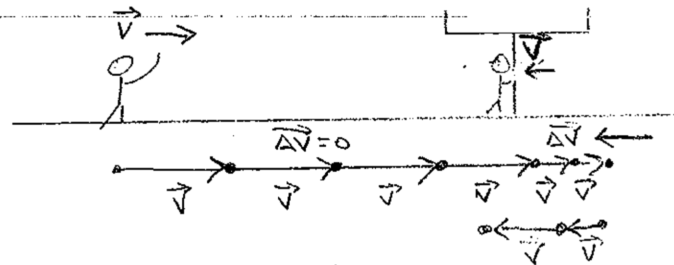
b. After the fuel stops burning, the rocket continues upward but with a velocity decreasing at a rate of about 10 m/s². Estimate the maximum height that the rocket reaches. What assumptions have you made while working through this problem?

$$y_2 = \frac{0-(60\frac{m}{s})^2}{2(-10 m/s^2)} = 180m$$

$$y_{MAX} = y_1 + y_2 = 60m + 180m = 240m$$

* Assume: Constant Acceleration, initial Velocity is zero, rocket is modeled as a point particle.

2.2.14 Regular problem While concentrating on catching the football, a wide receiver on a football team runs into the goalpost. He was originally moving at 10.0 m/s and bounced back at 2.0 m/s. A video of the collision indicates that it lasted 0.020 s. Determine the acceleration of the receiver during the collision. Indicate any assumptions you made. How will you model the receiver to solve the problem?



$$a = \frac{\Delta v}{\Delta t} = \frac{10\frac{m}{s} - (-2\frac{m}{s})}{0.02s} = \frac{12\frac{m}{s}}{0.02} = 600\frac{m}{s^2}$$

Assume: - Acceleration is constant during collision
- ignore interactions from other objects.

football player is modeled as a point particle.

2.2.15 Evaluate the solution

The problem: A firefighter slides a distance of 2.0 m at increasing speed for 2.0 s down a firepole (she holds on so she doesn't move too fast at the bottom). She bends her knees at the bottom and stops in 0.10 m. Determine her speed at the end of the slide and just before she contacts the floor. What is her acceleration while stopping?

Proposed solution:

$v = v_0 + at = 0 + (9.8\ \text{m/s}^2)(2.0\ \text{s}) = 19.6\ \text{m/s}$

$a = (v^2 - v_0^2)/2(x - x_0) = [0^2 - (19.6\ \text{m/s})^2]/2(0.10\ \text{m}) = -1920.8\ \text{m/s}^2$

a. Identify any errors in the solution.

The firefighter's acceleration was not 9.8 m/s². Her average speed was (2m)/(2s) = 1 m/s. Assuming constant acceleration, her final speed must be 2 m/s. So the acceleration while stopping was:

$$a = \frac{(v_f^2 - v_0^2)}{2(x - x_0)} = \frac{0 - (-2.0\frac{m}{s})^2}{2(-0.10m)} = 20\ m/s^2$$

b. Provide a corrected solution if there are errors.

Acceleration while stopping:

$$a = \frac{V_f^2 - V_o^2}{2(x-x_o)} = \frac{0-(-20\,\frac{m}{s})^2}{2(-0.10m)} = 20\,m/s^2$$

2.2.16 Evaluate the solution

The problem: You are driving at 20 m/s and slam on the brakes to avoid a goose walking across the road. You stop in 1.2 s. How far did you travel after hitting the brakes?

Proposed solution:

$(x - x_0) = vt = (20 \text{ m/s})(1.2 \text{ s}) = 24 \text{ m}$

a. Identify any missing steps and/or errors in the solution.

The average velocity is not 20 m/s it is

$$\frac{20\,\frac{m}{s} + 0}{2} = 10\,m/s$$

b. Provide a corrected solution if there are missing steps and/or errors.

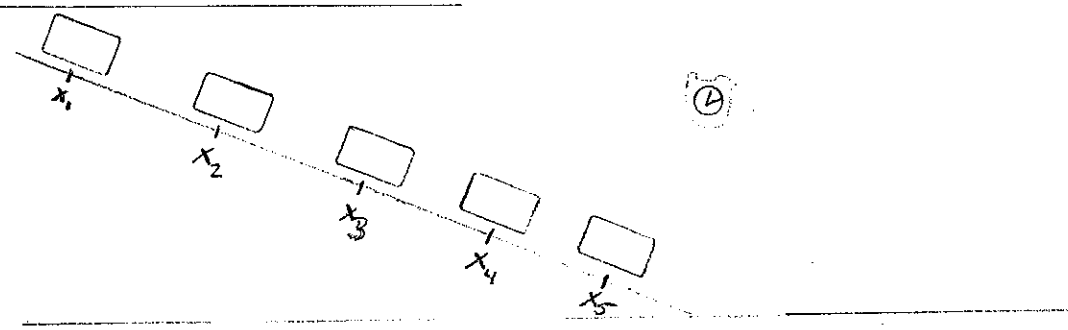
$$(x - x_o) = \bar{v}t = (10\,m/s)(1.2s) = 12m$$

2.2.17 Design an experiment You have a track tilted at a small angle, a toy car (not motorized), a stopwatch, and a meterstick.

a. Describe in detail an experiment that you can use to determine if the car moves at constant speed, constant acceleration, or changing acceleration along the track.

In order to determine the type of motion the toy car exhibits we will create a testable hypothesis. For our experiment we will start the cart at several different positions along the track and measure the time it takes to reach the bottom. Then we can plot the starting position versus clock reading. A linear graph implies constant velocity & a parabolic implies constant acceleration.

b. Sketch the apparatus. Write on the sketch what you will measure and what you will calculate.



c. List experimental uncertainties and how you will minimize them.

There are uncertainties in both quantities we measure, the distance and the clock reading. We can minimize them by performing many trails.

d. Perform the experiment; record the data in a table and use a best-fit function for the data to make a judgment.

Students should perform the experiment and include their data.

e. Write your judgment about the car's motion.

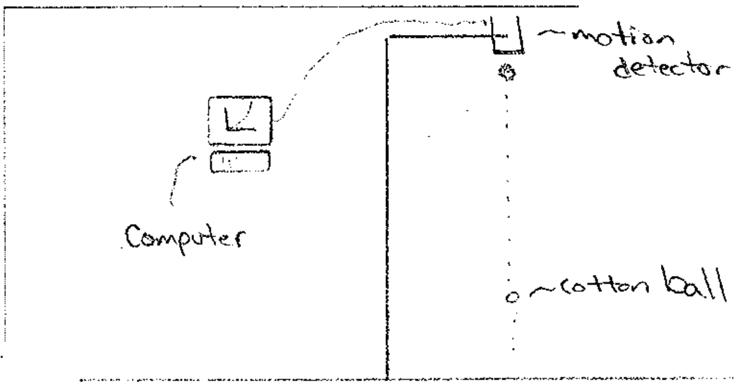
The data supports the constant acceleration model of motion.

2.2.18 Design an experiment You have a cotton ball, a stopwatch, and a meterstick.

a. Describe in detail an experiment that you can perform to determine whether the cotton ball falls with constant acceleration or changing acceleration.

For this experiment we will use a motion detector connected to a computer to plot the position vs time, and the velocity vs. time. We can make different predictions for each possible type of motion.

b. Sketch the apparatus.



activity continues ▶

c. Write the physical quantities that you will measure and the quantities that you will calculate.

The computer will measure position and clock reading. The computer then calculates velocity using the data collected.

d. List experimental uncertainties and how you will minimize them.

There are uncertainties in the position and clock reading measurements To minimize them we can increase the number of collections so that more data is taken.

e. Perform the experiment; record the data in a table and use a best-fit function for the data to make a judgment.

Students will perform the experiment and include their data.

f. Write your judgment about the ball's motion.